Letter

Neutron oscillation and baryogenesis from six dimensions

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Considering a six-dimensional geometry orbifolded on $S^1/Z_2 \times S^1/Z_2$ with quarks and leptons localized on orthogonal branes, we show that the construction admits observable $n - \bar{n}$ oscillation while naturally suppressing the proton decay rates. Consistent with other low-energy observables, the model also accommodates baryogenesis at $\mathcal{O}(10\ \text{TeV})$ scale.

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Baryon number (B) violation, a key ingredient for generating the observed baryon to photon ratio in the universe ($n_B/n_\gamma = (6.19 \pm 0.14) \times 10^{-10}$), is allowed in the Standard Model (SM) only through nonperturbative processes [1]. Presumably important for baryogenesis at temperatures near the electroweak scale [2], such processes are highly suppressed at low temperatures, and B is seemingly a good global symmetry. With $\not{B}$ and CP violation within the SM not being large enough to generate the n_B/n_γ, other sources must be explored.

The simplest gauge invariant and $\not{B}$ effective operator also violates lepton number (L) by one unit, and nonobservation of proton decay pushes the corresponding scale to well above 10^{15} GeV. On the other hand, if the leading operator were a $\Delta B = 2$ one, the scale could be much lower and be probed by looking for either $n - \bar{n}$ oscillation in nuclei, or annihilations brought about in collisions of cold neutrons against a target. The characteristic timescales are related through $T_{\text{nucl}} = \tau_{\text{free}}^2 R$, where R characterizes the strong interaction $\bar{n}$ annihilation time, and is, typically, $\mathcal{O}(100\ \text{MeV})$ [3]. Using different nuclei, experiments at SOUDAN-II (Fe_{56}) [4], Super-Kamiokande (O_{16}) [5], and SNO (deuteron) [6] have constrained τ_{free} to be larger than 1.3×10^8 s, 2.7×10^8 s, and 1.23×10^8 s respectively, each at 90% C.L. A correspondence with the underlying theory is best established [7] by considering the matrix element of the effective six-quark (dimension-9) operator *viz.* $\Delta m \equiv \tau_{\text{free}}^{-1} = \langle \bar{n} | \mathcal{O}_9 | n \rangle$. The transition probability for pure state $|n; t = 0\rangle$ to evolve to $|\bar{n}; t\rangle$ is given by $P(t) = (t/\tau_{\text{free}})^2 e^{-\lambda t}$ where $\lambda^{-1} = 880$ s is the mean life of a free neutron. The bound on τ_{free} implies $\Delta m \lesssim 6 \times 10^{-33}$ GeV, or, for an $\mathcal{O}(1)$ Wilson coefficient, a new physics scale ($\gtrsim$500 TeV [8]) much lower than the proton decay scale. A roadblock to a UV-complete model for $n - \bar{n}$ oscillation is that it, generically, needs two new fields with gauge symmetry allowing one of these to couple to a $\Delta L = 1$ current as well, thereby requiring an unnatural suppression for the said coupling.

In our quest for a well-motivated scenario that naturally circumvents all such constraints, we propose a six-dimensional space-time orbifolded on $S^1/Z_2 \times S^1/Z_2$ [9–11] and a highly-warped x_5—direction. With quarks and leptons localized on orthogonal branes, the geometry supports substantial $\Delta \mathcal{B} = 2$ while evading proton-decay constraints *without* any hierarchy/unnaturalness in the parameters. With successive warpings along the two compactified dimensions, ($x_4 \in [0, \pi R_y]$ and $x_5 \in [0, \pi r_z]$) that are individually Z_2-orbifolded with 4-branes sitting at each of the edges, the geometry is described by the line element [9]

$$ds_6^2 = b^2(x_5)[a^2(x_4)\eta_{\mu\nu}dx^\mu dx^\nu + dx_4^2] + dx_5^2,$$

where $\eta_{\mu\nu}$ is the flat metric. Denoting the fundamental scale in six dimensions by M_6 and the negative bulk cosmological constant by Λ_6, the total bulk-brane Lagrangian is, thus,

$$\begin{aligned}\mathcal{L} = &\sqrt{-g_6}(M_6^4 R_6 - \Lambda_6)\\ &+ \sqrt{-g_5}[V_1(x_5)\delta(x_4) + V_2(x_5)\delta(x_4 - \pi R_y)]\\ &+ \sqrt{-\tilde{g}_5}[V_3(x_4)\delta(x_5) + V_4(x_4)\delta(x_5 - \pi r_z)].\end{aligned}$$

The five-dimensional metrics $(g_5, \tilde{g}_5)$ are those induced on the appropriate 4-branes, and the brane potentials V_i encode the Israel junction conditions.

Generalizing from the restrictive case studied in Ref. [9], we admit a five-dimensional induced nonzero cosmological constant $\tilde{\Omega} < 0$ on the 4-branes, allowing these to be bent. The four-dimensional cosmological constant, though, is held to be zero (the generic case [10] only leads to algebraic complications). With this, the Einstein equations lead to

$$b(x_5) = b_1 \cosh(k|x_5| + b_2), \quad b_1 = \mathrm{sech}(k\pi r_z + b_2), \quad (1)$$

with $k = \sqrt{-\Lambda_6/10M_6^4}$. Similarly,

$$a(x_4) = \exp(-c|x_4|), \quad \text{where } c \equiv k b_1 R_y / r_z. \quad (2)$$

Unlike Ref. [9], we consider a very large b_2 [10,11] leading to $b(x_5) \propto \exp(k|x_5|)$, and one is forced to $c \ll k$, unless a large, and unpleasant, hierarchy between R_y and r_z is to be admitted. This also implies that the 4-brane tensions are pairwise almost equal and opposite ($V_1 = -V_2 \approx 0$ and $V_3 = -V_4 \approx -8M^4 k$) thereby ensuring the near vanishing of the induced cosmological constant on the 4-branes at the ends of the world. In this limit, the metric is nearly conformally flat, and, along with the AdS_6 bulk, resembles a generalization of the Randall-Sundrum geometry to one dimension higher.

In this space-time, the Lagrangian for a free massless fermion is given by

$$\begin{aligned}\mathcal{L}_{\mathrm{fermion}} &= \frac{1}{2} b^4(x_5)(\bar{Q} i \Gamma^a E_a^M D_M Q)\bar{\Delta}(x_4) \\ &\quad + b^5(x_5)(\bar{L} i \Gamma^a E_a^M D_M L)\delta(x_5) \\ \bar{\Delta}(x_4) &\equiv \delta(x_4) + \delta(x_4 - \pi R_y)\end{aligned}$$

where $Q(L)$ denote the quark (lepton) fields and E_a^M are the appropriate fünfbeins. The low-energy phenomenology is, of course, dictated by only the zero-modes of the fermions. Since the x_4—direction is nearly flat, so would be the wave profile of the leptonic zero mode. On the other hand, the quark zero mode is given, to a very good approximation, by an exponential function.

To induce $\not{B}$, we begin by introducing two colored scalar fields $\phi(3, 1, -1/3)$ and $\omega(6, 1, 2/3)$, localized, of course, on the two 4-branes at $x_4 = 0$ and $x_4 = \pi R_y$. With leptons being confined to the 4-brane at $x_5 = 0$, the scalar interactions are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{scal}} = \sqrt{2r_z}[&y_{ud}\phi\overline{u^c}P_R d + z_{dd}\omega\overline{d^c}P_R d \\ &+ \sqrt{4r_z R_y} y_{ue}\phi^*\overline{u^c}P_R e\delta(x_5) \\ &+ \lambda M\phi^2\omega]\overline{\Delta}(x_4) + \mathrm{H.c.}\end{aligned}$$

All the fermion fields here are $SU(2)$ singlets and we have listed just the relevant term of the scalar potential. Introduced for convenience, the scale M is equated to the mass of the heaviest scalar field. Since, post compactification, we would be interested only in the lightest KK-modes, the five dimensional fields ($\mathcal{F} = u, d, \phi, \omega$) could be decomposed as $\mathcal{F}(x_\mu, x_5) = (2r_z)^{-1/2}\mathcal{F}(x_\mu)\chi_{\mathcal{F}}(x_5)$, where the zero-mode wave functions $\chi_{\mathcal{F}}(x_5)$ satisfy the normalization condition $\int dx_5 b^s \chi_{\mathcal{F}}^2 = 1$, with ($s = 2, 3$) for scalars and fermions respectively, yielding canonically normalized four-dimensional fields. In contrast, thanks to the x_4-direction being nearly flat, the wave function for the lepton zero-mode is a trivial one. To obtain the effective four-dimensional theory, one needs to integrate over both x_5 and x_4 (given the smallness of the warping c, the second integration is essentially trivial), resulting in

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = \eta_{ud}\phi\overline{u^c}d + \eta_{ue}\phi^*\overline{u^c}e + \zeta_{dd}\omega^*\overline{d^c}d \\ + \rho M\phi^2\omega + \mathrm{H.c.},\end{aligned} \quad (3)$$

where

$$\begin{aligned}\zeta_{dd} &= (2r_z)^{-1} z_{dd} \int dx_5 b^2 \chi_\phi \chi_d^2 \\ \eta_{ud} &= (2r_z)^{-1} y_{ud} \int dx_5 b^2 \chi_\phi \chi_u \chi_d \\ \rho &= (2r_z)^{-1} \lambda \int dx_5 b^2 \chi_\phi^2 \chi_\omega \\ \eta_{ue} &= y_{ue} b^2(0)\chi_\phi(0)\chi_u(0).\end{aligned} \quad (4)$$

The relations above may include any possible flavor structure, a point that we return to later. Now, for fermions with vanishing bulk masses (an excellent approximation for the first and second generation fields), the wave profiles would be expected to be nearly identical, and the ratio $\zeta_{dd}/\eta_{ud} \approx z_{dd}/y_{ud}$.

The Wilson coefficients for proton-decay and neutron oscillation are now straightforward, viz.

$$C^p = \frac{\eta_{ud}\eta_{ue}}{m_\phi^2}, \qquad C^{nn} = \frac{\rho M}{m_\phi^2 m_\omega^4}\eta_{ud}^2 \zeta_{dd}. \quad (5)$$

Qualitative features of this result are best appreciated by assuming that all the masses are of the same order. Despite the fact that C^{nn} is fourth order in the couplings while C^p is only bilinear, $C^p \ll C^{nn}$ on account of the factor $b^2(0)$ in η_{ue}. Note that this relative suppression does not need any hierarchy, either of couplings or between masses. Rather, it is engendered dynamically on account of the warping.

The neutron oscillation rate is determined by $|\Delta m| = |\langle\overline{n}|H_{\mathrm{eff}}|n\rangle| = 8\xi^2 C^{nn}/3$ where the matrix element is computed using vacuum insertion approximation [8] with ξ parametrizing the reduced matrix element for the three quark operator. Lattice computations [12] give $\xi \approx 0.0096^{+6}_{-20}\ \mathrm{GeV}^3$, with this error being the main uncertainty in the calculation.

The observed limits on $|\Delta m|$ can now be translated to constraints on the parameter space. In view of the large dimensionality of the latter, it is instructive to make the simplifying assumption that $\eta_{ud} = \zeta_{dd} = \rho \equiv \kappa$ and

$m_\phi = m_\omega = M$. The saturation of the limit can then be expressed as

$$\frac{|\Delta m|}{|\Delta m|_{\max}} \approx \left[\frac{\kappa}{0.01}\right]^4 \left[\frac{M}{15\ \mathrm{TeV}}\right]^{-5}. \tag{6}$$

More precise values are listed in Table I for a few benchmark points. It is worth noting here that, for $kr_z \sim 8.5$ (a value that leads to $C^p \lesssim 10^{-30}$, thereby suppressing proton decay to rates well below the current limit) also implies that for anarchic bulk-couplings $z_{dd}, y_{ud}, \lambda \sim \mathcal{O}(1)$, the corresponding 4-dimensional ones $(\zeta_{dd}, \eta_{ud}, \rho)$, obtained on integrating over x_5, are all $\sim\mathcal{O}(10^{-2})$. Further suppression of the effective couplings could arise in two ways: (i) starting with smaller bulk couplings or (ii) localizing the light quark states away from the scalars. In contrast, the scenario with $\kappa \sim 1$ is achievable only for a flat extra dimension, at the cost of very large scalar masses and self-consistency of the treatment would imply that the scales of the KK-excitations be at least as large, thereby taking the entire scenario beyond detection.

We now examine other possible phenomenological consequences. For light (a few TeVs) ϕ or ω, resonance production at the LHC is possible, leading to a peak in the dijet invariant mass (while a lepton-jet decay is notionally available to the ϕ, it is highly suppressed). With the QCD background being very large, the only hope is to concentrate on high-p_T, high invariant-mass events. For $m_\phi = 3$ TeV and $\eta_{ud} = 10^{-2}$, the ϕ-production cross-section is $\lesssim 0.4$ fb (and somewhat lower for the ω) even without accounting for efficiencies. In other words, even in the most optimistic scenario, direct detection would have to wait. The only caveat to this would be to consider a hierarchy in the scalar masses whereby one of them could be made significantly lighter and brought into the current reach of the LHC.

Much more important are the flavor sector observables. While $n-\bar{n}$ oscillation needs only the $d-d-\omega$ coupling, it is conceivable that the $s-s-\omega$ coupling is unsuppressed as well. This would result in an effective flavor changing Hamiltonian of the form $\mathcal{H}^{\Delta S=2} = m_\phi^{-2}(\overline{s^c} P_R s)(\overline{d} P_L d^c)$, thereby contributing to K^0–$\overline{K^0}$ oscillation (which, within, the SM, proceeds through the charm-dominated box diagram). To bring this to the usual form, one has to effect Fierz rotations [13], both in the Dirac space and the color space, using $(6_c \otimes 6_c)_1 = (2/3)(1_c \otimes 1_c) + (1/2)(8_c \otimes 8_c)_1$, encapsulating both color-unsuppressed and suppressed contributions. Comparing with the SM contribution (which saturates the observed mass difference), we have

$$\frac{\mathrm{Re}(\zeta_{dd}\zeta_{ss}^*)}{8m_\omega^2} \lesssim \frac{G_F^2 M_W^2}{16\pi^2} \mathcal{F}^0 \sim 10^{-7}\ \mathrm{TeV}^{-2}, \tag{7}$$

where $\mathcal{F}^0$ includes the box-diagram computation [14] (note that the hadronic matrix element calculation is common to both). Assuming that the couplings are of similar size, for $m_\omega = 3$ TeV, this translates to $|\zeta_{dd}| < 0.0026$. In other words, the coupling values required for observable neutron-oscillation rates more than easily satisfy the bounds from kaon oscillations. This exercise, though, tells us that any introduction of hierarchy in the scalar masses needs to be approached carefully.

TABLE I. Lower limits on the mass parameter m_ϕ as a function of the common coupling.

κ	1	0.1	0.01	0.001
$m_\phi(\mathrm{TeV}) = m_\omega = M$	670	106	17	2.6
$m_\phi(\mathrm{TeV}) = m_\omega/3 = M/3$	345	55	9	1.4

As is well known, the creation of a baryon asymmetry needs not only $\not{B}$ and $\not{CP}$ (much larger than that we have in the SM) but also an accompanying epoch with out-of-equilibrium condition. To create an environment amenable for this, let us augment the model with the inclusion of a further copy of ϕ (henceforth called ϕ_1) and a singlet S (which, for simplicity, we consider to be localized on the same branes as the colored scalars). Similarly, we also assume, that the ϕ mass matrix is diagonal and that there is no substantial mass hierarchy between these. As for the potential in (3), it now includes additional terms such as $(i = 1, 2)$

$$-V \ni M[\rho_{ij}\phi_i\phi_j\omega + \tilde{\rho}_{ij}\phi_i^*\phi_j S] + \mathrm{H.c.}, \tag{8}$$

where, for the sake of simplicity, we retain only the zero-modes. While the terms in Eq. (8) could have extra factors, of S/M, these do not add anything to our discussion, and we omit them.

At the tree level, we forbid any Yukawa couplings for the new field ϕ_2 (this could be arranged simply by introducing a softly broken discrete symmetry). Thus, the dominant decay available to it would be one through ρ_{12} leading to $\phi_2 \to uddd$, or a final state of $B = 4/3$. On the other hand, $\phi_1 \to \bar{u}\,\bar{d}$ $(B = -2/3)$.

For $2m_\phi < m_S \lesssim 2m_\omega$, the dominant decay of the S would be $S \to \phi_i^*\phi_j$. While the tree-level diagram is obvious, one-loop corrections are brought about by the diagram in Fig. 1. The latter evidently has a nonzero

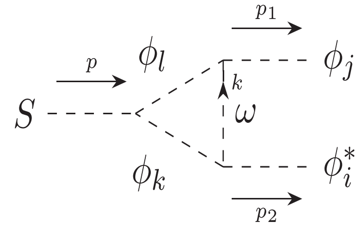

FIG. 1. One-loop diagram contributing to B-asymmetry generated from a decaying S.

absorptive part, receiving contributions from possibly three ways of cutting a pair of internal lines. For example, for $m_{\phi_1} \sim m_{\phi_2} \gtrsim m_\omega/2$, only the vertical cut in Fig. 1 contributes. Since a net baryon number ϵ_B (per S-decay) can arise only from the difference $\Gamma(S \to \phi_1 \phi_2^*) - \Gamma(S \to \phi_1^* \phi_2)$, we have (denoting $x_a \equiv m_a^2/m_S^2$)

$$\epsilon_B \approx \frac{M^2}{4\pi^2 m_S^2 \beta} \log\left(\frac{x_\omega - x_\phi + 1 + \beta}{x_\omega - x_\phi + 1 - \beta}\right) \mathcal{A}_\rho$$
$$\mathcal{A}_\rho \equiv Im[\tilde{\rho}_{12}^* (\rho^\dagger \tilde{\rho} \rho)_{12}] / \sum_{i,j} |\tilde{\rho}_{ij}|^2 \qquad (9)$$

where $\beta = (1 - 4x_\phi)^{1/2}$ and we have neglected terms proportional to the mass difference δm_ϕ^2.

Thus, the number density n_S of the decaying field is related to those for the (anti-)baryons through, $n_b - n_{\bar{b}} = n_S \epsilon_B B_q$, where $B_q = 2$ is the baryon number created per decay. Since the photons in the universe far outnumber the baryons, they dominate the entropy density s of the universe and we may write [15]

$$\frac{n_b - n_{\bar{b}}}{s} = \frac{n_S B_q \epsilon_B}{4\pi^2/45 g_* T^3} = \frac{180\zeta(3)}{\pi^4} \epsilon_B, \qquad (10)$$

where $g_* \approx 107$ is the weighted number of degrees of freedom operative at that temperature.

Using Eq. (10), the constraint on baryon asymmetry in the universe as, $\epsilon_B = 3.8 \times 10^{-11}$. This could be arranged quite easily in the model. In the parameter space where the nuetron-antineutron oscillation is observable, $m_\phi \sim m_\omega = 3$ TeV, and $M \sim m_S = 10$ TeV, the asymmetry required for baryogenesis is obtained for $\mathcal{A}_\rho \lesssim 2.2 \times 10^{-9}$. Note that the very structure of $\mathcal{A}_\rho$ implies cancellations between the nominal phases in the couplings. Consequently, the residual phase is naturally smaller. Indeed, such a phase is also stipulated by the ϵ' parameter in kaon oscillations.

To summarize, we have investigated baryon-number violation in a six-dimensional world compactified on $(S^1/Z_2) \otimes (S^1/Z_2)$ wherein one direction is highly warped with the other being almost flat (minuscule warping). Boasting stabilized moduli [10], such models are more natural than their flat extra dimension counterparts. As we have demonstrated here, augmenting the minimal SM extension by just two extra colored scalars allows for low-energy $\Delta B = 2$ processes, thereby offering the tantalizing prospect of an observable $n - \bar{n}$ oscillation signal. On the other hand, proton decay is dynamically suppressed (without the need for any hierarchy of couplings) on account of the leptons being localized on a different 4-brane and suffering a consequent warping at the intersection with the quark 4-brane. While, for natural sizes of Yukawa couplings, the masses of the scalars are just beyond the reach of the LHC, these are likely to be visible in the next generation of hadronic colliders or their presence inferred from more sensitive flavor probes. Most interestingly, a minor addition to the spectrum opens the possibility for multi-TeV scale baryogenesis. The parameter space amenable to reproducing the observed baryon asymmetry is large and no hierarchy or unnatural assumptions need be invoked. Furthermore, these scalars hold the prospect of generating strong phase transitions, signals of which are likely to be observable in the next generation of gravitational wave detectors.

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