

## Quantum-mechanical mechanism for reducing the cosmological constant

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 (Received 22 October 2022; accepted 15 November 2022; published 28 November 2022)

We exhibit a mechanism that dynamically adjusts the cosmological constant toward  $0^+$ . The adjustment is quantum mechanical, discharging the cosmological constant in random discrete steps. It renders de Sitter space unstable and triggers its decay toward Minkowski. Since the instability dynamically stops at  $\Lambda = 0$ , the evolution favors the terminal Minkowski space without a need for anthropics. The mechanism works for any quantum field theory coupled to gravity.

DOI: [10.1103/PhysRevD.106.L101701](https://doi.org/10.1103/PhysRevD.106.L101701)

Following the discussion [1,2] generalizing general relativity (GR) [3,4] to a theory of gravity on the multiverse, in this Letter we concentrate on the cosmological constant adjustment to zero. We focus on a greatly simplified limit of the theory of [1,2], with fixed Planck scale.<sup>1</sup> We allow dynamical variation of only the cosmological constant counterterm, mediated by a system of 4-forms and their membrane sources. This theory generalizes unimodular formulation of GR [5–13] by including charged membranes and corresponding boundary terms, which enforce local general covariance. Since the Planck scale is fixed, we can couple any quantum field theory (QFT) of matter to gravity minimally, as there is no chance for ghosts to arise in this limit [1,2]. This suffices to ensure de Sitter space is unstable to membrane nucleations, which completely cancel the cosmological constant to  $0^+$ . The huge numerical disparity between the QFT cutoff and the observation is irrelevant. It decays away.

This is how it works. Sans membranes, the cosmological constant is fixed, albeit completely arbitrary. It is not correlated to the local QFT scales in a calculable way [5–13]; it is set by initial conditions. However, with membranes, which source 4-forms, and by extension the cosmological constant, quantum discharge changes the *physical* cosmological constant  $\Lambda$  [14,15]. In a nested set of expanding bubbles bounded by membranes,  $\Lambda$  scans a wide range of values, which change randomly, both

increasing and decreasing relative to the exterior. This is a quantum random walk [1,2], and the variation of  $\Lambda$  essentially defines a toy model of eternal inflation [16]. On average,  $\Lambda$  decreases inside a sequence of membranes.

To make sure that the range of  $\Lambda$  comes arbitrarily close to zero without fine tuning, we invoke a system of two 4-forms that are degenerate on shell with  $\Lambda$ . We take their membrane charges such that their ratio is an irrational number. In this case the spectrum of values of  $\Lambda$  is a very fine discretuum [17]. As noted, the general dynamical drift is to decrease  $\Lambda$ . With the specific choice of membrane charge to tension ratio,

$$|q_j| = \frac{2M_{\text{Pl}}^4 |Q_j|}{3T_j^2} < 1, \quad (1)$$

the only processes that are kinematically allowed for the discharge favor the terminal value  $\Lambda \rightarrow 0^+$ . Near it, the discharges automatically damp down because the discharge rate  $\Gamma \simeq \exp\left(-\frac{24\pi^2 M_{\text{Pl}}^4}{\Lambda}\right)$  has an essential singularity at  $\Lambda \rightarrow 0^+$  [18]. This solves the “classic” cosmological constant problem [19–21] (sometimes also called the “old” cosmological constant problem).

The theory avoids the empty universe problem of [22] and admits inflation. The empty universe problem of [22] arose because the dynamical adjustment mechanism devised there employed a scalar field in permanent slow roll on an almost-linear potential, with obstacles to classical motion appearing only near  $\Lambda \rightarrow 0$ . To get there, the field had to dominate the cosmic contents eternally, supporting inflation all the way to almost Minkowski space. This meant that there was no reheating and no matter was produced in the terminal geometry.

Here we avoid this problem since the relaxation of  $\Lambda$  involves large successive jumps, which come from the charges  $Q_j$  being large, and the tiny terminal  $\Lambda$  comes from

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<sup>1</sup>For example, by decoupling the membranes of [1,2] that might change it.

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the irrational ratio of charges. As a result, the cosmological constant does not always dominate, but just sometimes [23]. The relaxation process is not a slow roll, but a quantum random walk. Thus, it is perfectly possible that the Universe selects the terminal vacuum well before the end of the last stage of inflation, which solves the usual cosmological problems and reheats the Universe. Therefore a “normal” late cosmology can be embedded in our framework. The cosmological constant problem reduces to “why now?,” for which the answers might involve late time physics. We leave the questions on how to embed inflation and model late acceleration for later.

Our action is a simplified version of the theory in [1,2], given explicitly in terms of the dual magnetic variables,

$$\begin{aligned}
S = & \int d^4x \left\{ \sqrt{g} \left( \frac{M_{\text{Pl}}^2}{2} R - M_{\text{Pl}}^2 (\lambda + \hat{\lambda}) - \mathcal{L}_{\text{QFT}} \right) \right. \\
& \left. - \frac{\lambda}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma} - \frac{\hat{\lambda}}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \hat{\mathcal{A}}_{\nu\lambda\sigma} \right\} \\
& + S_{\text{boundary}} - \mathcal{T}_A \int d^3\xi \sqrt{\gamma_A} - \mathcal{Q}_A \int \mathcal{A} \\
& - \mathcal{T}_{\hat{A}} \int d^3\xi \sqrt{\gamma_{\hat{A}}} - \mathcal{Q}_{\hat{A}} \int \hat{\mathcal{A}}. \tag{2}
\end{aligned}$$

$S_{\text{boundary}}$  is a generalization of the Israel-Gibbons-Hawking boundary action [24–26] to include boundary terms for the two gauge sectors [27,28],

$$\begin{aligned}
S_{\text{boundary}} = & \int d^3\xi \left( \left[ \frac{\lambda}{3} \epsilon^{\alpha\beta\gamma} \mathcal{A}_{\alpha\beta\gamma} \right] + \left[ \frac{\hat{\lambda}}{3} \epsilon^{\alpha\beta\gamma} \hat{\mathcal{A}}_{\alpha\beta\gamma} \right] \right) \\
& - \int d^3\xi \sqrt{\gamma} M_{\text{Pl}}^2 [K], \tag{3}
\end{aligned}$$

and [...] is the jump across a membrane. The charge terms are

$$\int \mathcal{A} = \frac{1}{6} \int d^3\xi \mathcal{A}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma}, \tag{4}$$

and likewise for  $\hat{\mathcal{A}}$ . Here  $\mathcal{T}_i$  and  $\mathcal{Q}_i$  are the membrane tension and charge, while  $\xi^\alpha$  are membrane coordinates and embedding maps are  $x^\mu = x^\mu(\xi^\alpha)$ . The winding direction of these maps sets the sign of the charge. We will take  $\mathcal{T}_i > 0$  to exclude negative local energy. Note that the first line of Eq. (2) is a minute generalization of unimodular formulation of GR [5–13]; our full action (2) generalizes it further by adding membranes.

Quantum mechanically, the membranes can nucleate in background fields [14,15]. Hence, these processes change the distribution of sources and the evolution of bubble interiors. The classical superselection sectors all mix up. This induces the evolution in the space of geometries due to the variation of  $\lambda$  and  $\hat{\lambda}$ . We remind the reader that here  $M_{\text{Pl}}^2$

is fixed. We also note that the charges  $\mathcal{Q}_A$  and  $\hat{\mathcal{Q}}_A$  have an irrational ratio,

$$\frac{\mathcal{Q}_{\hat{A}}}{\mathcal{Q}_A} = \omega \in \{\text{Irrational numbers}\}, \tag{5}$$

as in the irrational axion proposal [17].

Further note that (2) depends on the flux variables  $\lambda$  and  $\hat{\lambda}$  linearly, as opposed quadratically (the latter dependence being the common case as in [14,15] and follow-up work). This has crucial importance for ceasing decay of the cosmological term when it approaches zero. We note that, even if the higher-order corrections are included, since their weighing is by  $M_{\text{Pl}}$ , the linear terms remain dominant for sub-Planckian fluxes and the same behavior as we will uncover below remains. Finally, the higher-order corrections could come in with different coefficients for the two flux sectors. This would induce mutually irrational variation of fluxes even if the actual charge ratio were a rational number. We will keep this in mind as a possible explanation of the origin of our framework and for simplicity’s sake retain only the linear fluxes and irrational ratios below.

Now we can turn to studying the effects of quantum membrane discharge in the semiclassical limit. This means that we consider the dynamics described by the action (2) in Euclidean time, which controls the nucleation processes and their rates [18,29,30]. As explained in [1,2], we Wick rotate the action using  $t = -ix_E^0$ , defining the Euclidean action by  $iS = -S_E$  and restricting to locally maximally symmetric backgrounds. Those are the configurations with local  $O(4)$  symmetry that dominate in the semiclassical limit since they have minimal Euclidean action [18,29,30]. Therefore, we set  $\langle \mathcal{L}_{\text{QFT}}^E \rangle = \Lambda_{\text{QFT}}$ , with  $\Lambda_{\text{QFT}}$  the matter sector vacuum energy to an arbitrary loop order. The resulting Euclidean action is

$$\begin{aligned}
S_E = & \int d^4x_E \left\{ \sqrt{g} \left( -\frac{M_{\text{Pl}}^2}{2} R_E + M_{\text{Pl}}^2 (\lambda + \hat{\lambda}) + \Lambda_{\text{QFT}} \right) \right. \\
& \left. - \frac{\lambda}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma}^E - \frac{\hat{\lambda}}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \hat{\mathcal{A}}_{\nu\lambda\sigma}^E \right\} + S_{\text{boundary}} \\
& + \mathcal{T}_A \int d^3\xi_E \sqrt{\gamma_A} - \frac{\mathcal{Q}_A}{6} \int d^3\xi_E \mathcal{A}_{\mu\nu\lambda}^E \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma} \\
& + \mathcal{T}_{\hat{A}} \int d^3\xi_E \sqrt{\gamma_{\hat{A}}} - \frac{\mathcal{Q}_{\hat{A}}}{6} \int d^3\xi_E \hat{\mathcal{A}}_{\mu\nu\lambda}^E \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma}. \tag{6}
\end{aligned}$$

From the QFT/gravity couplings, it follows that  $\Lambda_{\text{QFT}} = \mathcal{M}_{\text{UV}}^4 + \dots \equiv M_{\text{Pl}}^2 \mathcal{H}_{\text{QFT}}^2$ , where  $\mathcal{M}_{\text{UV}}^4$  is the QFT UV cutoff and the ellipsis denotes subleading terms [31,32]. Thus, we can collect all the terms as  $\Lambda = M_{\text{Pl}}^2 (\mathcal{H}_{\text{QFT}}^2 + \lambda + \hat{\lambda}) = M_{\text{Pl}}^2 \lambda_{\text{eff}}$ . From here on we drop the index  $E$ .

The membranes serve as boundaries of regions with  $\Lambda_{\text{out/in}}$  [where out/in denote exterior (parent) and interior (offspring) of the membranes, respectively]. Both in and out have the metrics of the form  $ds_E^2 = dr^2 + a^2(r)d\Omega_3$ , where  $d\Omega_3$  is metric on a unit  $S^3$  and  $a$  solves  $(\frac{a'}{a})^2 - \frac{1}{a^2} = -\Lambda/3M_{\text{Pl}}^2$ , and the prime is the  $r$  derivative. On a membrane, the jump of the metric is controlled by the boundary conditions, which impose that  $a$ ,  $\mathcal{A}$ , and  $\hat{\mathcal{A}}$  are continuous, and the discontinuities are [1,2]

$$\lambda_{\text{out}} - \lambda_{\text{in}} = \frac{\mathcal{Q}_A}{2}, \quad \hat{\lambda}_{\text{out}} - \hat{\lambda}_{\text{in}} = \frac{\mathcal{Q}_{\hat{A}}}{2},$$

$$M_{\text{Pl}}^2 \left( \frac{a'_{\text{out}}}{a} - \frac{a'_{\text{in}}}{a} \right) = -\frac{\mathcal{T}_A + \mathcal{T}_{\hat{A}}}{2}. \quad (7)$$

We compactified notation here by writing the junction conditions as if  $A$  and  $\hat{A}$  membranes were nucleated simultaneously. Generally, in these equations one takes either  $A$  or  $\hat{A}$  terms.

Now to compute the membrane nucleation rates,  $\Gamma \sim e^{-S_{\text{bounce}}}$  [18,29,30], we need to construct the Euclidean instantons—i.e., a section of the parent and an offspring geometry glued together along a membrane as an interface. The bounce action is defined by  $S(\text{bounce}) = S(\text{instanton}) - S(\text{parent})$ . The instanton taxonomy was proffered in [14,15] for the theories with 4-form fluxes screening the cosmological constant [14,15,23,28,33]. A comprehensive related analysis for the case with linear flux dependence was given in [1,2]. Here we will merely use those results.

A key result in [1,2] is that when our Eq. (1) holds,  $|q_j| < 1$ , the only transitions that are allowed are one channel where  $dS \rightarrow dS$  (de Sitter) and one where  $dS \rightarrow \text{AdS}$  (Anti-de Sitter). The reason can be gleaned as follows. Combining the bulk equation for  $a'$  and the junction conditions on the membrane for either  $A$  or  $\hat{A}$  membrane processes, we find after a straightforward computation [1,2]

$$\zeta_{\text{out}} \sqrt{1 - \frac{\Lambda_{\text{out}} a^2}{3M_{\text{Pl}}^2}} = -\frac{\mathcal{T}_j}{4M_{\text{Pl}}^2} (1 - q_j) a,$$

$$\zeta_{\text{in}} \sqrt{1 - \frac{\Lambda_{\text{in}} a^2}{3M_{\text{Pl}}^2}} = \frac{\mathcal{T}_j}{4M_{\text{Pl}}^2} (1 + q_j) a. \quad (8)$$

Here  $\zeta_i = \pm$  designates two possible branches of the square root of  $(\frac{a'}{a})^2 - \frac{1}{a^2} = -\Lambda/3M_{\text{Pl}}^2$ , and fixing it is required to solve the junction conditions (7). It is now straightforward to check that the Eqs. (8) allow only  $(\zeta_{\text{out}}, \zeta_{\text{in}}) = (-, +)$  when both  $|q_j| < 1$  and both membrane tensions are positive. Further inspection shows that the parent geometry must be  $dS$ , whereas the offspring can be either  $dS$  or  $\text{AdS}$ . So it turns out that the conditions  $|q_j| < 1$  and  $\mathcal{T}_j > 0$  are extremely restrictive: only two nucleation channels are

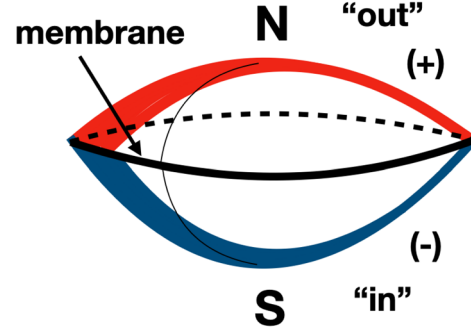


FIG. 1. A  $q_j < 1$  instanton comprising two sections of  $S^4$ .

available to both membrane systems. The other channels are either kinematically completely prohibited or are suppressed by infinite bounce action [1,2]. Basically, what happens is that due to the linear dependence of the junction conditions (8) on charges—instead of quadratic—other instantons are forbidden when  $|q_j| < 1$  [1,2]. This effect is actually a consequence of the gravitational vacuum stabilization found by Coleman and De Luccia [18], which now happens for all values of the curvature radius due to the linearity of (8) in  $\mathcal{Q}_j$ . Thus, the main process of interest to us describing  $dS \rightarrow dS$  transitions is given by the instanton of Fig. 1. When  $\Lambda_{\text{out}} > \Lambda_{\text{in}}$ , this describes a discharge, and its “time reversal” with  $\Lambda_{\text{out}} < \Lambda_{\text{in}}$  describes an upcharge. Both are possible, but the decrease of  $\Lambda$  is more likely.

To understand the quantum discharges, it is useful to solve Eqs. (8) for  $a^2$ ,

$$\frac{1}{a^2} = \frac{\Lambda_{\text{out}}}{3M_{\text{Pl}}^2} + \left( \frac{\mathcal{T}_j}{4M_{\text{Pl}}^2} \right)^2 (1 - q_j)^2$$

$$= \frac{\Lambda_{\text{in}}}{3M_{\text{Pl}}^2} + \left( \frac{\mathcal{T}_j}{4M_{\text{Pl}}^2} \right)^2 (1 + q_j)^2. \quad (9)$$

This shows there are two regimes of membrane nucleations for both  $A$  and  $\hat{A}$ . If  $a^2$  is comparable to de Sitter radii, then from Eq. (9)  $\sim (1 - \frac{\Lambda_j a^2}{3M_{\text{Pl}}^2})^{1/2} \ll 1$  and so the bounce action is approximately

$$S_{\text{bounce}} \simeq -\frac{12\pi^2 M_{\text{Pl}}^4 \Delta\Lambda}{\Lambda_{\text{out}} \Lambda_{\text{in}}},$$

$$\Delta\Lambda = \Lambda_{\text{out}} - \Lambda_{\text{in}} = \frac{1}{2} M_{\text{Pl}}^2 \mathcal{Q}_j. \quad (10)$$

Since  $|q_j| < 1$ , in this regime the discharge of the cosmological constant is fast because  $S_{\text{bounce}} < 0$ , as long as  $\Lambda_{\text{out}} \gg 3M_{\text{Pl}}^2 \left( \frac{\mathcal{T}_j}{4M_{\text{Pl}}^2} \right)^2$ . The cosmological constant decreases fast from near the Planckian scales. The reverse processes increasing  $\Lambda$  ( $\Delta\Lambda < 0$ ) have a positive action [sign reversed (10)] and so they are more rare. As claimed above, the dominant trend is to decrease  $\Lambda$ .

This ends when  $\Lambda < 3M_{\text{Pl}}^2 \left(\frac{T_j}{4M_{\text{Pl}}^2}\right)^2$ . For smaller cosmological constants, the discharge nucleations proceed via production of small bubbles, with the bounce action [2]

$$S_{\text{bounce}} \simeq \frac{24\pi^2 M_{\text{Pl}}^4}{\Lambda_{\text{out}}} \left(1 - \frac{8M_{\text{Pl}}^2 \Lambda_{\text{out}}}{3T_j^2}\right), \quad (11)$$

and  $S_{\text{bounce}} > 0$  because  $\Lambda < 3M_{\text{Pl}}^2 \left(\frac{T_j}{4M_{\text{Pl}}^2}\right)^2$ . This action has a remarkable property that it *diverges* as  $\Lambda_{\text{out}} \rightarrow 0$ . As a result, the bubbling rate  $\Gamma \sim e^{-S_{\text{bounce}}}$  has an essential singularity at  $\Lambda_{\text{out}} \rightarrow 0$ , where the rate goes to zero. Hence, when  $|q_j| < 1$  small cosmological constants become very long-lived, and the closer the geometry gets to a locally Minkowski space, the more stable it becomes to discharges. If it ends up with zero cosmological constant, further discharge stops.

To recapitulate, we have given a theory where the cosmological constant is unstable to quantum-mechanical, nonperturbative discharge of membranes, whose flux is degenerate with the cosmological constant due to covariance. The instability stops when  $\Lambda/M_{\text{Pl}}^4 \rightarrow 0$ . This feature is a consequence of Coleman and De Luccia's "gravitational stabilization" of flat space to nonperturbative instabilities, and it is operational when our Eq. (1) holds. For the theory (2) this holds throughout its range of validity. This suffices to relieve the cosmological constant problem. Let us explain how.

In our theory (2), as noted above, the total cosmological constant is

$$\Lambda_{\text{total}} = M_{\text{Pl}}^2 \lambda_{\text{eff}} = M_{\text{Pl}}^2 (\mathcal{H}_{\text{QFT}}^2 + \lambda + \hat{\lambda}). \quad (12)$$

Since  $\lambda$  and  $\hat{\lambda}$  change discretely, by  $\Delta\lambda_j = Q_j/2$ , we have  $\lambda_j = \lambda_{j0} + N_j \frac{Q_j}{2}$ . For simplicity, we absorb  $\lambda_{j0}$  into  $\mathcal{H}_{\text{QFT}}^2$ . This leaves us with, using Eq. (5),

$$\Lambda_{\text{total}} = M_{\text{Pl}}^2 \left( \mathcal{H}_{\text{QFT}}^2 + \frac{Q_A}{2} (N + \hat{N}\omega) \right). \quad (13)$$

Now, since we demand that  $\omega$  is irrational, there exist integers  $N$  and  $\hat{N}$  for any real number  $\rho$  such that  $N + \hat{N}\omega$  is arbitrarily close to  $\rho$  [17,34]. Therefore, integers  $N$  and  $\hat{N}$  exist such that  $N + \hat{N}\omega$  is arbitrarily close to  $-\frac{2\mathcal{H}_{\text{QFT}}^2}{Q_A}$ . As a consequence, there is a dense set of  $\Lambda_{\text{total}}$ , with values arbitrarily close to zero. In turn, this implies that, for any initial value of  $\Lambda$ , there exist many sequences of discharging membranes, in any order, which will arrive to  $N$  and  $\hat{N}$ , at which point the cosmological constant is arbitrarily close to zero, and the underlying nearly flat space is very long-lived. The key reason for this is the pole of the bounce action, Eq. (11), which occurs for the  $(-+)$  instantons of Fig. 1, which are the only ones allowed in our case because

of Eq. (1). As a consequence,  $\Lambda \rightarrow 0^+$  is the dynamical attractor.

This is captured by the semiclassical Euclidean partition function. Indeed, let us estimate  $Z = \int \dots DAD\hat{A}D\lambda D\hat{\lambda} Dg \dots e^{-S_E}$  by the semiclassical saddle point approximation result,

$$Z = \sum_{\text{instantons}} \sum_{\lambda, \hat{\lambda}} e^{-S_E(\text{instanton})}, \quad (14)$$

where we sum over classical extrema of the action. This means that we sum over the Euclidean instantons with any number of membranes included. Since  $O(4)$  invariant solutions minimize the action [18,29,30], in our case  $Z$  should be dominated by our instantons.

Without the explicit resummation, we can still get a feel for individual contributions. If we invert the bounce action,  $\mathcal{S}(\text{instanton}) = \mathcal{S}(\text{bounce}) + \mathcal{S}(\text{parent})$ , and recall that, without offspring, the instanton action is the parent action—i.e., the negative of the horizon area divided by  $4G_N$ ,  $\mathcal{S}(\text{parent}) = -24\pi^2 \frac{M_{\text{Pl}}^4}{\Lambda_{\text{out}}}$ —we can see that every time a transition occurs we add a bounce action for the process to the parent action. For example, consider a sequence of nested segments separated by membranes. By Eq. (11), a segment's contribution to the total action is  $\mathcal{S}(\text{instanton}) \lesssim -64\pi^2 M_{\text{Pl}}^6 / T_j^2$ . A long sequence may yield  $\mathcal{S}(\text{instanton}, n) \rightarrow -64\pi^2 M_{\text{Pl}}^6 \left( \frac{n_A}{T_A^2} + \frac{n_{\hat{A}}}{T_{\hat{A}}^2} \right) \rightarrow -24\pi^2 \frac{M_{\text{Pl}}^4}{\Lambda_{\text{terminal}}}$ . Thus, the total instanton action for a large sequence of nucleations of both types of membranes will be bounded by a number of contributions, which implies the total cannot exceed  $-24\pi^2 \frac{M_{\text{Pl}}^4}{\Lambda_{\text{terminal}}}$ . This happens since nucleations can go on until  $\Lambda_{\text{terminal}} \rightarrow 0^+$ . Beyond it, the processes that involve a dS  $\rightarrow$  AdS jump are allowed, but there can only be one such jump if it ever occurs: once a sequence ends up in AdS, membranes will not nucleate any further because  $|q_j| < 1$ . The dynamics stops, and AdS is a terminal sink [35].

As a consequence, the semiclassical partition function (14),  $Z \sim \sum e^{24\pi^2 \frac{M_{\text{Pl}}^4}{\Lambda} + \dots}$ , is dominated by configurations for which  $\Lambda \rightarrow 0$ , given that discharges cease in the Minkowski limit. Thus, our mechanism superexponentially favors<sup>2</sup>

$$\frac{\Lambda}{M_{\text{Pl}}^4} \rightarrow 0^+, \quad (15)$$

de Sitter is unstable, and quantum mechanics + GR dynamically relax  $\Lambda$  to zero. The final (near) Minkowski space is extremely long-lived.

<sup>2</sup>For earlier arguments, see Refs. [36–38].

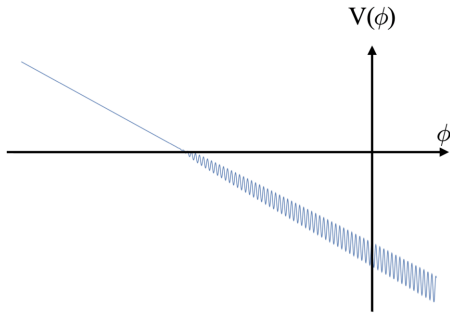


FIG. 2. A schematic of Abbott’s adjustment mechanism.

Our mechanism shares some features with the very insightful paper by Abbott [22], who designed a field-theoretic adjustment mechanism using a scalar with a potential given by a linear term and a strong-coupling-induced harmonic modulation as in Fig. 2. Because of the universality of gravity, the potential and the cosmological constant were degenerate, and thanks to an approximate shift pseudosymmetry the scalar was screening away the cosmological term. However, since the scalar evolution was purely classical slow roll, to adjust  $\Lambda$  to zero the scalar had to dominate the stress energy tensor forever—or at least until the cosmological constant was nearly zero. Only then did the harmonic modulations kick in and arrest the scalar. As a result, the universe was inflating forever, and no reheating was possible until the Hubble parameter relaxed to  $\simeq 10^{-34}$  eV—which means that the universe never reheated. This made the victory pyrrhic and led to the empty universe problem.

In our case, the empty universe problem of [22] is averted since the relaxation of  $\Lambda$  involves large successive jumps, because the charges  $Q_j$  are large. Every time a membrane is nucleated, the cosmological constant jumps by a large step  $\propto M_{\text{pl}}^2 Q_j$ . Since the charges are large, being only subject to the constraint (1), the cosmological constant

term can change down to its future terminal value faster than the age of the Universe now. The tiny terminal  $\Lambda$  comes not from small individual charges, but from their misalignment, which arises due to the irrational ratio of charges in Eq. (5). The tiny terminal value is not necessary, but it is superexponentially favored from all other values by evolution—because the small values are most long-lived. As a result, the cosmological constant does not always dominate, but just early on [23].

A related point is that, for exactly this same reason, inflation may also be embedded in this framework. Inflation should happen as the membrane evolutions stop, and the reason may be that by the time the universe starts to inflate in slow roll, the remaining relaxation of the net vacuum energy by slow roll is faster than that by quantum membrane nucleation. If after the universe exits slow roll inflation, there is a net large cosmological constant, that will not be a terminal state and the evolution will continue. The chances for finding the right history are further assisted by the possibility that even an empty universe could “restart” itself by a rare quantum jump. If it increases the cosmological constant, in subsequent evolution an inflationary stage could be found [39]. Thus, even a “rare” inflation can be found eventually [40]. Therefore, it seems that a normal cosmology can be embedded in our framework. Precisely what the likelihood is that inflation does occur after the membrane nucleations end is a question beyond this work. For now we must take solace from the fact that this is not excluded. Having a cosmological constant to be, most likely, tiny without deploying anthropics certainly helps ease the pain.

We would like to thank W. Buchmuller, G. D’Amico, and A. Hebecker for valuable comments and discussions. N.K. is supported in part by the DOE Award No. DE-SC0009999. A.W. is supported by the ERC Consolidator Grant No. STRINGFLATION under the HORIZON 2020 Grant Agreement No. 647995.

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