Medium modification of singly heavy baryons in a pion-mean-field approach

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We investigate how the masses of singly heavy baryons undergo changes in nuclear matter. The mass spectrum of the singly heavy baryons was successfully described in a pion mean field approach even with isospin symmetry breaking, based on which we extend the investigation to the medium modification of the singly heavy baryons. Since all dynamical parameters were determined by explaining the mass spectrum of the SU(3) light and singly heavy baryons in free space, we can directly implement the density-dependent functionals for the dynamical parameters, of which the density dependence was already fixed by reproducing the bulk properties of nuclear matter and medium modification of the SU(3) light baryons. We predict and discuss the density dependence of the masses of the singly heavy baryons.

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I. INTRODUCTION

Understanding hadrons in nuclear medium has been one of the most important issues in hadronic and nuclear physics, since it is deeply connected to the nonperturbative aspects of quantum chromodynamics (QCD): the restoration of chiral symmetry and quark confinement [1–4]. The quark condensate, an order parameter of spontaneous breakdown of chiral symmetry, is known to decrease in nuclear medium, which indicates that chiral symmetry tends to be restored as the nuclear density increases [1]. Experimentally, it has also been observed that the properties of the nucleon undergo a change in nuclei [5–10]. It implies that other baryons may be modified in nuclear matter. In the present work, we want to focus on how the masses of the singly heavy baryons change in nuclear matter. The heavy flavors in nuclei were already investigated right after the J/ψ was found [11–16]. The singly heavy baryons Λ_c and Σ_c in nuclear matter were examined in relativistic mean-field theory [17], the quark-meson coupling model [18–20], and QCD sum rules [21–24] (see also a recent review and references therein [25]). Recently, the SU(3) Skyrme model with a bound-state approach was applied to the masses of the singly heavy baryons in nuclear matter [26].

Recently, we investigated how the masses of the SU(3) baryon octet and decuplet undergo changes in nuclear medium, based on the medium-modified pion mean field approach [27]. We first examined baryonic matter including symmetric matter, asymmetric matter, neutron matter, and strange baryonic matter, taking empirical information on the bulk properties of nuclear matter as a guiding principle. By describing the various matters and masses of the octet and decuplet in nuclear medium, we were able to fix all density-dependent parameters. Thus, we can proceed to study the masses of the singly heavy baryons in nuclear medium with parameters already fixed. The pion mean field approach, also known as the chiral quark-soliton model (χQSM), was constructed by Witten's seminal idea [28]: in the large N_c (the number of colors) limit, the nucleon can be regarded as a state of N_c valence quarks bound by the pion mean field generated self-consistently by the presence of the N_c valence quarks. The same idea can be applied to the singly heavy baryons. If we take the limit of the infinitely heavy-quark mass ($m_Q \to \infty$), a heavy quark resided in a singly heavy baryon can be decoupled from the $N_c - 1$ valence quarks inside it. Thus, the heavy quark inside a singly heavy baryon is considered as a mere static color source and the quark dynamics inside it is governed by the light quarks. Since the heavy quark is infinitely heavy, the heavy-quark spin is conserved, which leads to the conservation of the light-quark spin. It is known as the

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heavy-quark spin symmetry. In this heavy-quark mass limit, the singly heavy baryon is independent of the heavy flavor, which is called the heavy-quark flavor symmetry [29–31]. In this picture, the singly heavy baryons are represented by a baryon antitriplet ($\bar{\mathbf{3}}$) and two baryon sextets ($\mathbf{6}$) with spin $1/2$ and $3/2$. Thus, the singly heavy baryons can be considered as a bound state of the $N_c - 1$ valence quarks with the single heavy quark detached. The heavy quark is required only for making the singly heavy baryon a color singlet.

Based on this idea, the pion mean field approach was directly extended to the singly heavy baryons [32]. It has successfully described various properties of the singly heavy baryons in free space [33–44] (see also a recent review [45]). As mentioned previously, using the pion mean field approach, we were able to describe how the masses of the baryon octet and decuplet are modified in nuclear medium [27]. The bulk properties of nuclear matter evaluated from the present approach were in good agreement with empirical and experimental data. We proceed now to describing the masses of the singly heavy baryons with both spin $1/2$ and $3/2$.

The paper is organized as follows: In the next section, we briefly review the general formalism for the pion mean field approach. In Sec. III, we show how to implement the density dependence into the dynamical parameters. In Sec. IV we present the numerical results and discuss them. The last section is devoted to the summary and conclusions of the present work. The explicit expressions for the baryon masses are presented in the Appendix.

II. GENERAL FORMALISM

The pion mean field approach allows one to describe both the light and singly heavy baryons on an equal footing. Replacing one light quark by a heavy quark with the infinitely heavy mass, we can construct a state for the singly heavy baryon [42]. We first define the normalization of the baryon state in the large N_c limit as

$$\langle B(p', J_3')|B(p, J_3)\rangle = 2M_B \delta_{J_3' J_3}(2\pi)^3 \delta^{(3)}(\boldsymbol{p}' - \boldsymbol{p}), \tag{1}$$

where M_B denotes the corresponding baryon mass. A state of the singly heavy baryon is then expressed as

$$\begin{aligned} |B, p\rangle &= \lim_{x_4 \to -\infty} \exp(i p_4 x_4) \mathcal{N}(\boldsymbol{p}) \\ &\quad \times \int d^3x \, \exp(i\boldsymbol{p}\cdot\boldsymbol{x})(-i\Psi_h^\dagger(\boldsymbol{x}, x_4)\gamma_4) J_B^\dagger(\boldsymbol{x}, x_4)|0\rangle, \\ \langle B, p| &= \lim_{y_4 \to \infty} \exp(-i p_4' y_4) \mathcal{N}^*(\boldsymbol{p}') \\ &\quad \times \int d^3y \, \exp(-i\boldsymbol{p}'\cdot\boldsymbol{y}) \langle 0| J_B(\boldsymbol{y}, y_4)\Psi_h(\boldsymbol{y}, y_4), \end{aligned} \tag{2}$$

where $\mathcal{N}(\boldsymbol{p})(\mathcal{N}^*(\boldsymbol{p}'))$ denotes the normalization factor depending on the initial (final) momentum. $J_B(x)$ and $J_B^\dagger(y)$ represent the Ioffe-type current of the $N_c - 1$ valence quarks [46] defined by

$$\begin{aligned} J_B(x) &= \frac{1}{(N_c-1)!}\epsilon_{\alpha_1\cdots\alpha_{N_c-1}} \Gamma^{f_1\cdots f_{N_c-1}}_{(TT_3Y)(JJ_3Y_R)} \\ &\quad \times \psi_{f_1\alpha_1}(x)\cdots\psi_{f_{N_c-1}\alpha_{N_c-1}}(x), \\ J_B^\dagger(y) &= \frac{1}{(N_c-1)!}\epsilon_{\alpha_1\cdots\alpha_{N_c-1}} \Gamma^{f_1\cdots f_{N_c-1}}_{(TT_3Y)(JJ_3'Y_R)} \\ &\quad \times (-i\psi^\dagger(y)\gamma_4)_{f_1\alpha_1}\cdots(-i\psi^\dagger(y)\gamma_4)_{f_{N_c-1}\alpha_{N_c-1}}, \end{aligned} \tag{3}$$

where $f_1\cdots f_{N_c-1}$ and $\alpha_1\cdots\alpha_{N_c-1}$ designate respectively the spin-isospin and color indices. The matrices $\Gamma_{(TT_3Y)(JJ_3Y_R)}$ carry the quantum numbers $(TT_3Y)(JJ_3Y_R)$ for the corresponding baryon. $\psi_{f_k\alpha_k}(x)$ denotes the light-quark field and $\Psi_h(x)$ stands for the heavy-quark field. In the limit of $m_Q \to \infty$, a singly heavy baryon satisfies the heavy-quark flavor symmetry. Then the heavy-quark field can be written as

$$\Psi_h(x) = \exp(-im_Q v\cdot x)\tilde{\Psi}_h(x), \tag{4}$$

where $\tilde{\Psi}_h(x)$ is a rescaled heavy-quark field almost on mass shell. It carries no information on the heavy-quark mass in the leading order approximation in the heavy-quark expansion. v denotes the velocity of the heavy quark [29–31].

We now prove that the normalization factor $\mathcal{N}^*(\boldsymbol{p}')\mathcal{N}(\boldsymbol{p})$ is correctly reduced to $2M_B$, which can be computed as

$$\begin{aligned} \langle B(p', J_3')|B(p, J_3)\rangle &= \frac{1}{\mathcal{Z}_{\mathrm{eff}}}\mathcal{N}^*(p')\mathcal{N}(p) \times \lim_{x_4\to-\infty}\lim_{y_4\to\infty} \exp\left(-iy_4p_4' + ix_4p_4\right) \times \int d^3x d^3y \, \exp(-i\boldsymbol{p}'\cdot\boldsymbol{y} + i\boldsymbol{p}\cdot\boldsymbol{x}) \\ &\quad \times \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}\tilde{\Psi}_h\mathcal{D}\tilde{\Psi}_h^\dagger J_B(y) \times \Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x) \\ &\quad \times \exp\left[\int d^4z\{\times(\psi^\dagger(z))^f_\alpha(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m})_{fg}\psi^{g\alpha}(z) + \Psi_h^\dagger(z)v\cdot\partial\Psi_h(z)\}\right] \\ &= \frac{1}{\mathcal{Z}_{\mathrm{eff}}}\mathcal{N}^*(p')\mathcal{N}(p)\lim_{x_4\to-\infty}\lim_{y_4\to\infty} \times \exp\left(-iy_4p_4' + ix_4p_4\right) \times \int d^3xd^3y \, \exp(-i\boldsymbol{p}'\cdot y + i\boldsymbol{p}\cdot\boldsymbol{x}) \\ &\quad \times \langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x)\rangle_0. \end{aligned} \tag{5}$$

Here, $\mathcal{Z}_{\rm eff}$ is the low-energy effective QCD partition function defined as

$$\mathcal{Z}_{\rm eff} = \int \mathcal{D}U \, \exp(-S_{\rm eff}). \tag{6}$$

$S_{\rm eff}$ is called the effective chiral action expressed as

$$S_{\rm eff} = -N_c \,{\rm Tr}\, \ln [i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}]. \tag{7}$$

$\langle \cdots \rangle_0$ in Eq. (5) expresses the vacuum expectation value of the baryon correlation function. M denotes the dynamical quark mass and the U^{γ_5} represents the chiral field defined by

$$U^{\gamma_5}(z) = \frac{1-\gamma_5}{2} U(z) + U^\dagger(z) \frac{1+\gamma_5}{2} \tag{8}$$

with

$$U(z) = \exp[i\pi^a(z)\lambda^a]. \tag{9}$$

$\pi^a(z)$ are the pseudo-Nambu-Goldstone (pNG) fields and λ^a the flavor Gall-Mann matrices. $\hat{m}$ is the mass matrix of current quarks $\hat{m} = {\rm diag}(m_{\rm u}, m_{\rm d}, m_{\rm s})$. The propagators of a light quark in the χQSM [46] is obtained to be

$$\begin{aligned} G(y,x) &= \left\langle y \left| \frac{1}{i\partial\!\!\!/ + iMU^{\gamma_5} + i\bar{m}} (i\gamma_4) \right| x \right\rangle \\ &= \Theta(y_4 - x_4) \sum_{E_n > 0} e^{-E_n(y_4 - x_4)} \psi_n(\boldsymbol{y}) \psi_n^\dagger(\boldsymbol{x}) \\ &\quad - \Theta(x_4 - y_4) \sum_{E_n < 0} e^{-E_n(y_4 - x_4)} \psi_n(\boldsymbol{y}) \psi_n^\dagger(\boldsymbol{x}), \end{aligned} \tag{10}$$

where $\Theta(y_4 - x_4)$ is the Heaviside step function. We introduce $\bar{m}$, which is the average mass of the up and down current quarks: $\bar{m} = (m_{\rm u} + m_{\rm d})/2$. It properly generates the Yukawa tail of the pion mean field, when we later solve the equation of motion. We define the one-body Dirac Hamiltonian as

$$H = \gamma_4 \gamma_i \partial_i + \gamma_4 M U^{\gamma^5} + \gamma_4 \bar{m} \mathbf{1}. \tag{11}$$

Solving the eigenvalue problem of H, we find the energy eigenvalues corresponding to the single-quark eigenstate

$$H\psi_n(\boldsymbol{x}) = E_n \psi_n(\boldsymbol{x}). \tag{12}$$

We now deal with the heavy-quark propagator in the limit of $m_Q \to \infty$:

$$G_h(y,x) = \left\langle y \left| \frac{1}{\partial_4} \right| x \right\rangle = \Theta(y_4 - x_4) \delta^{(3)}(\boldsymbol{y} - \boldsymbol{x}). \tag{13}$$

Using these quark propagators and taking the limit of $y_4 - x_4 = T \to \infty$, we evaluate the baryon correlation function $\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4) J_B^\dagger(x)\rangle_0$ as follows [46,47]:

$$\begin{aligned} &\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4) J_B^\dagger(x)\rangle_0 \\ &\quad \sim \exp[-\{(N_c - 1)E_{\rm val} + E_{\rm sea} + m_Q\}T] \\ &\quad = \exp[-M_B T], \end{aligned} \tag{14}$$

which cancels the term $\exp(-iy_4 p_4' + ix_4 p_4) = \exp[M_B T]$ in the large N_c limit. Therefore, we prove that the normalization factor becomes $\mathcal{N}^*(\boldsymbol{p}')\mathcal{N}(\boldsymbol{p}) = 2M_B$. Utilizing this normalization and Eq. (14), we derive the classical mass of the singly heavy baryon [34] as

$$M_B = (N_c - 1)E_{\rm val} + E_{\rm sea} + m_Q. \tag{15}$$

Before we proceed to compute the mass spectrum of the singly heavy baryons, we want to mention the ordering of the two limits: $N_c \to \infty$ and $m_Q \to \infty$. We first take the limit of $m_Q \to \infty$ and then we carry out $N_c \to \infty$. This ordering is compatible with the present pion mean field approach. If we had taken the ordering inversely, we would not have detached the heavy quark from the singly heavy baryons.

We restate $S_{\rm eff}$ in Eq. (7) in the following form:

$$S_{\rm eff}(U) = -N_c {\rm Tr}\, \ln D(U), \tag{16}$$

where the trace operator Tr runs over spacetime and all relevant internal spaces. The N_c stands for the number of colors, and $D(U)$ the one-body Dirac differential operator is defined by

$$D(U) = \gamma_4(i\partial\!\!\!/ + i\hat{m} + iMU^{\gamma_5}), \tag{17}$$

where ∂_4 is the time derivative in Euclidean space. The mass matrix of the current quarks $\hat{m}$ can be expressed in terms of the Gell-Mann matrices,

$$\hat{m} = m_1 \mathbf{1} + m_3 \lambda_3 + m_8 \lambda_8, \tag{18}$$

where

$$\begin{aligned} m_0 &= \frac{m_{\rm u} + m_{\rm d} + m_{\rm s}}{3}, \\ m_3 &= \frac{m_{\rm u} - m_{\rm d}}{2}, \\ m_8 &= \frac{m_{\rm u} + m_{\rm d} - 2m_{\rm s}}{2\sqrt{3}}. \end{aligned} \tag{19}$$

U^{γ_5} denotes the SU(3) chiral field,

$$U^{\gamma_5} = \exp[i\pi^a \lambda^a \gamma_5] = \frac{1+\gamma_5}{2} U + \frac{1-\gamma_5}{2} U^\dagger, \tag{20}$$

where $\pi^a(\boldsymbol{r})$ is the pNG field with flavor indices $a = 1, \ldots N_f^2 - 1$. N_f is the number of flavors. Since the hedgehog symmetry constrains the form of the classical pion field as $\boldsymbol{\pi}(\boldsymbol{x}) = \hat{\boldsymbol{n}} \cdot \boldsymbol{\tau} P(r)$, where $P(r)$ is called the profile function of the soliton, we keep only the pion fields π^a with $a = 1, 2, 3$.

We want to mention that the m_s term does not appear in the equation of motion. It is regarded as a small perturbation that breaks flavor SU(3) symmetry explicitly. We will explain the linear m_s corrections later. As pointed out in a recent work [48], the kaon clouds provide a better description for the SU(3) baryons with multiple strangeness such as the Ω^- and Ω_c. However, since we fix all the dynamical variables not by solving the equation of motion but by using the experimental data, the tail problem discussed in Ref. [48] does not enter in the present work.

Thus, we have the SU(2) chiral U field as $U_{\mathrm{SU(2)}} = \exp(i\hat{\boldsymbol{n}} \cdot \boldsymbol{\tau} P(r))$. We now embed the SU(2) soliton into SU(3) by Witten's ansatz [49],

$$U^{\gamma_5}(x) = \begin{pmatrix} U^{\gamma_5}_{\mathrm{SU(2)}}(x) & 0 \\ 0 & 1 \end{pmatrix}. \tag{21}$$

Since we consider the mean field approximation, we can carry out the integration over U in Eq. (6) around the saddle point ($\delta S_{\mathrm{eff}}/\delta\pi^a = 0$). This saddle-point approximation yields the equation of motion that can be solved self-consistently. The solution provides the self-consistent profile function $P_c(r)$, which is just the pion mean field. Compared to the SU(3) light baryons, it is weaker than that produced by the N_c valence quarks.

Since the classical U_{cl} field is not invariant under translation and rotation, we need to restore these symmetries such that we have the singly heavy baryons with correct quantum numbers. Thus, we perform the zero-mode quantization or the semiclassical quantization for the chiral soliton. Since the angular velocity is of order $1/N_c$ and strange quark mass is small in comparison with the soliton mass, we treat them perturbatively. Having carried out the zero-mode quantization, the effective chiral action has a form of

$$S_{\mathrm{eff}} = -N_c(i\partial\!\!\!/ + iMU_c^{\gamma_5} + i\gamma_4 R^\dagger \dot{R} + iR^\dagger \hat{m} R), \tag{22}$$

where $U_c^{\gamma_5}$ is a SU(3) classical soliton field given in Eq. (21), $R(t)$ belongs to a SU(3) rotational unitary group, and $R^\dagger \dot{R}$ is a angular velocity that is of order $1/N_c$. Since the angular velocity and the strange current quark mass are parametrically small, we expand the effective chiral action in Eq. (22) with respect to them. We consider the contribution of the strange current quark to linear order. A detailed formalism for the zero-mode quantization can be found in Refs. [47,50]. Having quantized the soliton, we obtain the collective Hamiltonian as

$$H_{\mathrm{coll}} = M_{\mathrm{cl}} + H_{\mathrm{coll}} + H_{\mathrm{sb}}, \tag{23}$$

where the rotational part of the collective Hamiltonian is given as

$$H_{\mathrm{rot}} = \frac{1}{2\bar{I}_1}\sum_{i=1}^{3} \hat{J}_i^2 + \frac{1}{2\bar{I}_2}\sum_{p=4}^{7} \hat{J}_p^2. \tag{24}$$

Here $\bar{I}_1$ and $\bar{I}_2$ have forms

$$\bar{I}_1 = \eta I_1, \qquad \bar{I}_2 = \eta I_2, \tag{25}$$

where I_1 and I_2 are the usual moments of inertia. Since we take a "model-independent" approach [51], we do not compute all the dynamical parameters such as I_1 and I_2 but determine them by using the experimental data on the mass splitting of the baryon octet and decuplet. In the case of the singly heavy baryons, we only know that $\bar{I}_1$ and $\bar{I}_2$ should be smaller than I_1 and I_2 because the pion mean field from the $N_c - 1$ valence quarks is weaker than that with the N_c ones. Thus, we fit η to the masses of the singly heavy baryons in free space [32]. The J_i are the generators of the SU(3) group of which the first three components are the ordinary spin operators. More details can be found in Refs. [52,53].

In representation $\mathcal{R} = (p, q)$, the eigenvalues of H_{rot} in Eq. (24) are given as

$$E^{\mathrm{rot}}_{(p,q)} = \left(\frac{1}{2\bar{I}_1} - \frac{1}{2\bar{I}_2}\right) J(J+1) + \frac{p^2 + q^2 + 3(p+q) + pq}{6\bar{I}_2} - \frac{3}{8\bar{I}_2} Y'^2, \tag{26}$$

where Y' denotes the right hypercharge. In the case of the SU(3) light baryons, the presence of N_c valence quarks imposes a constraint on the collective Hamiltonian: $Y' = N_c/3$, which selects allowed representations: the octet (**8**) and decuplet (**10**). Since the singly heavy baryon consists of the $N_c - 1$ valence quarks, the right hypercharge is constrained to be $Y' = (N_c - 1)/3$ [32] that allows the antitriplet ($\bar{\mathbf{3}}$) and sextet (**6**). The center masses for the baryon antitriplet and sextet are then given by

$$M^Q_{\bar{\mathbf{3}}} = M_{\mathrm{cl}} + \frac{1}{2\bar{I}_2}, \qquad M^Q_{\mathbf{6}} = M^Q_{\bar{\mathbf{3}}} + \frac{1}{\bar{I}_1}. \tag{27}$$

Note that the center masses are flavor independent.

To describe the mass splitting in a representation, it is essential to introduce the effects of isospin breaking and explicit flavor SU(3) symmetry breaking. They make degenerate baryons split. There are two independent origins of isospin symmetry breaking:

$$\Delta M_B = (\Delta M_B)_{\mathrm{H}} + (\Delta M_B)_{\mathrm{EM}}, \tag{28}$$

where the first term in Eq. (28) denotes the hadronic part, and the second corresponds to the electromagnetic (EM) one [54–56]. Thus, we have to consider both the hadronic and EM effects on the isospin symmetry breaking within the same theoretical framework. The isospin mass differences were already investigated within the pion mean field approach in Refs. [39,57].

Expanding the effective chiral action to the linear order of $\hat{m}$ and carrying out the quantization, we obtain the symmetry-breaking part of the collective Hamiltonian as

$$\begin{aligned} H_{\mathrm{sb}} &= (m_{\mathrm{d}} - m_{\mathrm{u}})\left(\frac{\sqrt{3}}{2}\bar{\alpha} D^{(8)}_{38}(R) + \beta \hat{T}_3 + \frac{\gamma}{2}\sum_{i=1}^{3} D^{(8)}_{3i}(R)\hat{J}_i\right) \\ &\quad + (m_{\mathrm{s}} - \bar{m}) \\ &\quad \times \left(\bar{\alpha} D^{(8)}_{88}(R) + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}}\sum_{i=1}^{3} D^{(8)}_{8i}(R)\hat{J}_i\right), \end{aligned} \tag{29}$$

where the first term arises from the isospin symmetry breaking to linear order, and the second term comes from the SU(3) symmetry breaking also to linear order. Once we introduce the isospin symmetry breaking, we need to include the contributions from the electromagnetic (EM) self-energies of the soliton [39,57]. $D^{(8)}_{ij}$ denote SU(3) Wigner functions. The parameters α, β, and γ are expressed as

$$\begin{aligned} \bar{\alpha} &= \frac{N_c - 1}{N_c}\alpha, \qquad \alpha = -\frac{2}{3}\frac{\Sigma_{\pi N}}{m_{\mathrm{u}} + m_{\mathrm{d}}} - \beta, \\ \beta &= -\frac{K_2}{I_2}, \qquad \gamma = 2\frac{K_1}{I_1} + 2\beta, \end{aligned} \tag{30}$$

where K_1 and K_2 designate the anomalous moments of inertia. $\Sigma_{\pi N}$ stands for the pion-nucleon sigma term. Note that α should be rescaled by $(N_c - 1)/N_c$, because the singly heavy baryon contains $N_c - 1$ valence quarks, which modify the pion mean field. More discussion of $\bar{\alpha}$, β, and γ can be found in Ref. [32].

In the limit of $m_Q \to \infty$, the spin 1/2 and 3/2 sextet states are degenerate. To remove the degeneracy, we have to introduce the hyperfine chromomagnetic interaction (spin-spin interaction) to order $1/m_Q$,

$$H^{\mathrm{HF}}_{LQ} = \frac{2}{3}\frac{\kappa}{m_Q M_{\mathrm{cl}}} S_{\mathrm{L}} \cdot S_{\mathrm{Q}} = \frac{2}{3}\frac{\varkappa}{m_q} S_L \cdot S_Q, \tag{31}$$

where the κ stands for the anomalous chromomagnetic moment. The operators S_L and S_Q designate respectively the spin operators for the soliton and heavy quark. Taking into account the hyperfine mass splitting, the center mass of the sextet in Eq. (27) can be decomposed into those for the spin 1/2 and spin 3/2:

$$\begin{aligned} M^Q_{\mathbf{6}_{1/2}} &= M^Q_{\mathbf{6}} - \frac{2}{3}\frac{\varkappa}{m_Q}, \\ M^Q_{\mathbf{6}_{3/2}} &= M^Q_{\mathbf{6}} + \frac{1}{3}\frac{\varkappa}{m_Q}. \end{aligned} \tag{32}$$

In addition to the EM self-energies of the soliton for the effects of the isospin symmetry breaking, we introduce the EM interaction between the soliton and the heavy quark, which can be formulated in the following expression:

$$H^{\mathrm{Coul}}_{LQ} = \alpha_{LQ}\hat{Q}_L\hat{Q}_Q, \tag{33}$$

where the $\hat{Q}_L$ and $\hat{Q}_Q$ represent charge operators acting on the soliton and heavy quark. The parameter α_{LQ} includes the expectation value of the inverse distance between the soliton and heavy quark, and the fine structure constants. We can fix it by reproducing the existing data on the masses of the singly heavy baryons [32].

Since almost all the dynamical parameters have already been fixed in the light baryon sector, and their density dependences have also been set up in the previous work [27], we will proceed directly to the masses of the singly heavy baryons in baryonic matter.

III. SINGLY HEAVY BARYONS IN BARYONIC MATTER

We now recapitulate the formalism with which we have described bulk properties of various baryonic matters, and the masses of the SU(3) light baryons [27]. We introduce three density-dependent free parameters λ, δ, and δ_s, which are respectively related to the normalized density of infinite nuclear matter, the parameter for isospin asymmetry, and that for the strangeness mixing. They are defined as

$$\lambda = \frac{\rho}{\rho_0}, \qquad \delta = \frac{N - Z}{A}, \qquad \delta_s = \frac{N_s}{A}, \tag{34}$$

where the ρ_0 stands for the normal nuclear matter density, N is the number of neutrons, Z the number of protons, A the baryon number, and N_s the number of baryons with the strangeness $s = |S|$. The strangeness is only an external free parameter, of which the fraction identifies strange matter. We introduce the strangeness-mixing parameter χ, which is defined as $\delta_s = s\chi$ such that we do not need to concern specific strange particles that consist of strange matter. Thus, by taking the nonzero value of χ, we can consider the strange matter.

Following Ref. [27], we have the following density-dependent classical mass, moments of inertia, effects of isospin and SU(3) symmetry breaking:

$$M^*_{\mathrm{cl}} = M_{\mathrm{cl}} f_{\mathrm{cl}}(\lambda, \delta, \delta_1, \delta_2, \delta_3), \tag{35}$$

$$\bar{I}^*_1 = \bar{I}_1 f_1(\lambda, \delta, \delta_1, \delta_2, \delta_3), \tag{36}$$

$$\bar{I}_2^* = \bar{I}_2 f_2(\lambda, \delta, \delta_1, \delta_2, \delta_3), \tag{37}$$

$$E_{\rm iso}^* = (m_d - m_u)\frac{K_{1,2}}{I_{1,2}} f_0(\lambda, \delta, \delta_1, \delta_2, \delta_3), \tag{38}$$

$$E_{\rm str}^* = (m_s - \bar{m})\frac{K_{1,2}}{I_{1,2}} f_s(\lambda, \delta, \delta_1, \delta_2, \delta_3), \tag{39}$$

where $f_{\rm cl}$, $f_{0,1,2}$, and f_s are given as the functions of the baryon density and other medium variables. They are explicitly written as

$$f_{\rm cl}(\lambda) = (1 + C_{\rm cl}\lambda), \tag{40}$$

$$f_{1,2}(\lambda) = (1 + C_{1,2}\lambda), \tag{41}$$

$$f_0(\lambda, \delta) = 1 + \frac{C_{\rm num}\lambda\delta}{1 + C_{\rm den}\lambda}, \tag{42}$$

$$f_s(\lambda, \delta_s) = 1 + g_s(\lambda)\delta_s, \tag{43}$$

$$g_s(\lambda) = s g(\lambda), \tag{44}$$

$$g(\lambda) = -\left(6\frac{K_2}{I_2} + \frac{K_1}{I_1}\right)^{-1} \times \frac{5(M_{\rm cl}^* - M_{cl} + E_{(1,1)1/2}^* - E_{(1,1)1/2})}{3(m_s - \hat{m})}. \tag{45}$$

Once these functions are plugged in the equations of state, the bulk properties of nuclear matter are well described up to the density $\sim 3\rho_0$. The parameters for the nuclear environment are fixed as follows [27]:

$$C_{\rm cl} = -0.0561, \qquad C_1 = 0.6434, \qquad C_2 = -0.1218,$$
$$C_{\rm num} = 65.60, \qquad C_{\rm den} = 0.60, \tag{46}$$

where $C_{\rm cl}$, C_1, and C_2 were determined by using the empirical data on the volume energy, pressure at the saturation point, and compressibility for symmetric nuclear matter. The volume energy is known to be $a_V = -16$ MeV from the semiempirical Bethe-Weizsäker formula [58,59]. The stability condition for nuclear matter requires the pressure to vanish, i.e., $P = 0$ near the saturation point. The compressibility for nuclear matter was predicted to be $K_0 \simeq (290 \pm 70)$ MeV within various frameworks [60–65], whereas a slightly lower value of K_0, i.e., $K_0 \sim (240 \pm 20)$, was suggested from the data on the energies of the giant monopole resonance in even-even $^{112-124}$Sn and $^{106,100-116}$Cd [66], and from earlier data on $58 \leq A \leq 208$ nuclei in Ref. [67].

The parameters $C_{\rm num}$ and $C_{\rm den}$ are relevant to the effects of isospin symmetry breaking. Since asymmetric nuclear matter appears when isospin symmetry is broken, The $C_{\rm num}$ and $C_{\rm den}$ play an essential role in reproducing the properties of asymmetric nuclear matter such as the symmetry energy $\varepsilon_{\rm sym}(\lambda)$ and the slope parameter $L_{\rm sym}$. The symmetry energy at the saturation density $\varepsilon_{\rm sym}(\lambda = 1)$ is known to be in the range of ∼30–34 MeV. The slope parameter taken from the experiments of ^{68}Ni, ^{120}Sn, and ^{208}Pb for the neutron skin thickness indicates that the heavier the nucleus is, the larger the value of $L_{\rm sym}$ [68] is observed. Thus, we choose the values of symmetry energy and slope parameter as $\varepsilon_{\rm sym} = 32$ MeV and $L_{\rm sym} = 60$ MeV. For more details about the medium functions and their parameters, we refer to Ref. [27].

The medium modification of the EM part of the isospin symmetry breaking is found to be small [69] and, therefore, we ignore it in the present work. However, the spin-spin interaction given in Eq. (31) contains the classical mass of the nucleon, which varies in nuclear medium. So, we consider its medium modification and redefine the ratio of κ and $M_{\rm cl}^*$ as $\varkappa^*$:

$$\varkappa^* = \frac{\kappa}{M_{\rm cl}^*}. \tag{47}$$

We neglect the medium dependence of κ, since it is only involved in lifting the degeneracy in the baryon sextet.

IV. RESULTS AND DISCUSSIONS

Since all the parameters were already fixed in the light-baryon sector, we can straightforwardly evaluate the masses of the baryon antitriplet and sextet. In Table I, we list the results for the masses of the singly charmed baryons. In the fourth column, we first present their numerical values in free space, which have been derived in Ref. [39]. They are in remarkable agreement with the experimental data. From the fifth column to the last one, we list the results for the medium-modified values for the masses of the singly charmed baryons. In the column, their results in symmetric nuclear matter are listed at the normal nuclear matter density. As expected from the previous work [27], the masses of the singly charmed baryons consistently decrease in nuclear matter. We now consider the mass modification in asymmetric nuclear matter with $\delta = 1$ set. Then, as shown in the sixth column, we find a very interesting aspect in the change of the Ξ_c masses. In the asymmetric nuclear matter, the proton and neutron undergo changes in a different manner: the proton mass starts to decrease as δ increases, whereas the neutron mass gets enhanced with larger values of δ. The down quarks outnumber the up quarks in asymmetric nuclear matter. If one puts a down quark in it, the Pauli exclusion principle brings about the repulsion between the down quarks. Thus, competition between up and down quarks will govern how the mass of a singly charmed quark is modified in asymmetric nuclear matter. It explains why the mass of Ξ_c^0 increases as δ increases

TABLE I. Masses of the singly charmed baryons in free space and in different baryonic matters at the normal nuclear matter density $\lambda = 1$. The experimental data are taken from the PDG [70]. In the fifth column, the results in symmetric nuclear matter ($\lambda = 1$) are listed, whereas, in the sixth and seventh columns, those in asymmetric matter ($\delta = 1$) and strange matter ($\chi = 0.15$) are respectively given. All the masses are given in unit of MeV.

				Baryonic matter at $\lambda = 1$		
Multiplet and spin	Baryon	Experimental	Free space	$\delta = 0, \chi = 0$	$\delta = 1, \chi = 0$	$\delta = 0, \chi = 0.15$
$\bar{3}_{1/2}$	Λ_c	2286.46 ± 0.14	2272.84	2268.71	2268.71	2264.49
	Ξ_c^+	2467.71 ± 0.23	2475.20	2472.97	2411.19	2475.09
	Ξ_c^0	2470.44 ± 0.28	2478.18	2472.16	2533.94	2474.27
$6_{1/2}$	Σ_c^{++}	2453.91 ± 0.14	2445.67	2372.37	2285.03	2368.51
	Σ_c^+	2452.9 ± 0.4	2444.65	2370.47	2370.47	2366.62
	Σ_c^0	2453.75 ± 0.14	2445.55	2370.50	2457.83	2366.64
	$\Xi_c^{\prime +}$	2578.2 ± 0.5	2579.83	2506.09	2462.42	2508.01
	$\Xi_c^{\prime 0}$	2578.7 ± 0.5	2580.73	2506.11	2549.78	2508.04
	Ω_c	2695.2 ± 1.7	2715.46	2641.28	2641.28	2648.99
$6_{3/2}$	Σ_c^{*++}	$2518.41^{+0.21}_{-0.19}$	2513.77	2444.52	2357.18	2440.66
	Σ_c^{*+}	2517.5 ± 2.3	2512.75	2442.62	2442.62	2438.77
	Σ_c^{*0}	2518.48 ± 0.20	2513.65	2442.64	2529.98	2438.79
	Ξ_c^{*+}	2645.10 ± 0.30	2647.93	2578.23	2534.57	2580.16
	Ξ_c^{*0}	2646.16 ± 0.25	2648.83	2578.26	2621.92	2580.18
	Ω_c^*	2765.9 ± 2.0	2784.52	2714.38	2714.38	2722.09

whereas Ξ_c^+ behaves opposedly in asymmetric nuclear matter. A similar propensity can also be observed in the baryon sextet, though it is not as prominent as in the baryon antitriplet. In the last column, we examine how the masses of the singly charmed baryons experience the medium modification in strange matter with $\chi = 0.15$. As discussed above, now the number of the strange quarks increases and hence a singly charmed baryon containing the strange quark may decrease less than the nonstrange ones. We observe this feature in the last column of Table I. We will later discuss the density dependences of the antitriplet and sextet masses quantitatively.

For completeness, we list the results for the mass modification of the singly bottom baryons in Table II.

TABLE II. Masses of the singly bottom baryons in free space and in different baryonic matters at the normal nuclear matter density $\lambda = 1$. The experimental data are taken from the PDG [70]. In the fifth column, the results in symmetric nuclear matter ($\lambda = 1$) are listed, whereas, in the sixth and seventh columns, those in asymmetric nuclear matter ($\delta = 1$) and strange matter ($\chi = 0.15$) are respectively given. All the masses are given in the unit of MeV.

				Baryonic matter at $\lambda = 1$		
Multiplet and spin	Baryon	Experimental	Free space	$\delta = 0, \chi = 0$	$\delta = 1, \chi = 0$	$\delta = 0, \chi = 0.15$
$\bar{3}_{1/2}$	Λ_b	5619.60 ± 0.17	5599.30	5595.17	5595.17	5590.95
	Ξ_b^0	5791.9 ± 0.5	5800.28	5798.05	5736.27	5800.17
	Ξ_b^-	5797.0 ± 0.6	5806.02	5800.00	5861.78	5802.11
$6_{1/2}$	Σ_b^+	5810.56 ± 0.25	5801.24	5729.83	5642.49	5725.98
	Σ_b^0	$\cdots$	5802.98	5730.69	5730.69	5726.84
	Σ_b^-	5815.64 ± 0.18	5806.64	5733.48	5820.81	5729.62
	$\Xi_b^{\prime 0}$	$\cdots$	5936.78	5864.93	5821.26	5866.85
	$\Xi_b^{\prime -}$	5935.02 ± 0.05	5940.44	5867.71	5911.38	5869.64
	Ω_b	6046.1 ± 1.7	6074.74	6002.46	6002.46	6010.16
$6_{3/2}$	Σ_b^{*+}	5830.32 ± 0.27	5821.54	5751.34	5664.00	5747.48
	Σ_b^{*0}	$\cdots$	5823.28	5752.20	5752.20	5748.35
	Σ_b^{*-}	5834.74 ± 0.30	5826.94	5754.98	5842.32	5751.13
	Ξ_b^{*0}	5952.3 ± 0.6	5957.08	5886.43	5842.77	5888.36
	Ξ_b^{*-}	5955.33 ± 0.13	5960.74	5889.22	5932.88	5891.14
	Ω_b^*	$\cdots$	6095.04	6023.96	6023.96	6031.67

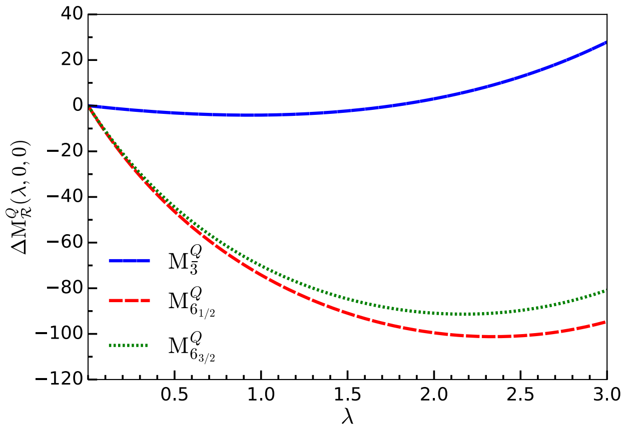

FIG. 1. Shifts of the center masses for the singly heavy baryons. The solid curve draws the mass shift of the baryon antitriplet. The dashed and dotted ones depict respectively the mass shifts for the baryon sextet with spin $1/2$ and spin $3/2$. The results are given in the unit of MeV.

Except for the spin-spin interaction that is proportional to $1/m_Q$, we respect in the current work the heavy-quark flavor symmetry. Thus, the changes of the masses of the singly bottom baryons are in conformity with those of the charmed baryons.

Figure 1 draws the mass shifts of the center masses, i.e., $\Delta M_c^{\mathcal{R}} = M_c^{\mathcal{R}*} - M_c^{\mathcal{R}}$, where the superscript $\mathcal{R}$ denotes the corresponding representation. The expressions for $M_c^{\mathcal{R}}$ are given in Eqs. (27) and (32), as functions of λ. Note that the center $\Delta M_c^{\bar{\mathbf{3}}}$ decreases as λ increases until $\lambda \approx 1.2$, and then gets enhanced. On the other hand, $\Delta M_c^{\mathbf{6}_{1/2}}$ ($\Delta M_c^{\mathbf{6}_{3/2}}$) is diminished rapidly until λ reaches around 2.2 (2.5) and then starts to increase. It implies that when the nucleons inside nuclear matter get more closely packed the repulsion overcomes the attractive interaction in the presence of the singly charmed baryons. The difference between the density dependences of the antitriplet and sextet can be understood as follows: the density dependences of $\bar{I}_1$ and $\bar{I}_2$ are different from each other. While $\bar{I}_1$ increases as λ

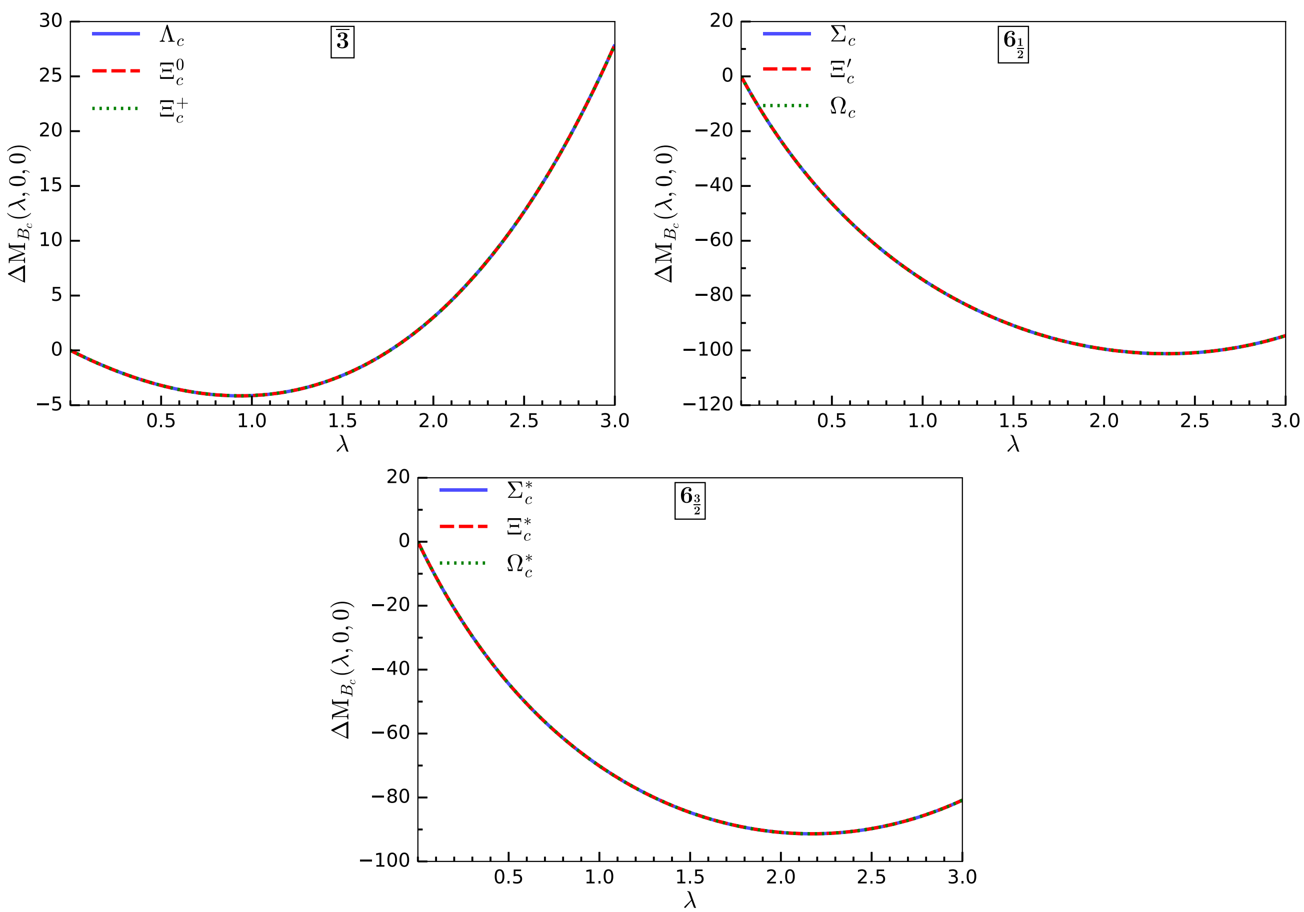

FIG. 2. Mass shifts of singly charmed baryons in symmetric nuclear matter ($\delta = 1, \chi = 0$). In the upper left panel, the λ dependences of the baryon antitriplet are drawn. In the right upper panel, those of the baryon sextet with spin $1/2$ are depicted, whereas in the lower panel, those of the baryon sextet with spin $3/2$ are shown. The results are given in the unit of MeV.

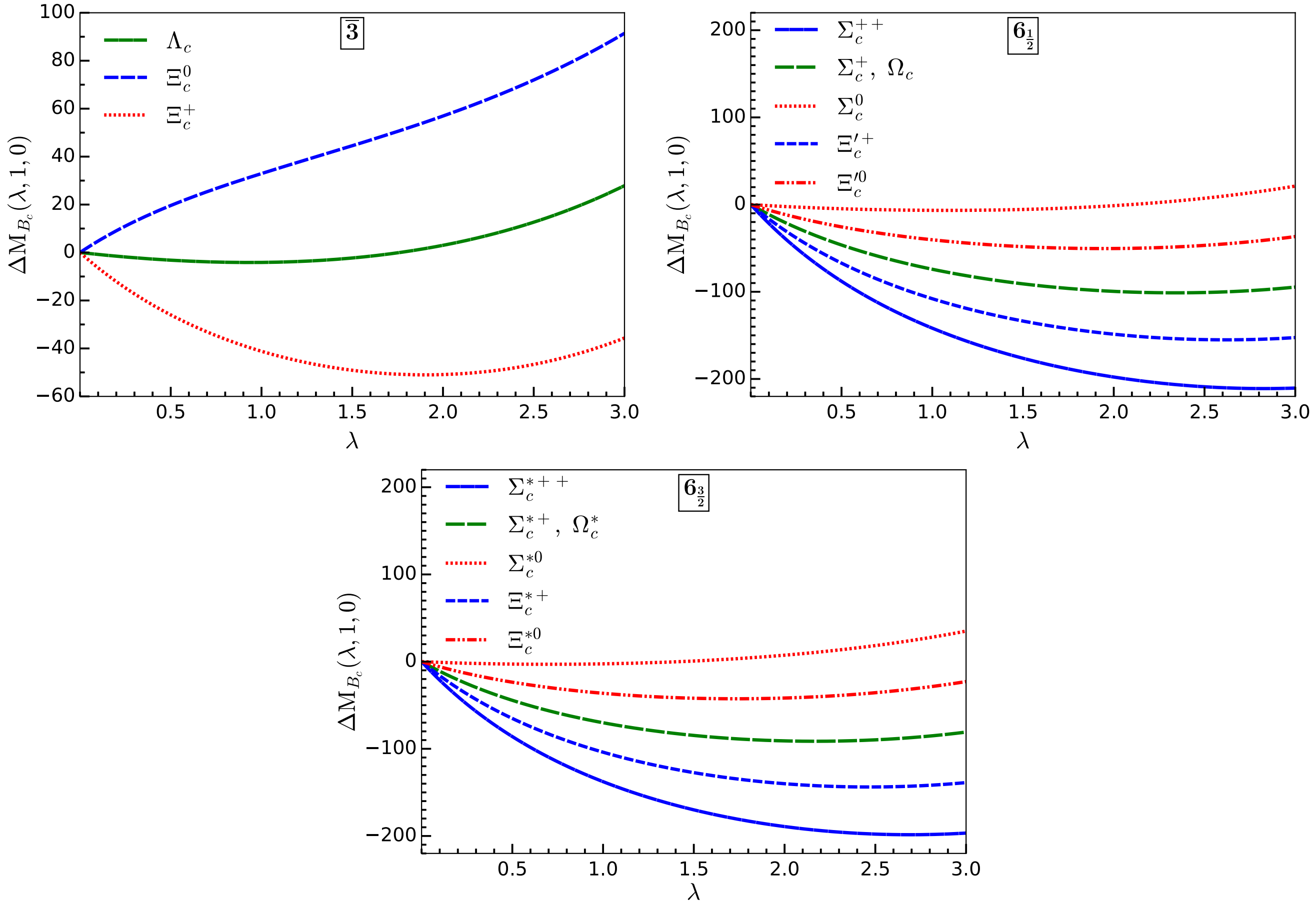

FIG. 3. Mass shifts of singly charmed baryons in asymmetric nuclear matter ($\delta = 1, \chi = 0$). In the upper left panel, the λ dependences of the baryon antitriplet, i.e., Λ_c, Ξ_c^0, and Ξ_c^+, are drawn in the solid curve, the dashed curve, and the dotted curve, respectively. In the right upper panel, those of the baryon sextet with spin $1/2$ are depicted, whereas in the lower panel, those of the baryon sextet with spin $3/2$ are shown. The results are given in the unit of MeV.

increases, $\bar{I}_2$ is lessened with the λ grown. $M_{\rm cl}$ decreases linearly as the nuclear density increases. When λ reaches around 1.2, the second term $1/2\bar{I}_2$ overtakes $M_{\rm cl}$, so that $M_{\bar{\mathbf{3}}}$ starts to increase. However, the second term for $M_{\mathbf{6}_{1/2}}$ and $M_{\mathbf{6}_{3/2}}$ in (27) is suppressed as λ increases. Thus, $M_{\mathbf{6}_{1/2}}$ and $M_{\mathbf{6}_{3/2}}$ follow the behavior of $M_{\rm cl}$. When λ further increases, the term with $\varkappa^*$ comes into play. In Fig. 2, we draw the mass shifts of the charmed baryon antitriplet and sextet, ΔM_{B_c}, in symmetric nuclear matter. The λ dependences of ΔM_{B_c} follow those of the center masses shown in Fig. 1. This is natural, because the effects of the flavor SU(3) symmetry breaking, which causes the mass splitting in the representations, are changed only in strange matter. This is the reason why the mass shift in each representation is degenerate.

In Fig. 3, we depict the mass shifts of the singly charmed baryons in asymmetric neutron matter with $\delta = 1$ and $\chi = 0$. The neutral and positively charged baryons generally show rather different behaviors as λ increases. The charmed baryons in the antitriplet exhibit the difference prominently. While Ξ_c^0 increases rather rapidly as λ increases, Ξ_c^+ decreases until λ reaches around $\lambda = 2.0$ in asymmetric nuclear matter. It indicates that the effects of isospin symmetry breaking stand out in neutron matter ($\delta = 1$). This has profound physical implications. The density-dependent function $f_0(\lambda, 1, 0)$ in Eq. (42) increases as δ grows. It contributes to the $\bar{\alpha}$, β, and γ in Eq. (30), so that d_3 and d_6 in Eq. (A8) become δ dependent. The terms containing d_3 d_6 in mass formulas in Eqs. (A1)–(A7) are proportional to the third component of the isospin operator, T_3, which brings about the isospin symmetry breaking. Thus, the differences between the neutral and positively charged baryons demonstrated in Fig. 3 arise from these terms. As explained above, the underlying physics in these differences comes from the Pauli exclusion principle.

Figure 4 illustrates how the masses of the singly charmed baryons are shifted as λ increases. Interestingly, the mass shifts of the singly charmed baryon show a general tendency: They first start to decrease as λ increases, and then increase when λ gets to some specific values. However, those of Ω_c and Ω_c^* monotonically fall off as λ increases. Inspecting Eqs. (A1)–(A7), we find that the terms with D_3 in the antitriplet and D_6 in the sextet cause respectively the mass splittings in the corresponding

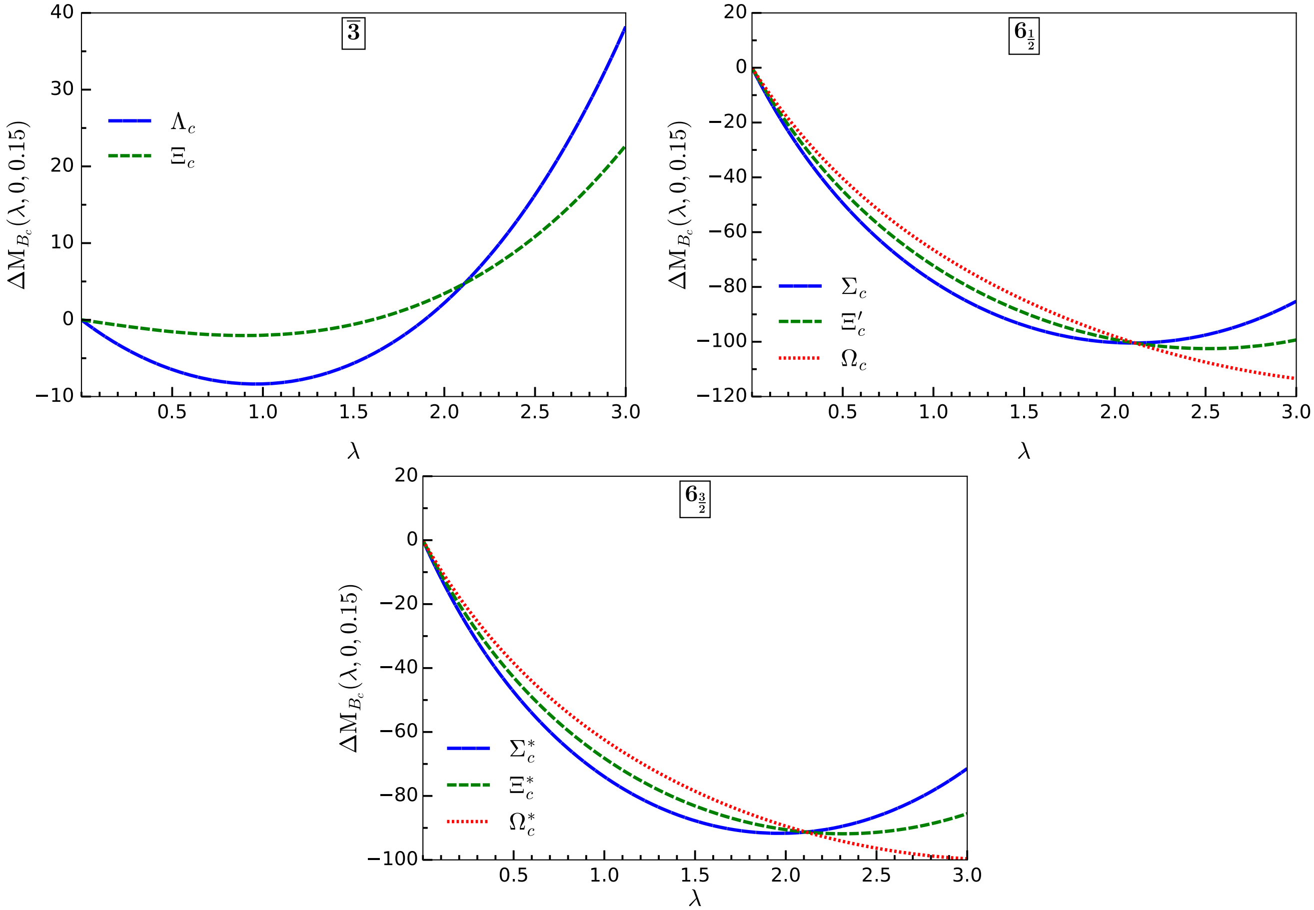

FIG. 4. Mass shifts of singly charmed baryons in strange matter ($\delta = 0$, $\chi = 0.15$). In the upper left panel, the λ dependences of the baryon antitriplet, i.e., Λ_c and Ξ_c, are drawn in the solid curve and dashed curve, respectively. In the right upper panel, those of the baryon sextet with spin $1/2$ are depicted, whereas in the lower panel, those of the baryon sextet with spin $3/2$ are shown. The results are given in the unit of MeV.

representations. We also observe that the λ dependences of the baryon sextet with spin $1/2$ are almost the same as those with spin $3/2$. Note that the sextet baryons with spin $1/2$ and $3/2$ are degenerate before we introduce the hyperfine interaction in Eq. (32). Though the parameter $\varkappa^*$ in Eq. (47) is also density dependent, its effect is marginal. The prefactor in the D_6 term of the Ω_c (Ω_c^*) baryons is $-4/3$, whereas the those of Σ_c (Σ_c^*) and Ξ_c' (Ξ_c^*) are respectively $+2/3$ and $-1/3$. This leads to the different λ dependences of the sextet baryons as shown in Fig. 4.

V. SUMMARY AND OUTLOOK

In the present work, we aimed at investigating the mass shifts of the singly heavy baryons within a pion mean field approach (χQSM) in various nuclear matters. In the limit of the infinite heavy-quark mass, the dynamics in a singly heavy baryon is governed by the light quarks whereas the heavy remains as the mere static color source with the heavy quark spin-flavor symmetry satisfied. The light quarks, which yield the right hypercharge $Y' = 2/3$, select the proper representations of the singly heavy baryons. This allows one to describe the light and singly heavy baryons on an equal footing. Since all the density-dependent variables had been determined in describing the bulk properties of nuclear matter and the mass shifts of the baryon octet and decuplet, we were able to evaluate those of the baryon antitriplet and sextet without fitting the parameters. Then, we first computed the medium-modified masses of the singly charmed baryons in symmetric nuclear matter. The center masses of the baryon antitriplet and sextet govern the density dependences of the singly charmed baryon masses. In the case of asymmetric nuclear matter, the neutral and positively charged baryons reveal different density dependences: The neutral baryons tend to increase as the nuclear density increases, whereas the positively charged ones decrease as the nuclear density grows. We explained the reason and discussed its physical implications. As a result, the effects of isospin symmetry breaking are more strengthened as the density increases in asymmetric nuclear matter. We also presented the mass shifts of the singly charmed baryons in strange matter.

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APPENDIX: EXPRESSIONS FOR THE MASSES OF THE SINGLY HEAVY BARYONS

The masses of the antitriplet baryon are expressed as

$$\begin{aligned} M_{\Lambda_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(0,1)} + m_{\mathrm{Q}} + \frac{2}{3}D_3 + \frac{1}{4}c_8, \\ M_{\Xi_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(0,1)} + m_{\mathrm{Q}} - \frac{1}{3}D_3 + d_3T_3 \\ &\quad + \frac{3}{4}\left(T_3 + \frac{1}{6}\right)c_8 - \hat{Q}_q\alpha_{LQ}T_3, \end{aligned} \tag{A1}$$

where the $E^{\mathrm{rot}}_{0,1}$ can be obtained from Eq. (26). The masses of the spin $1/2$ sextet baryon are given by the following expression:

$$\begin{aligned} M_{\Sigma_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{q}} - \frac{2}{3}\frac{\varkappa}{m_{\mathrm{Q}}} \\ &\quad + \frac{2}{3}D_6 + d_6T_3 + \frac{3}{10}\left(T_3 + \frac{1}{3}\right)c_8 \\ &\quad + \frac{1}{9}\left(T_3^2 + \frac{1}{5}T_3 - \frac{3}{5}\right)c_{27} + \hat{Q}_q\alpha_{LQ}T_3, \end{aligned} \tag{A2}$$

$$\begin{aligned} M_{\Xi'_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{Q}} - \frac{2}{3}\frac{\varkappa}{m_{\mathrm{Q}}} \\ &\quad - \frac{1}{3}D_6 + d_6T_3 + \frac{3}{10}\left(T_3 - \frac{1}{6}\right)c_8, \\ &\quad - \frac{2}{45}\left(T_3^2 + 2T_3 + \frac{1}{4}\right)c_{27} + \hat{Q}_q\alpha_{\mathrm{LQ}}T_3, \end{aligned} \tag{A3}$$

$$M_{\Omega_{\mathrm{Q}}} = M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{Q}} - \frac{2}{3}\frac{\varkappa}{m_{\mathrm{Q}}} - \frac{4}{3}D_6 + \frac{1}{5}c_8 - \frac{1}{45}c_{27}, \tag{A4}$$

where the $E^{\mathrm{rot}}_{2,0}$ can be obtained from Eq. (26). The masses of the spin $3/2$ baryon sextet mass can be written as the following expression:

$$\begin{aligned} M_{\Sigma^*_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{Q}} - \frac{1}{3}\frac{\varkappa}{m_{\mathrm{Q}}} \\ &\quad + \frac{2}{3}D_6 + d_6T_3 + \frac{3}{10}\left(T_3 + \frac{1}{3}\right)c_8 \\ &\quad + \frac{1}{9}\left(T_3^2 + \frac{1}{5}T_3 - \frac{3}{5}\right)c_{27} + \hat{Q}_q\alpha_{LQ}T_3, \end{aligned} \tag{A5}$$

$$\begin{aligned} M_{\Xi^*_{\mathrm{Q}}} &= M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{Q}} - \frac{1}{3}\frac{\varkappa}{m_{\mathrm{Q}}} \\ &\quad - \frac{1}{3}D_6 + d_6T_3 + \frac{3}{10}\left(T_3 - \frac{1}{6}\right)c_8 \\ &\quad - \frac{2}{45}\left(T_3^2 + 2T_3 + \frac{1}{4}\right)c_{27} + \hat{Q}_q\alpha_{LQ}T_3, \end{aligned} \tag{A6}$$

$$M_{\Omega^*_{\mathrm{Q}}} = M_{\mathrm{cl}} + E^{\mathrm{rot}}_{(2,0)} + m_{\mathrm{Q}} - \frac{1}{3}\frac{\varkappa}{m_{\mathrm{Q}}} - \frac{4}{3}D_6 + \frac{1}{5}c_8 - \frac{1}{45}c_{27}. \tag{A7}$$

Here $d_{3,6}$ and $D_{3,6}$ are defined as

$$\begin{aligned} D_3 &= (m_s - \hat{m})\left(\frac{3}{8}\bar{\alpha} + \beta\right), \\ D_6 &= (m_s - \hat{m})\left(\frac{3}{20}\bar{\alpha} + \beta - \frac{3}{10}\gamma\right), \\ d_3 &= (m_d - m_u)\left(\frac{3}{8}\bar{\alpha} + \beta\right), \\ d_6 &= (m_d - m_u)\left(\frac{3}{20}\bar{\alpha} + \beta - \frac{3}{10}\gamma\right). \end{aligned} \tag{A8}$$

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