Corrections to the instanton configuration as baryon in holographic QCD

Si-wen Li,* Hao-qian Li,† and Sen-kai Luo‡

Department of Physics, School of Science, Dalian Maritime University, Dalian 116026, China

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In this work, we first derive the corrections to the instanton configuration of the flavored gauge field in the D4–D8 model with generic flavor numbers. Then, as the instanton configuration on the D8-branes represents equivalently baryon in this model, keeping our corrections in hand, we systemically study the spectrum of baryon, heavy-light baryon, or heavy-light meson and find it is possible to fit the experimental data with the meson data in this model. We also briefly outline how to include the interaction of glueball and heavy-light meson or baryon with our corrections, evaluate numerically the decay rate of the heavy-light meson or baryonic matter involving glueball. Since it is possible to fit all the spectra with the same choice of the parameters to the experimental data, we believe our corrections improve the framework of D4–D8 model and the corrected instanton configuration is also useful to investigate other properties of baryon in holography.

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I. INTRODUCTION

QCD, as the characteristic strong coupling gauge theory, has been expected to be analyzed by the gauge-gravity duality and AdS/CFT since holography of gravity was proposed in the 1990s [1–4]. Along this direction, there are several holographic frameworks for QCD attracting great interests in the last two decades, e.g., [5–8]. Among these works, the D4–D8 model, i.e., the Witten-Sakai-Sugimoto model [9,10], as a top-down approach based on the underlying string theory, becomes famous and successful in the process of time because this model includes mostly all the fundamental elements of QCD in a very simple way so that it could reproduce various elementary features of QCD, e.g., deconfinement and chiral phase transition [11–18], baryon spectrum [19–24], glueball spectrum [25–29], the interaction of glueball and meson or baryon [30,31], the theta term [32–38].

Specifically, the D4–D8 model consists of N_c coincident D4-branes as colors and a stack of N_f pairs of probe D8- and anti D8-branes (D8/$\overline{\mathrm{D8}}$-branes) as flavors vertical to the D4-branes. The open string on the N_c D4-branes and N_f D8/$\overline{\mathrm{D8}}$-branes is respectively in the adjoint representation of $U(N_c)$ and $U(N_f)$ which is therefore identified as gluon and meson. The open string connecting N_c D4-branes and N_f D8/$\overline{\mathrm{D8}}$-branes is in the fundamental representation of $U(N_c)$ and $U(N_f)$ which is accordingly identified as the fundamental chiral quark. In the large N_c limit, the bulk geometry is described by the type IIA supergravity which can be solved by the bubble configuration of the D4-branes compactified on a circle with size of M_{KK}^{-1} since the dual theory will exhibit confinement in this geometry below the energy scale M_{KK}. The supersymmetry would also break down below M_{KK} when the periodic and antiperiodic condition is respectively imposed to the gauge boson and supersymmetric fermion along the circle, as it is illustrated in Fig. 1. Hence there are only two parameters in this model which are the energy scale M_{KK} and 't Hooft coupling λ. And it is possible to fit the meson spectrum in the D4–D8 model to the experimental data by setting $M_{KK} = 949$ MeV, $\lambda = 16.6$ as [9,10]. Note that this choice of the parameters is named as the meson data of the D4–D8 model in this work.

Following the idea in Witten's [39], baryon vertex, as a D4-brane wrapped on S^4, can be further introduced into the D4–D8 model. Analyzing the charge of D4- and D8-brane, the baryon vertex is recognized as the instanton solution of the gauge field on the D8-brane [40], hence the Hamiltonian of the collective modes, whose eigenvalue would be interpreted as the baryon spectrum, can be derived by additionally employing the idea of Skyrmions in the moduli space of instanton [41], as it is discussed in [19]. Besides, the glueball field in this model is identified as the bulk gravitational polarization since it is sourced by gauge invariant operator as the energy-momentum tensor of the gluon [25–29] in the dual theory. In this sense, when the bulk gravitational polarization is taken into account, there

*siwenli@dlmu.edu.cn
†lihaoqian@dlmu.edu.cn
‡luosenkai@dlmu.edu.cn

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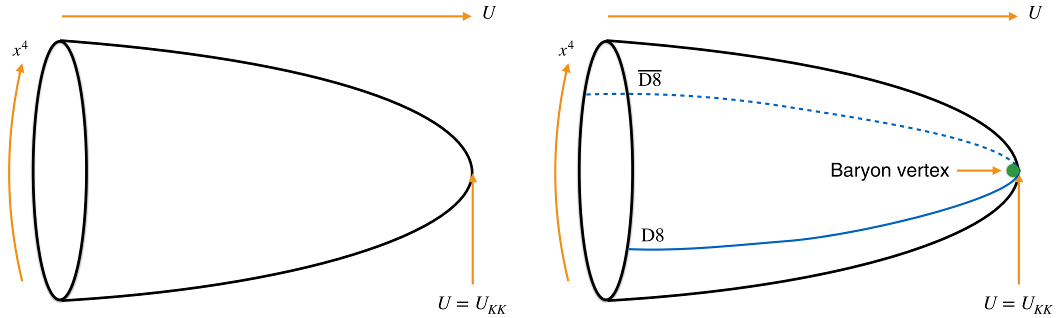

FIG. 1. The D-brane configurations in the D4–D8 model. Left: the bubble bulk geometry on $\{U, x^4\}$ plane produced by N_c coincident D4-branes. Right: the N_f pairs of $\mathrm{D8}/\overline{\mathrm{D8}}$-branes (blue line) and baryon vertex (green point) as probes in the bulk. The effective theory below the energy scale M_{KK} is confined and nonsupersymmetric.

must be open/close string interaction which is the interaction of bulk close string and the open strings on various D-branes (i.e., the flavor $\mathrm{D8}/\overline{\mathrm{D8}}$-branes and the baryon vertex), since the gravitational polarization is excited by the bulk closed string. Thus the effective action of the D8-brane will include the coupling terms of the bulk gravitational polarization and the world volume gauge field or instanton, which can be interpreted as the interaction of glueball and meson or baryon. Resultantly, when we derive the Hamiltonian of the collective modes of baryon, it will arise a time-dependent term describing the decay of baryon involving glueball as [30,31]. Altogether, the D4–D8 model could be treated basically as a holographic version of QCD.

Since the subject of this work is the corrections to holographic baryon in the D4–D8 model, our concern would be the instanton solution of the gauge field on the D8-brane and the associated Hamiltonian of the collective modes. The motivation of this work comes from [19,22–24,30,31]. In [19], it turns out the effective action of the D8-brane in the strong coupling limit (i.e., the 't Hooft coupling λ goes to infinity $\lambda \to \infty$) is pure Yang-Mills action, so that the instanton solution to the non-Abelian spatial part of the gauge field can be chosen as the $SU(2)$ Belavin–Polyakov–Schwarz–Tyupkin (BPST) solution which represents the Euclidean instanton. However the baryon spectrum based on the BPST instanton in [19] is unable to fit the experimental data of baryon when the meson data in this model is employed even if the framework in [19] is generalized into three-flavor case [20]. The same problem also appears in [22,23] where the meson data in the D4–D8 model is abandoned. The most likely reason could be that the derivation in [19,20,22,23] is strictly valid in the limit of $\lambda \to \infty$ while λ is a finite number in realistic QCD. To figure out this issue, [24] proposes a possible correction to the two-flavored instanton solution in this model. Since the two-flavored baryon spectrum is unrealistic, the baryon spectrum with the correction proposed in [24] may not fit the experimental data well enough when the meson data in this model is picked up. Nonetheless, in this work, we attempt to generalize the corrections to $SU(2)$ instanton into the case of $SU(N_f)$ instanton as [20]. Afterwards, we obtain the corrected baryon spectrum with generic flavor number N_f. Take into account the symmetries of isospin and angular momentum, we find the corrected baryon spectrum with $N_f = 3$ fits very well to the experimental data by picking up the meson data in this model. Moreover, when the heavy flavor is introduced into this model as [22,23,35], the corrected heavy-light baryon spectrum also fits well to the experimental data with the meson data in this model. And we also derive the $N_f = 2$ heavy-light baryon spectrum in order to fit the experimental data of heavy-light meson, since the heavy-light meson could be treated as a quasibaryon [42–44]. Picking up our corrections, the heavy-light meson spectrum with meson data in this model fits well to the experimental data of the lowest D-mesons. Finally, the bulk gravitational polarization is introduced with our corrections to the instanton, so it is able to describe the decay of the heavy-light meson involving the glueball in this framework.[1] Despite the corrections to the decay rate of the heavy-light baryon by following [30,31], it is out of reach to fit the experimental data exactly since the experimental data of glueball is less clear. Nonetheless, we overall believe our corrections to the instanton as baryon improve the framework of D4–D8 approach since it is able to fit both the spectra of meson and baryon with same choice of the parameters, even if the heavy flavor is included.

The outline of this paper is as follows. In Sec. II, we collect the essential parts of the D4–D8 model. In Sec. III, we derive our corrections to the BPST instanton solution with generic N_f as a generalization of [24]. Then compare our corrected baryon spectrum with the experimental data in the case of $N_f = 3$. In Sec. IV, we include the heavy

[1]Since the lightest glueball mass is evaluated around 1000 MeV by lattice QCD, it is mostly produced in the decay of baryon or baryonic meson, e.g., [45–47]. And it is also a motivation to include the interaction of glueball and baryon or heavy-light meson in the framework of D4–D8 model.

flavor in the D4–D8 model, obtain the heavy-light baryon spectrum with our corrections and fit the experimental data of the heavy-light meson. In Sec. V, we briefly outline how to describe the interaction of glueball and baryon or baryonic matter with our corrections, then evaluate the decay rate of heavy-light meson involving glueball numerically. The final section is the summary and discussion.

II. BARYON AS INSTANTON IN D4–D8 MODEL

In this section, let us collect the essential substance of the instanton configuration and the derivation of baryon spectrum in the D4–D8 model from [9,10,19,20].

A. The D4–D8 model

The D4–D8 model consists of N_c coincident D4-branes and a stack of N_f pairs of probe D8/$\overline{\text{D8}}$-branes in the large N_c limit. The bulk geometry is described by type IIA supergravity in 10-dimension (10d) given as [9],

$$\begin{aligned} ds^2 &= \left(\frac{U}{R}\right)^{3/2}[\eta_{\mu\nu}dx^\mu dx^\nu + f(U)(dx^4)^2] \\ &\quad + \left(\frac{R}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right], \\ e^\phi &= g_s\left(\frac{U}{R}\right)^{3/4}, \qquad F_4 = dC_3 = \frac{2\pi N_c}{\Omega_4}\epsilon_4, \\ f(U) &= 1 - \frac{U_{KK}^3}{U^3}. \end{aligned} \tag{2.1}$$

This gravity solution describes the bubble configuration of the spacetime ending on $U = U_{KK}$ as it is illustrated in Fig. 1. The D4-branes extend along $\{x^\mu, x^4\}$ where the index μ, ν runs over 0,1,2,3. The field ϕ, C_3 is respectively the dilaton and Ramond-Ramond 3-form in the type IIA superstring theory. Here ϵ_4, $\Omega_4 = 8\pi^2/3$ is the volume form, the volume of a unit S^4 and R refers to the radius of the bulk which relates to the string coupling g_s and string length l_s as $R^3 = \pi g_s N_c l_s^3$. Note that the direction x^4 is compactified on S^1 with a period δx^4 as $x^4 \sim x^4 + \delta x^4$, so, above the size δx^4, the supersymmetry is broken down in the low-energy effective theory on the D4-branes once the periodic and antiperiodic condition is imposed to the boson and fermion along S^1 [4]. In order to avoid the conical singularity at $U = U_{KK}$, we can define the Kaluza-Klein mass M_{KK} as,

$$M_{KK} = \frac{2\pi}{\delta x^4} = \frac{3U_{KK}^{1/2}}{2R^{3/2}}, \tag{2.2}$$

which specifies the dual theory is effectively four-dimensional confining Yang-Mills (YM) theory. By examining the dual theory on a probe D4-brane, the variables in terms of field theory can be expressed as,

$$\begin{aligned} R^3 &= \frac{1}{2}\frac{g_{\mathrm{YM}}^2 N_c l_s^2}{M_{KK}}, \qquad U_{KK} = \frac{2}{9}g_{\mathrm{YM}}^2 N_c M_{KK} l_s^2, \\ g_s &= \frac{1}{2\pi}\frac{g_{\mathrm{YM}}^2}{M_{KK} l_s}, \end{aligned} \tag{2.3}$$

where g_{YM} refers to the Yang-Mills coupling constant in the dual theory.

The N_f pairs of probe D8/$\overline{\text{D8}}$-branes embedded into the bulk geometry (2.1) are perpendicular and antipodal to the compactified direction x^4 as it is displayed in Fig. 1. The action of D8-brane is given by the Dirac-Born-Infeld (DBI) plus Wess-Zumino (WZ) action as

$$\begin{aligned} S_{\mathrm{D8}} &= S_{\mathrm{DBI}} + S_{\mathrm{WZ}}, \\ S_{\mathrm{DBI}} &= -T_8 \int_{\mathrm{D8}/\overline{\mathrm{D8}}} d^9x e^{-\phi}\mathrm{STr}\sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta})} \\ S_{\mathrm{WZ}} &= (2\pi\alpha')^3 T_8 \int_{\mathrm{D8}/\overline{\mathrm{D8}}} C_3 \mathrm{Tr}\mathcal{F}^3, \end{aligned} \tag{2.4}$$

where the index α, β runs over the D8-brane, $T_8 = (2\pi)^{-8} l_s^{-9}$ is the tension of the D8-brane and $\mathcal{F}$ refers to the $U(N_f)$ Yang-Mills gauge field strength on the D8-branes. Expand the DBI action up to quadratic term and integrate the WZ action by part, the action (2.4) becomes,

$$\begin{aligned} S_{\mathrm{D8}} &= S_{\mathrm{YM}}[\mathcal{A}] + S_{\mathrm{CS}}[\mathcal{A}], \\ S_{\mathrm{YM}}[\mathcal{A}] &= -\kappa \int d^4x dz \mathrm{Tr}\left[\frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2\right], \\ S_{\mathrm{CS}}[\mathcal{A}] &= \frac{N_c}{24\pi^2}\int \omega_5^{U(N_f)}(\mathcal{A}), \\ h(z) &= (1+z^2)^{-1/3}, \qquad k(z) = 1 + z^2 \end{aligned} \tag{2.5}$$

where the formulas are expressed in $M_{KK} = 1$ and the parameter κ is given as,

$$\kappa = a\lambda N_c, \qquad a = \frac{1}{216\pi^3}, \qquad \lambda = g_{\mathrm{YM}}^2 N_c. \tag{2.6}$$

We have used the dimensionless Cartesian coordinate z given by

$$U = U_{KK}(1+z^2)^{1/3}. \tag{2.7}$$

Here $\mathcal{A}$ refers to the $U(N_f)$ Yang-Mills gauge potential associated to $\mathcal{F}$ as $\mathcal{F} = d\mathcal{A} + i\mathcal{A}\wedge\mathcal{A}$ which does not have components along S^4 and is independent on S^4. The gauge Chern-Simons (CS) 5-form $\omega_5^{U(N_f)}(\mathcal{A})$ is given as,

$$\omega_5^{U(N_f)}(\mathcal{A}) = \mathrm{Tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right). \tag{2.8}$$

Therefore the following concern is to describe the baryon with the action presented in (2.5).

B. The classical instanton solution

According to the gauge-gravity duality, baryon in the D4–D8 model is recognized as the D4-brane wrapped on S^4 [39] presented in (2.1) which is illustrated in Fig. 1. On the other hand, by analyzing the charge of the D4- and D8-brane, in the D4–D8 model, baryon can be identified as the instanton configuration of the gauge field on the D8-branes [40]. Hence the instanton solution for the gauge field on D8-brane is the key to describe baryon in this model.

In order to obtain a low-energy solution representing a baryon for the gauge field on D8-branes, let us follow the steps in [19]. Specifically, we need an instanton solution for the D8-brane action (2.5) in the $1/\lambda$ expansion since the 't Hooft coupling λ is expected to be large in the dual theory. To carry out a systematic $1/\lambda$ expansion, we can rescale the coordinate $\{x^0, x^i, z\}$ and the gauge field $\mathcal{A}$ as,

$$\begin{aligned} x^M &\to \lambda^{-1/2}x^M, \qquad x^0 \to x^0 \\ \mathcal{A}_M &\to \lambda^{1/2}\mathcal{A}_M, \qquad \mathcal{A}_0 \to \mathcal{A}_0 \\ \mathcal{F}_{MN} &\to \lambda\mathcal{F}_{MN}, \qquad \mathcal{F}_{0M} \to \lambda^{1/2}\mathcal{F}_{0M}, \end{aligned} \tag{2.9}$$

where the indices denoted by capital letters M, N run over 1,2,3,z. Thus the Yang-Mills action in (2.5) can be written as,

$$\begin{aligned} S_{\mathrm{YM}} = &-aN_c \int d^4xdz \mathrm{Tr}\left[\frac{\lambda}{2}F_{MN}^2 \right. \\ &\left. + \left(-\frac{z^2}{6}F_{ij}^2 + z^2F_{iz}^2 - F_{0M}^2\right) + \mathcal{O}(\lambda^{-1})\right] \\ &- \frac{aN_c}{2}\int d^4xdz\left[\frac{\lambda}{2}\hat{F}_{MN}^2 \right. \\ &\left. + \left(-\frac{z^2}{6}\hat{F}_{ij}^2 + z^2\hat{F}_{iz}^2 - \hat{F}_{0M}^2\right) + \mathcal{O}(\lambda^{-1})\right], \end{aligned} \tag{2.10}$$

while the Chern-Simons action in (2.5) is invariant under this rescaling. Note that we have decomposed the $U(N_f)$ group as $U(N_f) \simeq U(1) \times SU(N_f)$ and correspondingly, the generator $\mathcal{A}$ of $U(N_f)$ is decomposed as,

$$\mathcal{A} = A + \frac{1}{\sqrt{2N_f}}\hat{A} = A^a t^a + \frac{1}{\sqrt{2N_f}}\hat{A}, \tag{2.11}$$

where $\hat{A}, A$ refers respectively to the generator of $U(1)$, $SU(N_f)$ and t^a $(a = 1, 2 \ldots N_f^2 - 1)$ are the normalized Hermitian bases of the $su(N_f)$ algebra satisfying

$$\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}. \tag{2.12}$$

In this convention, the Chern-Simons term in (2.5) can be derived as,

$$\begin{aligned} S_{\mathrm{CS}} = &\frac{N_c}{24\pi^2}\int \omega_5^{SU(N_f)}(A) \\ &+ \frac{N_c}{24\pi^2}\sqrt{\frac{2}{N_f}}\epsilon_{MNPQ}\int d^4xdz\left[\frac{3}{8}\hat{A}_0\mathrm{Tr}(F_{MN}F_{PQ}) \right. \\ &- \frac{3}{2}\hat{A}_M\mathrm{Tr}(\partial_0 A_N F_{PQ}) + \frac{3}{4}\hat{F}_{MN}\mathrm{Tr}(A_0 F_{PQ}) \\ &+ \frac{1}{16}\hat{A}_0\hat{F}_{MN}\hat{F}_{PQ} - \frac{1}{4}\hat{A}_M\hat{F}_{0N}\hat{F}_{PQ} \\ &\left. + (\text{total derivatives})\right]. \end{aligned} \tag{2.13}$$

Then the equations of motion can be obtained by varying the action (2.10) plus (2.13). For generic $N_f \geq 2$, the instanton solution can be obtained by employing the classical $SU(2)$ BPST solution as embeddable package [20] which is given as,

$$A_M^{\mathrm{cl}} = -if(\xi)g(x)\partial_M g^{-1}, \tag{2.14}$$

where

$$\begin{aligned} f(\xi) &= \frac{\xi^2}{\xi^2 + \rho^2}, \qquad \xi = \sqrt{(x^M - X^M)^2}, \\ g(x) &= \begin{pmatrix} g^{SU(2)}(x) & 0 \\ 0 & \mathbf{1}_{N_f - 2} \end{pmatrix}, g^{SU(2)}(x) \\ &= \frac{1}{\xi}[(z - Z)\mathbf{1}_2 - i(x^i - X^i)\tau^i]. \end{aligned} \tag{2.15}$$

We use $\mathbf{1}_N$ to denote the $N \times N$ identity matrix, and τ^i's are the Pauli matrices. The constants $X^M = \{X^i, Z\}$ and ρ refer respectively to the position and the size of the instanton which have already been rescaled as (2.9), hence the $U(1)$ part of the gauge field can be solved as,

$$\hat{A}_0^{\mathrm{cl}} = \sqrt{\frac{2}{N_f}}\frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2}\right], \quad \hat{A}_M^{\mathrm{cl}} = 0. \tag{2.16}$$

which leads to a nontrivial A_0 as,

$$A_0^{\mathrm{cl}} = \frac{1}{16\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2}\right]\left(\mathcal{P}_2 - \frac{2}{N_f}\mathbf{1}_{N_f}\right), \tag{2.17}$$

where $\mathcal{P}_2$ is a $N_f \times N_f$ matrix defined as $\mathcal{P}_2 = \mathrm{diag}(1, 1, 0, \ldots 0)$.

C. Lagrangian of the collective modes and baryon spectrum

In order to obtain the baryon spectrum, we need to derive the Lagrangian L of the collective coordinates $\mathcal{X}^\alpha$ in the moduli space of the one-instanton solution. For generic N_f, the collective coordinates $\mathcal{X}^\alpha$ consist of $\{X^M, \rho, y^a\}$, where $W = y^a t^a$ is the $SU(N_f)$ orientation of the instanton. The basic idea here is to approximate the classical soliton by slowly moving so that the collective coordinates $\mathcal{X}^\alpha$ are promoted to become time-dependent as $\mathcal{X}^\alpha(t)$ [41]. Thus the Lagrangian of the collective coordinates is expected to be the element of the world line with a potential in the moduli space as,

$$L(\mathcal{X}^\alpha) = \frac{m_X}{2}\mathcal{G}_{\alpha\beta}\dot{\mathcal{X}}^\alpha\dot{\mathcal{X}}^\beta - U(\mathcal{X}^\alpha) + \mathcal{O}(\lambda^{-1}), \tag{2.18}$$

where $\mathcal{G}_{\alpha\beta}$ refers to the metric of the moduli space and the potential $U(\mathcal{X}^\alpha)$ is the classical soliton mass given by $S[\mathcal{A}^{\mathrm{cl}}] = -\int dt U(\mathcal{X}^\alpha)$. By the approximation, the $SU(N_f)$ gauge field is also expected to be time-dependent by a gauge transformation,

$$\begin{aligned}
A_M(t,x) &= W(t)A_M^{\mathrm{cl}}(x,\mathcal{X}^\alpha)W(t)^{-1} - iW(t)\partial_M W(t)^{-1},\\
A_0(t,x) &= W(t)A_0^{\mathrm{cl}}(x,\mathcal{X}^\alpha)W(t)^{-1} + \Delta A_0,\\
\hat{A}_M(t,x) &= 0, \hat{A}_0(t,x) = \hat{A}_0^{\mathrm{cl}}(t,x),
\end{aligned} \tag{2.19}$$

where “cl” refers to the BPST instanton solution presented in Sec. II 2 with time-dependent $\mathcal{X}^\alpha(t)$ and the associated field strength becomes,

$$\begin{aligned}
F_{MN} &= W(t)F_{MN}^{\mathrm{cl}}W(t)^{-1},\\
F_{0M} &= W(t)\left(\dot{\mathcal{X}}^\alpha\frac{\partial}{\partial\mathcal{X}^\alpha}A_M^{\mathrm{cl}} - D_M^{\mathrm{cl}}\Sigma - D_M^{\mathrm{cl}}A_0^{\mathrm{cl}}\right)W(t)^{-1},\\
\hat{F}_{0M} &= \hat{F}_{0M}^{\mathrm{cl}}, \qquad \hat{F}_{MN} = \hat{F}_{MN}^{\mathrm{cl}},
\end{aligned} \tag{2.20}$$

where

$$\begin{aligned}
D_M^{\mathrm{cl}}A_0 &= \partial_M A_0 + i[A_M^{\mathrm{cl}}, A_0],\\
\Sigma &= W(t)^{-1}\Delta A_0 W(t) - i\dot{W}(t)^{-1}W(t).
\end{aligned} \tag{2.21}$$

Note that ΔA_0 must be determined by its equation of motion from (2.10) and (2.13) which is,

$$D_M^{\mathrm{cl}}\left(\dot{X}^N\frac{\partial}{\partial X^N}A_M^{\mathrm{cl}} + \dot{\rho}\frac{\partial}{\partial\rho}A_M^{\mathrm{cl}} - D_M^{\mathrm{cl}}\Sigma\right) = 0. \tag{2.22}$$

The exact solution for Σ can be found in [19,20]. Then the Lagrangian of the collective modes is given by

$$\begin{aligned}
S[\mathcal{A}] - S[\mathcal{A}^{\mathrm{cl}}] &= \int dt[L_{\mathrm{YM}}(\mathcal{X}^\alpha) + L_{\mathrm{CS}}(\mathcal{X}^\alpha)]\\
&= \int dt L(\mathcal{X}^\alpha)\\
S_{\mathrm{YM}}[\mathcal{A}] - S_{\mathrm{YM}}[\mathcal{A}^{\mathrm{cl}}] &= \int dt L_{\mathrm{YM}}(\mathcal{X}^\alpha),\\
S_{\mathrm{CS}}[\mathcal{A}] - S_{\mathrm{CS}}[\mathcal{A}^{\mathrm{cl}}] &= \int dt L_{\mathrm{CS}}(\mathcal{X}^\alpha).
\end{aligned} \tag{2.23}$$

Therefore we can obtain,

$$\begin{aligned}
L(\mathcal{X}^\alpha) &= -M + aN_c\mathrm{Tr}\int d^3x dz\bigg(\dot{X}^N F_{MN}^{\mathrm{cl}}\\
&\quad + \dot{\rho}\frac{\partial}{\partial\rho}A_M - \dot{X}^N D_M^{\mathrm{cl}}A_N^{\mathrm{cl}} - D_M^{\mathrm{cl}}\Sigma\bigg)^2 + \mathcal{O}(\lambda^{-1})\\
&= -M_0 + \frac{m_X}{2}\delta_{ij}\dot{X}^i\dot{X}^j + L_Z + L_\rho + L_{\rho W} + \mathcal{O}(\lambda^{-1}),
\end{aligned} \tag{2.24}$$

where

$$\begin{aligned}
L_Z &= \frac{m_Z}{2}(\dot{Z}^2 - \omega_Z^2 Z^2), \qquad L_\rho = \frac{m_\rho}{2}(\dot{\rho} - \omega_\rho^2\rho^2) - \frac{K}{m_\rho\rho^2},\\
L_{\rho W} &= \frac{m_\rho\rho^2}{2}\sum_a C_a[\mathrm{Tr}(-iW^{-1}\dot{W}t^a)]^2, a = 1,2 \ldots N_f^2 - 1
\end{aligned} \tag{2.25}$$

and

$$\begin{aligned}
M_0 &= 8\pi^2\kappa, \qquad m_X = m_Z = \frac{m_\rho}{2} = 8\pi^2\kappa\lambda^{-1},\\
K &= \frac{2}{5}N_c^2, \qquad \omega_Z^2 = 4\omega_\rho^2 = \frac{2}{3}.
\end{aligned} \tag{2.26}$$

Note that we have written the formulas in the unit $M_{KK} = 1$ and the metric of the moduli space can be obtained by comparing (2.24) with (2.18). C_a's are constants dependent on the $SU(N_f)$ instanton solution. For example, for $N_f = 2$, $C_{1,2,3} = 1$; for $N_f = 3$, $C_{1,2,3} = 1, C_{4,5,6,7} = 1/2$ and $C_8 = 0$. Accordingly, the collective modes, as baryon states, can be obtained by quantizing the Lagrangian (2.24), i.e., replace straightforwardly the derivative term by $\dot{X}^\alpha \to -\frac{i}{m_X}\partial_\alpha$. Afterwards, the quantized Hamiltonian associated to (2.24) is collected as,

$$\begin{aligned}
H &= M_0 + H_Z + H_\rho + H_{\rho W},\\
H_Z &= -\frac{1}{2m_Z}\partial_Z^2 + \frac{1}{2}m_Z\omega_Z^2 Z^2,\\
H_\rho &= -\frac{1}{2m_\rho}\frac{1}{\rho^\eta}\partial_\rho(\rho^\eta\partial_\rho) + \frac{1}{2}m_\rho\omega_\rho^2\rho^2 + \frac{K}{m_\rho\rho^2},\\
H_{\rho W} &= \frac{m_\rho\rho^2}{2}\sum_a C_a[\mathrm{Tr}(-iW^{-1}\dot{W}t^a)]^2 = \frac{2}{m_\rho\rho^2}\sum_a C_a(J^a)^2,
\end{aligned} \tag{2.27}$$

where $\eta = N_f^2 - 1$ and J^a's refer to the operators of the angular momentum of $SU(N_f)$. Therefore, the baryon spectrum can be obtained by evaluating the eigenvalues of the Hamiltonian (2.27).

III. CORRRECTIONS OF $\mathcal{O}(\lambda^{-1/3})$ TO THE HOLOGRAPHIC BARYON

As we have outlined that baryon in the D4–D8 model can be identified as the BPST instanton configuration on the D8-brane as its classical description, let us introduce a possible correction to the BPST solution presented in Sec. II as the deformed description of holographic baryon, then analyze the corrected baryon spectrum in this section.

A. Corrections to the classical solution

We start with equations of motion for the $SU(N_f)$ gauge fields A_0, A_M which are obtained by varying action (2.10) plus (2.13), as,

$$D_M F_{0M} + \frac{1}{64\pi^2 a}\sqrt{\frac{2}{N_f}}\epsilon_{MNPQ}\hat{F}_{MN}F_{PQ} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\left[F_{MN}F_{PQ} - \frac{1}{N_f}\mathrm{Tr}(F_{MN}F_{PQ})\right] + \mathcal{O}(\lambda^{-1}) = 0, \tag{3.1}$$

$$D_N F_{MN} + \mathcal{O}(\lambda^{-1}) = 0, \tag{3.2}$$

$$\partial_M \hat{F}_{0M} + \frac{1}{64\pi^2 a}\sqrt{\frac{2}{N_f}}\epsilon_{MNPQ}\left[\mathrm{Tr}(F_{MN}F_{PQ}) + \frac{1}{2}\hat{F}_{MN}\hat{F}_{PQ}\right] + \mathcal{O}(\lambda^{-1}) = 0, \tag{3.3}$$

$$\partial_N \hat{F}_{MN} + \mathcal{O}(\lambda^{-1}) = 0, \tag{3.4}$$

where the covariant derivative is defined as $D_M A_N = \partial_M A_N + i[A_M, A_N]$ in our convention. Then let us add the correction to the spatial part of $SU(N_f)$ BPST solution (2.14) first as,

$$\tilde{A}_M = A_M^{\mathrm{cl}} + \delta A_M, \qquad \tilde{F}_{MN} = F_{MN}^{\mathrm{cl}} + \delta F_{MN}, \tag{3.5}$$

where

$$\delta F_{MN} = D_M \delta A_N - D_N \delta A_M + i[\delta A_M, \delta A_N]. \tag{3.6}$$

In this sense, the equation of motion for $\tilde{A}_M$ takes the same formula as they are given in (3.2) after replacing D_M, F_{MN} by $\tilde{D}_M$, $\tilde{F}_{MN}$, so it leads to $\tilde{D}_N \tilde{F}_{MN} = 0$ or equivalently,

$$D_N D_N \delta A_M - 2i[F_{NM}^{\mathrm{cl}}, \delta A_N] = 0, \tag{3.7}$$

where we have imposed

$$D_N F_{MN}^{\mathrm{cl}} = 0, \qquad [D_N, D_M]\delta A_P = i[F_{NM}, \delta A_P]. \tag{3.8}$$

Besides, the instanton solution A_M^{cl} in (2.10) is gauged by $D_M A_M^{\mathrm{cl}} = 0$ which must remain as $\tilde{D}_M \tilde{A}_M = 0$. Thus we can obtain the gauge condition for δA_M as,

$$D_M \delta A_M = 0. \tag{3.9}$$

Solve the Eq. (3.7) with (3.9), we can obtain a solution for δA_M as,

$$\delta A_i = \frac{1}{2}\frac{B}{(\xi^2 + \rho^2)^2}\delta_{ij}t^j, \qquad \delta A_z = 0, \tag{3.10}$$

which is an embedding solution of the corrections to the $SU(2)$ case presented in [24]. The constant B must be determined by minimizing the classical Yang-Mills plus Chern-Simons action given in (2.10) (2.13).

Next, we need to solve the $U(1)$ part of Eq. (3.3) by picking up a correction $\delta\hat{A}_0$ and $\delta\hat{A}_M$. Due to $\hat{A}_M^{\mathrm{cl}} = 0$, we can simply choose

$$\delta\hat{A}_M = 0, \tag{3.11}$$

which leads to an equation for $\delta\hat{A}_0$ as,

$$\partial_M^2(\hat{A}_0^{\mathrm{cl}} + \delta\hat{A}_0) = \frac{1}{64\pi^2 a}\sqrt{\frac{2}{N_f}}\epsilon_{MNPQ}\mathrm{Tr}(\tilde{F}_{MN}\tilde{F}_{PQ}). \tag{3.12}$$

Using (2.14), (2.15), and (3.10), we can solve (3.12) as,

$$\begin{aligned}\delta\hat{A}_0 = -\sqrt{\frac{2}{N_f}}\Bigg\{&\frac{B}{32\pi^2 a}\frac{(z-Z)(3\rho^2+\xi^2)}{\rho^2(\rho^2+\xi^2)^3} \\ &+ \frac{B^2}{1536\pi^2 a}\frac{1}{\rho^4(\rho^2+\xi^2)^4}[9\rho^4 + \xi^4 - 4(z-Z)^2\xi^2 \\ &+ 4\rho^2\xi^2 - 16(z-Z)^2\rho^2] \\ &- \frac{B^3(z-Z)}{10240\pi^2 a\rho^8(\rho^2+\xi^2)^5}[11\rho^6 - 3\rho^2\xi^4 \\ &+ (\rho^2+\xi^2)^2(9\rho^2+2\xi^2)]\Bigg\}.\end{aligned} \tag{3.13}$$

Finally, by imposing (3.10)–(3.13) into (3.1), it leads to an equation for δA_0 as,

$$\tilde{D}_M \tilde{F}_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\left[\tilde{F}_{MN}\tilde{F}_{PQ} - \frac{1}{N_f}\mathrm{Tr}(\tilde{F}_{MN}\tilde{F}_{PQ})\right] = 0, \tag{3.14}$$

where its second part is calculated as,

$$\frac{1}{64\pi^2 a}\epsilon_{MNPQ}\left[\tilde{F}_{MN}\tilde{F}_{PQ}-\frac{1}{N_f}\mathrm{Tr}(\tilde{F}_{MN}\tilde{F}_{PQ})\right]$$
$$=\left[-\frac{3\rho^4}{2\pi^2 a(\xi^2+\rho^2)^4}+\frac{3(z-Z)\rho^2 B}{2\pi^2 a(\xi^2+\rho^2)^5}-\frac{10(z-Z)^2+2\xi^2-3\rho^2}{32\pi a^2(\xi^2+\rho^2)^6}B^2-\frac{3(z-Z)B^3}{64\pi^2 a(\xi^2+\rho^2)^7}\right]\left(\mathcal{P}_2-\frac{2}{N_f}\mathbf{1}_{N_f}\right). \tag{3.15}$$

Notice that

$$\tilde{D}_M\tilde{F}_{0M}=-D_M D_M A_0^{\mathrm{cl}}+D_M\delta F_{0M}+i[\delta A_M,F_{0M}^{\mathrm{cl}}]+i[\delta A_M,\delta F_{0M}]$$
$$\delta F_{0M}=-\partial_M\delta A_0+i[\delta A_0,A_M]+i[\delta A_0,\delta A_M], \tag{3.16}$$

so the Eq. (3.14) can be rewritten as,

$$D_M\delta F_{0M}+i[\delta A_M,F_{0M}^{\mathrm{cl}}]+i[\delta A_M,\delta F_{0M}]$$
$$=-\left[\frac{3(z-Z)\rho^2 B}{2\pi^2 a(\xi^2+\rho^2)^5}-\frac{10(z-Z)^2+2\xi^2-3\rho^2}{32\pi a^2(\xi^2+\rho^2)^6}B^2-\frac{3(z-Z)B^3}{64\pi^2 a(\xi^2+\rho^2)^7}\right]\left(\mathcal{P}_2-\frac{2}{N_f}\mathbf{1}_{N_f}\right). \tag{3.17}$$

It seems very difficult to search for a solution for (3.17) due to the presence of the commutators, however the instanton solution presented in Sec. II implies the commutation relationship $[A_0,A_M]=0$ and it must remain for $\tilde{A}_0$, $\tilde{A}_M$ as $[\tilde{A}_0,\tilde{A}_M]=0$. Therefore, all commutators should vanish in (3.17) which leads to the following ansatz for δA_0 as,

$$\delta A_0=Q(x^M)\left(\mathcal{P}_2-\frac{2}{N_f}\mathbf{1}_{N_f}\right). \tag{3.18}$$

In this sense, the Eq. (3.17) can be solved as,

$$\delta A_0=-\left\{\frac{B}{64\pi^2 a}\frac{(z-Z)(3\rho^2+\xi^2)}{\rho^2(\rho^2+\xi^2)^3}+\frac{B^2}{3072\pi^2 a}\frac{1}{\rho^4(\rho^2+\xi^2)^4}\times[9\rho^4+\xi^4-4(z-Z)^2\xi^2+4\rho^2\xi^2-16(z-Z)^2\rho^2]-\frac{B^3(z-Z)}{20480\pi^2 a\rho^8(\rho^2+\xi^2)^5}\times[11\rho^6-3\rho^2\xi^4+(\rho^2+\xi^2)^2(9\rho^2+2\xi^2)]\right\}\times\left(\mathcal{P}_2-\frac{2}{N_f}\mathbf{1}_{N_f}\right). \tag{3.19}$$

Afterwards, the classical mass of the soliton with the corrections $\delta\mathcal{A}$ can be evaluated by using $S[\mathcal{A}^{\mathrm{cl}}+\delta\mathcal{A}]=-\int(M+\Delta M)dt$, which, after some straightforward by very messy calculations, is,

$$M+\Delta M=\kappa\int d^3xdz\mathrm{Tr}\left[\frac{1}{2}\tilde{F}_{MN}^2+\lambda^{-1}\left(-\frac{z^2}{6}\tilde{F}_{ij}^2+z^2\tilde{F}_{iz}^2-\tilde{F}_{0M}^2\right)-\frac{\lambda^{-1}}{2}(\hat{F}_{0M}^{\mathrm{cl}}+\delta\hat{F}_{0M})^2\right]$$
$$-\frac{\kappa}{24\pi^2 a}\lambda^{-1}\int d^3xdz\epsilon_{MNPQ}\left[\sqrt{\frac{2}{N_f}}\frac{3}{8}(\hat{A}_0+\delta\hat{A}_0)\mathrm{Tr}(\tilde{F}_{MN}\tilde{F}_{PQ})+\frac{3}{4}\mathrm{Tr}(\tilde{A}_0\tilde{F}_{MN}\tilde{F}_{PQ})\right]+\mathcal{O}(\lambda^{-1})$$
$$=8\pi^2\kappa+\frac{\pi^2\kappa}{448\rho^{12}}B^4+\frac{8\pi^2\kappa}{\lambda}\left(\frac{\rho^2}{6}+\frac{Z^2}{3}+\frac{1}{320\pi^4 a^2\rho^2}-\frac{BZ}{12\rho^2}+\frac{3B^2}{640\rho^4}+\frac{B^2Z^2}{80\rho^6}-\frac{B^2}{7\times 2^{12}a^2\pi^4\rho^8}-\frac{B^3}{1920\rho^8}-\frac{B^4}{35\times 3\times 2^{10}\rho^{10}}-\frac{B^4Z^2}{21\times 2^9\rho^{12}}-\frac{11B^4}{7\times 3^3\times 2^{18}}\right), \tag{3.20}$$

which however is independent on N_f in our holographic setup and the terms of $\tilde{F}_{0M}^2,\frac{3}{4}\mathrm{Tr}(\tilde{A}_0\tilde{F}_{MN}\tilde{F}_{PQ})$ are absent in [24]. By minimizing (3.20), the constant B is obtained as,

$$B=4\times\left(\frac{7}{6}\right)^{1/3}Z^{1/3}\rho^{10/3}\lambda^{-1/3}+\mathcal{O}(\lambda^{-2/3}). \tag{3.21}$$

Thus $M,\Delta M$ is respectively evaluated as,

$$M=8\pi^2\kappa+\frac{8\pi^2\kappa}{\lambda}\left(\frac{\rho^2}{6}+\frac{Z^2}{3}+\frac{1}{320\pi^4 a^2\rho^2}\right),$$
$$\Delta M=-2\pi^2\kappa\lambda^{-4/3}\left(\frac{7}{6}\right)^{1/3}(\rho Z)^{4/3}, \tag{3.22}$$

B. Corrections to the baryon spectrum

In this section, picking up the corrections $\delta\mathcal{A}$ to the BPST solution, let us correct the baryon spectrum with $N_f=3$ for the realistic case.[2] As it is outlined in Sec. II, the Lagrangian of the collective modes can be obtained by

[2]Our calculation also covers the results in [24] where the corrected baryon spectrum with $N_f=2$ can be reviewed.

using (2.23), (2.24), thus the total quantized hamiltonian $H_{\rm tot}$ of the collective modes with the corrections can be obtained by repeating the calculations in Sec. II while we need to replace $\mathcal{A}^{\rm cl} \to \mathcal{A}^{\rm cl} + \delta\mathcal{A}, \mathcal{F}^{\rm cl} \to \mathcal{F}^{\rm cl} + \delta\mathcal{F}$ with $D_{0,M} \to \tilde{D}_{0,M}$. Resultantly, it leads to[3]

$$H_{\rm tot} = H + \Delta H + \mathcal{O}(\lambda^{-2/3}), \tag{3.23}$$

where, for $N_f = 3$,

$$\begin{aligned} H &= M_0 + H_Z + H_\rho \\ H_Z &= -\frac{1}{2m_Z}\partial_Z^2 + \frac{1}{2}m_Z\omega_Z^2 Z^2, \\ H_\rho &= -\frac{1}{2m_\rho}\frac{1}{\rho^8}\partial_\rho(\rho^8\partial_\rho) + \frac{1}{2}m_\rho\omega_\rho^2\rho^2 + \frac{K'}{m_\rho\rho^2}, \\ \Delta H &= \Delta M = -2\pi^2\kappa\lambda^{-4/3}\left(\frac{7}{6}\right)^{1/3}(\rho Z)^{4/3}. \end{aligned} \tag{3.24}$$

We have taken the (p, q) representation for the two $SU(3)_J$ and $SU(3)_I$ which refers respectively to the rotation and flavor (isospin) symmetry in the Hamiltonian as,

$$\begin{aligned} \sum_{a=1}^{8}(J_a)^2 &= \frac{1}{3}[p^2 + q^2 + qp + 3(p+q)], \\ \sum_{a=1}^{3}(J_a)^2 &= j(j+1), \\ K' &= \frac{N_c^2}{15} + \frac{4}{3}[p^2 + q^2 + qp + 3(p+q)] - 2j(j+1). \end{aligned} \tag{3.25}$$

Besides, for $N_f = 3$, we note that the baryon states with right spins should be selected by the constraint of the hypercharge,

$$J_8 = \frac{N_c}{2\sqrt{3}}, \tag{3.26}$$

from the Chern-Simons term. However the Chern-Simons term given in (2.5) is unable to reach this goal since $L_{\rm CS}$ in (2.23) would be vanished [20]. To figure out this problem, [20,48] proposed a new Chern-Simons term as,

$$S_{\rm CS}^{\rm new} = S_{\rm CS} + \frac{1}{10}\int_{N_5}{\rm Tr}(h^{-1}dh)^5 + \int_{\partial M_5}\alpha_4(dh, \mathcal{A}), \tag{3.27}$$

where $S_{\rm CS}$ refers to the Chern-Simons term given in (2.5), h is a $U(N_f)$ valued function and the 4-form α_4 is given as [48],

$$\alpha_4(dh, \mathcal{A}) = \frac{1}{2}{\rm Tr}\left[dh(\mathcal{A}^3 - \mathcal{A}\mathcal{F} - \mathcal{F}\mathcal{A}) + \frac{1}{2}dh\mathcal{A}dh\mathcal{A} + dh^3\mathcal{A}\right]. \tag{3.28}$$

Here, N_5 denotes a 5-dimensional manifold whose boundaries satisfies $\partial N_5 = \partial M_5 = M_{4,z=+\infty} - M_{4,z=-\infty}$ with the asymptotics of the gauge field on the D8-branes as,

$$\mathcal{A}|_{z\to\pm\infty} = h^\pm(d + \mathcal{A})h^{\pm -1}, h|_{\partial M_5} = (h^+, h^-), \tag{3.29}$$

where $\mathcal{A}$ is assumed to be regular on M_5 and produces no-boundary contributions. Accordingly, the constraint of the hypercharge (3.26) could be produced with the new Chern-Simons term (3.27).

Afterwards, the spectrum of the total Hamiltonian $H_{\rm tot}$ can be obtained approximately by solving the eigenequation of H with a perturbation ΔH given in (3.24) and the constraint (3.26). The eigenfunctions and values of H_Z are nothing but the eigenfunctions and values of harmonic oscillator while the eigenfunctions of H_ρ given by $\psi(\rho)$ could be solved as,

$$\begin{aligned} \psi(\rho) &= e^{-v/2}v^\beta\gamma(v), \qquad v = m_\rho\omega_\rho\rho^2, \\ \beta &= \frac{1}{4}\sqrt{(\eta-1)^2 + 8K'} - \frac{1}{4}(\eta - 1), \end{aligned} \tag{3.30}$$

where $\gamma(v)$ is hypergeometrical function satisfying the following hypergeometrical differential equation,

$$\left[v\frac{d^2}{dv^2} + \left(2\beta + \frac{\eta+1}{2} - v\right)\frac{d}{dv} + \left(\frac{E_\rho}{2\omega_\rho} - \beta - \frac{\eta+1}{4}\right)\right]\gamma(v) = 0. \tag{3.31}$$

So the eigenvalue E_ρ is solved as

$$E_\rho = \omega_\rho\left[2n_\rho + \frac{1}{2}\sqrt{(\eta-1)^2 + 8K'} + 1\right]. \tag{3.32}$$

Therefore the total spectrum E of H is given by (in the unit of M_{KK})

$$\begin{aligned} E = 8\pi^2\kappa &+ \omega_\rho\left[2n_\rho + \frac{1}{2}\sqrt{(\eta-1)^2 + 8K'} + 1\right] \\ &+ \omega_Z\left(n_Z + \frac{1}{2}\right), \qquad n_\rho, n_Z = 0, 1, 2, 3\ldots \end{aligned} \tag{3.33}$$

And using the standard method in quantum mechanics, the leading order correction to the spectrum (3.33) is given by

$$\Delta E = \langle \Delta H \rangle, \tag{3.34}$$

[3] By imposing our correction, ω_ρ may also contain a correction of $\mathcal{O}(\lambda^{-5/3})$ which has been neglected.

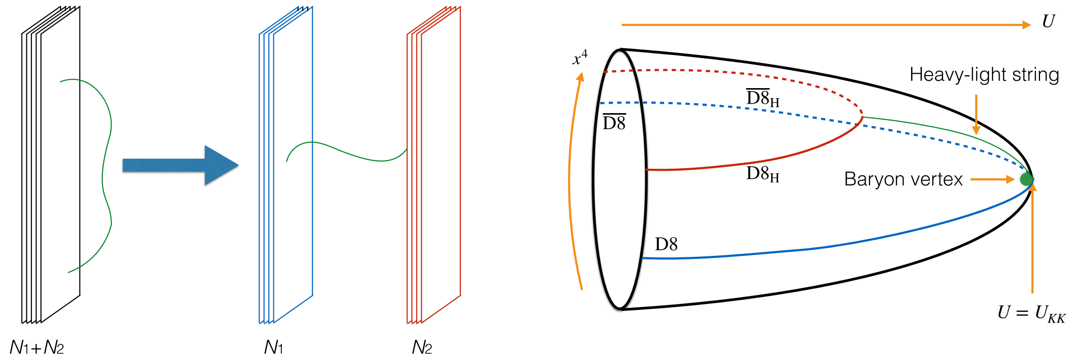

FIG. 2. Left: Higgs mechanism in string theory. A stack of $N_1 + N_2$ coincident D-branes move separately to become two stacks of N_1 and N_2 coincident D-branes. The $U(N_1 + N_2)$ gauge symmetry on the worldvolume breaks down into $U(N_1 \times U(N_2)$. The multiplets produced by the open string becomes massive. Right: Higgs mechanism in the D4–D8 model. The one pair of the heavy-avor D8/$\overline{\mathrm{D8}}$-branes are denoted by red which are separated from the other N_f pairs of the coincident D8/$\overline{\mathrm{D8}}$-branes (as light avors) denoted by blue. The heavy-light string is denoted by green. The multiplets produced by the heavy-light string becomes massive thus they can be identied as heavy-light mesons.

which leads to the approximated spectrum of H_{tot}. To simply compare our corrected baryon spectrum with the realistic QCD, we could set $N_c = 3$, so the constraint (3.26) requires that (p, q) must satisfy

$$p + 2q = 3 \times (\mathrm{integer}). \tag{3.35}$$

Therefore the allowed states with smaller (p, q), j and K' are given as,

$$(p,q) = (1,1), j = \frac{1}{2}, K' = \frac{111}{10}, (\mathrm{octet})$$
$$(p,q) = (3,0), j = \frac{3}{2}, K' = \frac{171}{10}, (\mathrm{decuplet})$$
$$(p,q) = (0,3), j = \frac{1}{2}, K' = \frac{231}{10}, (\mathrm{antidecuplet}). \tag{3.36}$$

Keeping above in mind, while the unit M_{KK} is not undetermined in this model, it is possible to compare the experimental data with our holographic baryon spectrum as $E + \Delta E$. In order to fit our baryon spectrum to the experimental data, we additionally notice that, on the other hand, the D4–D8 model is also able to give the meson spectrum which requests for the parameters $M_{KK} = 949$ MeV, $\lambda = 16.6$ (the only parameters in our theory) for a realistic matching [9]. Hence let us employ the same choice of M_{KK}, λ, as the meson data in this model, for the lowest octet and the decuplet or antidecuplet baryons $(n_\rho, n_Z) = (0,0)$, then the mass difference is evaluated with our corrections as,

$$M_{\mathbf{10}} - M_{\mathbf{8}} = 299.6 \ \mathrm{MeV},$$
$$M_{\mathbf{10}^*} - M_{\mathbf{8}} = 564.7 \ \mathrm{MeV}, \tag{3.37}$$

which is very close to the experimental data

$$M^{\mathrm{exp}}_{\mathbf{10}} - M^{\mathrm{exp}}_{\mathbf{8}} \simeq 292 \ \mathrm{MeV},$$
$$M^{\mathrm{exp}}_{\mathbf{10}^*} - M^{\mathrm{exp}}_{\mathbf{8}} \simeq 590.7 \ \mathrm{MeV}, \tag{3.38}$$

from the Particle Data Group (PDG) [49] and we have used the Θ^+ mass of 1530 MeV as the lowest antidecuplet baryon. As a comparison, we also list the mass difference of the lowest octet and the decuplet or antidecuplet baryons for same M_{KK}, λ without our $\mathcal{O}(\lambda^{-1/3})$ corrections as,

$$M_{\mathbf{10}} - M_{\mathbf{8}} = 366.6 \ \mathrm{MeV},$$
$$M_{\mathbf{10}^*} - M_{\mathbf{8}} = 688.0 \ \mathrm{MeV}, \tag{3.39}$$

which is coincident with [20] while it is quite far from the experimental data (3.38). In this sense, we believe our correction is reasonable to the framework of D4–D8 approach since both meson and baryon spectrum could be fit well.

IV. THE HEAVY FLAVOR

In this section, we will first outline how to include the heavy flavor in the D4–D8 model by employing the Higgs mechanism in string theory. Then let us obtain the heavy-light baryon spectrum with our corrections to the BPST instanton solution.

A. Higgs mechanism and the massive flavor in the D4–D8 model

Due to the vanished minimized size of the 4-8 string,[4] the fundamental quark in the D4–D8 model is massless [9]. So it is very necessary to include the heavy flavor in the D4–D8 model in order to describe quarks of heavy flavor.

[4]"4-8 string" refers to the open string connecting D4- and D8-branes.

To achieve this goal, let us follow the setup in [50,51] in which the Higgs mechanism in string theory is employed. Specifically, we can consider the configuration of two stacks of the separated D-branes with an open string connected them as it is illustrated in Fig. 2. In this configuration, the $U(N_1+N_2)$ symmetry on the world volume breaks down into $U(N_1)\times U(N_2)$ where N_1, N_2 refers to the number of the coincident D-branes in each stack. Thus the transverse modes of the D-brane acquire a nonzero vacuum expectation value (VEV) which is recognized as the separation of the D-branes. Therefore the multiplets produced by the open string connected the separated D-branes will be massive due to the VEV of the transverse modes [52,53], as the Higgs mechanism in the standard model of the particle physics.

Employing this configuration, another pair of D8/$\overline{\mathrm{D8}}$-branes as heavy flavor brane separated from the N_f coincident D8/$\overline{\mathrm{D8}}$-branes with an open string (heavy-light string) stretched between them can be introduced into the D4–D8 model. And this configuration can be identified as $N_1=N_f, N_2=1$ for a stack of D8/$\overline{\mathrm{D8}}$-branes as it is illustrated in Fig. 2, so that the gauge symmetry $U(N_f+1)$ on the D8/$\overline{\mathrm{D8}}$-branes breaks down to $U(N_f)\times U(1)$. Since the 8-8 string in the D4–D8 model is identified as meson, the massive multiplets produced by the open string (heavy-light string) stretched between the flavor branes is identified as the heavy-light meson. Besides, as our concern is to include the heavy flavor in baryon, we require that one endpoint of the heavy-light string is located at $U=U_{KK}$ where the baryon vertex lives in this model. Afterwards, the effective Lagrangian of the collective modes with heavy flavor can be obtained by following the steps in Sec. II. The notable derivation here is the Yang-Mills gauge field and its field strength now becomes an $(N_f+1)\times(N_f+1)$ matrix-valued field as,[5]

$$\mathcal{A}_\alpha\to\mathbf{A}_\alpha=\begin{pmatrix}\mathcal{A}_\alpha & \Phi_\alpha\\ \Phi_\alpha^\dagger & 0\end{pmatrix},$$
$$\mathcal{F}_{\alpha\beta}\to\mathbf{F}_{\alpha\beta}=\begin{pmatrix}\mathcal{F}_{\alpha\beta}+i\alpha_{\alpha\beta} & f_{\alpha\beta}\\ f_{\alpha\beta}^\dagger & i\beta_{\alpha\beta}\end{pmatrix}, \tag{4.1}$$

where $\mathcal{A}_\alpha$, $\mathcal{F}_{\alpha\beta}$ are $N_f\times N_f$ matrix-valued fields as we have specified in the previous sections. Φ_α is an $N_f\times 1$ matrix-valued field which is the multiplet created by the heavy-light string, i.e., the heavy-light meson field and[6]

$$\alpha_{\alpha\beta}=2\Phi_{[\alpha}\Phi_{\beta]}^\dagger,\qquad \beta_{\alpha\beta}=2\Phi_{[\alpha}^\dagger\Phi_{\beta]},$$
$$f_{\alpha\beta}=2\partial_{[\alpha}\Phi_{\beta]}+2iA_{[\alpha}\Phi_{\beta]}\equiv 2D_{[\alpha}\Phi_{\beta]}. \tag{4.2}$$

For non-Abelian excitation on the D-brane, the standard DBI action in (2.4) should include the dynamics of the transverse modes of the D-brane, which is given as (up to quadratic term),

$$S[\varphi^I]=-T_8\frac{(2\pi\alpha')^2}{4}\int d^9x\sqrt{-\det g}e^{-\phi}\mathrm{Tr}\{2D_\alpha\varphi^I D_\alpha\varphi^I + [\varphi^I,\varphi^J]^2\}, \tag{4.3}$$

where the index I, J runs over the transverse space of the D-brane. For the setup in the D4–D8 model with heavy flavor, φ^I is an $(N_f+1)\times(N_f+1)$ matrix-valued field with the covariant derivative $D_\alpha\varphi^I=\partial_\alpha\varphi^I+i[\mathbf{A}_\alpha,\varphi^I]$ and the transverse coordinate of the D8-brane consists only of x^4 so that $(2\pi\alpha')\varphi^I\to x^4$ is the only T-dualitied transverse coordinate of the D8-brane. According to [52,53], the moduli solution by the extrema of the potential contribution can be given by $[x^4,[x^4,x^4]]=0$, thus the moduli solution of x^4 for N_f D8/$\overline{\mathrm{D8}}$-branes separated from one pair of heavy-flavored D8/$\overline{\mathrm{D8}}$-branes can be chosen as,

$$\frac{x^4}{2\pi l_s}=\begin{pmatrix}-\frac{v}{N_f}\mathbf{1}_{N_f} & 0\\ 0 & v\end{pmatrix}, \tag{4.4}$$

where v refers to the VEV of x^4, which is proportional to the separation of the D8-branes in Fig. 2. Imposing (4.4) into (4.3), we can obtain a mass term for the heavy-light field Φ_α as [54],

$$S[x^4]=-\tilde{T}v^2\frac{(N_f+1)^2}{N_f^2}\int d^4xdzU^2(z)\times(g^{zz}\Phi_z^\dagger\Phi_z+g^{\mu\nu}\Phi_\mu^\dagger\Phi_\nu), \tag{4.5}$$

where $\tilde{T}=\frac{2}{3}T_8R^{3/2}U_{KK}^{1/2}\Omega_4g_s^{-1}$. Therefore it is clear that if the heavy-flavored D8/$\overline{\mathrm{D8}}$-brane is coincident to the N_f D8/$\overline{\mathrm{D8}}$-branes, i.e., $v=0$, the heavy-light field Φ_α becomes massless so that the $U(N_1+N_2)$ symmetry becomes restored, which means the action for x^4 (4.3) would be absent in the DBI action given in (2.4), as it is in the original model.

B. Corrections to the heavy-light baryon spectrum

Impose the replacement (4.1) into (2.10), and (2.13), one can obtain the effective action for the heavy-light field Φ_α in the large λ limit as (up to quadratic order of Φ_α),

[5]The last element in $\mathbf{A}_\alpha$ can be gauged to be zero by the gauge symmetry.

[6]In our notation, the index in the square brackets is ranked as $T_{[\alpha\beta]}=\frac{1}{2!}(T_{\alpha\beta}-T_{\beta\alpha})$. And the gauge field is Hermitian $\mathbf{A}_\alpha^\dagger=\mathbf{A}_\alpha$.

$$
\begin{aligned}
\mathcal{L}_H[\Phi_\alpha] &= aN_c\lambda\mathcal{L}_0[\Phi_\alpha] + aN_c\mathcal{L}_1[\Phi_\alpha] + \mathcal{L}_{\mathrm{CS}}[\Phi_\alpha] + \mathcal{O}(\lambda^{-1}),\\
\mathcal{L}_0[\Phi_\alpha] &= -(D_M\Phi_N - D_N\Phi_M)^\dagger(D_M\Phi_N - D_N\Phi_M)\\
&\quad + 2i\Phi_M^\dagger\mathcal{F}_{MN}\Phi_N,\\
\mathcal{L}_1[\Phi_\alpha] &= 2(D_0\Phi_M - D_M\Phi_0)^\dagger(D_0\Phi_M - D_M\Phi_0)\\
&\quad - 2i\Phi_0^\dagger\mathcal{F}^{0M}\Phi_M - 2i\Phi_M^\dagger\mathcal{F}^{M0}\Phi_0\\
&\quad + \frac{z^2}{3}(D_i\Phi_j - D_j\Phi_i)^\dagger(D_i\Phi_j - D_j\Phi_i)\\
&\quad - 2z^2(D_i\Phi_z - D_z\Phi_i)^\dagger(D_i\Phi_z - D_z\Phi_i)\\
&\quad - \frac{2i}{3}z^2\Phi_i^\dagger\mathcal{F}_{ij}\Phi_j - 2m_H^2\Phi_M^\dagger\Phi_M,
\end{aligned} \tag{4.6}
$$

and the CS term is

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CS}}[\Phi_\alpha] &= -\frac{N_c}{24\pi^2}(d\Phi^\dagger\mathcal{A}d\Phi + d\Phi^\dagger d\mathcal{A}\Phi + \Phi^\dagger d\mathcal{A}d\Phi)\\
&\quad + \frac{iN_c}{16\pi^2}(d\Phi^\dagger\mathcal{A}^2\Phi + \Phi^\dagger\mathcal{A}^2 d\Phi\\
&\quad + \Phi^\dagger\mathcal{A}d\mathcal{A}\Phi + \Phi^\dagger d\mathcal{A}\mathcal{A}\Phi)\\
&\quad + \frac{5N_c}{48\pi^2}\Phi^\dagger\mathcal{A}^3\Phi + \mathcal{O}(\Phi^4,\mathcal{A}),
\end{aligned} \tag{4.7}
$$

where the parameter m_H is the energy scale of the heavy flavor obtained by normalizing the mass term in (4.6) as $m_H = \frac{2}{3\sqrt{3}}\frac{N_f+1}{N_f}v$ and the associated equations of motion are obtained as,

$$
\begin{aligned}
&D_M D_M\Phi_N - D_N D_M\Phi_M + 2i\mathcal{F}_{MN}\Phi_M + \mathcal{O}(\lambda^{-1}) = 0,\\
&D_M(D_0\Phi_M - D_M\Phi_0) - i\mathcal{F}^{0M}\Phi_M\\
&\quad - \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\mathcal{K}_{MNPQ} + \mathcal{O}(\lambda^{-1}) = 0,
\end{aligned} \tag{4.8}
$$

where

$$
\begin{aligned}
\mathcal{K}_{MNPQ} &= i\partial_M\mathcal{A}_N\partial_P\Phi_Q - \mathcal{A}_M\mathcal{A}_N\partial_P\Phi_Q\\
&\quad - \partial_M\mathcal{A}_N\mathcal{A}_P\Phi_Q - \frac{5i}{6}\mathcal{A}_M\mathcal{A}_N\mathcal{A}_P\Phi_Q.
\end{aligned} \tag{4.9}
$$

The above equation of motion refers to the static wave function of heavy baryon which can be solved as $\Phi_\alpha = e^{\pm i m_H t}\phi_\alpha(x)$[7] as,

$$
\begin{aligned}
\phi_0 &= -\frac{1}{1024a\pi^2}\left[\frac{25\rho}{2(x^2+\rho^2)^{5/2}} + \frac{7}{\rho(x^2+\rho^2)^{3/2}}\right]\chi,\\
\phi_M &= \frac{\rho}{(x^2+\rho^2)^{3/2}}\sigma_M\chi,
\end{aligned} \tag{4.10}
$$

where χ refers to the $SU(N_f)$ spinor independent on x and σ_M is the embedded Pauli matrices as $\sigma_M/2 = (t_i, -\mathbf{1}_{N_f})$.

[7]The solution for Φ_α may contain a contribution of $\mathcal{O}(\lambda^{-2/3})$ when we use $\mathcal{A} = \mathcal{A}^{\mathrm{cl}} + \delta\mathcal{A}$. So it has been neglected since our concern is the correction of $\mathcal{O}(\lambda^{-1/3})$.

Then follow the steps in Sec. II and [19,22,23] with our corrections of the BPST solution, we could take the limit $m_H \to \infty$ to display mostly the contribution of the heavy flavor and simplify the calculation. So in the double limit $\lambda, m_H \to \infty$, the quantized Hamiltonian of the collective modes with heavy flavor is finally calculated with (4.10) as,

$$
\begin{aligned}
H_{\mathrm{HL}} &= H(\mathbf{K}) + (N_Q - N_{\bar{Q}})m_H + \Delta H + \mathcal{O}(\lambda^{-2/3}),\\
\mathbf{K} &= \frac{2N_c^2}{5}\left[1 - \frac{5\sqrt{6}+10}{6}\frac{N_Q - N_{\bar{Q}}}{N_c} + \frac{65}{36}\frac{(N_Q - N_{\bar{Q}})^2}{N_c^2}\right]\\
&\quad - \frac{N_c^2}{3}\left(1 - \frac{N_Q - N_{\bar{Q}}}{N_c}\right)^2\\
&\quad + \frac{4}{3}(p^2 + q^2 + pq) + 4(p+q) - 2j(j+1),
\end{aligned} \tag{4.11}
$$

where $N_Q, N_{\bar{Q}}, H(\mathbf{K})$ refers respectively to the numbers of the heavy flavor, antiheavy flavor and the Hamiltonian H in (3.24) by replacing K' to $\mathbf{K}$. Note that we have expressed all the formulas in the unit of M_{KK} which means m_H has been rescaled dimensionlessly as $m_H \to m_H M_{KK}$. Since the eigenfunctions and spectrum can be obtained by replacing K' to $\mathbf{K}$ in (3.30) (3.33), the corrections to the heavy-light spectrum can also be evaluated by using the standard method of quantum mechanics with ΔH as a perturbation.

Keeping these in hand, let us attempt to fit the experimental data of the heavy-light baryon. The lowest baryons with one heavy quark are characterized by $n_\rho = 0, 1$, $N_Q = 1$, $N_{\bar{Q}} = 0$ and $(p,q,j) = (0,1,0)$ for $\bar{\mathbf{3}}$ representation, $(p,q,j) = (2,0,1)$ for $\mathbf{6}$ representation due to their spin-1/2. The spin and parity of $\bar{\mathbf{3}}$ representation is $\frac{1}{2}^+$, so we can identify them as $\Lambda, \Xi(\bar{\mathbf{3}})$. The spin and parity of $\mathbf{6}$ representation is $J = \frac{1}{2}, \frac{3}{2}$, so we can identify them as $\Sigma, \Xi(\mathbf{6}), \Omega$ or $\Sigma^*, \Xi(\mathbf{6}), \Omega$. The parity of the baryon state can be identified as $(-1)^{n_Z}$ corresponding to the parity of the eigenfunction of H_Z in the holographic direction. Therefore, fitting the lowest $\bar{\mathbf{3}}$ representation by using the data of Particle Data Group $M^{\mathrm{exp}}_{\Lambda_c^+} \simeq 2286$ MeV, our

TABLE I. Mass spectrum of the lowest baryons with a single heavy flavor $N_Q = 1$, $N_{\bar{Q}} = 0$. The values of M and $\bar{M}$ are computed by the D4–D8 model with and without our $\mathcal{O}(\lambda^{1-/3})$ corrections respectively while M^{exp} refers to the corresponding experimental data. The parameter is set as $N_c = N_f = 3$ for realistic QCD and $M_{KK} = 949$ MeV, $\lambda = 16.6$ as the meson data in the D4–D8 model.

(MeV)	$\Lambda_c^+\,(\bar{\mathbf{3}})$	$\Xi_c^+\,(\bar{\mathbf{3}})$	$\Sigma_c^+\,(\mathbf{6})$	$\Xi_c^{\prime+}\,(\mathbf{6})$	$\Omega_c^0\,(\mathbf{6})$
$\bar{M}$	2286	3836	2603	4153	4928
M	2286	2451	2541	2567	2953
M^{exp}	2286	2468	2453	2576	2697

calculation reveals the mass of the lowest $\bar{\mathbf{3}}$ and $\mathbf{6}$ baryon with our corrections is very closed to the experimental data as it is illustrated in Table I. As a comparison, we also list the heavy-light baryon spectrum which is evaluated without our $\mathcal{O}(\lambda^{1-/3})$ corrections as the values of $\bar{M}$ in Table I. The notable point here is, our corrections greatly improve the heavy-light baryon spectrum when the meson data in this model as $M_{KK} = 949$ MeV, $\lambda = 16.6$ [9,10] is employed, so that the framework of the D4–D8 model can fit both the meson and baryon spectra to the experimental data, thus this model may become more consistent with our corrections.

V. THE INTERACTION OF GLUEBALL AND BARYONIC MATTERS

In this section, let us include the interaction of glueball and baryonic matters in the D4–D8 model with our corrections. We first outline the identification of glueball as the gravitational polarization in the bulk, then specify the interaction of glueball and baryonic matters with our corrections to the BPST solution.

A. Glueball as the gravitational polarization

According to gauge-gravity, the glueball field can be identified as the gravitational fluctuation in the D4–D8 model [25–29]. The basic idea is that as the bulk gravitational fluctuation is sourced by the operators in the dual field theory and the background geometry is produced by N_c D4-branes as colors, so the mass spectrum of the operators can be obtained by evaluating the pole of its correlation functions. Since the bulk geometry is dual to the pure Yang-Mills theory in holography, the operator, which sources the bulk gravitational fluctuation, must relate to the energy-momentum tensor of Yang-Mills theory thus it is gauge invariant. So this operator can be naturally identified as glueball and its spectrum is therefore identified as the mass of glueball in this model.

Recall the relation of the type IIA supergravity with N_c D4-branes solution and M-theory on $\mathrm{AdS}_7 \times S^4$ [4], the generic formulas of the gravitational fluctuations in the D4–D8 model can be chosen as the 11d gravitational polarization on AdS_7 given by [26–28],

$$\begin{aligned}
\delta G_{44} &= -\frac{r^2}{L^2} f(r) H_G(r) G(x),\\
\delta G_{\mu\nu} &= \frac{r^2}{L^2} H_G(r)\left[\frac{1}{4}\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3r_{KK}^6}{5r^6 - 2r_{KK}^6}\right)\frac{\partial_\mu\partial_\nu}{m_G^2}\right] G(x),\\
\delta G_{11,11} &= \frac{r^2}{4L^2} H_G(r) G(x),\\
\delta G_{rr} &= -\frac{L^2}{r^2}\frac{1}{f(r)}\frac{3r_{KK}^6}{5r^6 - 2r_{KK}^6} H_G(r) G(x),\\
\delta G_{r\mu} &= \frac{90 r^7 r_{KK}^6}{m_G^2 L^2 (5r^6 - 2r_{KK}^6)^2} H_G(r)\partial_\mu G(x),
\end{aligned} \tag{5.1}$$

where x refers to the coordinates $x^{0,1,2,3}$ in 4d spacetime, r is the radial coordinate in the holographic direction, m_G is the mass of the glueball. The 11d variables are related to the type IIA supergravity solution (2.1) by,

$$L = 2R, \qquad U = \frac{r^2}{2L}, \quad 1 + z^2 = \frac{r^6}{r_{KK}^6} = \frac{U^3}{U_{KK^3}}. \tag{5.2}$$

Perform the dimension reduction, the 10d metric (2.1) involving the 11d gravitational polarization (5.1) is given as,

$$\begin{aligned}
g_{\mu\nu} &= \frac{r^3}{L^3}\left[\left(1 + \frac{L^2}{2r^2}\delta G_{11,11}\right)\eta_{\mu\nu} + \frac{L^2}{r^2}\delta G_{\mu\nu}\right],\\
g_{44} &= \frac{r^3 f}{L^3}\left[1 + \frac{L^2}{2r^2}\delta G_{11,11} + \frac{L^2}{r^2 f}\delta G_{44}\right],\\
g_{rr} &= \frac{L}{rf}\left(1 + \frac{L^2}{2r^2}\delta G_{11,11} + \frac{r^2 f}{L^2}\delta G_{rr}\right),\\
g_{r\mu} &= \frac{r}{L}\delta G_{r\mu}, \qquad g_{\Omega\Omega} = \frac{r}{L}\left(\frac{L}{2}\right)^2\left(1 + \frac{L^2}{2r^2}\delta G_{11,11}\right),
\end{aligned} \tag{5.3}$$

with the dilaton,

$$e^{4\phi/3} = g_s^{4/3}\frac{r^2}{L^2}\left(1 + \frac{L^2}{r^2}\delta G_{11,11}\right). \tag{5.4}$$

Here and $H_G(r)$ is determined by the eigenequation

$$\begin{aligned}
&\frac{1}{r^3}\frac{d}{dr}\left[r(r^6 - r_{KK}^6)\frac{d}{dr}H_G(r)\right]\\
&\quad + \left[\frac{432 r^2 r_{KK}^{12}}{(5r^6 - 2r_{KK}^6)^2} + L^4 m_G^2\right] H_G(r) = 0,
\end{aligned} \tag{5.5}$$

where m_G is the eigenvalue. Impose (5.1) and (5.5) to the 11d gravity action for $\mathrm{AdS}_7 \times S^4$, we can obtain

$$\begin{aligned}
S_{11\mathrm{D}} &= \frac{1}{2\kappa_{11}^2}\left(\frac{L}{2}\right)^4 \Omega_4 \int d^7x\sqrt{-\det G}\left(\mathcal{R}_{11\mathrm{D}} + \frac{30}{L^2}\right)\\
&= \frac{1}{2}\int d^4x[(\partial_\mu G)^2 + m_G^2 G^2],
\end{aligned} \tag{5.6}$$

thus $G(x)$ can be identified as the massive scalar glueball field in this model, specifically, it refers to the lowest state of $J^{PC} = 0^{++}$. The eigenequation is numerically solve in [26–28] which leads to the eigenvalues of the glueball in holography as it is given in Table II.

TABLE II. The glueball mass spectrum in the D4–D8 model in the unit of $M_{KK} = 1$.

Excitation	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
Glueball mass m_G	0.901	2.285	3.240	4.149	5.041

B. Time-dependent perturbation for the collective modes

In order to include the interaction of glueball and baryonic matters, we need to derive Hamiltonian of the collective modes with the gravitational fluctuations and the steps we should follow has been given in Sec. II. Before this, we need to obtain the exact formula of the function $H_G(r)$ which is determined by (5.5). Fortunately, $H_G(r)$ can be solved analytically in the large λ expansion satisfying,

$$H_G''(z) + \left(\frac{1}{z} + \frac{2z}{\lambda}\right)H_G'(z) + \left(\frac{16}{3\lambda} + \frac{m_G^2}{\lambda}\right)H_G(z) + \mathcal{O}(\lambda^{-2}) = 0, \tag{5.7}$$

where we have expressed the formulas on z coordinate (2.7), imposed the rescaling (2.9) and rescaled m_G dimensionlessly as $m_G \to m_G M_{KK}$. Regularly, the solution to Eq. (5.7) is obtained as a hypergeometric function. In the large λ, we have

$$H_G(z) = \frac{\mathcal{C}}{M_{KK}}\left[1 - \frac{16+3m_G}{12\lambda}z^2 + \mathcal{O}(\lambda^{-2})\right], \tag{5.8}$$

where $\mathcal{C}$ is an integration constant. As a bulk fluctuation, the constant $\mathcal{C}$ should satisfy $\mathcal{C} \ll 1$. Picking up the gravitational fluctuations (5.3) (5.4), in the large λ expansion, the dilaton and the inverse of the induced metric on the D8-branes with gravitational fluctuations are calculated with the rescaling (2.9) as [up to $\mathcal{O}(\lambda^{-1})$],

$$\begin{aligned}
g^{\mu\nu} &= \frac{27}{8M_{KK}^3R^3}\left(1 - \frac{z^2}{2\lambda}\right)\eta^{\mu\nu} + \frac{\mathcal{C}}{M_{KK}^3R^3}\left[\frac{135}{32m_G^2}\frac{\partial^\mu\partial^\nu G(x)}{M_{KK}^2} - \frac{81}{64}G(x)\eta^{\mu\nu}\right. \\
&\quad \left. + \frac{27(22+3m_G)G(x)}{256}\frac{z^2}{\lambda}\eta^{\mu\nu} - \frac{45(38+3m_G)}{128m_G^2}\frac{\partial^\mu\partial^\nu G(x)}{M_{KK}^2}\frac{z^2}{\lambda}\right], \\
g^{zz} &= \frac{27}{8M_{KK}R^3}\left(1 + \frac{5z^2}{6\lambda}\right) + \frac{\mathcal{C}}{M_{KK}R^3}\left[\frac{189}{64} - \frac{9(202+21m_G)}{256}\frac{z^2}{\lambda}\right]G(x), \\
g^{\mu z} &= -\frac{45}{4m_G^2M_{KK}^2R^3}\frac{\partial^\mu G(x)}{M_{KK}}\frac{z}{\sqrt{\lambda}}\mathcal{C}, \\
e^{-\phi} &= \frac{3}{8}\sqrt{\frac{3}{2}}\left(4 - \frac{z^2}{\lambda}\right)g_s^{-1}M_{KK}^{3/2}R^{3/2} + \frac{3}{128}\sqrt{\frac{3}{2}}\left[-12 + \frac{19+3m_G}{\lambda}z^2\right]g_s^{-1}M_{KK}^{3/2}R^{3/2},
\end{aligned} \tag{5.9}$$

where we have additionally rescaled $G(x)$ as $G(x) \to G(x)M_{KK}$ so that $G(x)$ is dimensionless glueball field in the formulas. Note that $G(x)$ satisfies the equation of motion from action (5.6) which accordingly refers to the wave function for a free glueball as,

$$G(x) = \frac{1}{2}\left(e^{-ik_\mu x^\mu} + e^{ik_\mu x^\mu}\right), \tag{5.10}$$

thus it should remain under the rescaling (2.9), and so does $\partial_\mu G(x)$, $\partial_\mu\partial_\nu G(x)$ since the derivatives relate to the momentum k_μ of the glueball.

Then we insert the metric with the fluctuation (5.3) into the DBI action (2.4) up to quadratic term as,

$$\begin{aligned}
S_{\mathrm{DBI}} &= -T_8\int_{\mathrm{D8}/\overline{\mathrm{D8}}} d^9x e^{-\phi}\mathrm{STr}\sqrt{-\det\left(g_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta}\right)} \\
&= -T_8\Omega_4\int d^5x e^{-\phi}\sqrt{-\det g_{ab}}g_{\Omega\Omega}^2 \\
&\quad \times \left[1 + \frac{1}{4}(2\pi\alpha')^2\mathcal{F}_{ab}\mathcal{F}^{ab} + \ldots\right],
\end{aligned} \tag{5.11}$$

where the index a, b runs over $0, 1, 2, 3, z$. By imposing the rescaling (2.9), the effective Yang-Mills action is obtained as,

$$\begin{aligned}
S_{\mathrm{YM}} &= -\frac{1}{4}(2\pi\alpha')^2T_8\Omega_4\int d^5x e^{-\phi}\sqrt{-\det g_{ab}}g_{\Omega\Omega}^2\mathcal{F}_{ab}\mathcal{F}^{ab} \\
&\to -\frac{1}{4}(2\pi\alpha')^2T_8\Omega_4\int d^5x e^{-\phi}\sqrt{-\det g_{ab}}g_{\Omega\Omega}^2 \\
&\quad \times \left(2\lambda^{-1}\mathcal{F}_{0M}\mathcal{F}_{0N}g^{00}g^{MN} + 2\lambda^{-1}\mathcal{F}_{0N}\mathcal{F}_{M0}g^{0M}g^{N0}\right. \\
&\quad \left. + 4\lambda^{-1/2}\mathcal{F}_{0K}\mathcal{F}_{MN}g^{0M}g^{KN} + \mathcal{F}_{MN}\mathcal{F}_{KL}g^{MK}g^{NL}\right),
\end{aligned} \tag{5.12}$$

where the index M, N, K, L runs over $1, 2, 3, z$. The metric presented in (5.12) has been given in (5.3) and (5.9). So while the kinetic terms for the collective modes remain as they are given in Sec. II, the potential for the collective modes depending on the classical mass M of the soliton would be obtained from the onshell action (5.12) by recalling $S_{\mathrm{onshell}} = -\int M_{\mathrm{soliton}}dt$. Note the Chern-Simons term in (2.13) is independent on the metric, thus it is decoupled to metric fluctuations (5.1). Hence the classical soliton mass can be obtained after the metric, dilaton with the fluctuations (5.3) (5.9) and the instanton solution $\mathcal{A}^{\mathrm{cl}}$, $\delta\mathcal{A}$ presented in Sec. III are all plugged into (5.12). Although this calculation is very straightforward, the final result would be tediously messy. In order to get a compact result, let us consider the situation that the glueball is static or we are in the rest frame of the glueball. In this sense, the momentum of the glueball becomes $k_\mu = (m_G, \mathbf{0})$ (in the unit $M_{KK} = 1$), so that we have $\partial_0 \to -im_G$, $\partial_i \to 0$, $G(x) = G(t) = (e^{-im_G t} + e^{im_G t})/2$. Then (5.12) can be simplified in the leading order of δg_{ab}, as

$$S_{\mathrm{YM}} = S_{\mathrm{YM}}^{\mathrm{onshell}} + \delta S_{\mathrm{YM}}^{\mathrm{onshell}},$$

where $S_{\mathrm{YM}}^{\mathrm{onshell}}$ refers to the action in (2.10) and $\delta S_{\mathrm{YM}}^{\mathrm{onshell}}$ is the leading-order coupling term of the gauge field and bulk fluctuations calculated as,

$$\begin{aligned}\delta S_{\mathrm{YM}}^{\mathrm{onshell}} = \mathcal{C}\kappa \int d^4x dz \Bigg\{ & \Bigg[\frac{11}{32}\tilde{F}_{MN}^2 - \frac{5}{4}\tilde{F}_{iz}^2 + \frac{29}{16\lambda}\tilde{F}_{0M}^2 \\ & - \frac{5}{4\lambda}\tilde{F}_{0i}^2 + \left(\frac{89}{48} + \frac{9}{64}m_G\right)\frac{z^2}{\lambda}\tilde{F}_{iz}^2 \\ & - \left(\frac{55}{96} + \frac{11}{128}m_G\right)\frac{z^2}{\lambda}\tilde{F}_{ij}^2 + \frac{20z}{3m_G^2\lambda}F_{0i}F_{zi}\partial_0\Bigg]G(t) \\ & + \mathcal{C}\kappa\left[\frac{9}{32\lambda}(\hat{F}_{0M}^{\mathrm{cl}} + \delta F_{0M})^2 + \frac{5}{8\lambda}(\hat{F}_{0z}^{\mathrm{cl}} + \delta F_{0z})^2\right] \\ & \times G(t)\Bigg\}. \end{aligned} \tag{5.13}$$

After the integrating over x^i, z and using $\delta S_{\mathrm{YM}}^{\mathrm{onshell}} = -\int \Delta M_L dt$, we can obtain a fluctuation of the soliton mass as,

$$\begin{aligned}\Delta M_L = -\mathcal{C}\kappa\pi^2 G(t)\Bigg\{ & \left[\frac{1}{2} + \frac{11}{24}\left(\frac{7}{6}\right)^{1/3}(\rho Z)^{4/3}\lambda^{-4/3} \right.\\ & \left. - \frac{3^{1/3}7^{2/3}}{2^{5/3}}(Z\rho)^{2/3}\lambda^{-2/3}\right] \\ & + \lambda^{-1}\left(\frac{17}{12} - \frac{1}{16}m_G\right)(2Z^2 + \rho^2) + \frac{7}{320a^2\pi^4\rho^2\lambda}\Bigg\}, \end{aligned} \tag{5.14}$$

which implies a time-dependent term $\Delta H_L(t) = \Delta M_L(t)$ in Hamiltonian (2.27) would be presented when the bulk gravitational fluctuations are taken into account. Here we use subscript "L" in $\Delta M_L(t)$ to refer to that there is no contribution of heavy flavor to $\Delta M_L(t)$. As the bulk gravitational fluctuations are identified as the glueball field, the interaction of glueball and baryonic matters can be naturally included once the time-dependent term $\Delta H_L(t)$ is added to (2.27). And the decay of the baryonic matters involving glueball can be therefore evaluated with this time-dependent Hamiltonian.

Besides, we can further include the contributions of heavy flavor by using the replacement (4.1). In this sense, taking the double limit $\lambda, m_H \to \infty$, the variation $\delta S_{\mathrm{YM}}^{\mathrm{onshell}}$ corresponding to the fluctuation of the soliton is calculated as,

$$\begin{aligned}\delta S_{\mathrm{YM}}^{\mathrm{onshell}} &= -\int dt[\Delta M_L + \Delta M_H + \mathcal{O}(m_H^0)], \\ \Delta M_H(t) &= \frac{5}{2\lambda}\pi^2\kappa m_H^2 G(t)\left(1 - \frac{1}{8m_H a\pi^2\rho^2}\right)\mathcal{C}, \end{aligned} \tag{5.15}$$

where we have used subscript "H" in $\Delta M_H(t)$ to refer to the contribution of heavy flavor. Since the metric fluctuation (5.1) is taken into account, there is another contribution to the fluctuation of the soliton mass which comes from the action (4.3) for the transverse modes of the D8-brane as,

$$\begin{aligned}\delta S[x_4] = & -T_8\frac{(2\pi\alpha')^2}{4}\Omega_4 \\ & \times \int d^5x\sqrt{-\det g_{\mathrm{D8}}}e^{-\phi}(\delta g^{\alpha\beta} - \delta\phi g^{\alpha\beta})\Phi_\alpha^\dagger\Phi_\beta \\ = & -\frac{16}{27}m_H^2\kappa\lambda^{-1}\mathcal{C}\int d^4xdz \\ & \times\left[-\frac{81}{64} + \frac{27(22+3m_G)}{256}\frac{z^2}{\lambda}\right]G(t)\delta^{ij}\phi_i^\dagger\phi_j + \\ & -\frac{16}{27}m_H^2\kappa\lambda^{-1}\mathcal{C}\int d^4xdz \\ & \times\left[\frac{189}{64} - \frac{9(202+21m_G)}{256}\frac{z^2}{\lambda}\right]G(t)\phi_z^\dagger\phi_z, \end{aligned} \tag{5.16}$$

by picking up (4.10). So the mass fluctuation from $\delta S[x_4] = -\int \Delta M_{x^4} dt$ is exactly computed with (5.9) as,

$$\Delta M_{x^4}(t) = -\frac{5}{2\lambda}\pi^2\kappa m_H^2 G(t)\mathcal{C} + \mathcal{O}(\lambda^{-1}). \tag{5.17}$$

Thus the total contribution $\Delta M_{HL}(t)$ involving heavy flavor to the mass fluctuation is

$$\Delta M_{HL}(t) = \Delta M_{x^4}(t) + \Delta M_H(t) = -\frac{5}{16a\rho^2}m_H\kappa\lambda^{-1}G(t). \tag{5.18}$$

Keeping the Hamiltonian for baryonic matters (2.27) with our corrections (3.22) (4.11) in hand, we can compute the transition amplitude with the time-dependent perturbed Hamiltonian

$$\Delta H(t) = \Delta M_L(t) + \Delta M_{HL}(t). \tag{5.19}$$

in our system in order to evaluate the decay of the baryon involving glueball.

C. Decay of baryonic meson involving the glueball

In this section, let us evaluate the decay of the baryonic matters involving the glueball quantitatively with this model. To begin with, in experiment, there are some evidences that glueball may form in the decays of some heavy-light mesons [55,56] which behaves like a baryon. Accordingly, let us consider $N_f = 2$ for the case of baryonic heavy-light meson (with no antiheavy flavor). So the Hamiltonian in (4.11) involving one heavy flavor becomes,

$$\begin{aligned} H_{\mathrm{HL}} &= H^{N_f=2}(\mathbf{K}) + (N_Q - N_{\tilde{Q}})m_H + \Delta H \\ &\quad + \mathcal{O}(\lambda^{-2/3}), \\ H^{N_f=2}(\mathbf{K}) &= M_0 + H_\rho^{N_f=2}(Q) + H_Z + \Delta H, \\ H_\rho^{N_f=2}(Q) &= -\frac{1}{2m_\rho}\left[\frac{1}{\rho^3}\partial(\rho^3\partial_\rho) + \frac{1}{\rho^2}(\nabla_{S^3}^2 - 2Q)\right] \\ &\quad + \frac{1}{2}m_\rho\omega_\rho^2\rho^2, \\ H_Z &= -\frac{1}{2m_Z}\partial_Z^2 + \frac{1}{2}m_Z\omega_Z^2 Z^2, \\ \Delta H &= -2\pi^2\kappa\lambda^{-4/3}\left(\frac{7}{6}\right)^{1/3}(\rho Z)^{4/3}, \end{aligned} \tag{5.20}$$

where

$$Q = \frac{N_c}{40\pi^2 a} + \frac{N_Q}{8\pi^2 a}\left(\frac{N_Q}{3N_c} - \frac{3}{4}\right). \tag{5.21}$$

The eigenfunctions and energy spectrum of $H_\rho^{N_f=2}(Q)$ can be solve as

$$\begin{aligned} \psi(\rho) &= e^{-\frac{m_\rho\omega_\rho}{2}\rho^2}\rho^{\tilde{l}}F(-n_\rho, \tilde{l}+2; m_\rho\omega_\rho\rho^2)T^{(l)}(S^3), \\ E(l, n_\rho, n_Z) &= 8\pi^2\kappa + \sqrt{\frac{(l+1)^2}{6} + \frac{640}{3}a^2\pi^2Q^2} \\ &\quad + \frac{2(n_\rho + n_Z) + 2}{\sqrt{6}}, \end{aligned} \tag{5.22}$$

where $F(-n_\rho, \tilde{l}+2; m_\rho\omega_\rho\rho^2)$ is the hypergeometrical function, $T^{(l)}(S^3)$ is the spherical harmonic function on S^3 and $\tilde{l} = -1 + \sqrt{(l+1)^2 + 2m_\rho Q}$. Note that l is the quantum number of the angular momentum. The eigenfunctions and energy spectrum of $H^{N_f=2}(\mathbf{K})$ can be obtained approximately by using ΔH as perturbation. Afterwards, the decay rate of baryonic matters can be obtained by using the standard technique in quantum mechanics as,

$$\Gamma_{i\to f} = |\langle f|\Delta H(t)|i\rangle|^2\delta(E_f - E_i \pm m_G). \tag{5.23}$$

with the time-dependent term (5.19) in which the glueball field is involved.

To close this section, let us attempt to fit the parameters to the realistic QCD with $N_c = 3$. For the baryonic meson, we set $N_Q = 1, N_f = 2, l = 0, 1, n_Z = 1, 3, 5\ldots$ due to $J^P = 0^-, 1^-$ of the heavy-light meson. Then the mass difference of the lowest heavy-light meson states with distinct angular momentum is evaluated with our corrections as $(n_\rho = 0, n_Z = 1)$,

$$M^{l=1} - M^{l=0} = 0.171M_{KK} = 162\ \mathrm{MeV}, \tag{5.24}$$

where the meson data $M_{KK} = 949$ MeV, $\lambda = 16.6$ is also picked up. In experiment, the lowest heavy-light meson states with distinct angular momentum are D^{*0}, D^0 whose mass difference is

$$M_{D^{*0}} - M_{D^0} = 141\ \mathrm{MeV}, \tag{5.25}$$

which is close to our (5.24). Besides, we find the various decay processes among the lowest baryonic meson states $(l = 0, n_Z = 1)$, e.g.,

$$\begin{aligned} &1, |n_\rho = 3\rangle \to |n_\rho = 1\rangle + |m_G^{(n=0)}\rangle, \\ &2, |n_\rho = 5\rangle \to |n_\rho = 0\rangle + |m_G^{(n=1)}\rangle, \end{aligned} \tag{5.26}$$

satisfy the constraint (5.23) and the associated decay rates are computed in the limit $m_H \to \infty$ as,

$$\begin{aligned} \Gamma_1/M_{KK} &= 0.008\mathcal{C}m_H^2, \\ \Gamma_2/M_{KK} &= 0.003\mathcal{C}m_H^2. \end{aligned} \tag{5.27}$$

The parameter m_H can be chosen as $m_H = 0.129$ in order to fit the mass of D^{*0} in the heavy-light meson spectrum and the value of constant $\mathcal{C}$ can be chosen as it is suggested in [26], i.e., $\mathcal{C} = 144.545$ for $|m_G^{(n=0)}\rangle$; $\mathcal{C} = 114.871$ for $|m_G^{(n=1)}\rangle$. In this sense, the lowest decay rates can be evaluated as $\Gamma_1 = 0.002M_{KK}$, $\Gamma_1 = 0.004M_{KK}$. Altogether, we are able to describe the decay of heavy-light meson involving glueball in this holographic model while the exact property of glueball is less clear in experiment.

VI. SUMMARY AND DISCUSSION

In this work, we first derive the $\mathcal{O}(\lambda^{-1/3})$ corrections to the BPST instanton solution on the flavor brane in the D4–D8 model, which is a generalization of the $SU(2)$ case in [24] in the strong coupling limit. The corrections are obtained by solving the equations of motion for the gauge field on the D8-branes with the same gauge condition for $\mathcal{A}^{\mathrm{cl}}$, and minimizing the classical soliton mass. Then keeping our corrections in hand, we follow [19,20] in order to obtain the Hamiltonian of collective modes, which describes the excitation of baryon. Afterwards, the baryon states and spectrum are computed by solving the eigenequation of the Hamiltonian of the collective modes according to the gauge-gravity duality in this model. As the D4–D8 model is able to fit the meson spectrum on the other hand, we therefore employ the meson data in this model (i.e., the value for the unit M_{KK} and t' Hooft coupling constant λ is set as $M_{KK} = 949$ MeV, $\lambda = 16.6$ which is used to match the lowest meson spectrum) to fit the realistic baryon spectrum in QCD with $N_c = 3$, $N_f = 3$. Using the standard technique in quantum mechanics, we compute approximately the baryon spectrum with our corrections which is very close to the experimental data. Furthermore, the corrections to the heavy-light flavored baryon, in which the heavy flavor is introduced by employing the Higgs mechanism in string theory, is also taken into account in this work. So follow the same steps to obtain the Hamiltonian of the collective modes, we get the heavy-light baryon spectrum with our corrections and it matches very well to the experimental data with the same value of M_{KK}, λ. Besides, we finally display how to include the interaction of baryonic matter and glueball with our corrections. As the glueball is identified as the bulk gravitational polarization in this model, we obtain a fluctuation of the soliton mass due to the bulk gravitational polarization and correspondingly a time-dependent term arises in the Hamiltonian of the collective modes. Thus using the standard method for time-dependent Hamiltonian in quantum mechanics, it is possible to evaluate the decay rate of the baryonic matter involving the glueball. Accordingly, we consider the $N_f = 2$ heavy-light meson as the baryonic matter and evaluate the decay rate caused by the glueball field. Although the quantum mechanical description of the baryonic matter decay involving glueball is natural and simply in this model, the property of glueball is less clear in the current experiment so that we do not attempt to further fit the experimental data in this sector.

The remarkable point of this work is that, by picking up our corrections, it is possible to fit the lowest spectrum of two-flavor light meson, three-flavor baryon, and two-flavor heavy-light meson with same meson data, i.e., $M_{KK} = 949$ MeV, $\lambda = 16.6$ which are the only parameters in our theory. In this sense, this work is a good improvement of the D4–D8 framework and [19,20,22,23] (In [19,20,22,23], the infinitely large t' Hooft coupling constant λ is strictly necessary in theory thus it is unable to employ the meson data of M_{KK}, λ in the D4–D8 model). In addition, this work also introduces the next leading order correction to the baryon vertex through the instanton configuration in the large λ expansion, which is to equivalently consider the leading order interaction among the instantons. Therefore the instanton configuration with our corrections may be more close to the reality when they are employed to investigate the other features of baryonic matter such as its phase diagrams as [11,57]. And we will leave this part for the future work.

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