Stringent constraint on *CPT* violation with the synergy of T2K-II, NOνA extension, and JUNO

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Neutrino oscillation experiments have measured precisely at few percent levels the mass-squared differences (Δm^2_{21}, Δm^2_{31}) of three neutrino mass eigenstates, and the three leptonic mixing angles (θ_{12}, θ_{13}, θ_{23}) by utilizing both neutrino and antineutrino oscillations. The possible CPT violation may manifest itself in the difference of neutrino and antineutrino oscillation parameters, making these experiments promising tools for testing CPT invariance at unprecedented precision. We investigate empirically the sensitivity of the CPT test via the difference in mass-squared splittings ($\Delta m^2_{31} - \Delta \bar{m}^2_{31}$) and in leptonic mixing angles ($\sin^2\theta_{23} - \sin^2\bar{\theta}_{23}$) with the synergy of T2K-II, NOνA extension, and JUNO experiments. If the CPT symmetry is found to be conserved, the joint analysis of the three experiments will be able to establish limits of $|\Delta m^2_{31} - \Delta \bar{m}^2_{31}| < 5.3 \times 10^{-3}$ eV^2 and $|\sin^2\theta_{23} - \sin^2\bar{\theta}_{23}| < 0.10$ at 3σ confidence level (CL) on the possible CPT violation, extending substantially the current bound of these parameters. We find that with ($\Delta m^2_{31} - \Delta \bar{m}^2_{31}$), the dependence of the statistical significance on the relevant parameters to exclude the CPT conservation is marginal, and that, if the difference in the best-fit values of Δm^2_{31} and $\Delta \bar{m}^2_{31}$ measured by MINOS(+) and NOνA persists as the *true*, the combined analysis will rule out the CPT conservation at 4σ CL. With the ($\sin^2\theta_{23} - \sin^2\bar{\theta}_{23}$), the statistical significance to exclude CPT invariance depends strongly on the *true* value of $\theta_{23}(\bar{\theta}_{23})$ mixing angle. In the case of maximal mixing of θ_{23}, as indicated by the current T2K and NOνA measurements, the CPT conservation will be excluded at 3σ CL or higher if the difference in the best-fit values of θ_{23} and $\bar{\theta}_{23}$ remains as the *true*.

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I. *CPT* TEST WITH NEUTRINO

The CPT theorem, which connects three discrete symmetries: charge conjugation (C), parity (P), and time reversal (T), and has been theoretically proved in different ways [1–5], states that any Lorentz invariant local quantum field theory of point-particle must be CPT invariant. If it is discovered that CPT symmetry is not conserved, one of the three foundational assumptions (Lorentz invariance, Hamiltonian Hermiticity, and locality) must be sternly reconsidered. A consequence of the CPT invariance is that the particle and its antiparticle must have the same energy spectra. This important property opens a possibility for direct testing CPT invariance by comparing the mass spectra, or other properties such as lifetime or magnetic moment of a particle and its antiparticle. Reference [6] provides the latest results on Lorentz and CPT violation searches in the context of Standard Model extension. A summary of the model-independent CPT testing based on different properties of the different systems of particle and antiparticle can be found in Ref. [7]. In terms of relative precision, the most stringent constraint on the CPT test was achieved on the neutral kaon system [8]

$$\left|\frac{m(K^\circ) - m(\bar{K}^\circ)}{m_K}\right| < 6 \times 10^{-19} \text{ at } 90\% \text{ CL} \qquad (1)$$

As pointed out in Ref. [9], when expressed in terms of the mass-squared difference, the bound on the $K^\circ - \bar{K}^\circ$ mass difference does not appear to be formidable. From Eq. (1), one can get

$$|m^2(K^\circ) - m^2(\bar{K}^\circ)| < 0.3 \text{ eV}^2. \qquad (2)$$

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Comparing this to the two mass-squared differences of the three neutrino mass eigenstates [8], $m_{\nu 2}^2 - m_{\nu 1}^2 \approx 7.4 \times 10^{-5}$ eV^2 and $|m_{\nu 3}^2 - m_{\nu 2}^2| \approx 2.45 \times 10^{-3}$ eV^2, it becomes clear that neutrino measurements, rather than neutral kaons, provide the best constraint on the CPT test in terms of the mass-squared difference [9,10]. The aforementioned neutrino mass-squared differences come from measuring the neutrino oscillation, which is a macroscopic quantum phenomenon establishing that neutrinos are massive and thus beyond the Standard Model's description. It is worth noting that the neutrino mass spectrum cannot be calculated solely from neutrino oscillations, but must be combined with cosmological constraints and beta decay, as recently discussed in Ref. [11]. Neutrinos, unlike neutral B and K mesons, are neutral elementary particles, and it is intriguing that this particle could be a Majorana particle, where neutrino and antineutrino are indistinguishable in the conventional sense of the CPT invariant paradigm. The neutrino nature under the CPT-violating scenario has been explored in Ref. [12]. Here we focus on the phenomenological consequence of the CPT violation in the observable neutrino oscillation.

In context of three-flavor PMNS framework [13,14], for a given propagation distance L and matter density ρ, the probabilities $(P_{\nu_\alpha \to \nu_\beta}, P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta})$ for a neutrino and antineutrino at a specific energy $(E_\nu, E_{\bar{\nu}})$ oscillating from one flavor $(\nu_\alpha, \bar{\nu}_\alpha)$ to another flavor $(\nu_\beta, \bar{\nu}_\beta)$ are completely and commonly described with six oscillation parameters including three leptonic mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$, one Dirac CP-violating phase δ_{CP}, and two mass-squared differences $(\Delta m_{21}^2, \Delta m_{31}^2)$. Under CPT symmetry, the neutrino and antineutrino oscillation probabilities are well connected as follows:

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{CPT} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} = P_{\nu_\alpha \to \nu_\beta} = f(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\mathrm{CP}}, \Delta m_{21}^2, \Delta m_{31}^2).$$

If the CPT is violated in neutrino sector, the underlying sets of oscillation parameters in neutrino and antineutrino may differ. Empirically, we assume

$$P_{\nu_\alpha \to \nu_\beta} = f(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\mathrm{CP}}, \Delta m_{21}^2, \Delta m_{31}^2), \tag{3}$$

for describing the neutrino oscillations, and

$$P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} = f(\bar{\theta}_{12}, \bar{\theta}_{13}, \bar{\theta}_{23}, \bar{\delta}_{\mathrm{CP}}, \Delta \bar{m}_{21}^2, \Delta \bar{m}_{31}^2), \tag{4}$$

for antineutrino oscillations.

If there are observable differences in the parameters of the two sets, it may indicate a CPT violation in the lepton sector. Since the discovery of neutrino oscillations [15,16] at the end of the twentieth century, neutrino oscillation experiments [8] using both natural and man-made neutrino sources have transitioned into the precision measurement phase of three mixing angles and two mass-squared differences, and being explored three remained known unknowns including the neutrino mass ordering, whether CP is violated, and whether the mixing angle θ_{23} is maximal ($\theta_{23} = 45°$) or belong to a lower ($\theta_{23} < 45°$) or higher ($\theta_{23} > 45°$) octant. Each experiment is typically sensitive to a subset of the oscillation parameters but not the entire set. The experiments with solar neutrinos provide the most constraints on the $(\theta_{12}, \Delta m_{21}^2)$ parameters while the reactor-based long-baseline neutrino (R-LBL) experiments can measure precisely the $(\bar{\theta}_{12}, \Delta \bar{m}_{21}^2)$ parameters. The reactor-based short-baseline (order of 1 km) neutrino (R-SBL) experiments play a central role in measuring the $(\bar{\theta}_{13}, \Delta \bar{m}_{31}^2)$ parameters. The under-developing reactor-based medium-baseline neutrino (R-MBL) experiment JUNO, which will be discussed later, takes advantage of interference of oscillations at different wavelengths, huge statistics, and good energy resolution to achieve subpercent precision in measuring the $(\bar{\theta}_{12}, \Delta \bar{m}_{21}^2, \Delta \bar{m}_{31}^2)$ parameters. Experiments with the atmospheric neutrino and accelerator-based neutrino sources can precisely measure the $(\theta_{23}, \bar{\theta}_{23}, \Delta m_{31}^2, \Delta \bar{m}_{31}^2)$ parameters. Besides, this type of experiment is also sensitive to the $(\theta_{13}, \bar{\theta}_{13})$ parameters, but the precision of these parameters is much lower in comparison to the R-SBL experiment due to the statistical limit and their strong correlation with two known unknowns, CP-violating phase and neutrino mass ordering. Although there is some hint [17] of nonzero CP-violating phase δ_{CP}, precise measurement on this parameter is not possible until the next generation of the accelerator-based long-baseline (A-LBL) experiments. It is provided in Ref. [18] the most recent update at 3σ confidence level (CL) on the bounds of CPT violation on each individual parameter with global neutrino data.

$$\begin{aligned} &|\delta_{\nu\bar{\nu}}(\Delta m_{21}^2)| < 4.7 \times 10^{-5}\ \mathrm{eV}^2, \\ &|\delta_{\nu\bar{\nu}}(\Delta m_{31}^2)| < 2.5 \times 10^{-4}\ \mathrm{eV}^2, \\ &|\delta_{\nu\bar{\nu}}(\sin^2\theta_{12})| < 0.14, \\ &|\delta_{\nu\bar{\nu}}(\sin^2\theta_{13})| < 0.029, \\ &|\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})| < 0.19, \end{aligned} \tag{5}$$

where $\delta_{\nu\bar{\nu}}(X) = X - \bar{X}$ for the X neutrino oscillation parameter and the $\bar{X}$ antineutrino oscillation parameter. In this study, we focus on the synergy between two ongoing A-LBL experiments (T2K and NOνA) and one under-developing R-MBL experiment (JUNO) to explore the potential sensitivity to the measurement of $\delta_{\nu\bar{\nu}}(\Delta m_{31}^2)$ and $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ parameters. The A-LBL experiments utilize the highly intense beam of the almost pure muon neutrinos ν_μ and muon antineutrinos $\bar{\nu}_\mu$ for measuring the four transitions categorized into two channels, *appearance* channels $(\nu_\mu \to \nu_e, \bar{\nu}_\mu \to \bar{\nu}_e)$, and *disappearance* channels

$(\nu_\mu \to \nu_\mu, \bar{\nu}_\mu \to \bar{\nu}_\mu)$. While the *appearance* channels are sensitive to a wider subset of parameters and being explored for searching the *CP* violation in the lepton sector, measuring $(\nu_\mu \to \nu_e, \bar{\nu}_\mu \to \bar{\nu}_e)$ is not sufficient to test *CPT* directly since the corresponding *CPT*-mirrored processes are missing. The *disappearance* channels, on the other hand, are well-suited for testing *CPT* since they are two *CPT*-mirrored processes. We characterize the difference in the probabilities of the muon neutrino *disappearance* and muon antineutrino *disappearance*, $\mathcal{A}^{CPT}_{\mu\mu} = P_{\nu_\mu\to\nu_\mu} - P_{\bar{\nu}_\mu\to\bar{\nu}_\mu}$ as an observable measure of the *CPT*-violating effect.

The observable asymmetry $\mathcal{A}^{CPT}_{\mu\mu}$ may consist of two parts: *intrinsic CPT* asymmetry and *extrinsic CPT* asymmetry caused by differences in interactions between neutrinos and antineutrinos with the matter of the propagation medium [19–23]. Figure 1 illustrates the *CPT* asymmetries $\mathcal{A}^{CPT}_{\mu\mu}$ calculated in vacuum and in the matter presence at baselines of the T2K experiment (L = 295 km) and of the NOνA experiment (L = 810 km). Here we take the best-fit values of the mainly involved $(\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \theta_{23}, \bar{\theta}_{23})$ parameters from the recent T2K results [24] and of the others from the global data analysis [25], which are summarized in Table I. It is worthwhile to notice that our work with the muon-neutrino and muon-antineutrino disappearance data sample is insignificantly affected by the uncertainty in our understanding of $(\theta_{12}, \bar{\theta}_{12}, \theta_{13}, \bar{\theta}_{13}, \delta_{\rm CP}, \bar{\delta}_{\rm CP}, \Delta m^2_{21}, \Delta\bar{m}^2_{21})$ parameters. The primary driving parameters in this study are $(\theta_{23}, \bar{\theta}_{23}, \Delta m^2_{31}, \Delta\bar{m}^2_{31})$ parameters.

By comparing the in-vacuum and in-matter cases, it shows that the matter effect with NOνA is more visible than T2K due to the longer baseline. However, for both cases, at the peak of experimental neutrino spectra (0.6 GeV for T2K and 2.0 GeV for NOνA), the matter effect is marginal and thus the *CPT* test is relatively transparent. In addition, it is observed that 1% difference between the two mass-square splittings translates to approximately 1% difference in *CPT* asymmetry $\mathcal{A}^{CPT}_{\mu\mu}$. Regarding to $(\theta_{23}, \bar{\theta}_{23})$-dependence, since $P_{\nu_\mu\to\nu_\mu}$ and $P_{\bar{\nu}_\mu\to\bar{\nu}_\mu}$ up to the first order of approximation, are proportional to $\sin^2 2\theta_{23}$ and $\sin^2 2\bar{\theta}_{23}$, respectively. In Fig. 1, about 15% difference between $\sin^2\theta_{23}$ and $\sin^2\bar{\theta}_{23}$ is converted to 2% difference between $\sin^2 2\theta_{23}$ and $\sin^2 2\bar{\theta}_{23}$ and results in around 2% of the *CPT* asymmetry. Furthermore, it is worth noting that the possible *CPT* asymmetry, if happened, between the ν_μ and $\bar{\nu}_\mu$ *disappearances* does not depend on the Dirac *CP*-violating phase. This channel, on its own, is less sensitive to neutrino mass ordering. The scenario, where neutrino follows the *normal* ordering while the antineutrino follows the *inverted*

TABLE I. Values of nominal parameters, taken from the recent T2K measurements [24] of muon-neutrino and muon-antineutrino *disappearances* and from the global analysis of the neutrino oscillation data [25]. Our work utilizing the data samples of muon-neutrino and muon-antineutrino disappearance is insignificantly affected by uncertainty of (θ_{12}, $\bar{\theta}_{12}$, θ_{13}, $\bar{\theta}_{13}$, $\delta_{\rm CP}$, $\bar{\delta}_{\rm CP}$, Δm^2_{21}, $\Delta\bar{m}^2_{21}$) parameters.

Parameter	Value
$\sin^2\theta_{23}$	0.51
$\sin^2\bar{\theta}_{23}$	0.43
Δm^2_{31}	2.55×10^{-3} eV^2
$\Delta\bar{m}^2_{31}$	2.58×10^{-3} eV^2
$\sin^2\theta_{12}, \sin^2\bar{\theta}_{12}$	0.318
$\sin^2\theta_{13}, \sin^2\bar{\theta}_{13}$	0.022
$\delta_{\rm CP}, \bar{\delta}_{\rm CP}$	1.08π rad
$\Delta m^2_{21}, \Delta\bar{m}^2_{21}$	7.50×10^{-5} eV^2

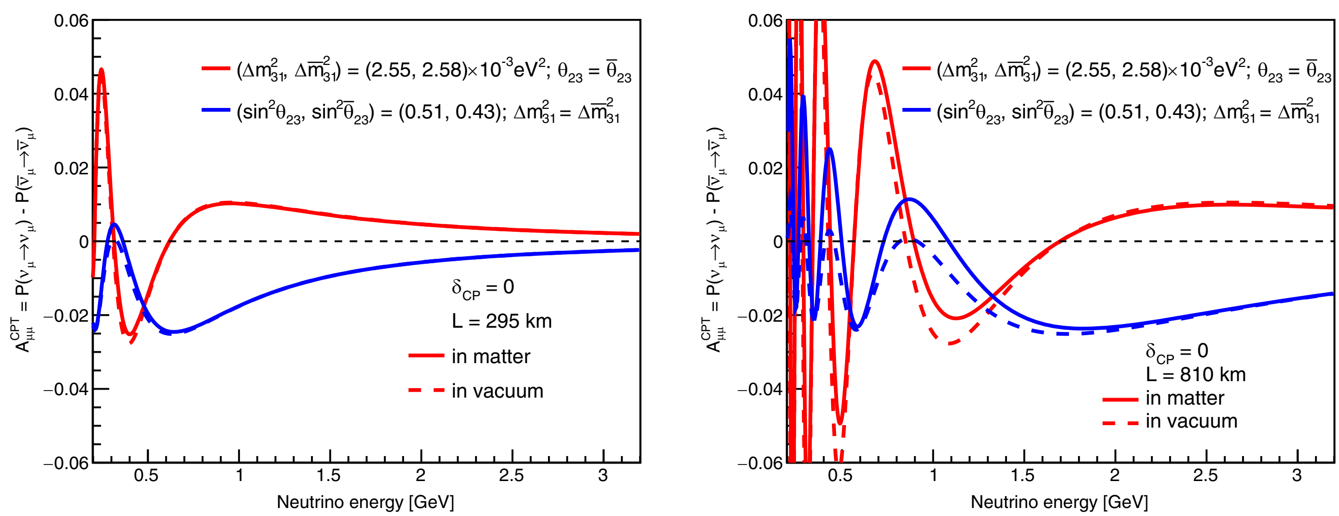


FIG. 1. *CPT* asymmetries in *disappearance* channels for T2K baseline L = 295 km (left) and NOνA baseline L = 810 km (right). The differences in solid lines and dashed lines indicate extrinsic *CPT* effects caused by matter.

TABLE II. Measurements of the $(\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \theta_{23}, \bar{\theta}_{23})$ parameters, which govern the muon neutrino and muon antineutrino *disappearances*, from different experiments: MINOS(+) [27,28], T2K [24], NOνA [29], Daya Bay [30]. *Normal* neutrino mass ordering is assumed.

	MINOS(+)	T2K	NOνA	Daya Bay
$\Delta m^2_{31}/10^{-3}$ eV^2	$2.48^{+0.08}_{-0.09}$	$2.55^{+0.08}_{-0.09}$	$2.56^{+0.07}_{-0.09}$	...
$\Delta\bar{m}^2_{31}/10^{-3}$ eV^2	$2.55^{+0.23}_{-0.25}$	$2.58^{+0.18}_{-0.13}$	$2.63^{+0.12}_{-0.13}$	$2.53^{+0.06}_{-0.06}$
$\sin^2\theta_{23}$	$0.43^{+0.20}_{-0.04}$	$0.51^{+0.06}_{-0.07}$	$0.51^{+0.06}_{-0.06}$	...
$\sin^2\bar{\theta}_{23}$	$0.41^{+0.05}_{-0.08}$	$0.43^{+0.21}_{-0.05}$	$0.41^{+0.04}_{-0.03}$	...

ordering or vice versa, implies the *CPT* violation. This scenario can be tested by comparing the neutrino mass ordering measured with neutrino-mode data samples in the A-LBL experiment to the measurement from the JUNO experiment. The work in Ref. [26] shows that under the *CPT*-invariant assumption, combining these three experiments will resolve the neutrino mass ordering completely. It will be exciting, however, if the A-LBL experiments and JUNO point separately to different mass orders with high statistical significance. In this study, we assume that neutrino and antineutrino masses are ordered similarly.

Table II summarize the measurements of the $(\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \theta_{23}, \bar{\theta}_{23})$ parameters with the first generation of the A-LBL experiment MINOS [27,28], on-going second generation T2K [24], NOνA [29], and precise constraint of the $\Delta\bar{m}^2_{31}$ parameter from the R-SBL experiment Daya Bay [30]. It is shown that Δm^2_{31} is measured with about 3% precision with the A-LBL experiments, while $\Delta\bar{m}^2_{31}$ is measured with about 10% precision, which can be complemented with the R-SBL experiment with 2.3% precision. For the mixing angle, the precision is varied among experiments due to the fact that we are unsure whether $(\sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$ is maximal or belong to a specific octant. The neutrino and antineutrino involved parameters agree within 1σ CL.

In this paper, we will investigate the prospects of testing the possible *CPT* violation via the applicably sensitive $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ parameters with the synergy of T2K-II, NOνA extension (for convenience, we will denote it NOνA-II from now on), and JUNO experiments. In particular, we focus on the use of data samples of the ν_μ and $\bar{\nu}_\mu$ *disappearance* channels from the T2K-II and NOνA-II experiments in combination with the *disappearance* of $\bar{\nu}_e$ collected by the JUNO experiment before 2028, where we expect the operational start of the next generation A-LBL experiments. The paper is organized as follows. We describe the simulation of T2K-II, NOνA-II and JUNO experiments in Sec. II. The possibly established bounds of the manifested quantities $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ of the *CPT* violation are presented in Sec. III. Further investigation into the potential significance of *CPT*-invariant exclusion and its robustness against the variation of the underlying physical parameters are discussed in Sec. IV. Finally, we conclude our study in Sec. V.

II. EXPERIMENTAL SIMULATION

The General Long Baseline Experiment Simulator (GLoBES) [31,32] is a sophisticated but flexible framework to simulate, explore the physic potentials of neutrino experiments and fit the experimental data. By default, GLoBES assumes that the oscillation parameters for neutrinos and antineutrinos in Eqs. (3) and (4) are identical or *CPT*-invariant. We extend the package to describe the neutrino and antineutrino oscillations independently.

For the oscillation probability formula, we follow the analytical expressions in Ref. [33]. Neutrino (antineutrino) oscillation in matter depends on nine variables, including six oscillation parameters listed in Eq. (3) for neutrino (or Eq. (4) for antineutrino), as well as neutrino energy E_ν, the propagation distance L, and the matter density ρ. For the *CPT* test, the oscillation parameters of neutrinos and antineutrinos can be treated independently, thus having twelve oscillation parameters as a complete set. However, for this particular study, since the A-LBL experiments have no sensitivity to the solar parameter, we keep $\theta_{12} = \bar{\theta}_{12}$; $\Delta m^2_{21} = \Delta\bar{m}^2_{21}$; $\theta_{13} = \bar{\theta}_{13}$; $\delta_{\rm CP} = \bar{\delta}_{\rm CP}$ and fixed practically. Four independent parameters of interests $(\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \theta_{23}, \bar{\theta}_{23})$ remains.

T2K [34] and NOνA [35] are two world-leading A-LBL experiments. For convenience, we denote T2K run up to 2027 by T2K-II and NOνA extension up to 2024 by NOνA-II. The similarity in experimental configuration and operating principle makes it interesting to have a joint fit between the two experiments [36,37]. Both experiments use intense muon (anti)neutrino beams created by accelerators to study oscillation phenomena. The off-axis technique adopted by both experiments can produce a narrow-band beam of neutrinos to enhance the sensitivity of neutrino oscillation measurements and mitigate the effect of possible bias in the neutrino energy reconstruction from their interaction products. The ability to focus either positive or negative particles (mainly pions and kaons) offers the A-LBL experiment a unique opportunity to operate in both neutrino-mode and antineutrino-mode. This important feature enables the testing of *CPT* invariance in the A-LBL experiments. JUNO [38] is a R-MBL experiment which studies electron antineutrino *disappearance* ($\bar{\nu}_e \to \bar{\nu}_e$). The experiment uses electron antineutrino flux produced from nuclear reactors to study neutrino oscillation at a medium baseline (about 50 km) to take advantage of the interference of two oscillation lengths, which are driven by two mass-squared splittings, $\Delta\bar{m}^2_{21}$ and $\Delta\bar{m}^2_{31}$. Achieving a neutrino energy resolution of less than 3% is essential for JUNO to resolve these two oscillation patterns and measure oscillation parameters $\sin^2\bar{\theta}_{12}, \Delta\bar{m}^2_{21}$

and $|\Delta\bar{m}^2_{31}|$ at precision less than 0.5% [39]. The JUNO experiment also can resolve neutrino mass hierarchy at 3σ CL after six years of operation. Combining data samples from JUNO and from the A-LBL experiments, T2K-II and NOνA-II, will definitely resolve the neutrino mass ordering [26].

We follow closely the experimental specifications for T2K-II, NOνA-II, and JUNO in the Ref. [26], except for some updates in T2K-II and JUNO. In original proposal [40], T2K-II is expected to operate until 2027, exposing 20×10^{21} protons-on-target (POT). According to the most recent plan [36], statistics may be cut in half. Thus, we use 10×10^{21} POT for T2K-II in this work. We also updated the T2K flux, which was released in 2020 [41]. For JUNO, a total thermal of 26.6 GWth [39] is used instead of 36 GWth as in the previous report. Also, the energy resolution is set at 2.9% [39] to reflect closely the JUNO's prospect.

In terms of the data samples for analysis, for T2K-II and NOνA-II, we used the *disappearance* channels only, with statistics equally divided into ν-mode and $\bar{\nu}$-mode. As we will show later in Sec. III, the *CPT* test on the $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ will be limited due to the precision of Δm^2_{31} measurement by the A-LBL experiment, and thus the bound established in this parameter can be elevated if we have more neutrino data. However, this scenario is unlikely since running an experiment in antineutrino mode is very important for the *CP* violation measurement. For JUNO, $\bar{\nu}_e$ *disappearance* data is used. We assume neutrino masses are in *normal* ordering throughout the study in Secs. III and IV. The study in Sec. III is done with the values of nominal parameters listed in Table I, in which we follow the measurements of T2K [24] for atmospheric parameters ($\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \sin^2\theta_{23}, \sin^2\bar{\theta}_{23}$) and global fit [25] for the rest.

The bounds and the sensitivities to rule out *CPT* invariant hypothesis with $\delta_{\nu\bar{\nu}}(X)$ parameter are explored. The χ^2 of individual experiment is calculated for given *true* values of X for neutrinos and $\bar{X}$ for antineutrinos, where X can be $\sin^2\theta_{23}$ or Δm^2_{31}. We use a log-likelihood χ^2 function for T2K-II and NOνA-II, while a Gaussian formula is used for JUNO due to its high statistics. The calculation of χ^2 is then minimized over the nuisance parameters and other oscillation parameters except for X and $\bar{X}$. The two-dimensional distributions of $\Delta\chi^2$ which is the sum of all the individual ones of the three experiments, are obtained. The minimum of $\Delta\chi^2$ as a function of $\delta_{\nu\bar{\nu}}(X) = X - \bar{X}$ is then found. The statistical significance of excluding *CPT* conservation is expressed as the squared root of the minimum $\Delta\chi^2$.

III. POSSIBLY ESTABLISHED BOUNDS OF $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ AND $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ ON *CPT* VIOLATION

In this study, assuming that *CPT* is exactly conserved or extremely small for detection, we estimate the expected bound of the two sensitive parameters, asymmetry in the mass-squared differences $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and asymmetry in the leptonic mixing angles $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$, on the possible *CPT* violation. In particular, $\Delta m^2_{31} = \Delta\bar{m}^2_{31} = 2.55\times10^{-3}$ eV^2 and $\sin^2\theta_{23} = \sin^2\bar{\theta}_{23} = 0.51$, which are the T2K's best-fit points with recent measurement [24], are assumed to be *true*. To compute the allowed region of the $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ parameters, we build up the χ^2 profiles on a two-dimensional grid points of neutrino and antineutrino corresponding parameters ($\Delta m^2_{31}, \Delta\bar{m}^2_{31}$) and ($\sin^2\theta_{23}$, $\sin^2\bar{\theta}_{23}$), respectively. The χ^2 profiles take into account the correlations among the oscillation parameters. The $\Delta\chi^2$ profiles are attained by subtracting to the minimum value of the according χ^2, which is essentially located at the *true* values.

Figure 2 shows 3σ CL allowed regions of pairs of parameters ($\Delta m^2_{31}, \Delta\bar{m}^2_{31}$) and ($\sin^2\theta_{23}, \sin^2\bar{\theta}_{23}$) under the assumption that *CPT* is conserved. Three different analyses are presented: (i) T2K-II only, (ii) a joint of T2K-II and

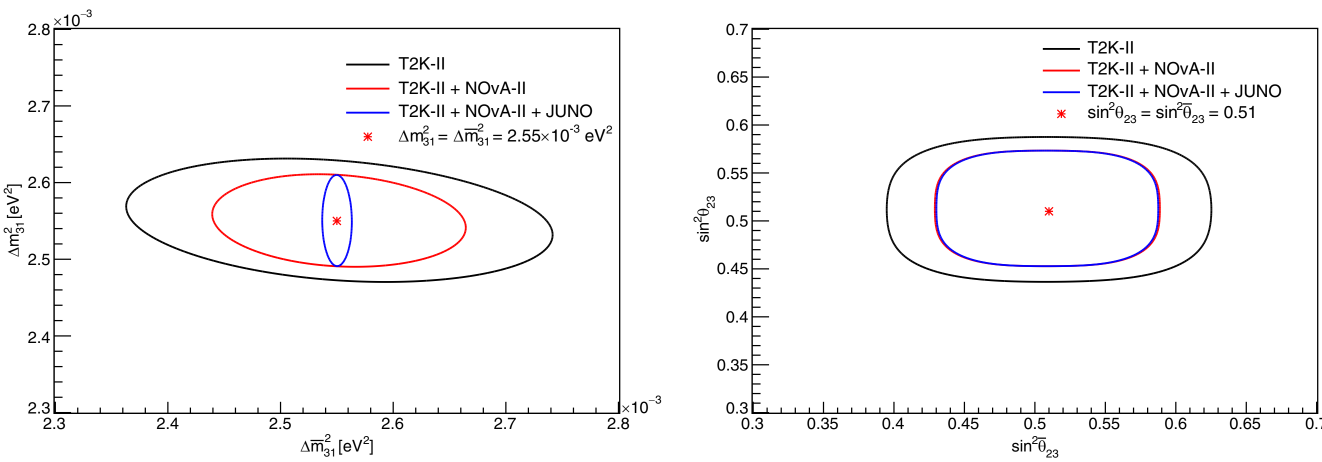


FIG. 2. The 3σ CL regions of Δm^2_{31} and $\Delta\bar{m}^2_{31}$ (left), $\sin^2\theta_{23}$ and $\sin^2\bar{\theta}_{23}$ (right). The black, red, and blue lines are for an analysis with T2K-II only, a joint of T2K-II and NOνA-II, and a joint of T2K-II, NOνA-II, and JUNO, respectively.

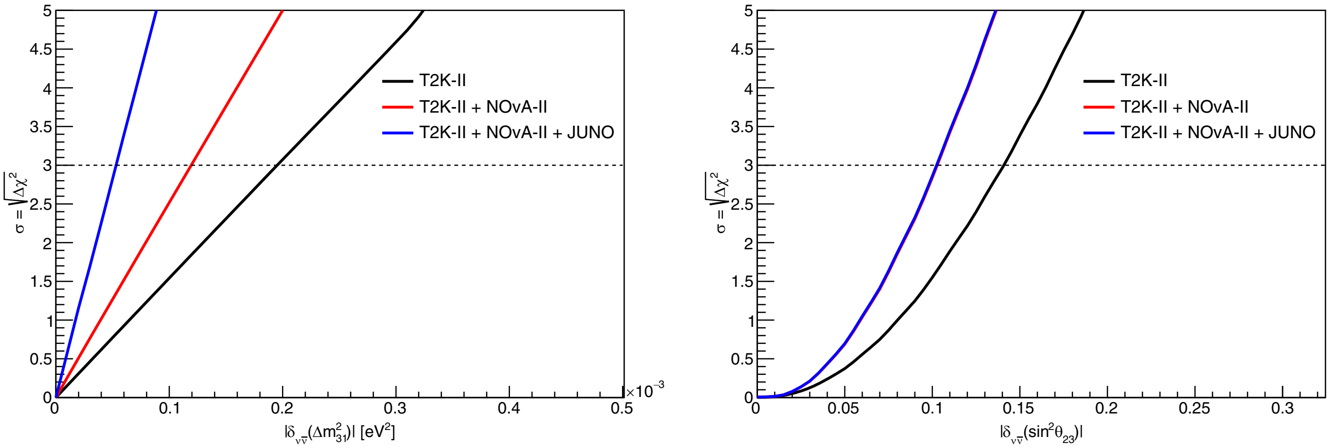


FIG. 3. The bounds on possible *CPT* violation manifested in the asymmetries of the mass-squared splittings $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})|$ (left) and of the leptonic mixing angles $|\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})|$ (right). The black, red, and blue lines correspond to an analysis with T2K-II only, a joint of T2K-II and NOνA-II, and a joint of T2K-II, NOνA-II, and JUNO, respectively.

NOνA-II, and (iii) a joint of T2K-II, NOνA-II, and JUNO. It is expected that a joint analysis of T2K-II and NOνA-II improves significantly the precision of four involved $(\Delta m^2_{31}, \Delta\bar{m}^2_{31}, \sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$ parameters while JUNO mainly contribute to the precision of $\Delta\bar{m}^2_{31}$.

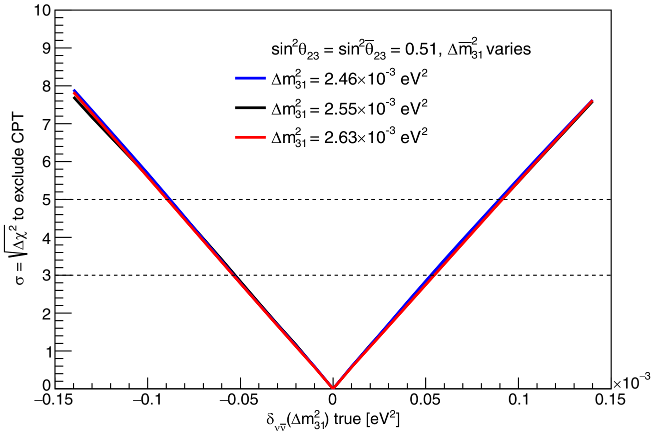
To answer for the question about the allowed parameter magnitudes in the mass-squared difference $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and the leptonic mixing angle $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$, projections of $\Delta\chi^2$ profiles on these two variables are constructed and depicted in Fig. 3. The upper limits of these two *CPT*-sensitive variables at 3σ CL are extracted and summarized in Table III. With total exposure of 10×10^{21} POT, T2K-II alone can set more stringent limits on the *CPT* violation search, if it will be not found, both with atmospheric mass-squared splitting $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})| \leq 2.0 \times 10^{-4}$ eV^2 and leptonic mixing angles $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23}) \leq 0.14$, than the combined data of current neutrino experiments. By adding NOνA-II, the 3σ CL limit on $|\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})|$ for *CPT* violation is reduced to 0.10, a 47% improvement over the current limit. Meanwhile, if no evidence of *CPT* violation is found, the potential bound on $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})|$ at 3σ CL will be expected to be 5.3×10^{-5} eV^2 for the combined analysis of the three experiments. This prospective bound on the possible *CPT* violation search is slightly better than the DUNE sensitivity [42], $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})| < 8.1 \times 10^{-5}$ eV^2 at 3σ *CL*.

TABLE III. The bounds on *CPT* violation with atmospheric mass-squared difference and mixing angle at 3σ CL for three analyses: T2K-II only, a joint of T2K-II and NOνA-II, a joint of T2K-II, NOνA-II, and JUNO.

	3σ CL upper limits	
Experiments	$\lvert\delta_{\nu\bar{\nu}}(\Delta m^2_{31})\rvert$	$\lvert\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})\rvert$
T2K-II	2.0×10^{-4} eV^2	0.14
T2K-II + NOνA-II	1.2×10^{-4} eV^2	0.10
T2K-II + NOνA-II + JUNO	5.3×10^{-5} eV^2	0.10

IV. SIGNIFICANCE OF *CPT* EXCLUSION: DEPENDENCE AND PROJECTION

Apparently if the analyses with real data shows the asymmetries of $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})|$ or $|\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})|$ larger than the corresponding upper limits presented in Table III, it would imply the *CPT* violation in the lepton sector. However, one raised question is whether these anticipated limits are affected by the *true* values of the underlying parameters, which can fluctuate from the current best-fit values. To investigate this issue, we performed the full joint analysis of T2K-II, NOνA-II, and JUNO under various assumptions of the involved parameters. In particular, for the potential effect on $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$, we examine the *CPT* sensitivity at three points $(2.46 \times 10^{-3}, 2.55 \times 10^{-3}, 2.63 \times 10^{-3}$ eV$^2)$ of Δm^2_{31}, taken as the T2K best-fit and $\pm 1\sigma$ shifted values, in combination with a variation of $\Delta\bar{m}^2_{31}$ such that $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})| < 0.15 \times 10^{-3}$ eV^2. In this case of study, $\sin^2\theta_{23} = \sin^2\bar{\theta}_{23} = 0.51$ is assumed to be *true*. In addition, we check the sensitivities of *CPT* violation on the $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ parameter at three shared values (0.44, 0.51, 0.57) of $(\sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$. For each case, the statistical significance to exclude the corresponding form of the *CPT* invariance is extracted as function of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and the results are shown in Fig. 4. It is observed that the *CPT* violation sensitivity manifested on the $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ parameter depend marginally on the central value of Δm^2_{31} and $\Delta\bar{m}^2_{31}$ in the current allowed range of this parameter. Also the dependence of the $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ sensitivity on the *true* value of the mixing parameter $(\sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$ is relatively small. Apparently, due to the octant degeneracy of $(\sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$ presented in the *disappearance*



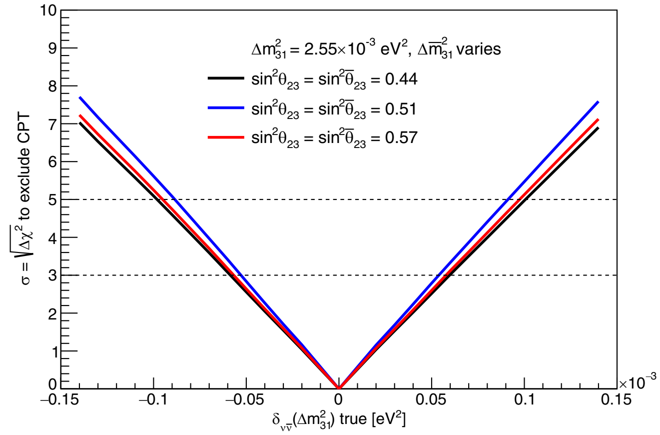


FIG. 4. Statistical significance to exclude *CPT* is computed as a function of *true* $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ under various scenarios of the involved parameters. The left plot is when Δm^2_{31} is examined at three different *true* values, while $\sin^2\theta_{23} = \sin^2\bar{\theta}_{23} = 0.51$ is assumed to be *true*. The right plot presents the *CPT* sensitivity of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ at different *true* values of $\sin^2\theta_{23}$ and $\sin^2\bar{\theta}_{23}$ while $\Delta m^2_{31} = 2.55\times10^{-3}$ eV^2 is assumed to be *true*.

probabilities of muon (anti)neutrinos, the significance of the *CPT* test is slightly worse than the case where $(\sin^2\theta_{23}, \sin^2\bar{\theta}_{23})$ is exactly equal or near the maximal mixing. The lower limit of *true* $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ magnitude to exclude the *CPT* at 3σ CL or higher significance is presented in Table IV. We find that if the deviation of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ from zero is greater than 6.0×10^{-5} eV^2 the *CPT* invariance will be excluded at 3σ CL for almost the entire currently-allowed range of the involved parameters. The range of possible $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ asymmetry to be explored significantly is slightly extended ($[5.36, 5.46]\times10^{-5}$ eV^2) if the mixing angle is near the maximal mixing. Due to the aforementioned octant degeneracy of the (anti)neutrino oscillation probabilities in the *disappearance* samples, the deviation of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ from zero must be moderately greater ($[5.77, 5.99]\times10^{-5}$ eV^2) for attaining a same level of significance to exclude the *CPT* invariance. To see how impressive the improvement in the *CPT* test sensitivity from this three-experiment combined analysis is, we project the statistical significance from the current measurements. As summarized in the Table II, the difference in mass-squared splitting at the best-fit values of $(\Delta m^2_{31}, \Delta\bar{m}^2_{31})$ measured by T2K [24] is $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})| = 3\times10^{-5}$ eV^2, well consistent within 1σ uncertainty of 20×10^{-5} eV^2. However, if this asymmetry persists as the *true*, it will correspond to 1.7σ CL exclusion of *CPT* conservation by the combined analysis of T2K-II, NOνA-II, and JUNO. If the level of asymmetrical $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ in the neutrino and antineutrino best-fit values of NOνA and MINOS(+), which is 7.0×10^{-5} eV^2, are assumed to be persisted as the *true*, the synergy of the three experiments can exclude *CPT* conservation at 4σ CL.

TABLE IV. Lower limits for the *true* $|\delta_{\nu\bar{\nu}}(\Delta m^2_{31})|$ magnitude to exclude *CPT* at 3σ CL are computed at different *true* values of the involved parameters.

	Shared values of $\sin^2\theta_{23}, \sin^2\bar{\theta}_{23}$		
Δm^2_{31} [eV^2]	0.44	0.51	0.57
2.46×10^{-3}	5.96×10^{-5}	5.36×10^{-5}	5.80×10^{-5}
2.55×10^{-3}	5.95×10^{-5}	5.39×10^{-5}	5.77×10^{-5}
2.63×10^{-3}	5.99×10^{-5}	5.46×10^{-5}	5.79×10^{-5}
	$\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ limit to exclude *CPT* at 3σ CL		

Regarding the sensitivity of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ on the *CPT* test, we examine and find that their dependence on the fluctuation of the $(\Delta m^2_{31}, \Delta\bar{m}^2_{31})$ parameters is relatively small while the dependence on the *true* value of ($\sin^2\theta_{23}$, $\sin^2\bar{\theta}_{23}$) is significant, as shown in Fig. 5. When the *true* value of $\sin^2\theta_{23}$ belongs to an octant, there exists a degenerated solution in the other octant. For example, when $\sin^2\theta_{23} = 0.44$, the *extrinsic CPT*-invariant solution of $\sin^2\bar{\theta}_{23} = 0.58$ (along with the *genuine* solution of $\sin^2\bar{\theta}_{23} = 0.44$). Similar behavior is observed when $\sin^2\theta_{23}$ values in the higher octant. The behavior is well-understood due to the dependence of muon (anti) neutrino *disappearance* probabilities on the $\sin^2 2\theta_{23}$ ($\sin^2 2\bar{\theta}_{23}$) rather than $\sin^2\theta_{23}$ ($\sin^2\bar{\theta}_{23}$). As summarized in Table V, to attain the same significance level to exclude the *CPT*, compared to the maximal case $\sin^2\theta_{23} = 0.51$, the magnitude of *true* $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ asymmetry in the non-maximal cases ($\sin^2\theta_{23} = 0.44$ and $\sin^2\theta_{23} = 0.57$) is required to be larger or smaller depending on whether the θ_{23} and $\bar{\theta}_{23}$ belong to the different or same octants, respectively. In particular, for $\sin^2\theta_{23} = 0.51$ as indicated by both T2K [24] and NOνA [29], the magnitude of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ asymmetry must be between [0.076, 0.084] to be discovered with 3σ CL T2K (NOνA) measured $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23}) = 0.08$ (0.10) respectively, and if it remains

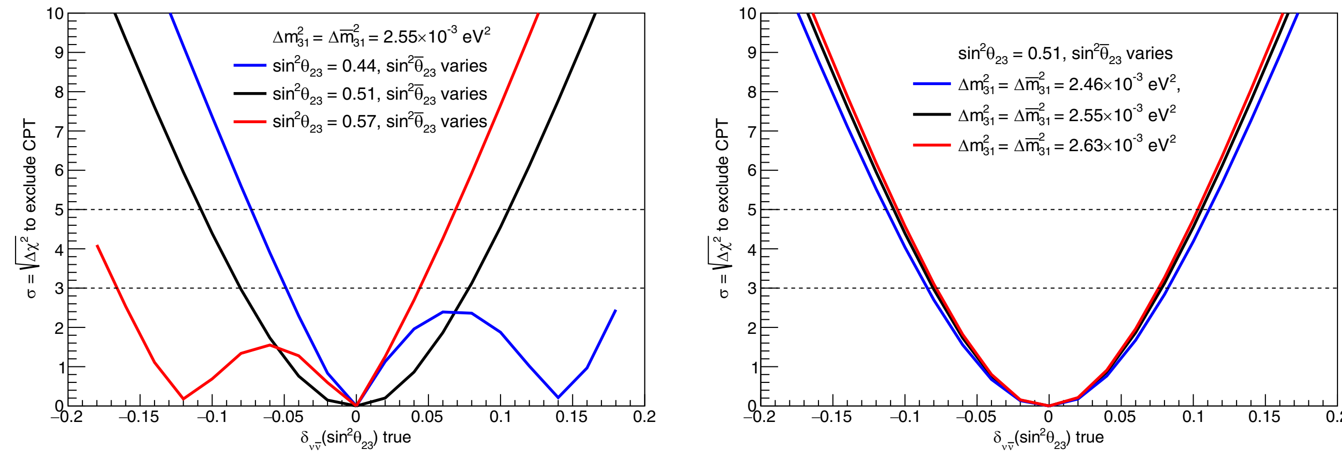


FIG. 5. Statistical significance to exclude *CPT* is computed as function of *true* $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ under various scenarios of the involved parameters. The left is when $\sin^2\theta_{23}$ is examined at three different *true* values while $\Delta m^2_{31} = \Delta\bar{m}^2_{31} = 2.55\times10^{-3}$ eV^2 is assumed. The right presents the *CPT* sensitivity of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ at different *true* values of Δm^2_{31} and $\Delta\bar{m}^2_{31}$ while $\sin^2\theta_{23} = 0.51$ is assumed to be *true*.

as *true* the *CPT* invariance will be excluded at 3σ or higher CL. If θ_{23} and $\bar{\theta}_{23}$ are in the same octant and relatively far off from the maximal values, the deviation of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ from zero must be greater than 0.051 in order to rule out *CPT* invariance at 3σ CL. If θ_{23} and $\bar{\theta}_{23}$ are in different octants, θ_{23} in lower octant and $\bar{\theta}_{23}$ in higher octant or vice versa, the magnitude of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ must be significantly higher, varying in the (0.165,0.190) range, to exclude *CPT* at the same 3σ statistical significance. The sensitivity to detect *CPT* violation via the $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ asymmetry is not good due to the aforementioned octant degeneracy in the muon (anti) neutrino *disappearance* samples. The sensitivity can be improved by adding the electron (anti)neutrino *appearance* samples from the A-LBL experiments. Figure 6 shows the sensitivity of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ on the *CPT* exclusion with a combination of both *disappearance* and *appearance* samples. It is observed that by adding the electron (anti)neutrino *appearance* samples, the statistical significance to exclude the *extrinsic CPT*-invariant solution is enhanced notably. Consequently, the sensitivity of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ to the *CPT* violation has improved. However, one must consider carefully when adding the electron (anti)neutrino *appearance* samples. The reason is that the probabilities of $\nu_e(\bar{\nu}_e)$ from $\nu_\mu(\bar{\nu}_\mu)$ depend not only on $\theta_{23}(\bar{\theta}_{23})$ but also on two known unknowns, *CP*-violating phase and mass ordering, which will complicate the interpretation of the experimental observation.

TABLE V. Lower limits for the *true* $|\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})|$ deviation from zero to exclude *CPT* at 3σ CL are computed at different *true* values of involved parameters. The $-(+)$ signs in each cell correspond to the negative (positive) value of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$.

	Shared values of Δm^2_{31}, $\Delta\bar{m}^2_{31}$ [eV^2]		
$\sin^2\theta_{23}$	2.46×10^{-3}	2.55×10^{-3}	2.63×10^{-3}
0.44	−0.051 (+0.190)	−0.049 (+0.187)	−0.048 (+0.186)
0.51	−0.084 (+0.082)	−0.080 (+0.078)	−0.078 (+0.076)
0.57	−0.169 (+0.047)	−0.166 (+0.044)	−0.165 (+0.043)
	$\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ limit to exclude *CPT* at 3σ CL		

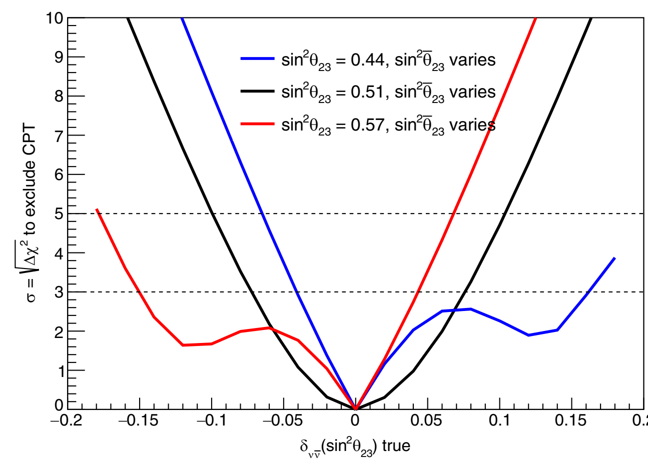


FIG. 6. Statistical significance to exclude *CPT* is computed as a function of *true* $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ under various scenarios of the involved parameters. Both muon (anti)neutrino *disappearance* samples and electron (anti)neutrino *appearance* samples from T2K-II and NOνA-II are used. The sensitivity is examined at three different *true* values of $\sin^2\theta_{23}$ values while $\Delta m^2_{31} = \Delta\bar{m}^2_{31} = 2.55\times10^{-3}$ eV^2 is assumed to be *true*.

V. CONCLUSION

In the paper, we presented the potential of timely combined analysis of the two on-going accelerator-based long-baseline experiments T2K-II, NOνA-II and a reactor-based medium-baseline JUNO experiment in testing *CPT* symmetry via the measurable asymmetry in the oscillation parameters of neutrinos and antineutrinos. The analysis is expected to happen by 2028, before the operational start of the next generation of accelerator-based long-baseline neutrino experiments, DUNE [43] and Hyper-Kamiokande [44]. In particular, we focus on the asymmetries in the mass-square splitting $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ and in the leptonic mixing angle $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$. The synergy of these three experiment will plausibly establish an unprecedented bound of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ to about 5.3×10^{-5} eV^2 at 3σ CL in case the CPT symmetry is conserved to be sensitive. This bound extends substantially the current bound of 2.5×10^{-4} eV^2 derived from the global neutrino data analysis. It is noteworthy to stress that this bound of $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ on the possible CPT violation is marginally dependent on the *true* values of the involved parameters, especially the ambiguity of the $\theta_{23}(\bar{\theta}_{23})$ values. The improvement of CPT sensitivity is very encouraging since if the difference between the best-fit values of Δm^2_{31} and $\Delta\bar{m}^2_{31}$ currently measured by NOνA and MINOS$(+)$ persists as the *true*, the statistical significance to exclude the CPT is about 4σ CL. For the testable asymmetry in the leptonic mixing angle $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$, the statistical significance of the CPT sensitivity depends strongly on their own *genuine* values, which rooted from the parameter degeneracy in the muon (anti)neutrino *disappearance* probabilities. In the case of CPT conservation, if the neutrino mixing angle θ_{23} is close to the maximal, as indicated by both T2K and NOνA current measurements, the combined analysis of the three experiments will potentially establish a limit of $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23}) = 0.10$, compared to the current bound of 0.19 attained from the global neutrino data analysis. Interestingly, if the difference in the best-fit values of $\sin^2\theta_{23}$ and $\sin^2\bar{\theta}_{23}$ measured recently by both T2K and NOνA persist as the *true*, the combined analysis of the two with their final data samples will indicate CPT violation with 3σ CL or higher.

Finally, it is important to emphasize that one cannot claim a CPT violation simply by observing sizable $\delta_{\nu\bar{\nu}}(\Delta m^2_{31})$ or $\delta_{\nu\bar{\nu}}(\sin^2\theta_{23})$ asymmetries because some nonstandard interactions, such as those discussed in Ref. [45], can mimic the effect. In any case, investigating the potential differences in the parameters governing neutrino and antineutrino oscillations is critical to revealing the new physics.

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