Letter

New recursion relations for tree-level correlators in anti–de Sitter spacetime

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We present for the first time classical multiparticle solutions in anti–de Sitter space (AdS) involving scalars, gluons, and gravitons. They are recursively defined through multiparticle currents which reduce to Berends-Giele currents in the flat space limit. This construction exposes a compact definition of tree-level boundary correlators using a general prescription that removes unphysical boundary contributions. Similarly to the flat space perturbiner, a convenient gauge choice leads to a scalar basis for all degrees of freedom, while the tensor structure is exclusively captured by field theory vertices. This provides a fully automated way to compute AdS boundary correlators to any multiplicity and cosmological wave function coefficients after Wick rotating to de Sitter space.

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I. INTRODUCTION

Quantum field theory (QFT) in curved spacetime is full of subtleties. For example, it is possible to construct an S-matrix in asymptotically flat backgrounds, but its definition is unclear in generic spacetimes. Thus, general techniques that we can export from flat to curved spaces are very welcome. Naturally, studying QFT in curved spacetime is also relevant for understanding quantum gravity. Of particular interest are backgrounds with nonzero cosmological constant, notably (anti–)de Sitter [(A)dS] space. Boundary correlators in these spacetimes are constrained by conformal Ward identities and reduce to bulk scattering amplitudes in the flat space limit. In AdS, this underlies the gauge-gravity duality between conformal field theory (CFT) and string theory [1]. In de Sitter (dS), this provides a powerful new set of tools for computing cosmological observables inspired by scattering amplitudes, which is now a very active area of research (see e.g., Ref. [2] for a recent review).

Recursive techniques have had a major impact on the understanding of flat space scattering amplitudes and are therefore valuable goals to pursue in curved space. For instance, the Britto-Cachazo-Feng-Witten (BCFW) recursion [3] was generalized to AdS in Refs. [4–6], while recursions for Witten diagrams were developed in Refs. [7–9] for scalars and more generally in Ref. [10]. However, they do not exhibit the same level of efficiency as flat space recursions and cannot be directly used to compute correlators involving more than one type of particle. Alternatively, the Berends-Giele (BG) recursion [11] (later extended and formalized in Ref. [12]) provides a clearer path for the computation of curved space correlators. In flat space, BG currents can be seen as tree-level amplitudes with one off-shell leg. Higher-point amplitudes are then built by connecting BG currents through field theory vertices. More recently, the BG recursion was partially extended to AdS embedding space [13,14], yielding a differential representation for boundary correlators with external scalars, although a practical extension to spinning particles remained elusive.

In this paper, we take this extra step and establish the AdS generalization of the so-called perturbiner method [15–17] (see also Refs. [18–33] for a number of recent applications). Based on a novel set of classical multiparticle solutions, we propose a robust framework to describe scalars, gluons, gravitons, and their interactions at tree level. Contrary to flat space, the multiparticle recursion in AdS momentum space is not algebraic and involves the inversion of differential operators in the radial coordinate. The key step here is a suitable gauge choice. Instead of the traditional axial gauge, we define a *boundary transversal gauge*. While equivalent at the linearized level, the latter lets us localize all the tensor structure into the vertices, exclusively working with scalar propagators.

The multiparticle currents are given by nested integrals in the radial coordinate and can be used to compute N-point tree-level boundary correlators. Because of the boundary, the usual BG prescription must be generalized to recover the permutation symmetry of the correlators. In Yang-Mills (YM), for example, this prescription makes the cyclicity of the color-ordered correlators manifest while removing unphysical boundary terms.

We start by discussing classical equations of motion in AdS, with the introduction of a convenient gauge choice for handling the multiparticle solutions. First, we look at the YM theory and the color-stripped perturbiner. Next, we analyze graviton multiparticle solutions and finalize with the discussion of scalars coupled to YM and gravity. In each case, we propose and verify the prescription for tree-level correlators. Along the way, we explain how to adapt our recursions to dS, where they compute coefficients of the cosmological wave function [34–36]. We then present some final remarks and natural directions to investigate next.

II. FIELD EQUATIONS IN AdS

We work with AdS_{d+1} with radius $\mathcal{R}$ in the Poincaré patch,

$$\tilde{g}_{mn}dx^m dx^n = \frac{\mathcal{R}^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \tag{1}$$

with $0 < z < \infty$. The spacetime indices $m, n, \ldots$ generically represent the radial direction z and the boundary directions. The latter are denoted by $\mu, \nu = 0, \ldots, d-1$, and $\eta_{\mu\nu}$ is the flat boundary metric (Lorentzian). We will often use the shorthand $(U \cdot V) = \eta^{\mu\nu}U_\mu V_\nu$ for boundary vectors. For dS, we take the boundary metric to be Euclidean and Wick rotate the radial coordinate, $z \to -i\eta$.

We start with a scalar field with mass m coupled to the curvature R via a constant parameter ξ, satisfying

$$g^{mn}\partial_m\partial_n\phi - g^{mn}\Gamma^p_{mn}\partial_p\phi = (\mathrm{m}^2 + \xi R)\phi. \tag{2}$$

The left-hand side is simply the curved d'Alembertian, with $\Gamma^p_{mn} = g^{pq}\Gamma_{mnq}$ denoting the Christoffel symbol

$$\Gamma_{mnp}[g] = \frac{1}{2}(\partial_m g_{np} + \partial_n g_{mp} - \partial_p g_{mn}). \tag{3}$$

In the rest of this paper, we take the free solutions to be eigenstates of the boundary momenta, denoted by k_μ. In the Poincaré patch ($g_{mn} = \tilde{g}_{mn}$), Eq. (2) is recast as

$$\mathcal{D}^2_k\phi = M^2\phi, \tag{4}$$

$$\mathcal{D}^2_k \equiv z^2\partial_z^2 + (1-d)z\partial_z - z^2k^2, \tag{5}$$

with $k^2 = (k \cdot k)$, and effective mass $M^2 = (\mathrm{m}\mathcal{R})^2 - \xi d(d+1)$. The solutions of (4) are Bessel functions (or Hankel functions for dS). Under proper boundary conditions and normalization, they are identified with (A) dS bulk-to-boundary propagators (see e.g., Refs. [37,38] for more details).

The curved Yang-Mills equations are given by

$$g^{np}\partial_p\mathbf{F}_{mn} = ig^{np}[\mathbf{A}_p, \mathbf{F}_{mn}] + \mathbf{J}_m + g^{np}(\Gamma^q_{mp}\mathbf{F}_{qn} + \Gamma^q_{np}\mathbf{F}_{mq}), \tag{6}$$

where $\mathbf{F}_{mn} = \partial_m\mathbf{A}_n - \partial_n\mathbf{A}_m - i[\mathbf{A}_m, \mathbf{A}_n]$ is the field strength, $\mathbf{A}_m$ is Lie algebra valued for some unspecified gauge group with generators T^a, and $\mathbf{J}_m$ generically denotes the coupling to other fields. We take $\mathbf{A}_\mu = (\mathcal{R}/z)A_\mu$ and $\mathbf{A}_z = (\mathcal{R}/z)\alpha$, such that the linearized version of (6) is rewritten as

$$(\mathcal{D}^2_k + d - 1)A_\mu = izk_\mu[z\partial_z + (2-d)]\alpha - z^2k_\mu(k \cdot A), \tag{7a}$$

$$k^2\alpha = i(1/z - \partial_z)(k \cdot A). \tag{7b}$$

Instead of the axial gauge $\alpha = 0$, we will choose the boundary transversal gauge,

$$\eta^{\mu\nu}\partial_\mu A_\nu = 0. \tag{8}$$

They are equivalent at the linearized level; when $k^2 \neq 0$, we have $\alpha = 0$, while for $k^2 = 0$, we set α to zero via a residual gauge symmetry.

Finally, we review Einstein's field equations with cosmological constant $\Lambda = d(1-d)/(2\mathcal{R}^2)$. In the presence of matter, with action S_{matter} and energy-momentum tensor

$$T_{mn} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{mn}}S_{\mathrm{matter}}, \tag{9}$$

they can be cast as

$$R_{mn} + \frac{d}{\mathcal{R}^2}g_{mn} = \kappa T_{mn} - \frac{\kappa}{(d-1)}g_{mn}g^{pq}T_{pq}, \tag{10}$$

with gravitational coupling κ, Ricci tensor R_{mn} given by

$$R_{mn} = \partial_p\Gamma^p_{mn} - \partial_n\Gamma^p_{mp} + \Gamma^p_{pq}\Gamma^q_{mn} - \Gamma^p_{nq}\Gamma^q_{mp}, \tag{11}$$

and scalar curvature $R = g^{mn}R_{mn}$.

The graviton dynamics can be accessed via a deformation of the background metric. Gravitons are parametrized here as

$$g_{mn} = \tilde{g}_{mn} + \frac{\mathcal{R}^2}{z^2}h_{mn}. \tag{12}$$

The analog of the boundary transversal gauge is

$$\eta^{\mu\nu}\partial_\mu h_{z\nu} = \frac{1}{2}\eta^{\mu\nu}\partial_z h_{\mu\nu} + \frac{d}{2z}h_{zz}, \tag{13a}$$

$$\eta^{\nu\rho}\partial_\rho h_{\mu\nu} = \frac{1}{2}\partial_\mu(\eta^{\nu\rho}h_{\nu\rho} + \beta h_{zz}), \tag{13b}$$

where β is a constant parameter. Then, the linearized version of (10) is given by

$$k^2 h_{zz} = 0, \tag{14a}$$

$$k^2 h_{z\mu} = \frac{i}{2z}(d - 2 - \beta z\partial_z)k_\mu h_{zz}, \tag{14b}$$

$$\mathcal{D}_k^2 h_{\mu\nu} = [(1-\beta)z^2 k_\mu k_\nu + \eta_{\mu\nu}(d - z\partial_z)]h_{zz} + [z^2\partial_z + (1-d)z](\partial_\mu h_{\nu z} + \partial_\nu h_{\mu z}). \tag{14c}$$

Like in Yang-Mills, the components h_{zz} and $h_{z\mu}$ vanish on shell ($k^2 \neq 0$) or via a residual gauge transformation ($k^2 = 0$).

With our gauge choice, the physical degrees of freedom both in YM and in gravity have scalar propagators, with interesting consequences in the context of multiparticle solutions.

III. MULTIGLUON SOLUTIONS AND CORRELATORS

We are now going to evaluate the multiparticle solutions of (6) through the ansatz:

$$\mathbf{A}_\mu(x,z) = \frac{\mathcal{R}}{z}\sum_I \mathcal{A}_{I\mu}(z)T^{a_I}e^{ik_I\cdot x}, \tag{15a}$$

$$\mathbf{A}_z(x,z) = \frac{\mathcal{R}}{z}\sum_I \alpha_I(z)T^{a_I}e^{ik_I\cdot x}, \tag{15b}$$

$$\mathbf{J}_m(x,z) = \sum_I \mathcal{J}_{Im}(z)T^{a_I}e^{ik_I\cdot x}. \tag{15c}$$

The word I denotes a sequence of letters $I = i_1 \ldots i_\ell$, where i is a single-particle label, with $k_I \equiv k_{i_1} + \cdots + k_{i_\ell}$ and $T^{a_I} = T^{a_{i_1}}\cdots T^{a_{i_\ell}}$. The boundary transversal gauge translates to $(k_I\cdot\mathcal{A}_I) = 0$. The single-particle solutions of (7), i.e., bulk-to-boundary propagators, are then associated to one-letter words, which we denote by $\mathcal{A}_{i\mu} = \varepsilon_{i\mu}\tilde{\phi}(z)e^{ik_i\cdot x}$ and $\alpha_i = 0$. The polarization $\varepsilon_{i\mu}$ is transversal, $(k_i\cdot\varepsilon_i) = 0$, and $\tilde{\phi}$ satisfies $(\mathcal{D}_i^2 + d - 1)\tilde{\phi} = 0$. We refer to $\mathcal{A}_{I\mu}$ and α_I as multiparticle currents. The specific form of $\mathbf{J}_m$ depends on the model, and we will see an explicit example later.

After plugging the above ansatz in (6), we obtain the multiparticle recursions

$$\begin{aligned}\frac{1}{z^2}(\mathcal{D}_I^2 + d - 1)\mathcal{A}_{I\mu} = {}& ik_{I\mu}[\partial_z + (2-d)/z]\alpha_I - \frac{\mathcal{R}}{z}\mathcal{J}_{I\mu}\\ &+ \frac{\mathcal{R}}{z}\sum_{I=JK}\{(k_{K\mu}\alpha_K + 2i\partial_z\mathcal{A}_{K\mu})\alpha_J + k_{K\mu}(\mathcal{A}_J\cdot\mathcal{A}_K) + \mathcal{A}_{K\mu}[i(\partial_z - d/z)\alpha_J - 2(k_K\cdot\mathcal{A}_J)] - (J\leftrightarrow K)\}\\ &+ \frac{\mathcal{R}^2}{z^2}\sum_{I=JKL}\{[\alpha_J\alpha_K\mathcal{A}_{L\mu} + (\mathcal{A}_J\cdot\mathcal{A}_K)\mathcal{A}_{L\mu} - (K\leftrightarrow L)] + [\alpha_K\alpha_L\mathcal{A}_{J\mu} + (\mathcal{A}_K\cdot\mathcal{A}_L)\mathcal{A}_{J\mu} - (J\leftrightarrow K)]\},\end{aligned} \tag{16}$$

and

$$\begin{aligned}k_I^2\alpha_I = {}& \frac{\mathcal{R}}{z}\sum_{I=JK}\{2\alpha_K(k_K\cdot\mathcal{A}_J) - 2\alpha_J(k_J\cdot\mathcal{A}_K)\\ &+ i(\mathcal{A}_J\cdot\partial_z\mathcal{A}_K) - i(\mathcal{A}_K\cdot\partial_z\mathcal{A}_J)\} + \frac{\mathcal{R}}{z}\mathcal{J}_{Iz}\\ &+ \frac{\mathcal{R}^2}{z^2}\sum_{I=JKL}\{\alpha_K(\mathcal{A}_J\cdot\mathcal{A}_L) - \alpha_L(\mathcal{A}_J\cdot\mathcal{A}_K) + (J\leftrightarrow L)\},\end{aligned} \tag{17}$$

with shorthand $\mathcal{D}_I^2 = \mathcal{D}_{k_I}^2$. The operation $I = JK$ (JKL) denotes a deconcatenation, which consists of all the *order preserving* ways of splitting the word I into JK (JKL). Note that $\mathcal{A}_{I\mu}, \alpha_I$ do not carry any color structure, which has been stripped off in (15).

The inversion of $(\mathcal{D}_I^2 - M^2)$ is defined via the Green's function $G_I(z,y)$, a bulk-to-bulk propagator, satisfying

$$(\mathcal{D}_I^2 - M^2)G_I = z^{d+1}\delta(z-y) \tag{18}$$

with appropriate boundary conditions. In particular, we have

$$(\mathcal{D}_I^2 - M^2)^{-1}\mathcal{O}(z) = \int\frac{dy}{y^{d+1}}G_I(z,y)\mathcal{O}(y), \tag{19}$$

and the recursion of $A_{I\mu}$, depicted in Fig. 1, is computed through nested integrals in the radial coordinate. Explicit expressions for G_I in AdS were derived in Ref. [5]. Wick rotating them to dS is subtle; see Refs. [38–40] for further details.

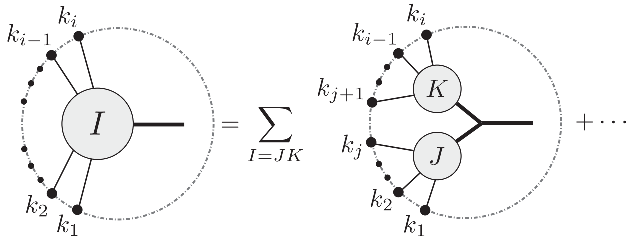

FIG. 1. Graphic representation of the cubic vertex deconcatenation $I = JK$ in the multiparticle recursion, with $I = 12\ldots i$. The dashed line denotes the AdS boundary. Thin (thick) lines denote bulk-to-boundary (bulk-to-bulk) propagators.

The prescription for computing N-gluon *color-ordered* correlators is defined to be

$$A(1,\ldots,N) = -\frac{1}{N}\int \frac{dz}{z^{d+1}}\eta^{\mu\nu}[\mathcal{A}_{1\mu}(\mathcal{D}^2_{2\ldots N}+d-1)\mathcal{A}_{2\ldots N\nu} + \mathrm{cyc}(1,\ldots,N)], \quad (20)$$

where boundary momentum conservation is implicit. Equation (20) effectively removes the bulk-to-bulk propagator from $\mathcal{A}_{2\ldots N\nu}$ (rightmost one in Fig. 1) and replaces it by a bulk-to-boundary propagator. This is a straightforward generalization of the usual Berends-Giele prescription [11]. The extra ingredients here are the integration over the radial coordinate and the explicit sum over the cyclic permutations of the N external legs, which is redundant in flat space. The latter removes unphysical boundary contributions that would otherwise break the cyclicity of the color-ordered correlators.

Let us now present a couple of examples. The three-point result is given by

$$A(1,2,3) = \mathcal{R}^3\varepsilon_{1\mu}\varepsilon_{2\nu}\varepsilon_{3\rho}V^{\mu\nu\rho}_{123}\int\frac{dz}{z^d}\tilde{\phi}_1\tilde{\phi}_2\tilde{\phi}_3, \quad (21)$$

with the usual polarization structure of Yang-Mills,

$$V^{\mu\nu\rho}_{123} = \eta^{\mu\nu}\eta^{\rho\sigma}(k_1-k_2)_\sigma + \mathrm{cyc}(1\mu,2\nu,3\rho). \quad (22)$$

The four-point correlator is given by

$$\begin{aligned} A(1,2,3,4) &= \Pi^0_{12|34}\int\frac{dz}{z^{d+1}}(\tilde{\phi}_1\overleftrightarrow{\partial}_z\tilde{\phi}_2)(\tilde{\phi}_3\overleftrightarrow{\partial}_z\tilde{\phi}_4) \\ &\quad + \Pi^1_{12|34}\int\frac{dz}{z^{d+1}}(z\tilde{\phi}_1\tilde{\phi}_2)(\mathcal{D}^2_{34}+d-1)^{-1}(z\tilde{\phi}_3\tilde{\phi}_4) \\ &\quad + [(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4)-(3\leftrightarrow 4)] \\ &\quad \times\int\frac{dz}{z^{d+1}}\tilde{\phi}_1\tilde{\phi}_2\tilde{\phi}_3\tilde{\phi}_4 - [(34)\to(23)], \end{aligned} \quad (23)$$

with $U\overleftrightarrow{\partial}_z V = U\partial_z V - V\partial_z U$. In the first line, the polarization structure is encoded in

$$\Pi^0_{12|34} = \frac{\mathcal{R}^4}{k^2_{34}}(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3\cdot\varepsilon_4), \quad (24)$$

while in the second line, we have

$$\begin{aligned} \Pi^1_{12|34} &= \mathcal{R}^4\frac{(k_1^2-k_2^2)(k_3^2-k_4^2)}{k^2_{34}}(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3\cdot\varepsilon_4) \\ &\quad + \mathcal{R}^4\eta^{\mu\nu}[2\varepsilon_{1\mu}(k_1\cdot\varepsilon_2)-k_{1\mu}(\varepsilon_1\cdot\varepsilon_2)-(1\leftrightarrow 2)] \\ &\quad \times[2\varepsilon_{3\nu}(k_3\cdot\varepsilon_4)-k_{3\nu}(\varepsilon_3\cdot\varepsilon_4)-(3\leftrightarrow 4)]. \end{aligned} \quad (25)$$

The third line in (23) is simply a four-point contact Witten diagram.

Because of the boundary transversal gauge, the correlators computed via (20) are expressed in terms of scalar-like factorization channels. The price to pay is the apparent introduction of spurious poles of the form k_{ij}^{-2}. The final expression, however, is equivalent to other results in the literature. For example, we match (23) with the results of [41] when $d = 3$.

IV. MULTIGRAVITON SOLUTIONS AND CORRELATORS

For the multiparticle solutions of (10), we start with an ansatz inspired by the parametrization (12),

$$g_{mn} = \tilde{g}_{mn} + \frac{\mathcal{R}^2}{z^2}\sum_I\mathcal{H}_{Imn}e^{ik_I\cdot x}. \quad (26)$$

The main difference with YM is the absence of the color structure, so we consider only the sum over ordered words $I = i_1\ldots i_\ell$, with $i_1 < i_2 < \ldots < i_\ell$.

The natural multiparticle ansatz for g^{mn} is

$$g^{mn} = \tilde{g}^{mn} - \frac{\mathcal{R}^2}{z^2}\sum_I\mathcal{I}^{mn}_I e^{ik_I\cdot x}. \quad (27)$$

Since the inverse metric satisfies $g^{mp}g_{np} = \delta^m_n$, the multiparticle currents in (27) are constrained to be

$$\mathcal{I}^{mn}_I = \tilde{g}^{mp}\mathcal{H}_{Ipq}\tilde{g}^{qn} - \frac{\mathcal{R}^2}{z^2}\sum_{I=J\cup K}\mathcal{I}^{mp}_J\mathcal{H}_{Kpq}\tilde{g}^{qn}. \quad (28)$$

The operation $I = J\cup K$ denotes the deshuffle, which means we consider all possible ways of splitting the ordered word I into two nonempty ordered words J and K. Equation (28) is responsible for packing the infinite number of vertices in gravity into a simple recursion [28]. In practice, the recursive structure is encoded in up to quintic interaction vertices, which is a vast improvement over standard diagrammatic techniques.

In terms of the multiparticle currents, gauge (13) reads

$$i\eta^{\mu\nu}k_{I\mu}\mathcal{H}_{Iz\nu} = \frac{1}{2}\eta^{\mu\nu}\partial_z\mathcal{H}_{I\mu\nu} + \frac{d}{2z}\mathcal{H}_{Izz}, \tag{29a}$$

$$i\eta^{\nu\rho}k_{I\rho}\mathcal{H}_{I\mu\nu} = \frac{i}{2}k_{I\mu}(\eta^{\nu\rho}\mathcal{H}_{I\nu\rho} + \beta\mathcal{H}_{Izz}), \tag{29b}$$

and the ansatz (26) solves Eq. (10) when the multiparticles currents satisfy

$$k_I^2\mathcal{H}_{Izz} = \frac{2\kappa}{(d-1)}[(d-2)\mathcal{T}_{Izz} - \eta^{\mu\nu}\mathcal{T}_{I\mu\nu}] - 2\mathcal{G}_{Izz}, \tag{30a}$$

$$k_I^2\mathcal{H}_{Iz\mu} = 2\kappa\mathcal{T}_{Iz\mu} - 2\mathcal{G}_{Iz\mu} + \frac{i}{2z}(d-2-\beta z\partial_z)k_{I\mu}\mathcal{H}_{Izz}, \tag{30b}$$

$$\begin{aligned}\mathcal{D}_I^2\mathcal{H}_{I\mu\nu} = {}& \frac{2\kappa z^2}{(d-1)}\eta_{\mu\nu}(\mathcal{T}_{Izz} + \eta^{\rho\sigma}\mathcal{T}_{I\rho\sigma}) - 2\kappa z^2\mathcal{T}_{I\mu\nu} \\ &+ [(1-\beta)z^2k_{I\mu}k_{I\nu} + \eta_{\mu\nu}(d - z\partial_z)]\mathcal{H}_{Izz} + 2z^2\mathcal{G}_{I\mu\nu} \\ &+ iz[z\partial_z + (1-d)](k_{I\mu}\mathcal{H}_{Iz\nu} + k_{I\nu}\mathcal{H}_{Iz\mu}).\end{aligned} \tag{30c}$$

$\mathcal{T}_{Imn}$ denotes the currents of the multiparticle expansion of the energy-momentum tensor, $T_{mn} = \sum_I \mathcal{T}_{Imn}e^{ik_I\cdot x}$. The interaction between gravitons and matter in AdS is captured by $\mathcal{G}_{Imn}$, which is fully displayed in the Supplemental Material [75]. By construction, the currents $\mathcal{H}_{Imn}$ are symmetric under the permutation of any single-particle labels. The single-particle solutions of (14) are again associated to one-letter words, which we denote by $\mathcal{H}_{i\mu\nu} = h_{i\mu\nu}\varphi(z)e^{ik_i\cdot x}$ and $\mathcal{H}_{iz\mu} = \mathcal{H}_{izz} = 0$. The boundary polarization $h_{i\mu\nu}$ is traceless ($\eta^{\mu\nu}h_{i\mu\nu} = 0$) and transversal ($\eta^{\nu\rho}k_{i\rho}h_{i\mu\nu} = 0$), and $\varphi(z)$ is a massless minimally coupled ($\xi = 0$) scalar.

Like in YM, the recursions in (30) present a characteristic feature of the boundary transversal gauge: the tensor structure of the correlator is relegated to the interaction vertices, and only scalar propagators appear. Moreover, the currents $\mathcal{H}_{Izz}$ and $\mathcal{H}_{Iz\mu}$, as well as $\eta^{\mu\nu}\mathcal{H}_{I\mu\nu}$ and $\eta^{\nu\rho}k_{I\rho}\mathcal{H}_{I\mu\nu}$, have a trivial propagator.

The generalization of the color-ordered correlators in (20) to gravity is given by

$$\begin{aligned}\mathcal{M}_N = {}& -\frac{1}{N}\kappa\int\frac{dz}{z^{d+1}}\eta^{\mu\rho}\eta^{\nu\sigma}\mathcal{H}_{1\mu\nu}(\mathcal{D}^2_{2\ldots N}\mathcal{H}_{2\ldots N\rho\sigma}) \\ &+ \mathrm{perm}(1\to 2\ldots N).\end{aligned} \tag{31}$$

The permutation in the last line makes the correlator manifestly symmetric in all N legs.

Since graviton correlators quickly grow in size, we present explicitly only the three-point case:

$$\begin{aligned}\mathcal{M}_3 = {}& \frac{\kappa}{4}h_{1\mu\nu}h_{2\rho\sigma}h_{3\gamma\lambda}\bigg\{V_{123}^{\mu\rho\gamma}V_{123}^{\nu\sigma\lambda}\int\frac{dz}{z^{d-1}}\varphi_1\varphi_2\varphi_3 \\ &-\frac{1}{3}\eta^{\nu\rho}\eta^{\sigma\gamma}\eta^{\lambda\mu}\int dz\partial_z\left[\frac{1}{z^{d-1}}\partial_z(\varphi_1\varphi_2\varphi_3)\right]\bigg\}.\end{aligned} \tag{32}$$

The first line is the well-known expression in terms of the cubic vertices of Yang-Mills (22). The second line encodes contact terms which have delta function support when Fourier-transformed to position space. Therefore, it vanishes for generic boundary positions of the operators. In momentum space, they are characterized by being analytic in at least two of the momenta [34]. The total derivative in (32) diverges when $z\to 0$, so we introduce a cutoff at $z = \epsilon$. After dropping the power-law divergent pieces, we find that

$$\mathcal{M}_{\mathrm{B}}^d \equiv \int dz\partial_z\left[\frac{1}{z^{d-1}}\partial_z(\varphi_1\varphi_2\varphi_3)\right] \propto \sum_{i=1}^{3}k_i^d, \tag{33}$$

in odd d, which can be removed by a redefinition of the bulk metric [34]. For even d, we obtain

$$\mathcal{M}_{\mathrm{B}}^d \propto \sum_{i=1}^{3}k_i^d\ln\left(\frac{1}{2}\epsilon k_i e^{\gamma_E}\right) + \cdots, \tag{34}$$

where the first term can also be removed by a redefinition of the metric [42] and the ellipsis denotes polynomials in the squares of momenta, known as ultralocal terms [43].

V. SCALARS, GLUONS, AND GRAVITONS

Now, we turn our attention to scalar theories. Since their classical multiparticle solutions have a very simple structure, we will focus on the more interesting cases with coupling to gluons and gravitons.

Consider first scalars in the adjoint representation of the gauge group. Their color-stripped multiparticle expansion is analogous to (15), given by

$$\phi = \sum_I \Phi_I(z)T^{a_I}e^{ik_I\cdot x}. \tag{35}$$

Single-particle states satisfy $(\mathcal{D}_i^2 - M^2)\Phi_i = 0$, and we consider a minimal coupling with gluons,

$$\partial_m\phi \to \partial_m\phi - i[\mathbf{A}_m, \phi], \tag{36}$$

such that $\mathbf{J}_m = [(i\partial_m\phi + [\mathbf{A}_m, \phi]), \phi]$ in (6).

In the gauge (8), Eq. (2) minimally coupled to YM yields the following recursion,

$$\frac{1}{z^2}(\mathcal{D}_I^2 - M^2)\Phi_I = \frac{\mathcal{R}}{z}\sum_{I=JK}[2\Phi_J(k_J\cdot\mathcal{A}_K) - i\left(\Phi_J\partial_z\alpha_K + 2\alpha_K\partial_z\Phi_J - \frac{d}{z}\Phi_J\alpha_K\right) - (J\leftrightarrow K)] + \frac{\mathcal{R}^2}{z^2}\sum_{I=JKL}[(\mathcal{A}_J\cdot\mathcal{A}_K)\Phi_L - (\mathcal{A}_J\cdot\mathcal{A}_L)\Phi_K + \alpha_J\alpha_K\Phi_L - \alpha_J\alpha_L\Phi_K + (J\leftrightarrow L)], \tag{37}$$

with color-ordered N-point correlators defined via

$$A(1,\ldots,N) = -\frac{1}{N}\int\frac{dz}{z^{d+1}}\Phi_1(\mathcal{D}_{2\ldots N}^2 - M^2)\Phi_{2\ldots N} + \mathrm{cyc}(1,\ldots,N). \tag{38}$$

As an example, we take the case of four external scalars exchanging gluons:

$$A(1,2,3,4) = \frac{\mathcal{R}}{2}\int\frac{dz}{z^d}\{\Phi_1\Phi_2[(k_1-k_2)\cdot\mathcal{A}_{34}] + i(\Phi_1\partial_z\Phi_2 - \Phi_2\partial_z\Phi_1)\alpha_{34}\} + \mathrm{cyc}(1,2,3,4). \tag{39}$$

For a conformally coupled scalar ($M^2 = 1-d$), this expression can be directly obtained from the YM result in (23) with the identifications $\tilde{\phi}_i \to \Phi_i$, $(k_i\cdot\varepsilon_j)\to 0$, and $(\varepsilon_i\cdot\varepsilon_j)\to 1$. The final result matches the form obtained in Ref. [41] for $d=3$.

When graviton excitations are considered, the color structure cannot be stripped off from the multiparticle currents, which would explicitly involve color indices. For simplicity, we will turn off the gluons and consider a colorless scalar

$$\phi = \sum_I \Phi_I(z)e^{ik_I\cdot x}, \tag{40}$$

as the multiparticle ansatz solving Eq. (2). We then obtain the recursion for Φ_I,

$$\frac{1}{z^2}(\mathcal{D}_I^2 - M^2)\Phi_I = \frac{\mathcal{R}^4}{z^4}\sum_{I=J\cup K}\Big\{\mathcal{I}_J^{mn}\partial_m\partial_n\Phi_K + \frac{z^2}{\mathcal{R}^2}\tilde{g}^{mn}\times\left[\tilde{g}^{pq}\Gamma_{Jmnq}\partial_p\Phi_K - \frac{2\xi}{(d-1)}\kappa\Phi_J\mathcal{T}_{Kmn}\right] - \tilde{\Gamma}_{mnq}(\tilde{g}^{mn}\mathcal{I}_J^{pq} + \tilde{g}^{pq}\mathcal{I}_J^{mn})\partial_p\Phi_K\Big\} + \ldots. \tag{41}$$

The ellipsis denotes contributions with higher-order deshuffles, which are spelled out in the Supplemental Material [75]. The current Γ_{Imnp} is defined through (3) as

$$\Gamma_{Imnp} = \Gamma_{mnp}\left[\frac{\mathcal{R}^2}{z^2}\mathcal{H}_I\right]. \tag{42}$$

The notation $\partial_p\mathcal{O}_I = i\delta_p^\mu k_{I\mu}\mathcal{O}_I + \delta_p^z\partial_z\mathcal{O}_I$ is implicit for any current $\mathcal{O}_I$. Finally, $\mathcal{T}_{Imn}$ denotes the multiparticle coefficients of the energy-momentum tensor:

$$T_{mn} = \partial_m\phi\partial_n\phi - \frac{1}{2}g_{mn}(g^{pq}\partial_p\phi\partial_q\phi + m^2\phi^2) + \xi\left(R_{mn} - \frac{1}{2}g_{mn}R\right)\phi^2 + \xi(g_{mn}g^{pq}\partial_p\partial_q - \partial_m\partial_n)\phi^2 - \xi(g_{mn}g^{pq}g^{rs}\Gamma_{pqr}\partial_s - g^{pq}\Gamma_{mnp}\partial_q)\phi^2. \tag{43}$$

The N-point scalar correlator is given by

$$A_N = -\frac{1}{N}\int\frac{dz}{z^{d+1}}\Phi_1(\mathcal{D}_{2\ldots N}^2 - M^2)\Phi_{2\ldots N} + \mathrm{perm}(1\to 2\ldots N). \tag{44}$$

We have explicitly checked that the four-point correlator matches the Witten diagram calculation modulo gauge-dependent contact terms for the case $M^2 = 0$ (see the Supplemental Material [75]). This case is of particular interest since it arises from the dimensional reduction of the four-point graviton amplitude in the flat space limit [13]. Note that theβ-dependent piece coming from the gauge choice (13) may be cast as a total derivative,

$$A_4|_\beta \propto \sum_{234=ij\cup k}\int dz\partial_z\Big\{z^{1-d}\Phi_1\partial_z\mathcal{H}_{ijzz}\Phi_k + \frac{(k_k^2 - k_1^2)}{k_{ij}^2}z^{1-d}[\mathcal{H}_{ijzz}(\Phi_1\partial_z\Phi_k - \Phi_k\partial_z\Phi_1)]\Big\} + \mathrm{perm}(1\to 234). \tag{45}$$

Once again, these boundary contributions correspond to contact terms with delta function support in position space.

VI. FINAL REMARKS

Inspired by the perturbiner method in flat space, we have derived the first classical multiparticle solutions for scalars, gluons, and gravitons in AdS_{d+1}. Their recursive character requires nested integrations in the radial coordinate, with bulk-to-bulk propagator insertions. Perhaps more noteworthy is the fact that in any of these theories we require only *scalar* bulk-to-bulk propagators. This follows from a special gauge choice, dubbed here boundary transversal gauge [see (8) for YM and (13) for Einstein gravity]. At the linear level, it is equivalent to the axial gauge. At the nonlinear level, however, the latter makes the perturbiner recursion impractical, introducing further differential operators in the radial coordinate.

Our recursive approach is equivalent to the Witten diagrammatic expansion in AdS momentum space up to contact terms with delta function support when Fourier-transformed to position space. In general, Witten diagrams capture the transverse traceless part of the dual CFT correlators. Ward identities can then be used to determine the remaining terms. They correspond to contact terms in position space and vanish for generic locations of the CFT operators [34,44,45].

Due to the (A)dS boundary, the usual flat space BG prescription had to be generalized. For Yang-Mills theory, we introduced a prescription that makes the cyclicity of color-ordered correlators manifest. We verified up to five points that this removes unphysical boundary contributions. For gravity, the prescription restores permutation invariance of the correlators. Finally, we analyzed scalars exchanging gluons and gravitons, obtaining novel formulas, which we matched against four-point Witten diagrams. They exhibit interesting new structures related to the double copy [46,47] and will be presented in Ref. [48]. We expect our framework to be more transparent to the color-kinematics duality, much in the same way that the flat space perturbiner could realize a Bern-Carrasco-Johansson (BCJ) gauge through a multiparticle gauge choice [19].

In summary, we have established an elegant tool for computing tree-level boundary correlators in (A)dS. Our results also provide a systematic construction of higher-point graviton correlators which is currently very challenging using Witten diagrams. Exploring whether our approach exposes some hidden structures in these correlators is therefore an important priority for future work. We plan to investigate the implications of our recursions for cosmology and the relation to other recent approaches based on the double-copy [13,14,37,44,49–59], factorization [41,60–63], unitarity [40,64–67], Mellin space [39,68], Witten diagrams [69,70], scattering equations in (A)dS [38,71–73], and geometric approaches [7].

One of the claims to fame of BG recursion in flat space is the first proof of the Parke-Taylor formula [74] for all tree-level Maximally Helicity Violating (MHV) amplitudes in YM [11]. In four-dimensional (A)dS, the natural analog of MHV amplitudes is tree-level all-plus correlators of gluons, which vanish in the flat space limit. It would be truly rewarding if the recursion relations we formulate in this paper could suggest all mutliplicity formulas for such correlators.

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