

Probing Lorentz-invariance-violation-induced nonthermal Unruh effect in quasi-two-dimensional dipolar condensates

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The Unruh effect states an accelerated particle detector registers a thermal response when moving through the Minkowski vacuum, and its thermal feature is believed to be inseparable from Lorentz symmetry: Without the latter, the former disappears. Here we propose to observe the analog circular Unruh effect using an impurity atom in a quasi-two-dimensional Bose-Einstein condensate (BEC) with dominant dipole-dipole interactions between atoms or molecules in the ultracold gas. Quantum fluctuations in the condensate possess a Bogoliubov spectrum $\omega_{\mathbf{k}} = c_0|\mathbf{k}|f(\hbar c_0|\mathbf{k}|/M_*)$, working as an analog Lorentz-violating quantum field with the Lorentz-breaking scale M_* , and the impurity acts as an effective Unruh-DeWitt detector thereof. When the detector travels close to the sound speed, observation of the Unruh effect in our quantum fluid platform becomes experimentally feasible. In particular, the deviation of the Bogoliubov spectrum from the Lorentz-invariant case is highly engineerable through the relative strength of the dipolar and contact interactions, and thus a viable laboratory tool is furnished to experimentally investigate whether the thermal characteristic of the Unruh effect is robust to the breaking of Lorentz symmetry.

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I. INTRODUCTION

One of the surprising fundamental consequences of relativistic quantum field theory is that the concept of particle number is observer dependent. A prominent paradigm is the so-called Unruh effect [1]: In the view of a uniformly accelerating observer, the Fock vacuum state of quantum field in the Minkowski spacetime appears as a thermal state rather than a zero-particle state. The corresponding characteristic temperature is proportional to the observer's acceleration a , given by $k_B T_U = \frac{\hbar a}{2\pi c}$. To produce a measurable temperature for fundamental quantum fields, extremely huge accelerations are required (e.g., smaller than 1 Kelvin even for accelerations as high as 10^{20} m/s²), and thus until now, the direct experimental confirmation of the Unruh effect still remains elusive.

Analog gravity [2,3] opens up a new route to study various phenomena predicted by relativistic quantum field theory, e.g., Hawking effect [4–21], cosmological particle production [22–34], and dynamical Casimir effect [35–43],

in a variety of electronic, acoustic, optical and even magnetic and superconducting settings. Recently, analog gravity program for observing the Unruh effect has been successfully theoretically put forward [44–56], and through a BEC system Hu *et al.* experimentally realized the analog Unruh effect relied on functional equivalence (i.e., simulating two-mode squeezed mechanics) [57]. Furthermore, in the quantum field theory, the contributions from the trans-Planckian modes as seen by an inertial observer are indispensable for deriving the Unruh effect with Bogoliubov transformation method. This particular feature makes the Unruh effect a potentially important arena for understanding and exploring implications of trans-Planckian physics [58–66], and even the detecting means to probe some candidate theories of quantum gravity that may modify the trans-Planckian modes significantly. This modification usually accompanied by the breaking of Lorentz symmetry may even challenge the equivalence between the Unruh effect and the Hawking effect since the latter appears to be robust to high energy modifications of the dispersion relation [67] while the former is not so immune and will lose its conventional thermal interpretation [58–60,62,64] (see more details in the following). It is

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thus of great interest to experimentally investigate the consequences of trans-Planckian physics in a microscopically well-understood setup in a regime, that is inaccessible for quantum fields in real relativistic scenarios.

In this paper, we propose to study the interplay between the Unruh effect and trans-Planckian physics with an experimentally accessible platform consisting of a dipolar BEC [68] and an immersed impurity [25,69]. From the perspective of analog, the density fluctuations in the condensate possessing trans-Planckian spectra leading to strong departures from Lorentz invariance [31,70,71], resemble Lorentz-violating quantum field (LVQF), while the impurity, analogously dipole coupled to the density fluctuations, is modeled as an Unruh-DeWitt detector coupled to the LVQF. We will show that for the Lorentz-invariant (LI) spectrum the Unruh-DeWitt detector for an accelerated circular path indeed experiences a similar thermal response, while yields significant changes of this standard thermal feature when the spectra strongly deviate from Lorentz invariance. As far as we know, this represents the first example within analog gravity where Unruh effect without thermality caused by the breaking of Lorentz symmetry can become experimentally manifest.

II. LORENTZ-VIOLATING QUANTUM FIELD AND ANALOG UNRUH-DEWITT DETECTOR IN DIPOLAR BEC

As schematically shown in Fig. 1, we establish the connection between an impurity immersed in a quasi-two-dimensional (quasi-2D) dipolar BEC with the Unruh-DeWitt detector model [1,72], inspired by the seminal atomic quantum dot idea introduced in Refs. [25,69,73].

We begin with the Lagrangian density of an interacting Bose gas comprising atoms or molecules of mass m ,

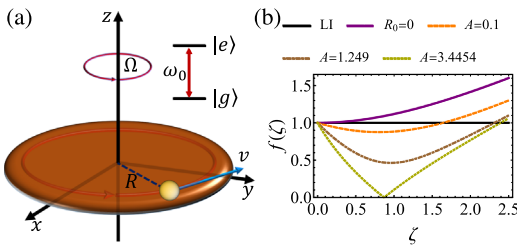


FIG. 1. (a) A two-level atom immersed in a quasi-2D dipolar BEC, moving along a circular trajectory with the radius of the orbit R , an angular velocity Ω and the corresponding linear speed v . (b) The dimensionless function f shown in (2) as a function of $\zeta = \hbar c_0 k / M_*$. For the LI case, $f = 1$. $R_0 = 0$ denotes the contact interaction case, where f is independent of A . For DDI dominance, $R_0 = \sqrt{\pi/2}$, f dips below 1 for an interval of ζ . Note that f becomes negative when $A > A_c = 3.4454$, which means the spectrum of quasiparticle becomes unstable.

$$\mathcal{L} = \frac{i\hbar}{2} (\Psi^* \partial_t \Psi - \partial_t \Psi^* \Psi) - \frac{\hbar^2}{2m} |\nabla \Psi|^2 - V_{\text{ext}} |\Psi|^2 - \frac{1}{2} |\Psi|^2 \int d^3 \mathbf{R}' V_{\text{int}}(\mathbf{R} - \mathbf{R}') |\Psi(\mathbf{R}')|^2, \quad (1)$$

where $\mathbf{R} = (\mathbf{r}, z)$ are spatial 3D coordinates. The interaction reads $V_{\text{int}}(\mathbf{R} - \mathbf{R}') = g_c \delta^3(\mathbf{R} - \mathbf{R}') + 3g_d \{ [1 - 3(z - z')^2 / |\mathbf{R} - \mathbf{R}'|^2] / |\mathbf{R} - \mathbf{R}'|^3 \} / 4\pi$, where $g_c = 4\pi \hbar^2 a_c / m_B$ represents the contact interaction strength, with a_c being the s -wave scattering length; the dipole-dipole interaction (DDI) strength $g_d = \mu_0 \mu_m^2 / 3$, with μ_0 and μ_m being the permeability of vacuum and the magnetic dipole moment that is polarized to the z direction, respectively. Moreover, the gas is trapped by an external potential, $V_{\text{ext}}(\mathbf{R}) = m_B \omega^2 \mathbf{r}^2 / 2 + m_B \omega_z^2 z^2 / 2$, and is strongly confined along the z axis, with aspect ratio $\kappa = \omega_z / \omega \gg 1$ over the whole time evolution. As a result of that, the motion of the Bose gas along the z axis is frozen to the ground state with a Gaussian form, $\rho_z(z) = (\pi d_z^2)^{-1/2} \exp[-z^2 / d_z^2]$, with $d_z = \sqrt{\hbar / m_B \omega_z}$. Therefore, the whole system effectively reduces to a quasi-2D one which can ensure stability in the DDI-dominated regime [70]. Finally, we may assume that the Bose gas is condensed to the zero-momentum state with an area density ρ_0 .

Within the Bogoliubov theory of small excitations on top of the condensate [74,75], density fluctuations in Heisenberg representation can be written as $\delta \hat{\rho}(t, \mathbf{r}) = \sqrt{\rho_0} \int [d^2 \mathbf{k} / (2\pi)^2] (u_{\mathbf{k}} + v_{\mathbf{k}}) [\hat{b}_{\mathbf{k}}(t) e^{i\mathbf{k} \cdot \mathbf{r}} + \hat{b}_{\mathbf{k}}^\dagger(t) e^{-i\mathbf{k} \cdot \mathbf{r}}]$ which closely resemble the quantum field in terms of bosonic operators $\hat{b}_{\mathbf{k}}(t) = \hat{b}_{\mathbf{k}} e^{-i\omega_{\mathbf{k}} t}$, satisfying the usual Bose commutation rules $[\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger] = (2\pi)^2 \delta^2(\mathbf{k} - \mathbf{k}')$. Note that $u_{\mathbf{k}} = (\sqrt{\mathcal{H}_{\mathbf{k}}} + \sqrt{\mathcal{H}_{\mathbf{k}} + 2\mathcal{A}_{\mathbf{k}}}) / 2 (\mathcal{H}_{\mathbf{k}}^2 + 2\mathcal{H}_{\mathbf{k}} \mathcal{A}_{\mathbf{k}})^{1/4}$ and $v_{\mathbf{k}} = (\sqrt{\mathcal{H}_{\mathbf{k}}} - \sqrt{\mathcal{H}_{\mathbf{k}} + 2\mathcal{A}_{\mathbf{k}}}) / 2 (\mathcal{H}_{\mathbf{k}}^2 + 2\mathcal{H}_{\mathbf{k}} \mathcal{A}_{\mathbf{k}})^{1/4}$ are Bogoliubov parameters, and the quasiparticle frequency $\omega_{\mathbf{k}} = \sqrt{\mathcal{H}_{\mathbf{k}}^2 + 2\mathcal{H}_{\mathbf{k}} \mathcal{A}_{\mathbf{k}}}$ with $\mathcal{H}_{\mathbf{k}} = k^2 / 2m$ and $\mathcal{A}_{\mathbf{k}} = \rho_0 V_{\text{int},0}^{2D}(k)$ [71]. Here $k = |\mathbf{k}|$ and the Fourier transformation of DDI $V_{\text{int},0}^{2D}(k) = g_0^{\text{eff}} (1 - \frac{3R_0}{2} k d_z w[\frac{k d_z}{\sqrt{2}}])$, with $w[x] = \exp[x^2] (1 - \text{erf}[x])$, an effective contact coupling $g_0^{\text{eff}} = \frac{1}{\sqrt{2\pi} d_z} (g_c + 2g_d)$, and the dimensionless ratio $R_0 = \sqrt{\pi/2} / (1 + g_c / 2g_d)$. The parameter R_0 could be tunable via Feshbach resonance [76–79] and rotating polarizing field [80,81], ranging from $R_0 = 0$ (when $g_d / g_c \rightarrow 0$, i.e., contact dominance), to $R_0 = \sqrt{\pi/2}$ (when $g_d / g_c \rightarrow \infty$, i.e., DDI dominance).

The density fluctuations described above closely resemble a LVQF with a explicit dispersion relation given by

$$\omega_{\mathbf{k}} = c_0 k \sqrt{1 - \frac{3R_0}{2} \sqrt{A} \zeta w \left[\sqrt{\frac{A}{2}} \zeta \right] + \frac{\zeta^2}{4}} = c_0 k f(\zeta), \quad (2)$$

where $\zeta = \hbar c_0 k / M_*$, $c_0 = \sqrt{g_0^{\text{eff}} \rho_0 / m_B}$ is the speed of sound, $A = g_0^{\text{eff}} \rho_0 / \hbar \omega_z$ represents the effective chemical potential as measured relative to the transverse trapping, and $M_* = m_B c_0^2$ is the analog energy scale of Lorentz violation. This dispersion relation (2) is approximately Lorentz invariant ($f(\zeta) \simeq 1$) for $\zeta \ll 1$. By appropriately setting the relevant parameters A and R_0 , the dispersion could be analogously superluminal ($f(\zeta) > 1$) and subluminal ($f(\zeta) < 1$). In Fig. 1, we plot the function $f(\zeta)$ shown in (2) to see how the Lorentz invariance is violated in this dispersion. For the DDI dominance, $R_0 = \sqrt{\pi/2}$, the analogous subluminal spectrum develops a roton minimum for sufficiently large A , and the Lorentz invariance is strongly broken near $\zeta \simeq 0.9$ [71].

In order to probe the analog LVQF in the dipolar BEC, we use an impurity as the analog Unruh-DeWitt detector, which consists of a two-level atom (1 and 2) and its motion is supposed to be externally imposed by a tightly confining and relatively moving trap potential, so that its only degrees of freedom are the internal ones. Furthermore, the impurity is assumed to be controlled by a driving of a monochromatic external electromagnetic field at the frequency ω_L close to resonance with $1 \rightarrow 2$ transition $\omega \simeq \omega_{21}$, with a Rabi frequency ω_0 . Then the Hamiltonian of the whole system can be written as

$$\begin{aligned}
 H(t) = & \int d^2 \mathbf{k} \hbar \omega_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \hbar \omega_{21} |2\rangle \langle 2| \\
 & - \left(\frac{\hbar \omega_0}{2} e^{-i\omega_L t} |2\rangle \langle 1| + \text{H.c.} \right) \\
 & + \sum_{s=1,2} g_s \hat{\rho}(\mathbf{r}_A(t)) |s\rangle \langle s|, \quad (3)
 \end{aligned}$$

where the last term denotes the collisional coupling between the impurity and Bose gas, and $\hat{\rho}(\mathbf{r}_A) = \hat{\psi}^\dagger(\mathbf{r}_A) \hat{\psi}(\mathbf{r}_A) \simeq \rho_0 + \delta \hat{\rho}(\mathbf{r}_A)$ represents the field density operator of the Bose gas with $\mathbf{r}_A(t)$ being the time-dependent position of the impurity.

In the rotated $|g, e\rangle = (|1\rangle \pm |2\rangle) / \sqrt{2}$ basis, the Rabi frequency ω_0 determines the splitting between the $|g, e\rangle$ states, while the detuning $\delta = \omega_L - \omega_{21}$ gives a coupling terms. In such case, the impurity immersed in the condensate is collisionally coupled to the Bose gas via two channels [48,82,83]: The first term resembles the interaction of a static charge to an external scalar potential and can be canceled through proper tuning of the interaction constants $g_{1,2}$ (e.g., via Feshbach resonance [76–79]), while the second term resembles a standard electric-dipole coupling. Choosing proper detuning δ to exactly compensate the coupling to the average density, we can finally find the impurity-fluctuations interaction Hamiltonian

$$H_{\text{int}} = g_- (e^{i\omega_0 \tau} \sigma^+ e^{-i\omega_0 \tau} \sigma_-) \delta \hat{\rho}(\mathbf{r}_A(\tau), t_A(\tau)), \quad (4)$$

reproducing the usual Unruh-DeWitt detector-field interaction with $g_- = (g_1 - g_2)/2$ satisfying $\hbar \delta / 2 + g_- \rho_0 = 0$. However, here the LVQF is coupled to the detector.

Note that when $\zeta = \hbar c_0 k / M_* \ll 1$ the density fluctuations resemble massless scalar field with spectrum, $\omega_{\mathbf{k}} \simeq c_0 k$, the linearly moving impurity remains unexcited when its velocity satisfying $v < c_0$, while behaves dramatically differently when moving at a supersonic speed $v \gtrsim c_0$. Specifically, although the ‘‘charge neutrality’’ of the impurity rules out Bogoliubov-Cherenkov emission [84,85], the anomalous Doppler effect may induce it to be excited from its ground state while emitting Bogoliubov phonon and still conserving energy. This is the analog Ginzburg emission for superluminal moving particles [86,87], occurring in BEC. When the spectrum breaks the Lorentz symmetry satisfying $\omega_{\mathbf{k}} = c_0 k f(\zeta)$: If $0 < f(\zeta) < 1$ for an interval of ζ , the impurity would get excited when its velocity exceeds the critical $v_c = c_0 f_c(\zeta)$ with $f_c(\zeta) = \inf f(\zeta)$ [83]. This particular property provides us a potential effective tool to constrain on the possible families of modified dispersion relations with the experimental results of the Relativistic Heavy Ion Collider [88]. We will in the following consider a circularly moving impurity to observe the circular Unruh effect [89–91], and in particular examine whether the circular Unruh effect is robust to the breaking of Lorentz symmetry.

III. SPONTANEOUS EXCITATION OF THE CIRCULARLY MOVING DETECTOR

If the detector moves with a circular trajectory $(c_0 t(\tau), \mathbf{x}(\tau)) = (c_0 \gamma \tau, R \cos(\Omega \gamma \tau), R \sin(\Omega \gamma \tau), 0)$, with constant radius R , the angular velocity Ω , the usual relativistic factor $\gamma = 1 / \sqrt{1 - R^2 \Omega^2 / c_0^2}$, and the corresponding acceleration $a = \Omega^2 \gamma^2 R$, we find the transition rate of the detector from its ground state to excited state [92]

$$\begin{aligned}
 \mathcal{P}(\omega_0) = & \frac{g_-^2 \rho_0 m_B}{2 \hbar^3} \int_0^\infty d\zeta \frac{\zeta^2}{\gamma f(\zeta)} \sum_{m=-\infty}^\infty J_m^2(\tilde{M} \zeta) \\
 & \times \delta \left(\zeta f(\zeta) - \frac{1}{\tilde{M}} \left(m v - \frac{\tilde{E}}{\gamma} \right) \right), \quad (5)
 \end{aligned}$$

where $\tilde{M} = R M_* / \hbar c_0$, $\tilde{E} = R \omega_0 / c_0$, and $v = R \Omega / c_0$ are dimensionless parameters. Note that we focus on the detector’s speed in the preferred Lorentz frame satisfying $0 \leq v < 1$.

For the LI scenario where $R_0 = 0$ and $\omega_{\mathbf{k}} = c_0 k \sqrt{1 + \zeta^2 / 4} \approx c_0 k$ for $k \ll k_c = M_* / c_0 \hbar$, we find in the ultrarelativistic limit, $\gamma \gg 1$, the equilibrium population of the upper level relative to the lower is [92]

$$\frac{\mathcal{P}(\omega_0)}{\mathcal{P}(-\omega_0)} = \frac{12 c_0^2 \omega_0^2}{5 a^2} \exp \left(- \sqrt{\frac{24}{5}} \frac{c_0 \omega_0}{a} \right), \quad (6)$$

when the energy splitting of the detector is not too small $\omega_0 \gg a/c_0$ [89], which leads to an effective temperature

$$T_{\text{eff}} = \frac{\sqrt{5}\hbar a}{2\sqrt{6}k_B c_0}. \quad (7)$$

This temperature is higher by a factor $\sqrt{5/6}\pi$ than the Unruh temperature for the linear acceleration, and higher by a factor $\sqrt{5/2}$ than the Unruh temperature for the real massless scalar field case as a result of the Bogoliubov transformation for the quasiparticles.

If $(\zeta f(\zeta))' > 0$, we can find in the limit, $\tilde{M} \rightarrow \infty$, the transition rate (5) behaves quite differently for $0 < v < f_c$ and $f_c < v < 1$, where $f_c = f(\zeta_c)$ is a global minimum at $\zeta = \zeta_c > 0$. Specifically, for the former, the transition rate $\mathcal{P}(\omega_0) \rightarrow \mathcal{P}_0(\omega_0)$, given by

$$\mathcal{P}_0(\omega_0) = \Gamma_0 \sum_{m=\lceil \frac{\tilde{E}}{\gamma} \rceil}^{\infty} \frac{1}{\gamma} \left(m v - \frac{\tilde{E}}{\gamma} \right)^2 J_m^2 \left(m v - \frac{\tilde{E}}{\gamma} \right), \quad (8)$$

where $\Gamma_0 = \frac{g^2 \rho_0}{2\hbar M_* R^2}$. Note this is the response for the analog massless scalar field, and thus no low-energy Lorentz violation can be seen when $v < f_c$. While for the latter there is a correction to the former case, $\mathcal{P}(\omega_0) \rightarrow \mathcal{P}_0(\omega_0) + \Delta\mathcal{P}$, with

$$\Delta\mathcal{P} = \frac{g^2 \rho_0 m_B}{2\pi \hbar^3 \gamma} \int_{\zeta_-}^{\zeta_+} d\zeta \frac{\zeta}{f(\zeta) \sqrt{v^2 - f^2(\zeta)}}, \quad (9)$$

where $\zeta_- \in (0, \zeta_c)$ and $\zeta_+ \in (\zeta_c, \infty)$ are unique solutions to $f(\zeta) = v$ in the respective intervals. This correction

means the detector sees a low-energy Lorentz violation when $v > f_c$, and alternatively the departure from the standard prediction of Unruh effect appears as a consequence of the Lorentz violation.

IV. EXPERIMENTAL IMPLEMENTATION

Recent experimental advances have allowed for groundbreaking observations of strongly dipolar BEC, its excitation spectrum displaying roton minimum, and dynamics of impurity immersed in BEC [93–104]. These experiments hold promise to realize our experimental scenario proposed above. Specifically, we can consider a single ^{87}Rb atom immersed in a BEC of Dy atom [93,95,98] which possesses a magnetic dipole moment of $10\mu_B$ with μ_B being the Bohr magneton. The condensate density is assumed to be $\rho_0 \sim 4.4 \times 10^3 \mu\text{m}^{-2}$; The observer trajectory radius $R \sim 10 \mu\text{m}$; A typical trap frequency $\omega_z = 2\pi \times 10^3 \text{ Hz}$, the corresponding harmonic oscillator width is $d_z \simeq 0.25 \times 10^{-6} \text{ m}$. Figure 2 displays the transition rate in (5), and clearly shows the spontaneous excitation of the detector as a result of the circular acceleration in a Minkowski vacuum. This would be viewed as the circular Unruh effect: Thermal bath is predicted for an accelerated detector moving through the inertial vacuum. In addition, the transition rates clearly show the deviation from the LI field case, occurring for strongly dipolar interactions. Specifically, when the field slightly deviates from the LI case, e.g., $A = 0.1$ case, their corresponding transition rates share similar behaviors. When this deviation becomes stronger, the excitation rates increase and behave sharply differently compared with the LI case, especially in the high velocity and low energy spacing regimes. However, for the low velocity case and

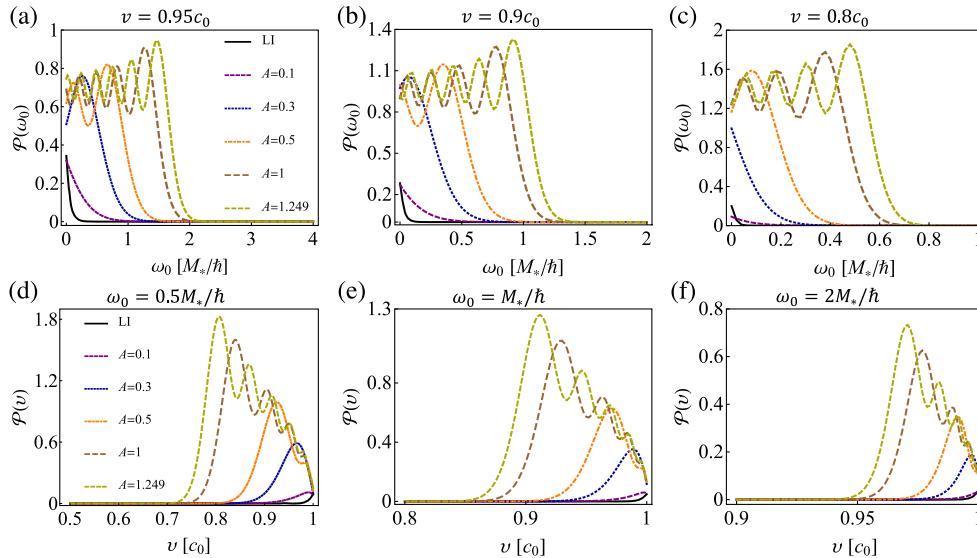


FIG. 2. Transition rates shown in (5) for different effective chemical potential A , in units of $\frac{g_0^2 \rho_0 m_B}{2\hbar^3}$: (a), (b), (c) as a function of the detector's energy spacing ω_0 with fixed velocity; (d), (e), (f) as a function of the velocity v with fixed energy spacing of the detector. Here we assume that the DDI dominance case ($R = \sqrt{\pi/2}$) is valid.

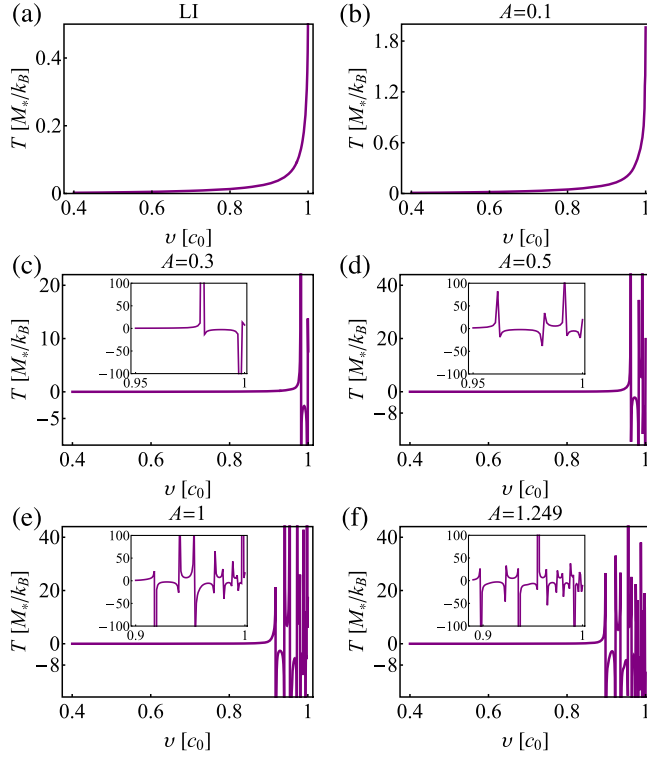


FIG. 3. The temperature defined in (10) as a function of the detector's velocity v with fixed energy spacing of the detector, $\omega_0 = M_*/\hbar$. Insets are the details of the corresponding oscillation parts. Here we assume that the DDI dominance case ($R = \sqrt{\pi/2}$) is valid.

large energy spacing of the detector, the excitation rates change slightly in the presence of Lorentz violation (i.e., they are not so sensitive to the breaking of Lorentz symmetry), since in such cases the detector is harder to excite.

To further check whether the Unruh effect is robust to the breaking of Lorentz symmetry, we naively define the Unruh temperature T to estimate the fluctuations sampled by the impurity, using the Einstein's detailed balanced condition,

$$T = \hbar\omega_0 k_B^{-1} \ln^{-1} \left(\frac{\mathcal{P}(-\omega_0)}{\mathcal{P}(\omega_0)} \right). \quad (10)$$

This temperature is independent of ω_0 for uniformly linearly accelerated detectors, given by $T_U = \frac{\hbar a}{2\pi k_B c}$, since whose response function satisfies the Kubo-Martin-Schwinger condition [105–107]. For the circular acceleration cases, such definition of T may depend on ω_0 , however, keeps monotonous increase via the effective acceleration parameter for the LI case [108]. We here plot this temperature registered by the impurity coupled to the analog LVQF as shown in Fig. 3. For the LI case, the temperature increases monotonously with the increase of

the detector's speed v as expected. If the spectrum of the field deviates from the LI case slightly, e.g., $A = 0.1$ case, the present temperature behaves similarly as the LI case but with a larger magnitude. Remarkably, when the spectra of the analog LVQF strongly deviate from the LI spectrum, the temperature first increases with the increase of the detector's speed and then oscillates if the detector's speed exceeds a critical value which depends on the degree of the deviation. The counterintuitive oscillation phenomenon means that Lorentz violation may break the thermal characteristic of Unruh effect, and even cause the analog anti-Unruh effect [109]: *Unruh temperature decreases with the increase of acceleration*. Besides, the effective negative temperature [110,111] occurs during the oscillation, which implies that as a result of Lorentz violation, the corresponding detector's state is characterized by an inverted occupation distribution, where excited state is populated more than ground state.

V. CONCLUSIONS

We present a concrete experimental proposal to test how the Lorentz violation affects the circular Unruh effect using an impurity immersed in a dipolar BEC. We find that if the spectra of quantum field deviate from the LI case strongly, the transition rates and the predicted temperature of the analog Unruh-DeWitt detector behave quite differently compared with the LI case, and the Lorentz violation even more may induce the counter-intuitive anti-Unruh effect on certain conditions. Our preliminary estimates indicate that the proposed experimental implementation of the analog circular Unruh effect and its interaction with the Lorentz-breaking physics is within reach of current state-of-the-art ultracold-atom experiments.

Our proposed quantum fluid platform may also allow us in the experimentally accessible regime to explore open questions concerning Unruh effect [47], and why its robustness to high energy modifications of the dispersion relation [64] behaves differently from that of its equivalence principle dual—Hawking effect [67]. In addition, two impurities could be used as detectors to explore correlations harvest from the quantum vacuum of analog quantum fields [112].

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