Nonleptonic decays of $\Xi_{cc} \to \Xi_c \pi$ with $\Xi_c - \Xi_c'$ mixing

Chia-Wei Liu[*] and Chao-Qiang Geng

School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China and University of Chinese Academy of Sciences, 100190 Beijing, China

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Aiming on testing the $\Xi_c - \Xi_c'$ mixing, we study the decays of $\Xi_{cc} \to \Xi_c \pi$ with $\Xi_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^{+})$, $\Xi_c = (\Xi_c^{(\prime)+}, \Xi_c^{(\prime)0})$ and $\pi = (\pi^+, \pi^0)$. The soft-meson limit is considered along with the pole model, and the baryon matrix elements are evaluated by the bag model with and without removing the center-of-mass motion (CMM). We find that the four-quark operator matrix elements are about two times larger once the unwanted CMM is removed. We obtain that $\mathcal{R} = \mathcal{B}(\Xi_{cc}^{+} \to \Xi_c'^{+}\pi^+)/\mathcal{B}(\Xi_{cc}^{+} \to \Xi_c^{+}\pi^+) = 0.87^{+0.17}_{-0.11}$ and 1.45 with and without removing the CMM, where the former is close to the lower bound and the later is well consistent with $\mathcal{R} = 1.41 \pm 0.17 \pm 0.10$ measured at LHCb. In addition, we show that after including the mixing the up-down asymmetry of $\alpha(\Xi_{cc}^{+} \to \Xi_c^{(\prime)0}\pi^+)$ flips sign. Explicitly, we obtain that $\alpha(\Xi_{cc}^{+} \to \Xi_c'^{+}\pi^0) = 0.52$ and $\alpha(\Xi_{cc}^{+} \to \Xi_c^{0}\pi^+) = 0.31$ with and without the CMM corrections, respectively, which are all negative if the mixing is absence. As a bonus, a positive value of $\alpha(\Xi_{cc}^{+} \to \Xi_c'^{0}\pi^+)$ in experiments can also serve as the evidence of the W-exchange contributions.

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I. INTRODUCTION

The baryon wave functions are the precondition in evaluating the decay quantities. It has been shown that the large $SU(3)$ flavor $[SU(3)_F]$ breaking effect in the singly charmed baryon semileptonic decays can be traced back to the $\Xi_c - \Xi_c'$ mixing [1–4], given as

$$|\Xi_c\rangle = \cos\theta_c|\Xi_c^{\bar{\mathbf{3}}}\rangle + \sin\theta_c|\Xi_c^{\mathbf{6}}\rangle,$$
$$|\Xi_c'\rangle = \cos\theta_c|\Xi_c^{\mathbf{6}}\rangle - \sin\theta_c|\Xi_c^{\bar{\mathbf{3}}}\rangle, \tag{1}$$

where $\Xi_c^{(\prime)} = \Xi_c^{(\prime)+,0}$ are the physical baryons and $\Xi_c^{\bar{\mathbf{3}}(\mathbf{6})}$ correspond to the antitriplet (sextet) charmed baryons. At the limit of the $SU(3)_F$ symmetry, the physical baryons shall have definite $SU(3)_F$ representations, i.e., $\theta_c = 0$. From the mass relations, we have found that [5]

$$\theta_c = \pm 0.137(5)\pi, \tag{2}$$

with the sign unfixed. In the decays involving Ξ_c, the mixing should be considered seriously, as its effects are shown to be sizable [4]. It particular, it can be attributed to the nonzero signals of $\Xi_c^+ \to \Xi'^0(1530)\pi^+$ observed at Belle [6], which are unexpected in the previous studies in the literature [7–10]. If the mixing is further confirmed, it would undoubtedly reshape our knowledge of the baryon spin-flavor structures.

Recently, the LHCb Collaboration reported the ratio [11]

$$\mathcal{R}(\Xi_c^{++} \to \Xi_c^+\pi^+) = 1.41 \pm 0.17 \pm 0.10, \tag{3}$$

where $\mathcal{R}(\Xi_{cc} \to \Xi_c\pi) \equiv \mathcal{B}(\Xi_{cc} \to \Xi_c'\pi)/\mathcal{B}(\Xi_{cc} \to \Xi_c\pi)$, and the first and second uncertainties are systematic and statistical, respectively. It provides an ideal place to examine the mixing as it affects both the denominator and numerator of $\mathcal{R}$. In the literature [12–17] before the experiments, the ratio deviates largely to the value in Eq. (3). In this work, we will show that the responsible mechanism is precisely the $\Xi_c - \Xi_c'$ mixing. On the other hand, combing several experiments, we have [18–20]

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+\pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \to \Lambda_c^+K^-\pi^+\pi^+)} = 0.35 \pm 0.20. \tag{4}$$

By using $\mathcal{B}(\Xi_{cc}^{++} \to \Lambda_c^+K^-\pi^+\pi^+) > \mathcal{B}(\Xi_{cc}^{++} \to \Sigma_{cc}^{++}\bar{K}^{*0})$ $\mathcal{B}(\bar{K}^{*0} \to K^-\pi^+)$, $\mathcal{B}(\Xi_{cc}^{++} \to \Sigma_c^{++}\bar{K}^{*0}) = 5.61\%$ [21] and $\mathcal{B}(\bar{K}^{*0} \to K^-\pi^+) = 2/3$, we obtain

$$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+\pi^+) > 0.59\%, \tag{5}$$

at 1σ confidence level. In addition, we have

$$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+\pi^+) = (1.33 \pm 0.74)\%, \tag{6}$$

[*]chiaweiliu@ucas.ac.cn

by assuming that the decay of $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ contributes solely by $\Xi_{cc}^{++} \to \Sigma_{cc}^{++} \bar{K}^{*0}$.

In the theoretical aspect, it is known that a trustworthy method in evaluating the charm quark baryonic decays has not been given yet, since the charm quark is neither heavy nor light enough to apply the heavy quark or $SU(4)_F$ symmetry. Nevertheless, it has been shown in Ref. [22] that the pole model conjunction with the current algebra and soft-meson limit can explain well the experimental data of $\Lambda_c^+ \to BP$, with B and P the octet baryons and pseudoscalar mesons, respectively. As a phenomenological study, focusing on the mixing effects, we shall follow their methodology for the formalism. For the baryon wave functions, we will examine both the mixing effects and the center-of-mass motion (CMM) corrections of the bag model. Very recently, it was shown that the bag model is well consistent with the experimental data of $\mathcal{B}(\Xi_Q \to \Lambda_Q \pi^-)$ once the CMM is removed [23–26], where $\Lambda_Q = (\Lambda_c^+, \Lambda_b^0)$ for $\Xi_Q = (\Xi_c^0, \Xi_b^-)$.

This work is organized as follows. In Sec. II, we briefly recall the formalism of the pole model and current algebra. In Sec. III, we give the baryon wave functions and their matrix elements with and without the CMM. In Sec. IV, we give the numerical results. We conclude this study in Sec. V.

II. FORMALISM

In general, the amplitude of $\mathcal{B}_i \to \mathcal{B}_f \pi$ is decomposed as

$$i\bar{u}_f(A - B\gamma_5)u_i, \tag{7}$$

where $u_{i(f)}$ is the Dirac spinor of the initial (final) baryon and $A(B)$ is referred to as the parity violating (conserving) amplitude. In the pole approximation, the nonfactorizable amplitudes read as [17]

$$\begin{aligned} A^{\rm pole} &= -\sum_{\mathcal{B}_n^*}\left[\frac{g_{\mathcal{B}_f\mathcal{B}_n^*\pi} b_{n^*i}}{M_i - M_n^*} + \frac{b_{fn^*} g_{\mathcal{B}_n^*\mathcal{B}_i\pi}}{M_f - M_n^*}\right], \\ B^{\rm pole} &= \sum_{\mathcal{B}_n}\left[\frac{g_{\mathcal{B}_i\mathcal{B}_n\pi} a_{ni}}{M_i - M_n} + \frac{a_{fn} g_{\mathcal{B}_n\mathcal{B}_i\pi}}{M_f - M_n}\right], \end{aligned} \tag{8}$$

where $\mathcal{B}_n^{(*)}$ are the parity-even (-odd) intermediate baryons; $M_{i,f,n}^{(*)}$ correspond to the masses of $\mathcal{B}_{i,f,n}^{(*)}$,

$$\begin{aligned} \langle \mathcal{B}_2|\mathcal{H}_{\rm eff}|\mathcal{B}_1\rangle &= \bar{u}_2(a_{21} + b_{21}\gamma_5)u_1, \\ \langle \mathcal{B}_n^*|\mathcal{H}_{\rm eff}|\mathcal{B}_1\rangle &= b_{n^*1}\bar{u}_n u_1, \end{aligned} \tag{9}$$

$\mathcal{B}_{1,2} \in \{\mathcal{B}_i, \mathcal{B}_f, \mathcal{B}_n\}$; and $\mathcal{H}_{\rm eff}$ represents the effective Hamiltonian. The baryon-baryon-pion couplings of $g_{\mathcal{B}_1\mathcal{B}_n^{(*)}\pi}$ are extracted by the Goldberg-Treiman relations

$$\begin{aligned} g_{\mathcal{B}_1\mathcal{B}_2\pi} &= \frac{\sqrt{2}}{f_\pi}(M_1 + M_2) g_{\mathcal{B}_1\mathcal{B}_2}^{A(\pi)}, \\ g_{\mathcal{B}_n^*\mathcal{B}_2\pi} &= \frac{\sqrt{2}}{f_\pi}(M_n^* - M_2) g_{\mathcal{B}_n^*\mathcal{B}_2}^{A(\pi)}, \end{aligned} \tag{10}$$

where f_π is the pion decay constant, the axial vector couplings of $g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}$ are defined by

$$\langle \mathcal{B}'|A^\mu(\pi)|\mathcal{B}\rangle = \overline{u'}(g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}\gamma^\mu - i\bar{g}_2\sigma^{\mu\nu}q_\nu + \bar{g}_3 q^\mu)\gamma_5 u, \tag{11}$$

$u^{(\prime)}$ is the Dirac spinor of $\mathcal{B}^{(\prime)}$, $A^\mu(\pi^+) = \bar{d}\gamma^\mu\gamma_5 u$, $A^\mu(\pi^0) = \frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d)$, and $\mathcal{B}^{(\prime)} \in \{\mathcal{B}_i, \mathcal{B}_f, \mathcal{B}_n, \mathcal{B}_n^*\}$. Note that $\bar{g}_{2,3}$ are irrelevant to this work.

To overcome the unknown baryon wave functions of $\mathcal{B}_n^*$, we use the soft-meson limit and $[Q_5^\pi + Q^\pi, \mathcal{H}_{\rm eff}] = 0$.[1] The amplitudes of $\Xi_{cc} \to \Xi_c \pi$ are summarized as[2] [17]

$$\begin{aligned} A(\Xi_{cc}^{++} \to \Xi_c^{(\prime)+}\pi^+) &= \zeta(f_\pi^2 a_1 f_1^{(\prime)} M_-^{(\prime)} - c_- a^{(\prime)}), \\ A(\Xi_{cc}^{+} \to \Xi_c^{(\prime)0}\pi^+) &= \zeta(f_\pi^2 a_1 f_1^{(\prime)} M_-^{(\prime)} + c_- a^{(\prime)}), \\ A(\Xi_{cc}^{+} \to \Xi_c^{(\prime)+}\pi^0) &= \sqrt{2}\zeta c_- a^{(\prime)}, \end{aligned} \tag{12}$$

and

$$\begin{aligned} B(\Xi_{cc}^{++} \to \Xi_c^{(\prime)+}\pi^+) &= \zeta\left(-f_\pi^2 a_1 g_1^{(\prime)} M_+^{(\prime)} - 2c_- a^{(\prime)}\frac{M_{cc}}{M_-^{(\prime)}} g_{\Xi_{cc}^{+}\Xi_{cc}^{++}}^{A(\pi^+)}\right), \\ B(\Xi_{cc}^{+} \to \Xi_c^{(\prime)0}\pi^+) &= \zeta\left(-f_\pi^2 a_1 g_1^{(\prime)} M_+^{(\prime)} + c_- a\frac{M_c + M_c^{(\prime)}}{M_-} g_{\Xi_c^{(\prime)0}\Xi_c^{+}}^{A(\pi^+)} + c_- a'\frac{M_c' + M_c^{(\prime)}}{M_-'} g_{\Xi_c^{(\prime)0}\Xi_c'^{+}}^{A(\pi^+)}\right), \\ B(\Xi_{cc}^{+} \to \Xi_c^{(\prime)+}\pi^0) &= \sqrt{2}\zeta c_-\left(-2a^{(\prime)}\frac{M_c}{M_-^{(\prime)}} g_{\Xi_{cc}^{+}\Xi_{cc}^{+}}^{A(\pi^0)} + a\frac{M_c + M_c^{(\prime)}}{M_-} g_{\Xi_c^{(\prime)+}\Xi_c^{+}}^{A(\pi^0)} + a'\frac{M_c' + M_c^{(\prime)}}{M_-'} g_{\Xi_c^{(\prime)+}\Xi_c'^{+}}^{A(\pi^0)}\right), \end{aligned} \tag{13}$$

[1] The charge operators are defined as $Q^\pi = \int d^3x(q^\dagger\sigma_i q)/2$ and $Q_5^\pi = \int d^3x(q^\dagger\gamma_5\sigma_i q)/2$, where $q = (u,d)^T$ and $\sigma_i = \sigma_3, (\sigma_1 \pm i\sigma_2)/\sqrt{2}$ for $\pi = \pi^0, \pi^\pm$, respectively. The commutation relations come from the fact that the left-handed and right-handed currents commute.

[2] We note that the amplitudes of the charm baryon nonleptonic two-body decays (196 in total) are compactly expressed by five of the topological tensor invariants within the current algebra [27].

where

$$\zeta = \frac{G_F}{f_\pi \sqrt{2}} V_{cs} V_{ud}^*, \qquad c_- = \frac{1}{2}(c_1 - c_2), \qquad M_\pm^{(\prime)} = M_{cc} \pm M_c^{(\prime)}; \tag{14}$$

M_{cc} and $M_c^{(\prime)}$ are the masses of Ξ_{cc} and $\Xi_c^{(\prime)}$, respectively; G_F is the Fermi constant; a_1 is the effective Wilson coefficient; and V_{cs} and V_{ud} are the Cabibbo-Kobayashi-Maskawa matrix elements. The information of the baryon wave functions is encapsulated in a, f_1, and g_1, defined by[3]

$$\langle \Xi_c^{(\prime)+} | O | \Xi_{cc}^+ \rangle = \langle \Xi_c^{(\prime)+} | 2(u^\dagger L^\mu d)(s^\dagger L_\mu c) | \Xi_{cc}^+ \rangle = \bar{u}_c (a^{(\prime)} + b^{(\prime)} \gamma_5) u_{cc}, \tag{15}$$

$$\begin{aligned} \langle \Xi_c^{(\prime)+} | \bar{s} \gamma^\mu c | \Xi_{cc}^{++} \rangle &= \bar{u}_c \left(f_1^{(\prime)}(\omega^{(\prime)}) \gamma^\mu - i f_2^{(\prime)}(\omega^{(\prime)}) \frac{\sigma^{\mu\nu}}{M_{cc}} q_\nu + f_3^{(\prime)}(\omega^{(\prime)}) \frac{q^\mu}{M_{cc}} \right) u_{cc}, \\ \langle \Xi_c^{(\prime)+} | \bar{s} \gamma^\mu \gamma_5 c | \Xi_{cc}^{++} \rangle &= \bar{u}_c \left(g_1^{(\prime)}(\omega^{(\prime)}) \gamma^\mu - i g_2^{(\prime)}(\omega^{(\prime)}) \frac{\sigma^{\mu\nu}}{M_{cc}} q_\nu + g_3^{(\prime)}(\omega^{(\prime)}) \frac{q^\mu}{M_{cc}} \right) \gamma_5 u_{cc}, \end{aligned} \tag{16}$$

with $L^\mu = \gamma^0 \gamma^\mu (1 - \gamma_5)$ and $u_{c(c)}$ the Dirac spinor of $\Xi_{c(c)}$. Since $\Xi_c^{\bar{\mathbf{3}}}$ and $\Xi_c^{\mathbf{6}}$ do not have definite masses for $\theta_c \neq 0$, we define the variables

$$\omega^{(\prime)} = \frac{1 + v^2}{1 - v^2} = \frac{M_{cc}^2 + M_c^{(\prime)2} - M_\pi^2}{2 M_c^{(\prime)} M_{cc}}, \tag{17}$$

with v the speed of the baryons in the Breit frame. Throughout this work, we employ the isospin symmetry so that $\Xi_{cc}^{++} (\Xi_c^+)$ and $\Xi_{cc}^+ (\Xi_c^0)$ have the same masses and form factors. In addition, we have

$$g^{A(\pi^+)}_{\Xi_{cc}^+ \Xi_{cc}^{++}} = -\frac{1}{2} g^{A(\pi^0)}_{\Xi_{cc}^+ \Xi_{cc}^+}, \qquad g^{A(\pi^+)}_{\Xi_c^{\prime 0} \Xi_c^{(\prime)+}} = \frac{1}{2} g^{A(\pi^0)}_{\Xi_c^{\prime +} \Xi_c^{(\prime)+}}, \qquad g^{A(\pi^+)}_{\Xi_c^0 \Xi_c^{(\prime)+}} = \frac{1}{2} g^{A(\pi^0)}_{\Xi_c^+ \Xi_c^{(\prime)+}}. \tag{18}$$

The above results are the general ones under the soft-meson limit, and the unknown parts of the baryon wave functions are absorbed in the form factors and $a^{(\prime)}$.

Plugging the mixing of Eq. (1) into Eq. (16), we arrive at

$$\begin{aligned} f_1 &= \cos\theta_c f_1^{\bar{\mathbf{3}}}(\omega) + \sin\theta_c f_1^{\mathbf{6}}(\omega), & g_1 &= \cos\theta_c g_1^{\bar{\mathbf{3}}}(\omega) + \sin\theta_c g_1^{\mathbf{6}}(\omega), \\ f_1{}' &= \cos\theta_c f_1^{\mathbf{6}}(\omega') - \sin\theta_c f_1^{\bar{\mathbf{3}}}(\omega'), & g_1{}' &= \cos\theta_c g_1^{\mathbf{6}}(\omega') - \sin\theta_c g_1^{\bar{\mathbf{3}}}(\omega'), \\ a &= \cos\theta_c a(\bar{\mathbf{3}}) + \sin\theta_c a(\mathbf{6}), & a' &= \cos\theta_c a(\mathbf{6}) - \sin\theta_c a(\bar{\mathbf{3}}), \end{aligned} \tag{19}$$

where $(f_1^{\bar{\mathbf{3}}}(\omega^{(\prime)}), f_1^{\mathbf{6}}(\omega^{(\prime)})$, $(g_1^{\bar{\mathbf{3}}}(\omega^{(\prime)}), g_1^{\mathbf{6}}(\omega^{(\prime)})$ and $(a(\bar{\mathbf{3}}), a(\mathbf{6}))$ are calculated by taking $(\Xi_c^{(\prime)} = \Xi_c(\bar{\mathbf{3}}), \Xi_c^{(\prime)} = \Xi_c(\mathbf{6}))$ in Eqs. (15) and (16). Similarly, the axial vector couplings are modified as

$$\begin{aligned} g^{A(\pi^+)}_{\Xi_c^{\prime 0} \Xi_c^{\prime +}} &= \cos^2\theta_c g^A_{\mathbf{66}} - \sin(2\theta_c) g^A_{\mathbf{6}\bar{\mathbf{3}}}, \\ g^{A(\pi^+)}_{\Xi_c^0 \Xi_c^{\prime +}} = g^{A(\pi^+)}_{\Xi_c^{\prime 0} \Xi_c^+} &= \cos(2\theta_c) g^A_{\mathbf{6}\bar{\mathbf{3}}} + \frac{1}{2} \sin(2\theta_c) g^A_{\mathbf{66}}, \\ g^{A(\pi^+)}_{\Xi_c^0 \Xi_c^+} &= \sin(2\theta_c) g^A_{\mathbf{6}\bar{\mathbf{3}}} + \sin^2\theta_c g^A_{\mathbf{66}}, \end{aligned} \tag{20}$$

with

$$\langle \Xi_c^0(\mathbf{R}_2) | \bar{d} \gamma^\mu \gamma_5 u | \Xi_c^+(\mathbf{R}_1) \rangle = \bar{u}_{\mathbf{R}_2} (g^A_{\mathbf{R}_2 \mathbf{R}_1} \gamma^\mu - i \bar{g}_2 \sigma^{\mu\nu} q_\nu + \bar{g}_3 q^\mu) \gamma_5 u_{\mathbf{R}_1} \tag{21}$$

and $\mathbf{R}_{1,2} = (\bar{\mathbf{3}}, \mathbf{6})$. Finally, the decay widths and up-down asymmetries are given by

[3]We use the Fierz transformation to sort O_- defined in Ref. [17].

$$\Gamma = \frac{\mathbf{p}_f}{8\pi}\frac{(M_i+M_f)^2-M_\pi^2}{M_i^2}(|A|^2+\kappa^2|B|^2),$$
$$\alpha = \frac{2\kappa\mathrm{Re}(A^*B)}{|A|^2+\kappa^2|B|^2}, \tag{22}$$

where $\mathbf{p}_f$ is the magnitude of the pion 3-momentum and $\kappa = \mathbf{p}_f/(E_f+M_f)$ with $E_f=\sqrt{\mathbf{p}_f^2+M_f^2}$.

III. BARYON WAVE FUNCTIONS AND MATRIX ELEMENTS

The bag model provides approximations of the hadron wave functions, aiming to reconcile two very different ideas in QCD [28–30]. Inside the bag, quarks move freely as a result of the asymptotic freedom but cannot penetrate the bag due to the QCD confinement. One of the great advantages of the bag model is that the parameters are fitted from the mass spectra. Consequently, the model provides fixed predicted results, which can be tested by the experiments. In this work, we calculate the baryon matrix elements by the bag models with and without removing the CMM, referred to as the homogeneous bag (HB) and static bag (SB) approaches, respectively.

The baryon wave functions concerned by this work are given as

$$|\Xi_{cc},\updownarrow\rangle = \int\frac{1}{2\sqrt{3}}\epsilon^{\alpha\beta\gamma}q^\dagger_{a\alpha}(\vec{x}_1)c^\dagger_{b\beta}(\vec{x}_2)c^\dagger_{c\gamma}(\vec{x}_3)\Psi^{abc}_{A_\updownarrow(ucc)}(\vec{x}_1,\vec{x}_2,\vec{x}_3)[d^3\vec{x}]|0\rangle,$$
$$|\Xi_c^{\bar{\mathbf{3}}},\updownarrow\rangle = \int\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}q^\dagger_{a\alpha}(\vec{x}_1)s^\dagger_{b\beta}(\vec{x}_2)c^\dagger_{c\gamma}(\vec{x}_3)\Psi^{abc}_{A_\updownarrow(qsc)}(\vec{x}_1,\vec{x}_2,\vec{x}_3)[d^3\vec{x}]|0\rangle,$$
$$|\Xi_c^{\mathbf{6}},\updownarrow\rangle = \int\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}q^\dagger_{a\alpha}(\vec{x}_1)s^\dagger_{b\beta}(\vec{x}_2)c^\dagger_{c\gamma}(\vec{x}_3)\Psi^{abc}_{S_\updownarrow(qsc)}(\vec{x}_1,\vec{x}_2,\vec{x}_3)[d^3\vec{x}]|0\rangle, \tag{23}$$

where $q_{a\alpha}\in\{u_{a\alpha},d_{a\alpha}\}$, the latin (greek) letters are the color (Dirac spinor) indices, and Ψ describe the spatial distributions of the quarks. In the SB approach, Ψ read as [30]

$$\Psi^{abc(\mathrm{SB})}_{A\updownarrow(q_1q_2q_3)}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \frac{\mathcal{N}}{\sqrt{2}}(\phi^a_{q_1\uparrow}(\vec{x}_1)\phi^b_{q_2\downarrow}(\vec{x}_2)-\phi^a_{q_1\downarrow}(\vec{x}_1)\phi^b_{q_2\uparrow}(\vec{x}_2))\phi^c_{q_3\updownarrow}(\vec{x}_3),$$
$$\Psi^{abc(\mathrm{SB})}_{S\uparrow(q_1q_2q_3)}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \frac{\mathcal{N}}{\sqrt{6}}(2\phi^a_{q_1\uparrow}(\vec{x}_1)\phi^b_{q_2\uparrow}(\vec{x}_2)\phi^c_{q_3\downarrow}(\vec{x}_3)-\phi^a_{q_1\downarrow}(\vec{x}_1)\phi^b_{q_2\uparrow}(\vec{x}_2)\phi^c_{q_3\uparrow}(\vec{x}_3)$$
$$-\phi^a_{q_1\uparrow}(\vec{x}_1)\phi^b_{q_2\downarrow}(\vec{x}_2)\phi^c_{q_3\uparrow}(\vec{x}_3)), \tag{24}$$

where $\mathcal{N}$ is the normalization constant,

$$\phi_{q\updownarrow}(\vec{x}) = \begin{pmatrix}\omega_{q_+}j_0(\mathbf{p}_q|\vec{x}|)\chi_\updownarrow\\ i\omega_{q_-}j_1(\mathbf{p}_q|\vec{x}|)\hat{x}\cdot\vec{\sigma}\chi_\updownarrow\end{pmatrix}\quad \text{for } |\vec{x}|<R, \tag{25}$$

R is the bag radius, $\mathbf{p}_q$ is the magnitude of the quark 3-momentum, $\omega_{q_\pm}=\sqrt{E_q\pm M_q}$ with M_q the quark mass and $E_q=\sqrt{\mathbf{p}_q^2+M_q^2}$, $j_{0,1}$ are the spherical Bessel functions, $\chi_\uparrow=(1,0)^T$, and $\chi_\downarrow=(0,1)^T$.

The baryon wave functions in Eq. (24) are localized and cannot be momentum eigenstates according to the Heisenberg principle. In other words, the baryons at rest must be invariant under the spatial translations and so cannot be localized. Another way to see the problem is that the spatial wave functions (3-momenta) of the quarks in the SB are untangled. Therefore, we have $\langle(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)^2\rangle = \langle\mathbf{p}_1^2\rangle+\langle\mathbf{p}_2^2\rangle+\langle\mathbf{p}_3^2\rangle>0$, where $\mathbf{p}_i$ is the 3-momentum of the ith quark with $\langle\mathbf{p}_i\rangle=0$ and $\langle\mathbf{p}_i\mathbf{p}_j\rangle=\langle\mathbf{p}_i\rangle\langle\mathbf{p}_j\rangle$ for $i\neq j$. To overcome the problem, the baryon wave functions shall be distributed uniformly over the three-dimensional space, while the quarks shall be entangled in the spatial wave functions. The simplest way to do the job is to linearly superpose the wave functions over the three-dimensional space [31]

$$\Psi^{(\mathrm{HB})}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \int d^3\vec{x}_\Delta\Psi^{(\mathrm{SB})}(\vec{x}_1-\vec{x}_\Delta,\vec{x}_2-\vec{x}_\Delta,\vec{x}_3-\vec{x}_\Delta), \tag{26}$$

where $\Psi^{(\mathrm{SB})}$ are the ones given in Eq. (24). With this trick, the translational invariance of the baryons is recovered since

$$\Psi^{(\mathrm{HB})}(\vec{x}_1+\vec{d},\vec{x}_2+\vec{d},\vec{x}_3+\vec{d}) = \Psi^{(\mathrm{HB})}(\vec{x}_1,\vec{x}_2,\vec{x}_3), \tag{27}$$

where $\vec{d}$ is an arbitrary 3-vector, and the equality can be proven by taking $\vec{x}_\Delta\to\vec{x}_\Delta+\vec{d}$ in Eq. (26). From Eq. (27), it is clear that the quarks are no longer constrained in the specific region. However, the quarks are bounded and entangled in the sense that

$$\Psi^{(\mathrm{HB})}(\vec{x}_1, \vec{x}_2, \vec{x}_3) = 0, \quad \text{for } |\vec{x}_i - \vec{x}_j| > 2R, \tag{28}$$

for $i, j \in \{1, 2, 3\}$, which can be derived by $\phi_{q\updownarrow}(\vec{x}) = 0$ for $\vec{x} > R$. As the baryons are invariant under the spatial translations, we conclude that the CMM is removed.

With the baryon wave functions, the calculations of the baryon matrix elements are straightforward. The results of the SB approach can be found in Ref. [17], while the form factors of the HB approach are given in Ref. [32].

Here, we sketch the method of calculating $a(\bar{\mathbf{3}})$ and $a(\mathbf{6})$ in the HB approach. To diminish the directional dependencies in Eq. (15), we trace over the baryon spins

$$a(\mathbf{R}) = \frac{1}{2}(\langle \Xi_c^+(\mathbf{R}), \uparrow | O | \Xi_{cc}^+, \uparrow\rangle + \langle \Xi_c^+(\mathbf{R}), \downarrow | O | \Xi_{cc}^+, \downarrow\rangle), \tag{29}$$

with the normalization of $\bar{u}_{c(c)} u_{c(c)} = 1$. By using the anticommutation relations among the quark operators

$$\{q_{a\alpha}(\vec{x}), q^\dagger_{b\beta}(\vec{x}')\} = \delta_{ab}\delta_{\alpha\beta}\delta^3(\vec{x} - \vec{x}'), \tag{30}$$

we arrive at [26]

$$\sum_{J_z = \updownarrow} \langle \Xi_c^+(\mathbf{R}), J_z | (u^\dagger L^\mu d)(s^\dagger L_\mu c) | \Xi_{cc}^+, J_z\rangle = \mathcal{N}_c \mathcal{N}_{cc} \int d^3\vec{x}_\Delta \mathcal{D}_c(\vec{x}_\Delta) \Upsilon^{\mathbf{R}}(\vec{x}_\Delta), \tag{31}$$

where $\mathcal{N}_{c(c)}$ is the normalization constant of $\Xi_{c(c)}$,

$$\begin{aligned}
\mathcal{D}_c(\vec{x}_\Delta) &= \int d\vec{x} \phi_c^\dagger(\vec{x}^+) \phi_c(\vec{x}^-), \\
\Upsilon^{\mathbf{R}}(\vec{x}_\Delta) &= \sum_{[\lambda]} \mathcal{F}([\lambda], \mathbf{R}) \int d^3\vec{x} \phi^\dagger_{u\lambda_4}(\vec{x}^+) L_\mu \phi_{d\lambda_2}(\vec{x}^-) \phi^\dagger_{s\lambda_3}(\vec{x}^+) L^\mu \phi_{c\lambda_1}(\vec{x}^-),
\end{aligned} \tag{32}$$

$[\lambda] = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, $\vec{x}^\pm = \vec{x} \pm \vec{x}_\Delta/2$, and $\mathcal{F}$ are the spin-flavor overlappings, given as

$$\begin{aligned}
\sum_{[\lambda]} \mathcal{F}([\lambda], \bar{\mathbf{3}})(\lambda_1 \otimes \lambda_2 \otimes \lambda_3 \otimes \lambda_4) &= \frac{\sqrt{6}}{2}(\uparrow\downarrow\uparrow\downarrow - \downarrow\uparrow\uparrow\downarrow - \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow), \\
\sum_{[\lambda]} \mathcal{F}([\lambda], \mathbf{6})(\lambda_1 \otimes \lambda_2 \otimes \lambda_3 \otimes \lambda_4) &= \frac{1}{3\sqrt{2}}[(\uparrow\downarrow + \downarrow\uparrow)(\uparrow\downarrow + \downarrow\uparrow) + 2\uparrow\uparrow\uparrow\uparrow + 2\downarrow\downarrow\downarrow\downarrow].
\end{aligned} \tag{33}$$

From Eq. (33), it is easy to deduce that

$$\begin{aligned}
\sum_{[\lambda]} \mathcal{F}([\lambda], \mathbf{R})(\chi^\dagger_{\lambda_3}\chi_{\lambda_1})(\chi^\dagger_{\lambda_4}\chi_{\lambda_2}) &= \mathcal{C}^{\mathbf{R}}_{\mathrm{unflip}}, \\
\sum_{[\lambda]} \mathcal{F}([\lambda], \mathbf{R})(\chi^\dagger_{\lambda_3}\sigma_i\chi_{\lambda_1})(\chi^\dagger_{\lambda_4}\chi_{\lambda_2}) &= 0, \\
\sum_{[\lambda]} \mathcal{F}([\lambda], \mathbf{R})(\chi^\dagger_{\lambda_3}\sigma_i\chi_{\lambda_1})(\chi^\dagger_{\lambda_4}\sigma_j\chi_{\lambda_2}) &= \delta_{ij}\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}},
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
(\mathcal{C}^{\bar{\mathbf{3}}}_{\mathrm{unflip}}, \mathcal{C}^{\bar{\mathbf{3}}}_{\mathrm{flip}}) &= (\sqrt{6}, -\sqrt{6}), \\
(\mathcal{C}^{\mathbf{6}}_{\mathrm{unflip}}, \mathcal{C}^{\mathbf{6}}_{\mathrm{flip}}) &= \left(\sqrt{2}, \frac{\sqrt{2}}{3}\right),
\end{aligned} \tag{35}$$

and $\sigma_{i,j}$ are the Pauli matrices. The second and third equations of Eq. (34) are due to the fact that we have traced over the baryon spins so the matrix elements cannot depend on specific directions.

We decompose Υ into several pieces,

$$\Upsilon^{\mathbf{R}}(\vec{x}_\Delta) = \int d^3\vec{x} \sum_{k=1,2,3,4} \Gamma^{\mathbf{R}}_k(\vec{x}_\Delta, \vec{x}), \tag{36}$$

with

TABLE I. Results of the form factors in the HB approach (The uncertainties are smaller than the ones obtained in Ref. [32] because a smaller range of the bag radii is considered.).

$f_1^{\bar{3}}(\omega)$	$f_1^{\bar{3}}(\omega')$	$f_1^{6}(\omega)$	$f_1^{6}(\omega')$	$g_1^{\bar{3}}(\omega)$	$g_1^{\bar{3}}(\omega')$	$g_1^{6}(\omega)$	$g_1^{6}(\omega')$
0.480(17)	0.593(17)	0.277(10)	0.342(10)	0.152(5)	0.188(5)	0.439(16)	0.542(15)

$$
\begin{aligned}
\Gamma_1^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= \sum_{[\lambda]}\mathcal{F}([\lambda],\mathbf{R})\phi^\dagger_{u\lambda_4}(\vec{x}^+)\phi_{d\lambda_2}(\vec{x}^-)\phi^\dagger_{s\lambda_3}(\vec{x}^+)\phi_{c\lambda_1}(\vec{x}^-),\\
\Gamma_2^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= \sum_{[\lambda]}\mathcal{F}([\lambda],\mathbf{R})\phi^\dagger_{u\lambda_4}(\vec{x}^+)\gamma_5\phi_{d\lambda_2}(\vec{x}^-)\phi^\dagger_{s\lambda_3}(\vec{x}^+)\gamma_5\phi_{c\lambda_1}(\vec{x}^-),\\
\Gamma_3^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= -\sum_{[\lambda]}\mathcal{F}([\lambda],\mathbf{R})\phi^\dagger_{u\lambda_4}(\vec{x}^+)V_i\phi_{d\lambda_2}(\vec{x}^-)\phi^\dagger_{s\lambda_3}(\vec{x}^+)V_i\phi_{c\lambda_1}(\vec{x}^-),\\
\Gamma_4^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= -\sum_{[\lambda]}\mathcal{F}([\lambda],\mathbf{R})\phi^\dagger_{u\lambda_4}(\vec{x}^+)V_i\gamma_5\phi_{d\lambda_2}(\vec{x}^-)\phi^\dagger_{s\lambda_3}(\vec{x}^+)V_i\gamma_5\phi_{c\lambda_1}(\vec{x}^-),
\end{aligned}
\tag{37}
$$

where $V_i=\gamma_0\gamma_i$ with $i=1,2,3$. Plugging Eq. (34) into Eq. (37), we obtain

$$
\begin{aligned}
\Gamma_1^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= \mathcal{C}^{\mathbf{R}}_{\mathrm{unflip}}(u_u^+u_d^-+v_u^+v_d^-\hat{x}^+\cdot\hat{x}^-)(u_s^+u_c^-+v_s^+v_c^-\hat{x}^+\cdot\hat{x}^-)-\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}}\frac{(\vec{x}_\Delta\times\vec{x})^2}{(r^+r^-)^2}v_u^+v_d^-v_s^+v_c^-,\\
\Gamma_2^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= -\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}}(u_u^+v_d^-\hat{x}^- - v_u^+u_d^-\hat{x}^+)(u_s^+v_c^-\hat{x}^- - v_s^+u_c^-\hat{x}^+),\\
\Gamma_3^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= -\frac{\mathcal{C}^{\mathbf{R}}_{\mathrm{unflip}}}{\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}}}\Gamma_2^{\mathbf{R}}(\vec{x}_\Delta,\vec{x})-2\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}}(u_u^+v_d^-\hat{x}^-+v_u^+u_d^-\hat{x}^+)\cdot(u_s^+v_c^-\hat{x}^-+v_s^+u_c^-\hat{x}^+),\\
\Gamma_4^{\mathbf{R}}(\vec{x}_\Delta,\vec{x}) &= -\mathcal{C}^{\mathbf{R}}_{\mathrm{flip}}[3u_u^+u_d^-u_s^+u_c^-+v_u^+v_d^-v_s^+v_c^-(2+(\hat{x}^+\cdot\hat{x}^-)^2)\\
&\quad -(u_u^+u_d^-v_s^+v_c^-+v_u^+v_d^-u_s^+u_c^-)\hat{x}^+\cdot\hat{x}^-]+\mathcal{C}^{\mathbf{R}}_{\mathrm{unflip}}v_u^+v_d^-v_s^+v_c^-\frac{(\vec{x}_\Delta\times\vec{x})^2}{(r^+r^-)^2},
\end{aligned}
\tag{38}
$$

with the abbreviation

$$
\phi_q(\vec{x}^\pm)=\begin{pmatrix} u_q^\pm\chi \\ iv_q^\pm(\hat{x}^\pm\cdot\vec{\sigma})\chi\end{pmatrix}. \tag{39}
$$

Collecting Eqs. (29), (31), (32), (37), and (38), now we are able to calculate $a(\mathbf{R})$. Note that the formalism is reduced to the SB approach by eliminating the $\vec{x}_\Delta$ integral

$$
a(\mathbf{R})=\Upsilon^{\mathbf{R}}(0). \tag{40}
$$

To compare with the SB approach [17], we rescale the parameters as

$$
a(\bar{\mathbf{3}})=16\sqrt{6}\pi X_2, \qquad a(\mathbf{6})=-\frac{16\sqrt{2}\pi}{3}X_1. \tag{41}
$$

IV. NUMERICAL RESULTS

In crunching the numbers, we take the bag model parameters [33]

$$
\begin{aligned}
&M_{u,d}=0, \qquad M_s=0.28\ \mathrm{GeV},\\
&M_c=1.655\ \mathrm{GeV}, \qquad R=(5.0\pm0.1)\ \mathrm{GeV}^{-1}.
\end{aligned}
\tag{42}
$$

In the HB model, the axial vector couplings and $X_{1,2}$ are found to be

$$
\begin{aligned}
(g^{A(\pi^+)}_{\Xi_{cc}^+\Xi_{cc}^{++}}, g^A_{\mathbf{66}}, g^A_{\mathbf{6\bar{3}}}) &= (-0.259, 0.522, -0.453),\\
X_2 &= (3.52\pm0.22)10^{-4}\ \mathrm{GeV}^3,\\
X_1 &= (-2.44\pm0.08)10^{-6}\ \mathrm{GeV}^3,
\end{aligned}
\tag{43}
$$

while g_1 and f_1 are summarized in Table I. The overlappings of X_2 and X_1 are twice and one-half larger than those of the SB approach [17], and the same tendencies are found in the heavy-flavor-conserving decays [26]. We emphasize that $X_1\propto M_s$ due to the Körner-Pati-Woo theorem [34]. As a consequence, the calculated X_1 from the bag model shall not be fully trusted, as M_s is difficult to determine. Nevertheless, X_1 can be taken as zero in practice, so the final results are little affected.

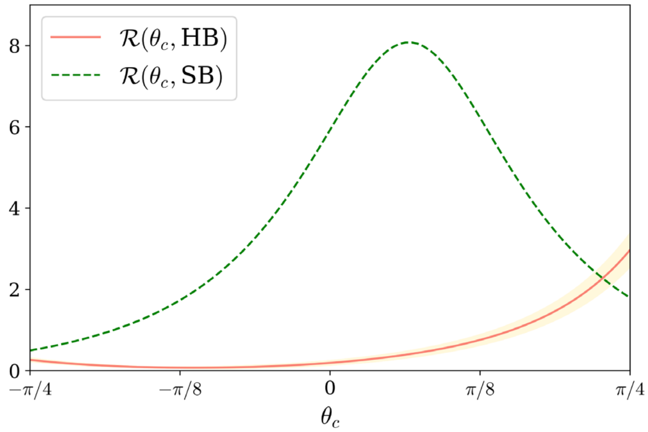

FIG. 1. $\mathcal{R}(\Xi_c^{++} \to \Xi_c^+ \pi^+)$ vs θ_c.

The mixing largely modifies $\mathcal{R}(\Xi_c^{++} \to \Xi_c^+ \pi^+)$, as shown in Fig. 1. Particularly, with $\theta_0 \equiv 0.142\pi$, we find that

$$\begin{aligned} \mathcal{R}(0, \mathrm{SB}) &= 6.74, & \mathcal{R}(\theta_0, \mathrm{SB}) &= 5.39, \\ \mathcal{R}(-\theta_0, \mathrm{SB}) &= 1.45, & \mathcal{R}(0, \mathrm{HB}) &= 0.19 \pm 0.05, \\ \mathcal{R}(\theta_0, \mathrm{HB}) &= 0.87^{+0.17}_{-0.11}, & \mathcal{R}(-\theta_0, \mathrm{HB}) &= 0.07. \end{aligned} \tag{44}$$

Because of the large difference in $X_{1,2}$, the HB and SB approaches predict very different ratios. However, they both require $\theta_c \neq 0$ to explain the experiments. With $\theta_c = -\theta_0$, the SB approach is in good agreement with Eq. (3), whereas with $\theta_c = \theta_0$, the HB approach shows accordance with the experimental lower bound.

We list the results of the branching fractions and up-down asymmetries in Table II along with those in the literature, where we have normalized the branching fractions by $(\tau(\Xi_{cc}^{++}), \tau(\Xi_{cc}^{+})) = (2.56, 0.45) \times 10^{-13}$ s [35,36]. In the literature, Ref. [15] adopts the covariant quark model up to three-loop calculations; Ref. [13] employs the pole model, but only the parity even pole is considered; Ref. [37] calculates the W-exchange contributions by the light-cone sum rule with the heavy quark effective theory; and Refs. [14,16,38] consider only the factorizable parts of the amplitudes. In the table, the quoted values of Ref. [13] are calculated by the nonrelativistic quark model (N) and heavy quark effective theory (H) with the flavor-independent pole, and the ones of Refs. [16,38] are given by $\theta_c = 0.090 \pm 0.013\pi$ (M) and $\theta_c = 0$ (N) with the light-front quark model. The results of Ref. [17] are essentially the ones of the SB approach with $\theta_c = 0$. Remarkably, Refs. [17,15] show good accordance, which indicates their treatments for $\theta_c = 0$ are reliable. However, they are inconsistent with the experimental data of $\mathcal{R}(\Xi_c^{++} \to \Xi_c^+ \pi^+)$. We believe that such deviations are caused by the $\Xi_c - \Xi_c'$ mixing. As shown in the table, after considering the mixing, both $\mathcal{B}$ and $\mathcal{R}$ are compatible with the current experimental data. To test our theory, we recommend the future experiments on $\mathcal{R}(\Xi_c^+ \to \Xi_c^{0(+)} \pi^{+(0)})$, found to be

TABLE II. The calculated branching fractions and up-down asymmetries (in units of %) along with the ones in the literature. All the branching fractions are normalized by $(\tau(\Xi_{cc}^{++}), \tau(\Xi_{cc}^{+})) = (2.56, 0.45) \times 10^{-13}$ s. For Ref. [13], we quote the results of the flavor-independent pole, and the parentheses of (N) and (H) indicate the form factors are calculated by the nonrelativistic quark model and heavy quark effective theory, respectively. For Refs. [16,38], (U) and (M) are the results with and without the $\Xi_c - \Xi_c'$ mixing, respectively.

	HB $\theta_c = \theta_0$			Cheng *et al.* [17]			Gutsche *et al.* [15]			Sharma and Dhir [13]				
	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$ (N)	$\mathcal{B}$ (H)	α (N)	α (H)	$\mathcal{R}$
$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	10.3(24)	−30	$0.87^{+0.17}_{-0.11}$	0.69	−41	6.74	0.71	−57	4.77	6.66	9.30	−99	−99	0.82 (N)
$\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+$	8.91(68)	−96		4.65	−84		3.39	−93		5.46	7.51	−78	−79	0.81 (H)
$\Xi_{cc}^{+} \to \Xi_c^0 \pi^+$	8.12(55)	−52	0.25	3.84	−31	0.40				0.59	0.95	55	34	0.39 (N)
$\Xi_{cc}^{+} \to \Xi_c'^0 \pi^+$	2.05(17)	97		1.55	−73					1.49	2.12	65	65	0.45 (H)
$\Xi_{cc}^{+} \to \Xi_c^+ \pi^0$	8.58(104)	−37	0.23	2.38	−25	0.07				0.50				0.11
$\Xi_{cc}^{+} \to \Xi_c'^+ \pi^0$	1.94(24)	52		0.17	−3					0.054				

	SB $\theta_c = -\theta_0$			Shi *et al.* [37]		Gerasimov *et al.* [14]		Ke *et al.* [16,38]				
	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$	$\mathcal{R}$	$\mathcal{B}$	$\mathcal{R}$	$\mathcal{B}$ (U)	$\mathcal{B}$ (M)	α (U)	α (M)	$\mathcal{R}$
$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	2.24	−93	1.45	6.22(194)	1.42(78)	7.01	0.83	3.48(46)	2.14(18)	−44(1)	9(7)	0.56(18) (U)
$\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+$	3.25	−63		8.55(62)		5.85		1.96(24)	3.0(1)	−98(1)	−99(1)	1.41(20) (M)
$\Xi_{cc}^{+} \to \Xi_c^0 \pi^+$	2.26	31	1.17			1.23	0.85	0.61(8)	0.38(3)	−44(1)	9(7)	0.56(18) (U)
$\Xi_{cc}^{+} \to \Xi_c'^0 \pi^+$	2.64	−99				1.04		0.35(4)	0.53(2)	−98(1)	−99(1)	1.41(20) (M)
$\Xi_{cc}^{+} \to \Xi_c^+ \pi^0$	2.01	−5	0.25									
$\Xi_{cc}^{+} \to \Xi_c'^+ \pi^0$	0.51	−65										

$$\mathcal{R}(\Xi_c^+ \to \Xi_c^0\pi^+) = 0.25(\mathrm{SB}), \qquad 1.17(\mathrm{HB}),$$
$$\mathcal{R}(\Xi_c^+ \to \Xi_c^+\pi^0) = 0.23(\mathrm{SB}), \qquad 0.25(\mathrm{HB}). \qquad (45)$$

It is interesting to point out that the sign of $\alpha(\Xi_{cc}^+ \to \Xi_c^0\pi^+)$ is flipped by the mixing in the SB approach. Under the factorization ansatz, the decays of $\Xi_{cc}^{++} \to \Xi_c^{(\prime)+}\pi^+$ and $\Xi_{cc}^+ \to \Xi_c^0\pi^+$ behave identically; i.e., they have the same decay widths and up-down asymmetries as shown explicitly in Refs. [14,16,38]. Therefore, the experimental measurements of $\mathcal{B}$ and α up on these decays may clarify the W-exchange contributions. Especially, we recommend the future measurements on $\alpha(\Xi_{cc}^+ \to \Xi_c^{(\prime)0}\pi^+)$, as it is essentially negative in the factorization ansatz with $\theta_c = 0$. It is interesting to point out that the sign of $\alpha(\Xi_{cc}^0 \to \Xi_c^0\pi^+)$ is flipped after the mixing is considered in both the SB approach and Ref. [38].

Unfortunately, with the experimental value in Eq. (3), the HB and SB models suggest opposite signs of θ_c as shown in Eq. (44). In $\Xi_c^0 \to \Lambda_c^+\pi^-$ and $\Xi_b^- \to \Lambda_b^0\pi^-$, where the soft-meson limit is trustworthy,[4] it has been found that the HB approach is much more suitable than the SB one [23–26]. More importantly, the HB wave functions are self-consistent on the contrary of the SB ones. However, the computed $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+\pi^+)$ with the HB is much larger than Eq. (6), indicating that the branching fractions might be overestimated. Viewing on the successes of the SB approach in the Λ_c^+ decays [22], it is likely that the CMM and finite $\mathbf{p}_f$ corrections compensate each other. Accordingly, the sign of θ_c shall be negative, suggested by the SB model. We note that the semileptonic decays of $\Xi_{cc} \to \Xi_c e^+\nu_e$ are ideal places to determine the sign of θ_c, as they are uncontaminated by the W-exchange contributions. Nonetheless, the experiments are subjected to the difficulties imposed by the chargeless neutrinos.

[4]The $\mathbf{p}_f$ in $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ and $\Xi_c^0 \to \Lambda_c^+\pi^-$ are 0.96 and 0.11 GeV, respectively.

V. CONCLUSION

We have studied the $\Xi_c - \Xi_c'$ mixing effects in $\Xi_{cc} \to \Xi_c\pi$ with the soft-meson limit. The bag model has been employed for the baryon matrix elements with and without removing the CMM. We have found that the CMM corrections are sizable, as found in the heavy-flavor-conserving decays. The branching fractions and up-down asymmetries have been calculated, and special attention has been given to $\mathcal{R}$. In particular, for $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$, we have obtained that $(\mathcal{B},\mathcal{R}) = (10.3(24)\%, 0.87^{+0.17}_{-0.11})$ and $(2.24\%,\ 1.45)$ with and without removing the CMM, respectively, which are consistent with the current experimental data. To test our theory, we recommend the future experiments to examine $\mathcal{R}(\Xi_{cc}^+ \to \Xi_c^0\pi^+)$, which have been computed as 0.25 and 1.17 in the HB and SB approaches, respectively. To probe the W-exchange contributions, we recommend the measurement on $\alpha(\Xi_{cc}^+ \to \Xi_c^{(\prime)+}\pi^+)$, as they are negative in the factorization ansatz but 0.31(0.52) in the SB (HB) approach.

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