

# High energy sphalerons for baryogenesis at low temperatures

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We discuss baryogenesis in scenarios where the Universe is reheated to temperatures  $\lesssim 100$  GeV by the decay of long-lived massive particles into energetic SM particles. Before its thermalization, the center-of-mass energy in collisions between such a particle and a particle from the ambient plasma can be higher than the typical sphaleron mass, even if the temperature of the plasma itself is much lower. Optimistic estimates for the high energy enhancement of the sphaleron cross section suggest that successful baryogenesis is possible for reheating temperatures as low as 0.1–1 GeV. With a simple extension of the SM, sufficient baryon production should be achieved by enhancing the W-boson coupling even if more pessimistic results for the sphaleron rate are correct. In both cases, if the two to many sphaleron reaction is significant enough for the baryogenesis, the same process can be probed in collider and cosmic-ray experiments. Complementing such experimental tests, a significantly improved understanding of the two to many sphaleron rate in the nonperturbative coupling regime is mandatory to determine whether this scenario is viable. Finally, we briefly discuss the possible origin of the required  $CP$  violation.

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## I. INTRODUCTION

Many scenarios of string [1–5] or supersymmetric theories [6–11], (see, e.g., [12–17] for discussions in light of current data or in the context of axions [18–35]) predict relatively low temperatures for the final reheating before the start of standard cosmology. This is due to the existence of intermediate-mass particles with Planck suppressed couplings to the standard model (SM) particles. Featuring such small couplings they are often long-lived. Due to the Hubble expansion, they then become nonrelativistic and eventually dominate the energy density of the Universe. Finally, they decay to reheat the Universe with a relatively low reheating temperature, often below the electroweak scale. This is often viewed as a problem for baryogenesis (see, however [36–49] for examples of mechanisms to achieve the required matter-antimatter asymmetry at potentially quite low temperatures that use additional sources of baryon number violation c.f. Refs. [50–53]; alternatively, cold baryogenesis in low-scale hybrid inflation models was considered in [54–57]). Indeed, below the critical temperature  $T_c \sim 150$  GeV in the broken phase the sphaleron/instanton effect, which is the only baryon number violating process within the Standard Model, is very strongly suppressed. Nevertheless, we want

to argue that even in such a scenario, successful baryogenesis may be achieved in a rather minimal setup.

From the viewpoint of the Sakharov conditions [58] the late-time decays of heavy particles also provide an opportunity: they are inherently an out-of-equilibrium process.<sup>1</sup> Therefore, let us focus on the short period soon after a long-lived particle decays into energetic SM particles. During the first steps of the thermalization of an energetic SM particle, the center-of-mass energy between it and a particle in the (already thermalized) ambient plasma can easily be higher than  $M_{\text{sph}}$ , even when the reheating temperature  $\lesssim 100$  GeV. In such energetic collisions, it may be easier to produce suitable sphalerons and therefore fulfill Sakharov's condition of a sufficiently effective baryon number violating process.

Earlier work in similar directions includes [60], where the authors considered the possibility that in each decay of the long-lived particles the decay products first heat a small asymmetric region where the plasma temperature is much higher than 100 GeV. Thus less suppressed symmetric-phase sphalerons can be active in this region. Another option is to use symmetric-phase sphalerons at the early stage of reheating [61] (when the plasma temperature is actually higher), the resulting asymmetry is, however, typically diluted by  $(T_R/100 \text{ GeV})^5$ . In contrast, we will consider the direct sphaleron enhancement by the scattering of an energetic particle from the decay and a plasma particle.

<sup>1</sup>See Refs. [37–44,46,51–53,59] for baryogenesis using this, too.

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At present, calculations of highly energetic sphaleron rates are still plagued by uncertainties. Therefore, we first quantify how much enhancement of the sphaleron effect is needed for successful baryogenesis. The relevant cross sections turn out to be of a size that may allow for tests at the LHC, future colliders, and cosmic-ray experiments. Turning to concrete calculations of the sphaleron rate we find that more optimistic estimates give values that allow for successful baryogenesis. If more pessimistic estimates turn out to be correct, we can still obtain successful baryogenesis by considering a simple BSM extension of the gauge group that enhances the gauge interaction and thus the sphaleron rate at high energies.

Finally, turning to the last Sakharov condition— $CP$  violation—we briefly discuss some possibilities how this may be realized in the present context in Appendix A.

## II. BRIEF REVIEW OF OUT-OF-EQUILIBRIUM SPHALERONS

The electroweak instanton/sphaleron is the only source of baryon number violation within the SM. As a tunneling process between vacua with different Chern-Simons numbers it is expected that the instanton effect is exponentially suppressed. A sphaleron, being externally supplied with enough energy,  $\gtrsim M_{\text{sph}} \sim 7$  TeV, does not have this suppression since it is a classical saddle-point solution connecting the different vacua. Here,  $m_W(\alpha_w)$  is the weak boson mass (coupling constant). Therefore, cosmological models with a reheating temperature  $T_R \ll M_{\text{sph}}$  are usually argued not to have efficient baryogenesis via sphalerons.

The instanton/sphaleron-induced cross section for highly energetic quarks has been studied in the context of collider and cosmic ray phenomenology (cf., e.g., [62–69]), but also for a better conceptual understanding of the sphaleron itself. Unfortunately, the size of the cross section for this nonperturbative process is still under debate. Let us briefly review the current status. The instanton effect can be estimated from the 't Hooft operator [70]  $\mathcal{L}_{\text{eff}} \sim e^{-\frac{2\pi}{\alpha_w} \frac{Q^9 L^3}{v^{14}}} + \text{H.c.}$ , with  $Q$  and  $L$  being the left-handed quarks and leptons, and  $v \sim 170$  GeV the Higgs vacuum expectation value (VEV). Importantly, [71,72] pointed out that by emitting multiple bosons in 2 to many processes  $QQ \rightarrow 7\bar{Q} + 3\bar{L} + n_Z Z + n_W W + n_H H$  the cross section can be enhanced exponentially in  $N = n_Z + n_W + n_H$ , fitting the picture that sphalerons are more efficient than instantons at high energy scales since  $N$  can be larger.

The inclusive cross section is often given in terms of the so-called holy grail function,  $F$ , specifying the exponential suppression,

$$\sigma_{\text{sph}} \approx \frac{1}{E_{\text{cm}}^2} \exp\left(-\frac{4\pi}{\alpha_w} F(E_{\text{cm}})\right), \quad (1)$$

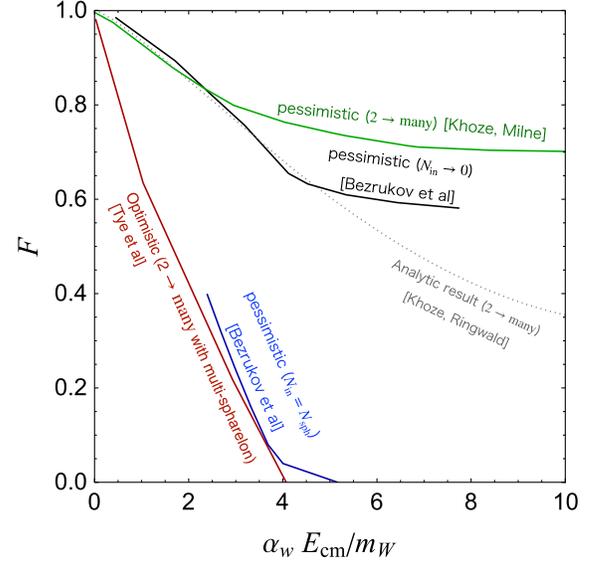


FIG. 1. Different estimations of the holy grail function depending on  $E_{\text{cm}}\alpha_w/m_W$ . The optimistic estimation (red) is from [73–75] (see also [76]). The pessimistic ones are from [77,78] (black) and [79] (green). We also show the low energy analytic result [80] (gray dotted) and a result with  $N_{\text{in}} = N_{\text{sph}} = 1/\alpha_w$  translated from [77,78] (blue). While the latter is in principle a pessimistic estimate, we note that applying it to  $\alpha_w = \mathcal{O}(1)$  and  $N_{\text{in}} = N_{\text{sph}} = 2$  might be optimistic (see the discussion below Eq. (12)).

where  $E_{\text{cm}}$  is the center-of-mass energy. The holy grail function has been calculated by various authors and using a range of different methods and limits. We summarize the different estimations in Fig. 1 as functions of  $\alpha_w E_{\text{cm}}/m_W$ .

## III. BROKEN PHASE SPHALERON FROM REHEATING

For concreteness, we discuss the thermalization process during reheating caused by a modulus decay. Our conclusions do not change if we replace the modulus by the inflaton, a long-lived fermion, e.g., gravitino/Peccei-Quinn fermion, or interaction with macroscopic objects, as, e.g., a bubble wall collision with the ambient plasma [48,81,82] or with another bubble wall [83], as long as they provide energetic daughter SM particles<sup>2</sup> and reheat the Universe.

Let  $\phi$  be the long-lived modulus. A simple case is that  $\phi$  decays into a pair of energetic SM particles charged under  $SU(2)_L$ <sup>3</sup>

<sup>2</sup>In the bubble wall case, especially for the wall-wall collisions, the scenario of [60] may also be interesting for the sphaleron process, since the temperature may locally exceed 100 GeV.

<sup>3</sup>Precisely speaking, considering the chirality suppression, the decay of a scalar to SM fermions should be three-body with large  $m_\phi$ . Alternatively, the mother particle could also be a vector boson. On the order of magnitude level this does not change our conclusions.

$$\phi \rightarrow \chi\bar{\chi} \equiv LL, Q\bar{Q}, W\bar{W}, ZZ, \text{ or/and } hh. \quad (2)$$

If the non-relativistic modulus dominates the Universe, the reheating temperature is given by  $T_R \equiv (g_* \frac{\pi^2}{90})^{-1/4} \sqrt{M_{\text{pl}} \Gamma_\phi}$ . Here  $M_{\text{pl}} \approx 2.4 \times 10^{18}$  GeV is the reduced Planck scale,  $g_*$  (and  $g_{s*}$  used below) is the number of relativistic degrees of freedom (for entropy) in the SM (we use the numbers from Ref. [84]). With  $\Gamma_\phi = m_\phi^3/M^2$ ,  $m_\phi$  being the mass of the modulus and  $M$  the typical scale of the coupling to the SM particles via a dimension 5 term, we obtain  $T_R \approx 20 \text{ MeV} (\frac{M_{\text{pl}}}{M}) (\frac{11}{g_*})^{1/4} (\frac{m_\phi}{100 \text{ TeV}})^{3/2}$ . Thus, if  $m_\phi \ll 10^8$  GeV,  $T_R$  is below the weak scale if  $M \sim M_{\text{pl}}$ .

Let us focus on what happens within a Hubble time around  $T = T_R$ , where the energy density in the moduli and in radiation is roughly equal. In general,  $F$  depends on  $E_{\text{cm}}$  and the sphaleron contribution during reheating may depend on the behavior of  $F$ . However, for large enough  $E_{\text{cm}}$ ,  $F$  is thought to be approximately constant. Thus, we consider a simplified situation where this holds. We will briefly revisit this assumption below.

Around the relevant time we can estimate the total number density of highly energetic SM particles,  $\chi$ , produced in the decay of the moduli, as,

$$n_\chi \sim 2n_\phi \frac{\Gamma_\phi}{H} \sim \frac{g_* \pi^2 T_R^4}{15m_\phi}, \quad (3)$$

with  $H$  being the Hubble parameter at that time. In the last approximation, we have used  $\Gamma_\phi \sim H^4$  and the definition of the end of reheating,  $n_\phi m_\phi \sim g_* \pi^2 T_R^4/30$ .

Now, let us look at the thermalization of the energetic SM particles,  $\chi$ . To this end, we focus on a single  $\phi$  particle. At this time, the Universe is comprised of the other  $\phi$  particles who will decay later, ambient thermalized plasma from the  $\phi$  decays at earlier times, and the energetic  $\chi\bar{\chi}$  from the  $\phi$  decay of interest. Initially  $\chi$  carries an energy,  $E_\chi \simeq \frac{m_\phi}{2}$ . Thermalization proceeds via the gauge interactions. The timescale on which  $\chi$  loses an  $\mathcal{O}(1)$  fraction of its energy, is  $t_{\text{th}}$ . This is estimated [60,85–90] (see also Refs. [91,92]) from the Landau-Pomeranchuk-Migdal effect [93,94] to be,

$$t_{\text{th}} \simeq t_{\text{LPM}} \simeq \left( \alpha_\chi^2 \sqrt{\frac{T_R}{E_\chi}} T_R \right)^{-1}. \quad (4)$$

<sup>4</sup>We note that this is not in conflict with the out-of-equilibrium nature of the decay of  $\phi$ , since  $m_\phi \gg T_R$ . The produced  $\chi$  quickly lose the energy due to the thermalization and accordingly the inverse decay  $\chi\bar{\chi} \rightarrow \phi$  is highly suppressed. Similarly there is no significant washout from because the plasma has energy much smaller energy/temperature.

Here,  $\alpha_\chi = \alpha_3$  or  $\alpha_e$ , i.e., the coupling constants for the gluon and the photon, depending whether  $\chi$  is a left-handed quark or charged lepton. The scattering rate [denominator in Eq. (4)] also receives contributions of order  $\alpha_2^3 T_R^3/m_{W,Z}^2 \times m_{W,Z}/E_\chi$  from the W/Z Bremsstrahlung. If this rate is larger than the denominator on the r.h.s of (4), we should use it in the evaluation of  $t_{\text{th}}$ . A similar t-channel  $2 \rightarrow 2$  scattering process obtained by exchanging a soft W-boson is particularly important when  $\chi$  is a neutrino. In this case, the neutrino is translated into a charged lepton at a timescale (see e.g., [95])  $t_\nu \approx (\alpha_2^3 T_R^3/m_W^2)^{-1}$ . After this the resulting charged lepton quickly thermalizes. In addition, if the sphaleron reaction is too fast, we need to consider its effect on the thermalization. Since Eq. (4) will be dominant in most of the parameter region of interest, we will mainly focus on that timescale in our discussion. In the result of Fig. 2, we also include the other contributions. The important thing is that during  $t_{\text{th}}$  we have energetic SM particles,  $\chi$ , whose collision center-of-mass energy can be much larger than  $T_R$ .

The most significant process for the baryon number violation is that  $\chi$  scatters with the ambient plasma. The typical center-of-mass energy for these interactions is

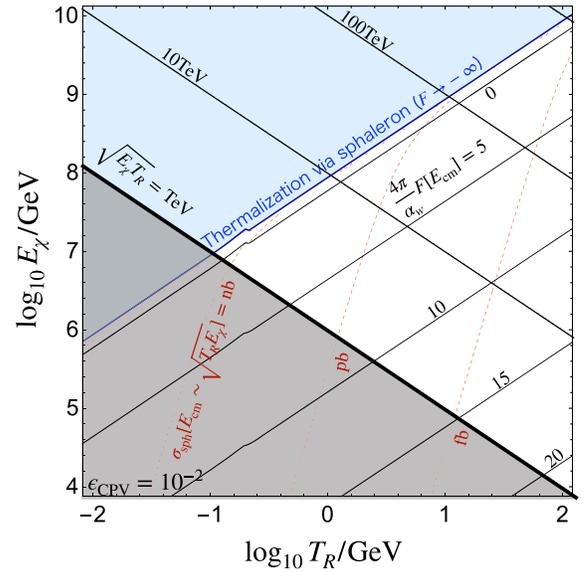


FIG. 2. Contours for the required sphaleron cross section (red dashed lines) and holy grail function  $4\pi F/\alpha_w$  (thinner black lines) at the center-of-mass energy  $E_{\text{cm}} \sim \sqrt{T_R E_\chi}$  for successful baryogenesis with  $\epsilon_{\text{CPV}} = 10^{-2}$  as a function of  $T_R$  and  $E_\chi (\approx m_\phi/2)$ . Here, we assume  $\chi$  is a charged lepton. We show the contours of  $E_{\text{cm}} \sim \sqrt{E_\chi T_R}$  in black bold lines. The shaded region below the former may be already excluded by collider searches. Above the blue solid line, the sphaleron dominates the thermalization and the baryon asymmetry is too small. This analysis is only valid when  $E_{\text{cm}}$  is in the regime that  $F$  is almost a constant.

$E_{\text{cm}} \sim \sqrt{E_\chi T_R}$ . For each  $\chi$ , the probability to interact via a sphaleron can be estimated as,

$$P_{\text{sph}} \simeq \langle \sigma_{\text{sph}} n_r \rangle \times t_{\text{th}} \sim \frac{1}{E_\chi} T_R^2 e^{-\frac{4\pi}{\alpha_w} F(T_R E_\chi)} \times t_{\text{th}}. \quad (5)$$

Here,  $\langle \rangle$  denotes the thermal average, and  $n_r$  is the number density of the ambient plasma particles that are charged under the  $SU(2)_L$ .<sup>5</sup> Then we obtain the sphaleron events per unit volume per unit Hubble time as

$$n_{\text{sph}} \simeq P_{\text{sph}} n_\chi. \quad (6)$$

Normalizing this to the entropy density,  $s = \frac{2g_{s*} \pi^2 T_R^3}{45}$ , we obtain [using Eq. (4)],

$$\frac{n_{\text{sph}}}{s} \sim 3 \times 10^{-5} \frac{g_\star}{g_{s\star}} e^{-\frac{4\pi}{\alpha_w} F(\sqrt{T_R E_\chi})} \times \left( \frac{10^7 \text{ GeV}}{m_\phi} \right)^{3/2} \left( \frac{T_R}{10 \text{ GeV}} \right)^2 \left( \frac{\alpha_e}{\alpha_\chi} \right)^2. \quad (7)$$

In the case where the sphaleron interaction dominates the thermalization of  $\chi$ , it is this rate that determines,  $t_{\text{th}}^{-1} \sim t_{\text{sph}}^{-1} \equiv \langle \sigma_{\text{sph}} n_r \rangle$ . This leads to

$$\frac{n_{\text{sph}}}{s} \simeq \frac{3g_\star}{4g_{s\star}} \frac{T_R}{E_\chi}, \quad (8)$$

which is independent of  $\sigma_{\text{sph}}$  as long as the reaction is faster than the other thermalization processes.

The baryon to entropy ratio should satisfy [96]  $\frac{\Delta n_B}{s} \simeq \epsilon_{\text{CPV}} \frac{n_{\text{sph}}}{s} \simeq 8.7 \times 10^{-11}$  with  $\epsilon_{\text{CPV}}$  being a model-dependent parameter representing the amount of the  $CP$  violation, which will be discussed in Appendix A. Effectively it is the number of baryons produced per sphaleron interaction. Usually, we expect  $\epsilon_{\text{CPV}} < 1$ . In Fig. 2 we show the required sphaleron cross section (red dashed) and values for the holy grail function (black) that allow for a sufficient baryon asymmetry in the  $T_R - E_\chi (\simeq m_\phi/2)$  plane (for  $\epsilon_{\text{CPV}} = 10^{-2}$ ). We have taken into account the various thermalization processes assuming that  $\chi$  is a charged lepton, and using  $t_{\text{th}}^{-1} = t_{\text{LPM}}^{-1} + t_{\text{sph}}^{-1}$ . The lower shaded region corresponding to  $\sqrt{T_R E_\chi} \lesssim \text{TeV}$  may be inconsistent with the collider search [68,69]. In the blue region, the sphaleron interaction dominantly thermalizes  $\chi$  and not enough baryons are generated. The figure in the Appendix B depicts the case where  $\chi$  is a neutrino.

Exploring the parameter space, we find that successful baryogenesis is possible even for temperatures around

<sup>5</sup>The thermal average of the reaction is  $\langle \sigma_{\text{sph}} n_r \rangle \simeq \frac{g \log \epsilon}{48(E_\chi T_R)} T_R^3$  with  $g$  being the number of scatterers in the plasma,  $\epsilon$  the IR cutoff in the angular integral, and  $g \log \epsilon = \mathcal{O}(10)$ .

$\sim 100$  MeV, if the sphaleron cross sections are large enough. If we adopt the most optimistic estimation of Fig. 1,  $F \approx 0$  for  $E_{\text{cm}} \gtrsim 9$  TeV. In this case, our scenario allows for  $T_R \lesssim 100$  GeV with  $\epsilon_{\text{CPV}} \gtrsim 10^{-6}$ . As mentioned before, realistically  $F$  is not a simple constant. In the optimistic scenario of Fig. 1,  $F$  changes rapidly when  $E_{\text{cm}} < 9$  TeV. If  $\sqrt{T_R E_\chi} \lesssim 9$  TeV, the reaction during reheating with the dilute plasma temperature  $T > T_R$  is actually more important despite the later dilution. It is dominant at  $T \sim T_*$  satisfying  $\sqrt{T_* \cdot E_\chi} \sim 9$  TeV if the reheating lasts long enough. For this case we obtain  $\frac{\Delta n_B}{s} \simeq \epsilon_{\text{CPV}} \left( \frac{T_R}{T_*} \right)^5 \frac{n_{\text{sph}}}{s} \Big|_{T_R \rightarrow T_*}$  if  $T_R < T_*$ .<sup>6</sup> From this we may obtain additional allowed parameter space.

However, if we adopt the pessimistic estimations in Fig. 1, we get  $4\pi/\alpha_w F = \mathcal{O}(100)$ , and the scenario is difficult in the minimal setup. Therefore, let us briefly consider BSM effects that may sufficiently enhance the sphaleron such that successful baryogenesis is also possible in the pessimistic case.

#### IV. BSM MODEL WITH FASTER SPHALERONS

Let us consider a simple renormalizable model to enhance the sphaleron rate. The key idea is to enhance the gauge coupling for the SM fermion at a high energy scale. One possibility is  $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$  broken by a bifundamental Higgs field. (See also models with a bifundamental Higgs for GUT breaking [97,98] and generating axion/ALP potentials [99–103].) The Lagrangian is given as

$$\mathcal{L} = -\sum_i \frac{1}{4g_i^2} F_i F_i + \mathcal{L}_{H_b} + \mathcal{L}_{\text{SM}}(A_2), \quad (9)$$

with  $\mathcal{L}_{H_b}$  being the Lagrangian for a bifundamental Higgs field, which transforms as  $H_b \rightarrow U_1^\dagger H_b U_2$ .  $F_i$  ( $U_i$ ) is the field strength (gauge transformation) for the gauge field  $A_i$  of  $SU(2)_i$ . We omit the Lorentz indices.  $A_1$  ( $A_2$ ) does not (does) appear in  $\mathcal{L}_{\text{SM}}[A_2]$  which is the SM Lagrangian with the usual  $SU(2)_L$  fields and coupling,  $A_L$ ,  $g_L$ , replaced by  $A_2$ ,  $g_2$ .

$\langle H_b \rangle = v_b \delta_{ij}$  spontaneously breaks  $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$ . For simplicity, we further restrict the field space with the condition,

$$H_b = \sigma_2 H_b^* \sigma_2, \quad (10)$$

such that the number of real degrees of freedom is four, three of which are eaten by the gauge bosons (see Appendix C for more details on the vacuum structure.)

<sup>6</sup>When  $T_* \gtrsim 100$  GeV, the usual sphaleron in the thermal plasma is active and we also need to consider the effect of [61].

There exists a heavy ( $A_H$ ) with mass  $M_H^2 = v_b^2(g_1^2 + g_2^2)$  and an up to this point massless ( $A_L$ ) gauge boson,

$$A_L \equiv \cos\theta A_1 + \sin\theta A_2, \quad A_H \equiv \sin\theta A_1 - \cos\theta A_2, \quad (11)$$

with  $\sin\theta = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$ ,  $\cos\theta = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$ . The light  $A_L$  corresponds to the usual weak bosons and couples to the SM fermions with strength,

$$g_L = \sin\theta g_2 \quad g_H = \cos\theta g_2. \quad (12)$$

If we have  $g_2 \gg g_1$ , we get  $g_L \simeq g_1$ ,  $A_L \simeq A_1$ , as well as  $A_H \simeq A_2$  and  $g_H \simeq g_2$ . Thus we can have a strong coupling of the SM fermions/Higgs to  $A_2$ . To evade collider bounds we need, very roughly,  $M_H \gtrsim \mathcal{O}(1)$  TeV. Moreover, we note that the severe indirect bound from flavor/ $CP$  violation are evaded because our model conserves the  $CP$  and flavor accidentally.<sup>7</sup>

Importantly, when  $E_{\text{cm}} \gg M_H$ , the strongly coupled  $SU(2)_2$  also contributes to the sphaleron rate, significantly enhancing it. For instance, the suppression factor  $4\pi/\alpha_2 \sim 10$  in Eq. (1) for  $\alpha_2 \sim 1$ . Thus even we take  $F = 1$  the required amount of  $4\pi F/\alpha_2$  for baryogenesis may be achieved. In this case, one can have an efficient baryon number violation with  $T_R \gtrsim 100$  GeV (10 GeV, 1 GeV) and  $m_\phi = \text{TeV-PeV}$  with  $\epsilon_{\text{CPV}} = 10^{-6}(10^{-3}, 1)$ .

Moreover, for large  $\alpha_2$  we may be closer to the situation  $2 \sim N_{\text{in}} \sim N_{\text{out}} \sim 1/\alpha_2$  that gives the blue curve in Fig. 1 that corresponds to a nearly unsuppressed rate with  $F \approx 0$ . Alternatively, we may consider that the exponential suppression originates from the ratio of the size from the sphaleron,  $\sim 1/m_W$  over its Compton wavelength,  $\sim 1/M_{\text{sph}} \propto \alpha_w/m_W$ , cf., [104]. Following this argument, with a large coupling  $\alpha_2$ , the exponential suppression may be absent.<sup>8</sup>

If the result given in [77,78] is applicable to  $\alpha_2 \sim 1$  with fermions, we could have successful baryogenesis with  $T_R \gtrsim 10$  GeV, (100 MeV, 10 MeV) with  $m_\phi = \text{TeV-EeV}$  and  $\epsilon_{\text{CPV}} = 10^{-6}(10^{-3}, 1)$ .

Another efficient, but not renormalizable, model to enhance the sphaleron rate may be a large volume extra-dimension where the  $SU(2)_L$  gauge boson lives in the bulk. In this case, the small instanton of the  $SU(3)$  can be significantly enhanced in the context of the heavy QCD axion [105,106]. However, the  $CP$ -violating higher dimensional terms should be somehow made small to solve the strong  $CP$  problem [107,108]. In the context of our  $SU(2)$  sphalerons, the  $CP$ -violating nature is good news since it naturally satisfies one of the conditions for baryogenesis.

<sup>7</sup>The  $CP$ -safe structure is violated if we remove the condition of Eq. (10) (See Appendix A).

<sup>8</sup>We would like to thank the referee for pointing out this explanation, as well as several other valuable suggestions improving our paper.

## V. CONCLUSIONS

Depending on whether more optimistic or more pessimistic estimates for the sphaleron rate at high energies are correct, successful baryogenesis via sphalerons induced by scattering energetic decay products on the ambient plasma may be achievable for reheating temperatures as low as  $\sim 100$  MeV with or without a BSM modification. This scenario for baryogenesis requires a very fast out-of-equilibrium sphaleron rate. This is true both in the minimal as well as in the BSM scenario. Probing this sphaleron process in collider or cosmic-ray experiments is also a test of our scenario. In addition, in the BSM models we may search for the predicted heavy gauge bosons. The  $\phi$  decays may also produce additional feebly-coupled out-of-equilibrium BSM particles that could also be probed in suitable Earth-bound experiments [20–22,27,28,34,35,95,109–113].

That said, a further understanding of the sphaleron interaction with multiple incoming particles in the regime of large coupling and including fermions is quite important for clarifying the allowed parameter region of this simple scenario.

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## APPENDIX A: $CP$ VIOLATION

So far we have simply parametrized the necessary  $CP$  violation by employing a free parameter  $\epsilon_{\text{CPV}}$  in the sphaleron rate. Let us now list some exemplary scenarios how such a  $CP$  violation could arise. In fact, there are various viable models for  $CP$  violation e.g., [37–44,46,48,114,115]. In principle, these should also work in our scenario of sphalerons being triggered in the broken phase.

*Baryogenesis via  $CP$ -violating sphaleron reactions* We can directly produce the baryon asymmetry by including the  $CP$  violation during the thermalization. For instance, we can slightly extend Eq. (9) without imposing the restriction of the field space. Note that we can have the couplings  $(\lambda'|H|^2 + A' + \lambda''|H_b|^2) \det(H_b^2) + \text{h.c.}$  with  $H$  being the Higgs doublet that gives masses to the SM fermions. In this case, the couplings  $\lambda'$ ,  $A'$ ,  $\lambda''$  generically involve  $CP$  phases that cannot be removed by the redefinition of the  $H_b$  phase. (They are absent if we impose the restriction.) Using naïve dimensional analysis we expect very roughly that  $\epsilon_{\text{CPV}} \sim \max[|\Im[A'\lambda'^*]/M_{\text{sph}}|, |\Im[A'\lambda''^*]/M_{\text{sph}}|, |\Im[\lambda'\lambda''^*]|]$ . In this case, we may need to care about the low energy CPV since the model is no longer accidentally  $CP$  conserving. This however can be made safe from the current bound of the electric dipole moments (EDMs). For instance, the dominant contribution to the electron EDM should be generated at the 2loop level via the Baa-Zee diagram

composed by  $H_b$ ,  $H$ , and  $A_H$  internal lines. The contribution is of the order  $d_e \sim e \frac{\Im[\lambda'(\lambda'')^*] g_H^2 m_e v_d^2}{(16\pi^2)^2 M_H^4}$ , which can be easily made to be below the current bound and it may be tested in the future. Here we have assumed that this perturbative estimation is justified and that the highest mass scale in the loop is  $M_H$ . Again, the flavor violation effect is highly suppressed since the model is still minimally flavor violating.

With the field space restriction another possibility is to introduce another heavy Higgs doublet,  $\tilde{H}$ , whose VEV is small. If it transforms via  $(2, -1/2)$  under  $SU(2)_1$  and  $U(1)_Y$ , the Higgs potential can have a contribution  $V \supset A\tilde{H}^* \cdot H_b \cdot H + \tilde{\lambda}(\tilde{H}^*)^2 H^2$ . By noting that the phase of  $H_b$  is fixed by the reality condition (10), the phase  $\arg \tilde{\lambda} A^*$  cannot be removed by field-redefinitions. Then very roughly we get  $\epsilon_{\text{CPV}} \sim \frac{\Im[\tilde{\lambda} A^*]}{M_{\text{sph}}}$ .

Interestingly, in this setup the flavor structure of the SM may be partially explained by noting that some families may be charged under  $SU(2)_1$  while some are charged under  $SU(2)_2$ . For instance, let us consider that only the 3rd generation fermions are charged under  $SU(2)_2$  and thus obtain mass via the VEV of  $H$ . Then the first two generation masses are naturally suppressed by the mixing between  $\tilde{H}$  and  $H$ . The flavor structure is similar to the two Higgs doublet model specifying the top quark in Ref. [116]. The only difference is that the flavor-dependent discrete symmetry in the reference is replaced by the gauge symmetry in our case. Therefore the flavor structure is quite similar: i.e., we have suppressed flavor-changing neutral currents among the first two generation quarks as well as an electron EDM which is generated at the 2-loop level [116].<sup>9</sup> In particular, with a BSM mass scale around 1–10 TeV the EDM may be probed in the not too distant future. If the sphaleron is enhanced by a large  $\alpha_2$ , there is a small instanton contribution in the low energy physics. In this case we have an effective coupling  $\frac{Q^3 L}{v_b^2} e^{-2\pi/\alpha_2 [v_b]}$ , for the 3rd generation fermions. Via CKM mixing  $V_C$ , we then obtain an operator that involves only first generation quarks, e.g.,  $(V_C)_{t,d}(V_C)_{t,d}(V_C)_{t,b}\epsilon_b(V_C^*)_{u,b} \frac{d_t d_t u_t \nu_t}{v_b^2} e^{-2\pi/\alpha_2 [v_b]}$  with  $\epsilon_b = \mathcal{O}(y_b^2/16\pi^2)$  being the order of magnitude estimate of the RG effect and  $y_b$  the bottom quark Yukawa coupling. The effective suppression scale for this higher dimensional operator is around the GUT scale for  $\alpha_2 \sim 0.2$  and  $v_b \sim 10$  TeV. Thus proton decay may be a useful tool to probe this scenario. On the other hand, it is already excluded for  $\alpha_2 \sim 1$ . We can also use different charge assignments. For example if the first two generations are charged under  $SU(2)_2$  the proton does not decay. This is because then the operator generated by the instanton is  $\frac{Q^6 L^2}{v_b^6} e^{-2\pi/\alpha_2 [v_b]}$ , which preserves a  $Z_2$  baryon number symmetry, which stabilizes the proton. This model can also explain the small

masses of the first two generations if  $\tilde{H}$  is the SM-like Higgs field.

*Leptogenesis* One can also first produce a lepton asymmetry from the  $\phi$  decays. Later this is transferred into the baryon asymmetry by the broken phase sphalerons discussed in this Letter. This is the famous idea of leptogenesis in the symmetric phase [50–53]. One can easily produce the lepton asymmetry from lepton flavor oscillations [59,114,115,117]. Alternatively, we can assume that  $\phi$  carries lepton number and employ the Affleck-Dine mechanism [36,118]. The SM leptons are then obtained from a coupling  $\phi H L H L$ . In any case, we need a much larger neutrino/lepton asymmetry  $\sim \epsilon_{\text{CPV}}$  than the baryon asymmetry because only a small fraction can be transferred by the sphalerons induced by highly energetic particles [cf. Eq. (C)]. This is in contrast to the conventional symmetric phase leptogenesis which predicts that the two asymmetries are comparable. The predicted large neutrino asymmetry may be probed, e.g., in future CMB experiments [119,120].

*Spontaneous baryogenesis (may not work)* We may also consider a coupling of  $\frac{\alpha_w}{2\pi} \frac{\mathcal{O}[t]}{\Lambda^d} \text{tr} W \tilde{W}$  with  $\mathcal{O}$  being a time-dependent operator made up of single or several fields,  $1/\Lambda^d$  the higher dimensional coupling, and  $d$  the dimension of  $\mathcal{O}$ . Then in the single sphaleron transition, we get the amplitude  $\mathcal{M} e^{\pm i(\frac{\mathcal{O}(t+\delta t)}{\Lambda^d})} \simeq \mathcal{M} e^{\pm i(\frac{\mathcal{O}(t)+\dot{\mathcal{O}}\delta t}{\Lambda^d})}$  for particle/antiparticle process with  $\delta t$  being a small change of time and expanding  $\mathcal{O}$  in time up to linear order. This amplitude time-dependence feeds into the energy momentum conservation and thus the cross section  $\sigma_{\text{sph}}[E_{\text{cm}}] \rightarrow \sigma_{\text{sph}}[E_{\text{cm}} \pm \frac{\dot{\mathcal{O}}}{\Lambda^d}]$  for the particle/antiparticle transition. In this way we obtain a  $CP$  violating effect  $\epsilon_{\text{CPV}} \simeq \frac{\dot{\mathcal{O}}}{\Lambda^d} \frac{\partial}{\partial E_{\text{cm}}} \log \sigma_{\text{sph}}$ . A rough estimate tells us  $\epsilon_{\text{CPV}} \sim \dot{\mathcal{O}}/(\Lambda^d E_{\text{cm}})$ . This is similar to the spontaneous baryogenesis picture [121] (cf. also [122–137] for a variety of interesting implementations) in which the baryon asymmetry is proportional to the chemical potential  $\mu = \frac{\dot{\mathcal{O}}}{\Lambda^d}$ . However, this also requires  $\dot{\mathcal{O}} \sim E_{\text{cm}} \Lambda^d$  in order to have efficient  $CP$  violation. Assuming  $\Lambda \gtrsim E_{\text{cm}}$ , this implies that there is a large kinetic energy density contribution from the operator  $\gtrsim E_{\text{cm}}^4$  (at least for  $d = 1$ ). This is inconsistent with the assumption in our setup that the Universe has an energy density  $T_R^4 \ll E_{\text{cm}}^4$ . Thus the spontaneous baryogenesis scenario may not work.

## APPENDIX B: SPHALERONS FROM NEUTRINO SCATTERING

In the main text we have shown results for the situation where the moduli decay into charged leptons. Another interesting situation arises if the decay is into neutrinos. This is shown in Fig. 3, where  $\epsilon_{\text{CPV}} = 1$  (top panel),  $10^{-3}$  (middle panel),  $10^{-6}$  (bottom panel). Here, we use

<sup>9</sup>The nonperturbative instanton effect.

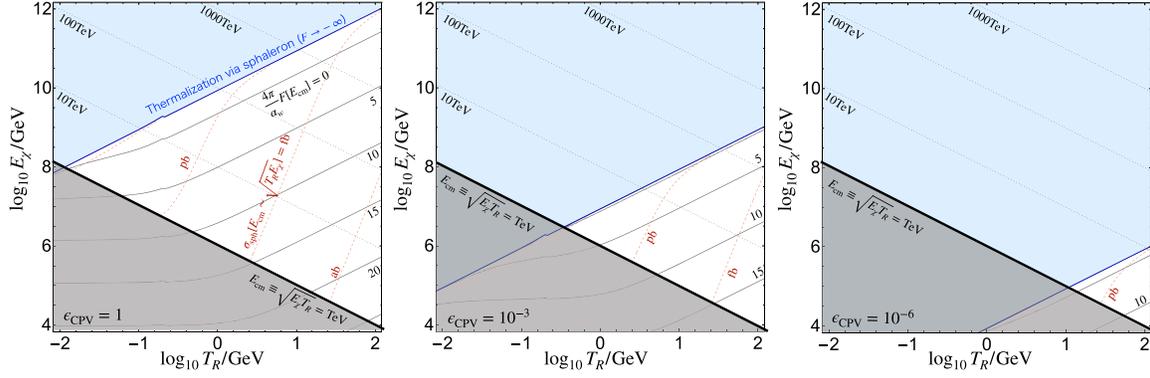


FIG. 3. Same as Fig. 2 with  $\chi$  being a neutrino with  $\epsilon_{\text{CPV}} = 1, 10^{-3}, 10^{-6}$  from left to right.

$t_{\text{th}}^{-1} = (t_{\nu} + t_{\text{LPM}})^{-1} + t_{\text{sph}}^{-1}$  because only after a soft W-boson exchange, the photon interaction becomes active. The contour is flat in the low temperature regime because the thermalization timescale for a neutrino is determined by the W boson emission,  $t_{\nu}$ . This gives a longer time for the sphaleron reaction to work.

### APPENDIX C: VACUUM STRUCTURE OF THE BIFUNDAMENTAL HIGGS

Here we discuss the vacuum structure by stabilizing the potential for the bifundamental  $SU(2)_1 \times SU(2)_2$  Higgs. Let us start by commenting on what happens if we do not impose the restriction (10). From gauge invariance, the potential is then given by

$$V = -\mu_H^2 \text{tr}[H_b^\dagger \cdot H_b] + \lambda_b (\text{tr}[H_b^\dagger \cdot H_b])^2 + \lambda_{\text{tr}} \text{tr}[H_b^\dagger \cdot H_b \cdot H_b^\dagger \cdot H_b] + A' \det(H_b) + \lambda'' (\det H_b) |H_b|^2 + \text{H.c.} \quad (\text{C1})$$

Here  $\lambda_b, \lambda_{\text{tr}}, \lambda''$  denote the dimensionless parameters and  $\mu_H, A'$  are the dimensionful parameters. The first line is invariant under an  $SO(8)$  symmetry. Here,  $\lambda''$  and  $A'$  may be complex. The second line breaks  $SO(8)$  explicitly down to  $U(1) \times SU(2)_1 \times SU(2)_2$ . Finally, the third line breaks the  $U(1)$  explicitly. Therefore, the would-be Nambu-Goldstone (NG) bosons from the symmetry breaking of  $SO(8) \rightarrow SU(2)_1 \times SU(2)_2$  obtain masses via the explicit breakings. In addition, the third line also contributes to the  $CP$  violation, as discussed above. In general, we also have the potential terms  $\delta V = \lambda''' \det[H_b]^2 + \lambda'''' \det H_b \det H_b^\dagger + \lambda_p |H|^2 \text{tr}[H_b^\dagger H_b] + \lambda'_p H^\dagger \cdot H_b^\dagger \cdot H_b \cdot H + \lambda' \det[H_b] |H|^2 + \text{H.c.}$  with additional dimensionless couplings. Including them does not significantly change the discussion as long as the additional couplings are subdominant.

To check the spectrum, we expand

$$H_b = \mathbf{1}T_0 + \sum_i \sigma_i T_i, \quad (\text{C2})$$

by using the Pauli matrices. Here  $T_{0,1,2,3}$  are the complex fields. We then find

$$H_b^\dagger H_b = \mathbf{1} \left( |T_0|^2 + \sum_i |T_i|^2 \right) + \sum_i \sigma_i (T_i^* T_0 + T_0^* T_i + i\epsilon_{ijk} T_j^* T_k). \quad (\text{C3})$$

If we take  $\mu_H^2 > 0, \lambda_b > 0$  and neglect the other terms the potential becomes

$$V_1 = -2\mu_H^2 \left( |T_0|^2 + \sum_i |T_i|^2 \right) + 4\lambda_b \left( |T_0|^2 + \sum_i |T_i|^2 \right)^2. \quad (\text{C4})$$

Thus  $\langle H_b \rangle = \langle T_0 \rangle \mathbf{1} = \frac{1}{2} \sqrt{\frac{\mu_H^2}{\lambda_b}} \mathbf{1}$  without loss of generality. At this point, we have 7 NG modes.

Next let us introduce  $\lambda_{\text{tr}} \neq 0$ , but  $|\lambda_{\text{tr}}| \lesssim \lambda_b/2$ . Then we can see that the potential features an additional term

$$V_2 = 2\lambda_{\text{tr}} \left( (|T_0|^2 + \sum_i |T_i|^2)^2 + \sum_i (T_i^* T_0 + T_0^* T_i + i\epsilon_{ijk} T_j^* T_k)^2 \right). \quad (\text{C5})$$

Now from the first term the VEV is shifted to  $\langle T_0 \rangle = \sqrt{\mu_H^2 / (\lambda_b + \lambda_{\text{tr}}/2)} / 2$ . We note that the second term in Eq. (C5) provides a mass term for the  $CP$ -even triplet

$$V_2 \supset 2\lambda_{\text{tr}} |\langle T_0 \rangle|^2 \sum_i (T_i^* + T_i)^2. \quad (\text{C6})$$

The  $CP$ -odd triplets are the would-be NG bosons that are eaten by the heavy gauge bosons in the  $SU(2)_1 \times$

$SU(2)_2 \rightarrow SU(2)_L$  breaking. To achieve that the remnant  $SU(2)$  remains unbroken, i.e.,  $T_i^* + T_i$  does not have a VEV, we need  $\lambda_{\text{tr}} > 0$ . We also note that given  $\lambda_{\text{tr}} = \mathcal{O}(1)$  the triplet is as heavy as  $\langle T_0 \rangle = v_b$ .

The determinant terms not only give a VEV shift and the mass shift to the  $CP$ -even triplet Higgs but it also gives the mass to the neutral NG boson associated with the  $U(1)$  symmetry, under which,  $H_b \rightarrow e^{ia} H_b$ . We get the potential for the axion  $a$  from the term as

$$(A' v_b^2 + \lambda'' v_b^4) e^{ia/v_b} + \text{H.c.} \quad (\text{C7})$$

In summary, having spontaneous symmetry breaking by our bifundamental Higgs and a potential that consists only of the first row of Eq. (C1), we obtain 7 NG modes. They are a neutral “axion” and 2 triplet Higgs fields under  $SU(2)_L$ . The  $CP$ -odd triplet is eaten by the gauge boson. The second row of Eq. (C1) provides mass for the  $CP$ -even triplet Higgs. The third row finally provides masses to the  $CP$ -even triplet Higgs and the “axion.” Our discussion in the main part corresponds to the case that the triplet Higgs and the axion are heavy. If they are lighter they may lead to an interesting phenomenology in accelerator experiments.

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