Charges, conserved quantities, and fluxes in de Sitter spacetime

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We discuss the definition of conserved quantities in asymptotically locally de Sitter spacetimes. One may define an analog of holographic charges at future and past infinity and at other Cauchy surfaces I_t as integrals over the intersection of timelike surfaces C and the Cauchy surface I_t. In general, the charges Q^t defined on the Cauchy surface I_t depend on C, but if gravitational flux is absent the charges are independent of C. On the other hand, if there is a net gravitational flux entering or leaving the spacetime region bounded by C_1, C_2 and two Cauchy surfaces then $\Delta Q^t(C_1, C_2) = Q^t(C_1) - Q^t(C_2)$ changes by the same amount.

I. INTRODUCTION

Isolated systems with no dynamical gravity have conserved energy and momentum originating from translational invariance. These charges may be obtained from a conserved local energy-momentum tensor via appropriate integrals over a Cauchy surface. If a system is open there may be flux of energy and momentum through its boundary, which is again encoded by the energy-momentum tensor. A prime example of flux is that of electromagnetic radiation originating from a localized source and propagating out to infinity. When gravity is dynamical the corresponding symmetries are local and as such the corresponding charges would be zero if there are no boundaries or asymptotic regions.

The study of (conserved) charges in gravitational theories and of gravitational radiation has a long history going back to (at least) [1–5]. Diffeomorphism invariance implies that the charges should be defined by surface integrals at infinity, but due to the infinite volume of spacetime any such definition requires a subtraction, potentially making the charges ambiguous. Most of the initial work concerned asymptotically flat gravity, and in this context charges were defined relative to flat spacetime. With such asymptotics one may also define outgoing gravitational radiation through null infinity, as illustrated in Fig. 1. In the presence of a cosmological constant the asymptotic structure changes and new issues arise.

With negative cosmological constant, the AdS/CFT correspondence suggests that one should be able to

formulate the problem in exactly the same way as when gravity is nondynamical. Indeed, one can show that such spacetimes are always equipped with a covariantly conserved symmetric tensor which may be used to construct conserved charges. This tensor is identified with the (expectation value of the) energy-momentum tensor of the dual conformal field theory (CFT) in the AdS/CFT correspondence [6]. Asymptotically locally anti–de Sitter (ALAdS) spacetimes have a timelike conformal boundary and an associated boundary conformal structure. In the neighborhood of the conformal boundary the metric admits the following Fefferman-Graham form [7],

$$ds^2 = \frac{l_{\text{AdS}}^2}{\rho^2}(d\rho^2 + g_{ab}(\rho, x)dx^a dx^b), \qquad (1)$$

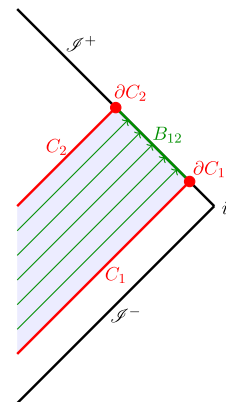where l_{AdS} is the curvature radius and in four dimensions:

FIG. 1. Penrose diagram illustrating the flux of gravitational radiation through null infinity, $\mathscr{I}^+$, in an asymptotically flat spacetime. The hypersurfaces C_1, C_2 are taken to be null ("instants of retarded time") although they could also have been chosen to be spacelike.

$$g_{ab} = g_{(0)ab} + \rho^2 g_{(2)ab} + \rho^3 g_{(3)ab} + \cdots. \quad (2)$$

Throughout this paper for concreteness we specialize to four dimensions. Most of the discussion generalizes straightforwardly to other dimensions, modulo issues associated with the holographic conformal anomaly in odd dimensions [8]. Here $g_{(0)}$ is the representative of the boundary conformal structure and the holographic energy momentum tensor is given by

$$T_{ab} = \frac{3l_{\text{AdS}}}{16\pi G} g_{(3)ab}, \quad (3)$$

where G is Newton's constant. If the boundary conformal structure admits conformal Killing vectors ζ^a then the energy-momentum tensor may be used to construct associated conserved charges via a surface integral over a spacelike surface at infinity,

$$Q_\xi = \int_{\partial C} ds_a T^a_{\ b} \zeta^b, \quad (4)$$

where ∂C is the intersection of the bulk spacelike surface C with the conformal boundary. It was shown in [9] using Noether's method and covariant phase space methods that (4) are the bulk gravitational charges. These charges are well defined due to the boundary counterterms introduced by holographic renormalization [6,8,10,11]. In fact the counterterms are also required for well posedness of the variational problem when the boundary conformal structure is fixed [9].

The purpose of this paper is to address the analogous issues for spacetimes with positive cosmological constant. ALdS spacetimes share a number of properties with ALAdS spacetimes, but there are also important differences. The conformal boundary of ALdS spacetimes is spacelike and typically has two causally connected components, one at past infinity and one at future infinity (in contrast the AdS conformal boundary is typically connected[1]). Near each component the structure of the spacetime is related by analytic continuation to that of a corresponding ALAdS spacetime [13], with results analogous to (1), (2), (3) (recovering the asymptotic expansion first presented in [14]). ALdS spacetimes are spatially compact so one cannot define conserved charges as integrals at spatial infinity. Nevertheless, explicit solutions (for example the Schwarzschild dS solution) have integration constants that traditionally are interpreted as the values of conserved charges (like mass and angular momentum). These charges are associated with conformal Killing vectors of the conformal structure at future infinity and may be computed by integrals of the energy momentum tensor at future infinity. However, they are not conserved in the usual sense, i.e., under time evolution. Instead these charges are conserved

under spatial translations if there is no gravitational radiation, as we discuss below.

The discussion of gravitational charges in de Sitter space-times goes back to at least [15], and there has been renewed interest in this topic since our Universe appears to have positive cosmological constant and gravitational waves have been observed. Recent works include those of Ashtekar et al. [16–21] (which builds upon earlier work [22,23]) that discuss charges and radiation for linearized fields on dS, and Chruściel et al. [24–26] where different approaches to the computation of energy and linear fluxes between null cones in dS are considered, see also [27–31]. A number of other approaches to examining radiative bodies in dS can be found in [32–37]. As outlined above, there are many similarities between dS and AdS and works that use the similarity to define charges and fluxes include [38–46].

Our work significantly develops understanding of charges and fluxes in de Sitter. Earlier works were focused on asymptotically de Sitter spacetimes but our analysis holds for the general class of asymptotically locally de Sitter spacetimes. Restricting to asymptoticially de Sitter does not allow for gravitational waves propagating to the conformal boundary and therefore misses important physical effects. In our work divergences associated with the conformal boundary are treated systematically with holographic renormalization i.e., covariant boundary counterterms. This approach leads to unambiguous results for changes that are intrinsic to the spacetime rather than defined relative to a reference spacetime and hold for general asymptotically locally de Sitter spacetimes.

Another key difference with previous works is that we focus on the qualitative new issues that arise in de Sitter, relative to AdS and flat space. One such key difference is the observation that there are contributions to charges from both end points of the timelike surfaces used to define the charges. In previous literature contributions from inner boundaries were often set to be zero by imposing falloff conditions on fields but we do not need to impose any such restrictions. We explain that one needs to consider fluxes through box regions in generic dynamical situations and we give a precise characterization of this gravitational flux. In particular, we obtain a quantity that is conserved under time evolution in the absence of fluxes and equals the flux when gravitational flux is present.

II. COVARIANT PHASE SPACE FORMALISM

The main tool we use in our analysis is the covariant phase space [47–52] (see also the recent works [53,54] and references therein, and [55] for an alternative but equivalent formalism). The key object is the $(d-1)$-form symplectic current ω. Starting from a diffeomorphism covariant Lagrangian d-form $\mathbf{L}(\psi)$, the symplectic current is given by

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[1]See [12] for general results regarding the Euclidean AdS case.

$$\omega(\psi, \delta_1\psi, \delta_2\psi) = \delta_1\Theta(\psi, \delta_2\psi) - \delta_2\Theta(\psi, \delta_1\psi), \quad (5)$$

where Θ is the total derivative in the on shell variation of $\mathbf{L}(\psi)$, $\delta\mathbf{L} = d\Theta$ (ψ denotes the metric and other fields). A crucial property of $\boldsymbol{\omega}$ is that it is closed on shell,

$$d\omega = 0. \quad (6)$$

Consider now on shell configurations ψ whose variations $\delta\psi$ are also on shell. For a diffeomorphism generated by ξ the corresponding Hamiltonian H_ξ is given by

$$\delta H_\xi = \int_C \omega(\psi, \delta\psi, \mathcal{L}_\xi\psi), \quad (7)$$

with C a $(d-1)$-dimensional slice, which for many applications is a Cauchy surface, and $\mathcal{L}_\xi$ is the Lie derivative. Given such a diffeomorphism we can associate a $(d-1)$-form Noether current, $\mathbf{J}(\xi) = \Theta(\psi, \mathcal{L}_\xi\psi) - i_\xi\mathbf{L}$, where i_ξ is the interior product. The current $\mathbf{J}$ is closed on shell and is thus locally exact: $\mathbf{J}(\xi) = d\mathbf{Q}(\xi)$ with $(d-2)$-form Noether charge $\mathbf{Q}$, the variation of H_ξ may be written as

$$\delta H_\xi = \int_{\partial C} (\delta\mathbf{Q}(\xi) - i_\xi\Theta(\psi, \delta\psi)), \quad (8)$$

where ∂C is the boundary of C.

One can integrate this equation to obtain H_ξ if and only if the right-hand side is a total variation, requiring

$$\int_{\partial C} i_\xi\Theta(\psi, \delta\psi) = \delta\int_{\partial C} i_\xi\mathbf{B}(\psi). \quad (9)$$

Note that one can also express the condition for existence of H_ξ in terms of $[\delta_1, \delta_2]H_\xi = 0$:

$$\int_{\partial C} i_\xi\omega(\psi, \delta_1\psi, \delta_2\psi) = 0. \quad (10)$$

The difference between Hamiltonians at two different Cs is given by

$$\delta H_\xi|_{\partial C_2} - \delta H_\xi|_{\partial C_1} = -\int_{B_{12}} \omega(\psi, \delta\psi, \mathcal{L}_\xi\psi), \quad (11)$$

where B_{12} is a hypersurface with boundary $\partial C_1 \sqcup \partial C_2$. If (10) is satisfied by virtue of $\omega|_{B_{12}} = 0$, then Eq. (11) tells us that H_ξ is independent of the slice C. We now review the application of this formalism to ALAdS spacetimes, before discussing ALdS spacetimes.

III. ASYMPTOTICALLY LOCALLY ANTI–DE SITTER:

This case was fully analyzed in [9]. The slice C in (7) is a bulk spacelike surface that ends on the boundary of ALAdS. Considering Dirichlet conditions,[2] where the conformal

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[2]Other boundary conditions may be obtained by adding finite boundary terms that implement this change of boundary condition and tracking their contributions, see also [45,46].
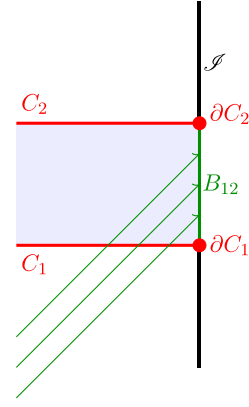

FIG. 2. Illustrating flux in an ALAdS spacetime. If flux arrives in B_{12}, then the gravitational charge at C_2 will differ from that in C_1. The conformal boundary $\mathscr{I}$ is a timelike hypersurface and the C_1, C_2 are spacelike.

class $[g_{(0)}]$ is kept fixed, and a $[g_{(0)}]$ that admits conformal Killing vectors, the integrability criterion (10) is automatically satisfied by virtue of $\omega(g_{\mu\nu}, \delta g_{\mu\nu}, \mathcal{L}_\xi g_{\mu\nu})|_{B_{12}} = 0$, where here ξ is an asymptotic conformal Killing vector; i.e., it approaches the conformal Killing vectors ζ at the boundary, see Appendix B of [9] for the specific fall-off conditions) Then the Wald Hamiltonian integrates to (4).

When there is gravitational radiation from the interior, it will reach the conformal boundary at finite proper time. As a result, radiative AdS spacetimes have a time-dependent boundary conformal structure, as was confirmed recently in [56]. In such cases there are no conserved charges because the boundary conformal structure does not admit conformal Killing vectors. However, if the gravitational flux arrives in finite time intervals, so that the boundary geometry outside these intervals admits conformal Killing vectors, there will be piecewise constant gravitational charges, and the difference in their values between two instances of time will account for the gravitational flux arriving at the conformal boundary, see Fig. 2. More generally, if the spacetime admits a limit where conserved quantities exist, then one can discuss the flux of such quantities away from that limit. An example is the Robinson-Trautmann solution which we will discuss below in the context of $\Lambda > 0$.

IV. ASYMPTOTICALLY LOCALLY DE SITTER:

We now consider ALdS spacetimes with Dirichlet boundary conditions, i.e., we keep fixed a conformal class at each end. Different boundary conditions may be implemented as discussed in footnote 2. One may adapt straightforwardly all steps of $\Lambda < 0$ case to $\Lambda > 0$; in most formulas this amounts to differences in signs. However, there are important conceptual differences. The analog of the surface C in (7) we used in the ALAdS case is now timelike, and because the conformal boundary has two components, one at future infinity and another at past
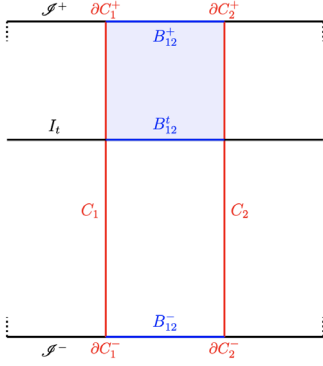
FIG. 3. Timelike C_1, C_2 intersecting spacelike conformal boundaries $\mathscr{I}^\pm$ in ALdS spacetime.

infinity, such a C will have two ends. Let us consider the case where both past and future infinity admit conformal Killing vectors and a hypersurface C that extends from future infinity to past infinity. Let ξ be a bulk vector that is an asymptotic conformal Killing vector near both past and future conformal infinity, approaching a boundary conformal Killing vector $\zeta_\pm^b$ at $\mathscr{I}^\pm$. Then

$$H_\xi[C] = Q_\xi^+ - Q_\xi^-, \tag{12}$$

where $Q_\xi^\pm[C] = \int_{\partial C^{(\pm)}} ds_a T^a_{(\pm)b} \zeta_\pm^b$; $T^a_{(\pm)b}$ is the holographic energy momentum tensor, and $\partial C^\pm$ are the two ends of C. Now consider two such surfaces C_1 and C_2, see Fig. 3. Then (11) implies

$$\Delta Q_\xi^+(C_1, C_2) = \Delta Q_\xi^-(C_1, C_2) = 0, \tag{13}$$

where $\Delta Q_\xi^\pm(C_1, C_2) = Q_\xi^\pm[C_1] - Q_\xi^\pm[C_2]$. The first equality in (13) follows from $\omega(g_{\mu\nu}, \delta g_{\mu\nu}, \mathcal{L}_\xi g_{\mu\nu})|_{B_{12}^\pm} = 0$ and the second from applying Stokes's theorem to $\Delta Q_\xi^\pm$ independently at $B_{12}^\pm$, using the explicit formulas for $Q_\xi^\pm$. It follows that while the values of the charges may change from $\mathscr{I}^-$ to $\mathscr{I}^+$, their difference is zero. Recall that $\mathscr{I}^\pm$ are Cauchy surfaces and consider any other Cauchy surface I_t. Then as we now argue, $\Delta Q_\xi^t = \Delta Q_\xi^-$, where $\Delta Q_\xi^t(C_1, C_2) = Q_\xi^t[C_1] - Q_\xi^t[C_2]$, and $Q_\xi^t[C] = \int_{C \cap I_t} ds_a 2\pi_b^a \xi^b$, with π_{ab} as the conjugate momentum of the induced metric at I_t (here, for simplicity, we assume that we deal with pure gravity). In other words, ΔQ_ξ^t is conserved under time evolution, i.e., independent of the Cauchy surface I_t. To see this, note that since ω is closed, its integral over the boundary of the region enclosed by $C_1, C_2, I_t, \mathscr{I}^-$ is zero, so ΔQ_ξ^- must be equal to ΔQ^t plus any flux passing through B_{12}^t. However, there cannot be such flux, since any flux would eventually reach future infinity and as a result the conformal class at $\mathscr{I}^+$ would not admit conformal Killing vectors. Since we assume that future infinity admits conformal Killing vectors, there cannot be gravitational flux in the interior, and we find $\Delta Q_\xi^- = \Delta Q_\xi^t$. So while Q_ξ^t may depend in t, $\Delta Q_\xi^t = 0$ for all

t. One may also check that known solutions such as Kerr–de Sitter have $\Delta Q_\xi = 0$.

In our discussion we consider timelike slices C that extend from the future infinity to past infinity. One may also consider hypersurfaces that start and end at future infinity. Such C may be obtained by using the portion of the hypersurface C_1 from future infinity till I_t, then join B_{12}^t and return to future infinity using (the time reverse of) C_2.

Let us now consider the effects of radiation; i.e., we consider the case when $\mathscr{I}^+$ admits no conformal Killing vectors. In this case the same argument implies that ΔQ_ξ^+ will differ from ΔQ_ξ^t by the amount of net radiation in the region bounded by $\mathscr{I}^+, C_1, C_2$ and I_t. In many previous discussions the contribution from I_t is either ignored or argued to be zero by postulating suitable decay rates in the deep interior. Assuming $\Delta Q_\xi^t = 0$, any nonzero ΔQ_ξ^+ will be due to gravitational radiation at $\mathscr{I}^+$, captured explicitly via the *future flux formula* [39,43,45]

$$\Delta Q_\xi^+(C_1, C_2) = -\int_{B_{12}^+} \boldsymbol{F}_{\xi_{(0)}}, \tag{14}$$

where

$$\boldsymbol{F}_{\xi_{(0)}} = \left(-\frac{1}{2}\sqrt{g_{(0)}} T^{ab} \mathcal{L}_{\xi_{(0)}} g_{ab}^{(0)}\right)\boldsymbol{\epsilon}_3. \tag{15}$$

Using our formalism, we can also consider the case of $\Delta Q_\xi^t \neq 0$, i.e., account for radiation passing through the interior hypersurface I_t. The *past flux formula* is

$$\Delta Q_\xi^t(C_1, C_2) = -\int_{B_{12}^t} \boldsymbol{G}_\xi, \tag{16}$$

where

$$\boldsymbol{G}_\xi = (-\sqrt{\gamma}\pi^{ab} L_\xi \gamma_{ab})\boldsymbol{\epsilon}_3. \tag{17}$$

In summary, the effect of gravitational radiation is to spoil the conservation of spacelike separated charges. Combining Eqs. (14) and (16), we see that the explicit difference is given by

$$\Delta Q^+ - \Delta Q^t = -\int_{B_{12}^+} \boldsymbol{F}_{\xi_{(0)}} + \int_{B_{12}^t} \boldsymbol{G}_\xi. \tag{18}$$

V. ROBINSON-TRAUTMAN

To conclude we discuss an example of a class of radiative dS solutions: the Robinson-Trautman (RT) family. RT [57] describes an isolated system which relaxes to equilibrium by radiating excess energy via gravitational waves, becoming Schwarzschild dS. The solution exists for any value of the cosmological constant, and here we discuss the $\Lambda > 0$ case. The metric is

$$ds^2 = -\Phi_\Lambda du^2 - 2dudr + 2r^2 P^{-2} d\zeta d\bar{\zeta}, \quad (19)$$

where $P = P(u, \zeta, \bar{\zeta})$ and

$$\Phi_\Lambda = \Delta \ln P - 2r(\ln P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3} r^2. \quad (20)$$

$\Delta = 2P^2 \partial^2 / \partial \zeta \partial \bar{\zeta}$ and P satisfies the *Robinson-Trautman* equation:

$$(\ln P)_{,u} + \frac{1}{12M} \Delta\Delta(\ln P) = 0. \quad (21)$$

Using results from [58], a representative of the conformal class at $\mathscr{I}^+$ is

$$ds^2_{(0)} = g_{(0)ab} dx^a dx^b = dy^2 + \frac{6}{\Lambda} \hat{P}^{-2} d\zeta d\bar{\zeta}, \quad (22)$$

where $\hat{P}$ denotes the boundary value of P. This metric generically possesses no conformal Killing vectors and thus we cannot define charges. However, as $y \to \infty$ the solution approaches Schwarzschild dS which has an isometry related to mass, and therefore we may discuss its flux. Let us consider a vector field which in this limit is associated with *Bondi "time" translation* along the boundary (note that it is spacelike). The vector generating these translations is

$$\xi^a_{(0)} \partial_a = \frac{\hat{P}_0}{\hat{P}} \partial_y, \quad (23)$$

where $\hat{P}_0 = 1 + \zeta\bar{\zeta}/2$ is the limit of $\hat{P}$ as $y \to \infty$. The normalization of this vector relates to the RTdS metric (19) needing to undergo a coordinate transformation to be expressible in Bondi gauge [59]. Exploiting results from [58], one can show that

$$Q^+_\xi = \frac{2m}{\kappa^2} \int_{\partial C^+} \left(\frac{P_0}{P}\right)^3 d\mu_0, \quad (24)$$

where $d\mu_0$ is the area element on the unit S^2. This quantity is analogous to the *Bondi mass*, $\mathcal{M}_B$, of RT in the asymptotically flat setting [4]. It has been previously observed [58,60,61] that the quantity defined in Eq. (24) is monotonically decreasing regardless of the sign of Λ. Our construction thus illustrates explicitly how the Bondi mass of the asymptotically flat RT solution arises in the current context.

Since we have not used an asymptotic conformal Killing vector to construct $\mathcal{M}_B$, we should expect to discover additional flux through $B^+_{12} \subset \mathscr{I}^+$ when comparing the difference in Bondi mass between the hypersurfaces C_1 and C_2. Applying Eqs. (14) and (15) to the RT solution, we find the following equation for the flux through B^+_{12}:

$$\Delta Q^+_\xi = \mathcal{M}_B|_{\partial C^+_1} - \mathcal{M}_B|_{\partial C^+_2} = -\int_{B^+_{12}} \mathbf{F}_{\xi_{(0)}}, \quad (25)$$

where

$$\mathbf{F}_{\xi_{(0)}} = -\frac{2m}{\kappa^2} \partial_y \left(\frac{P_0}{P}\right)^3 \sqrt{\mathring{\sigma}} \boldsymbol{\epsilon}_3 \quad (26)$$

and $\mathring{\sigma}_{AB} dx^A dx^B = 2P_0^{-2} d\zeta d\bar{\zeta}$ is the metric on the unit round S^2. This flux formula gives

$$\mathcal{M}_B|_{\partial C^+_1} - \mathcal{M}_B|_{\partial C^+_2} \le 0 \quad (27)$$

by monotonicity of the expression defined in Eq. (24) [60,61]. We note that the existence of this monotonically decreasing charge is unlikely to be a feature of all ALdS spacetimes, where charges may obey flux-balance laws similar to those proposed in [44]. However, this analysis proves that all metrics in the RT(A)dS family possess such a charge.

It would be interesting to compute ΔQ^t_ξ and confirm monotonicity in u. For this we would need to know the form of Bondi time translation vector field in the deep interior of the RT spacetime. We leave this computation to future work.

VI. CONCLUSIONS

We have discussed the definition of charges in asymptotically locally de Sitter spacetimes. We show that the difference of charges $\Delta Q^t_\xi(C_1, C_2)$ defined using timelike surfaces C_1 and C_2 at the Cauchy surface I_t is zero if gravitational flux is absent. If there is gravitational flux the corresponding quantities differ by the net amount of flux. We illustrated the latter with the example of Robinson-Trautmann dS and derived a bound on the flux (27). It would be interesting to exemplify our discussion with further explicit examples, investigate possible connections with work on gravitational wave memory [62–69], and analyze possible relevance to present and future gravitational wave observations. Covariant phase space has been instrumental in formulating in generality and obtaining a deep understanding of the first law of black hole thermodynamics [49,50]. Thermodynamics of black holes with de Sitter asymptotics is still poorly understood and we believe the framework we develop here and the issues we uncover are important in this context as well.

ACKNOWLEDGMENTS

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