

Gravitational susceptibility of QGP

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We use $\mathcal{N} = 2^*$ and cascading gauge theory holographic models to extract the general features of the gravitational susceptibility κ of strongly coupled nonconformal quark-gluon plasma. We show that in theories with a relevant coupling constant the gravitational susceptibility is renormalization scheme dependent. We propose to use its temperature derivative, i.e., $\frac{d\kappa}{d\ln T}$, as a scheme-independent characteristic of a quark gluon plasma. Although κ is a thermodynamic quantity, its critical behavior can be drastically distinct in the vicinity of seemingly identical thermal phase transitions.

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I. INTRODUCTION AND SUMMARY

Modern relativistic hydrodynamics [1] is a widely accepted framework to analyze strongly coupled quark-gluon plasma (QGP) produced in high energy heavy-ion collisions [2–6]. It is an effective theory of the conservation law of the fluid stress-energy tensor¹ $T^{\mu\nu}$,

$$\nabla_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \sum_{n=0}^{\infty} T_{(n)}^{\mu\nu}, \quad (1.1)$$

organized as an expansion in the gradients of its four-velocity u^μ , $T_{(n)}^{\mu\nu} \sim \nabla^n u$. It is deemed to be applicable close to equilibrium and in weakly curved background spacetimes.² Specifically,³ at zero order in the gradients, the stress-energy tensor is that of the thermal equilibrium of the theory in Minkowski space-time,

$$T_{(0)}^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu},$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu, \quad g_{\mu\nu} u^\mu u^\nu = -1, \quad (1.2)$$

where the pressure P is related to the energy density ϵ via the equilibrium equation of state $P = P_{eq}(\epsilon)$, and $g^{\mu\nu}$ is

¹We consider uncharged fluids here. The general hydrodynamic treatment must include the conservation of all conserved four-currents J_i^μ of the theory.

²As in most effective theories, the series expansion in (1.1) is asymptotic and has zero radius of convergence [7,8].

³We are using the Landau-Lifshitz frame.

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the background space-time metric tensor. Additionally, the local temperature T and the entropy density s are introduced as

$$\epsilon + P = sT, \quad d\epsilon = Tds. \quad (1.3)$$

At the first-order in the velocity gradients constitutive relations between the stress-energy tensor $T_{(1)}^{\mu\nu}$ and the four-velocity require two-independent transport coefficients—the shear η , and the bulk ζ viscosities:

$$T_{(1)}^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha,$$

$$\sigma^{\mu\nu} \equiv \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\Delta_{\alpha\beta}\nabla_\gamma u^\gamma\right). \quad (1.4)$$

The sensitivity of the fluid to the space-time background curvature arises at the second-order in the velocity gradients. At the second-order in the gradient expansion, there are 5 second-order transport coefficients, if the fluid is conformal [9], and 15 coefficients for a general nonconformal theory [10]. In this paper we will be interested in the gravitational coupling of the general hydrodynamics, so we present only the relevant terms of the Romatschke classification [10]:

$$T_{(2),grav}^{\mu\nu} = \kappa(R^{\langle\mu\nu\rangle} - 2u_\alpha u_\beta R^{\alpha\langle\mu\nu\rangle\beta}) + 2\kappa^* u_\alpha u_\beta R^{\alpha\langle\mu\nu\rangle\beta}$$

$$+ \Delta^{\mu\nu}(\zeta_5 R + \zeta_6 u^\alpha u^\beta R_{\alpha\beta}), \quad (1.5)$$

where

$$R^{\mu(\nu\alpha)\beta} \equiv \frac{1}{2}R^{\mu\kappa\sigma\beta}\left(\Delta_\kappa^\nu\Delta_\sigma^\alpha + \Delta_\sigma^\nu\Delta_\kappa^\alpha - \frac{2}{3}\Delta^{\nu\alpha}\Delta_{\kappa\sigma}\right) \quad (1.6)$$

is constructed from the curvature tensor $R^{\mu\alpha\beta}$ of the background metric $g^{\mu\nu}$. Of the four gravitational transport coefficients $\{\kappa, \kappa^*, \zeta_5, \zeta_6\}$ only the gravitational susceptibility κ is independent: non-negativity of the entropy current divergence requires [11–13]

$$\begin{aligned}\kappa^* &= \kappa - \frac{T}{2} \frac{d\kappa}{dT}, \\ \zeta_5 &= \frac{1}{2} \left(c_s^2 T \frac{d\kappa}{dT} - c_s^2 \kappa - \frac{\kappa}{3} \right), \\ \zeta_6 &= c_s^2 \left(3T \frac{d\kappa}{dT} - 2T \frac{d\kappa^*}{dT} + 2\kappa^* - 3\kappa \right) - \kappa + \frac{4}{3} \kappa^* + \frac{\lambda_4}{c_s^2},\end{aligned}\quad (1.7)$$

where c_s is the speed of the sound wave

$$c_s^2 = \frac{dP}{d\epsilon}, \quad (1.8)$$

and λ_4 is the second-order nonlinear transport coefficient appearing in $T_{(2)}^{\mu\nu}$ as [10]

$$T_{(2)}^{\mu\nu} = \dots + \lambda_4 \nabla^{(\mu} \ln s \nabla^{\nu)} \ln s + \dots \quad (1.9)$$

Both κ and λ_4 are thermodynamic quantities and can be extracted from the Euclidean (correspondingly) 2- and 3-point correlation functions of the stress-energy tensor⁴ [16]

$$\begin{aligned}\kappa &= \lim_{k_z \rightarrow 0} \frac{\partial^2}{\partial k_z^2} G_E^{xy,xy}(k)|_{k_0=0}, \\ \lambda_4 &= -2\kappa^* + \kappa - \frac{c_s^4}{2} \lim_{p^x, q^y \rightarrow 0} \frac{\partial^2}{\partial p_x \partial q_y} G_E^{tt,tt,xy}(p, q)|_{p_0, q_0=0}.\end{aligned}\quad (1.10)$$

In this paper we will be interested in the gravitational susceptibility κ of a QGP. Being a thermodynamic coefficient, it can, in principle, be computed from the corresponding gauge theory lattice implementation [16]. Instead, we use the holographic correspondence [17,18] and extract κ from the retarded correlation function of the gauge theory stress-energy tensor [9]. We focus on two examples of holographic models:

- (i) the mass-deformed $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills theory also known as $\mathcal{N} = 2^*$ gauge theory [19,20];
- (ii) the $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ cascading gauge theory [21].

Both theories are nonconformal—in the former, the scale invariance is broken explicitly by the mass terms for the bosonic and the fermionic components of the $\mathcal{N} = 2$

hypermultiplet; in the latter, the scale invariance is broken spontaneously through the dimensional transmutation of the gauge couplings. Our holographic models are examples of top-down,⁵ rather than phenomenological, holography.

Before we report our result, we review what is known in the literature.

- (i) The gravitational susceptibility of $\mathcal{N} = 4$ $SU(N)$ SYM in the planar limit and at infinitely large 't Hooft coupling constant was computed in [9]

$$\begin{aligned}\kappa \Big|_{\mathcal{N}=4} &= \frac{T^2 N^2}{8} \Rightarrow \frac{4\pi^2 \kappa T}{s} \Big|_{\mathcal{N}=4} = 1, \\ \frac{2\pi^2 T^2}{s} \frac{d\kappa}{dT} \Big|_{\mathcal{N}=4} &= 1,\end{aligned}\quad (1.11)$$

where we also presented two benchmark quantities that would allow for comparison with other models.

- (ii) The finite 't Hooft coupling corrections for the $\mathcal{N} = 4$ QGP κ were evaluated in [25]

$$\frac{4\pi^2 \kappa T}{s} \Big|_{\mathcal{N}=4} = 1 - \frac{265}{8} \zeta(3) (g_{\text{YM}}^2 N)^{-3/2} + \dots \quad (1.12)$$

- (iii) For weakly coupled $SU(N)$ gauge theory [26]

$$\frac{4\pi^2 \kappa T}{s} \Big|_{SU(N), \text{free}} = \frac{5}{2}. \quad (1.13)$$

- (iv) κ was determined directly to the leading order in lattice perturbation theory for QCD QGP in [27].
- (v) Using large- N QFT techniques, the computation of κ were performed for the $O(N)$ model for any coupling value in [28].
- (vi) κ was computed in certain phenomenological non-conformal models in [29]. However, the validity of the results presented there should be verified with the implementation of the holographic renormalization—at least for the class of models we consider here the proper treatment of the holographic renormalization, including the finite counterterms and the corresponding issue of the scheme dependence, is crucial to obtain correct results.

We now summarize our results:

- (i) It is well known that in a quantum field theory (QFT) the expectation value of the stress-energy tensor is renormalization scheme dependent. From the holographic perspective, renormalization of the boundary

⁴See also [14,15].

⁵While some observables, e.g., the ratio of the shear viscosity to the entropy density, are universal in all holographic models in the supergravity approximation [22], certain exotic phase transitions are ubiquitous in phenomenological holography, but not in string theory [23,24].

correlation functions is sensitive to finite counterterms—one can crudely think that the energy density and the pressure of $T_{(0)}^{\mu\nu} = \text{diag}\{\epsilon, P, P, P\}$ in (1.2) are defined up to additive constants. If the theory, as well as its regularization and the renormalization, preserves supersymmetry, some of the finite counterterms can be fixed requiring the vanishing of the stress-energy tensor expectation value in a supersymmetric vacuum state, i.e., as $T \rightarrow 0$, see [30] for example. For a QFT in curved space-time and/or with time-dependent relevant couplings, there are even more possibilities for finite counterterms and thus the renormalization scheme dependence.⁶ These new counterterms cannot be fixed requiring Minkowski space-time supersymmetry. As we explicitly show in Sec. II, holographic models with relevant couplings for dimension $\Delta = \{2, 3\}$ operators—e.g., the mass terms $\{m_b^2, m_f\}$ for the bosons and fermions—introduce the scheme dependence for κ . Specifically, the gravitational susceptibility in such models is defined up to arbitrary constants $\{\delta_f, \delta_b\}$:

$$\kappa \rightarrow \kappa + \delta_f m_f^2 + \delta_b m_b^2. \quad (1.14)$$

The corresponding finite counterterms involve the boundary curvature tensor, and thus are insensitive to Minkowski supersymmetry. From (1.14) it is clear that the renormalization scheme independent quantity is $\frac{d\kappa}{d \ln(T)}$; to this end we propose to characterize the gravitational susceptibility of QGPs with \mathcal{R}_κ ,

$$\mathcal{R}_\kappa \equiv 2\pi^2 \frac{T}{s} \frac{d\kappa}{d \ln T} = 4\pi^2 \frac{T}{s} (\kappa - \kappa^*), \quad (1.15)$$

where the normalization is chosen with (1.11) in mind.

- (ii) Supersymmetry tames somewhat the value of \mathcal{R}_κ (1.15): in supersymmetric $\mathcal{N} = 2^*$ theory with $m_b^2 = m_f^2$,

$$\mathcal{R}_\kappa|_{\mathcal{N}=2^*, m_b=m_f} \in \left[1, \frac{5}{4}\right], \quad \text{as } \frac{m_b^2}{T^2} \in [0, +\infty), \quad (1.16)$$

and in $\mathcal{N} = 1$ supersymmetric cascading gauge theory \mathcal{R}_κ grows as

$$\mathcal{R}_\kappa|_{\text{cascading}} \in [1, 2.23(1)], \quad \text{as } T \in [T_{\chi\text{SB}}, +\infty), \\ T_{\chi\text{SB}} = 0.541(9)\Lambda, \quad (1.17)$$

where Λ is the strong coupling scale of the cascading gauge theory. The results reported are for the

cascading QGP with the unbroken chiral symmetry—this phase becomes perturbatively unstable to chiral symmetry breaking fluctuations below $T_{\chi\text{SB}}$ [32]. We cannot use holography to compute κ in the confining phase of the theory, which occurs, as a large- N suppressed first-order phase transition, for $T < T_c = 0.614(1)\Lambda$ [33]. The deconfined phase of the cascading gauge theory with spontaneously broken chiral symmetry is unstable to energy density fluctuations (the sound waves) [34], thus we do not report the susceptibility in this phase as well.

- (iii) $\mathcal{N} = 2^*$ gauge theory with $m_b \neq 0$ and $m_f = 0$ completely breaks the supersymmetry. Here we find

$$\mathcal{R}_\kappa|_{\mathcal{N}=2^*, m_f=0} \in [1, -\infty), \quad \text{as } \frac{m_b^2}{T^2} \in [0, 5.4(1)]. \quad (1.18)$$

The thermal deconfined states of the theory exist only for $T > T_{\text{crit}} = 2.3(3)m_b$. In the vicinity of the critical point, i.e., as $T \rightarrow T_{\text{crit}} + 0$, the speed of the sound waves vanishes and the specific heat diverges [35]

$$c_s^2 \propto \pm \sqrt{T - T_{\text{crit}}}, \quad c_V \propto \pm \frac{1}{\sqrt{T - T_{\text{crit}}}}, \quad (1.19)$$

and we further find, see Sec. IV,

$$\mathcal{R}_\kappa|_{\mathcal{N}=2^*, m_f=0} \propto \mp \frac{1}{\sqrt{T - T_{\text{crit}}}}. \quad (1.20)$$

The signs in (1.19) and (1.20) correlate: there is an additional deconfined phase of the theory (the lower signs), which is however unstable to sound waves, $c_s^2 < 0$.

- (iv) Cascading gauge theory plasma in the chirally symmetric phase has an identical critical point to that of $\mathcal{N} = 2^*$ (1.19). The analogous (terminal) temperature here is $T_u = 0.537(3)\Lambda$ [36], and as $T \rightarrow T_u + 0$ we have

$$c_s^2 \propto \pm \sqrt{T - T_u}, \quad c_V \propto \pm \frac{1}{\sqrt{T - T_u}}. \quad (1.21)$$

Once again, there are two deconfined phases, both existing only for $T > T_u$, that join at the terminal temperature T_u . Interestingly, we find that despite identical critical thermodynamics of $\mathcal{N} = 2^*$ and the cascading QGP, the gravitational susceptibility of the cascading gauge theory plasma at criticality is very different (see Sec. III)

$$\mathcal{R}_\kappa|_{\text{cascading}} = \underbrace{\text{const}}_{>0} \mp \propto \sqrt{T - T_u}. \quad (1.22)$$

As we explore in more details in Sec. III, as $T \rightarrow T_u + 0$,

⁶See a discussion of this issue in the holographic context in [31].

$$\frac{\kappa}{T^2} \Big|_{\text{cascading}} = C_0 + C_1(T - T_u) \pm C_2(T - T_u)^{3/2} + \dots, \quad (1.23)$$

where $C_2 > 0$. Note that κ of the cascading gauge theory plasma is scheme independent as the theory lacks relevant couplings.

Within our computational framework, see Sec. II, we will also have access to the *shear* relaxation time τ_π . This second-order transport coefficient enters $T_{(2)}^{\mu\nu}$ as [9]

$$T_{(2)}^{\mu\nu} = \dots + \eta\tau_\pi \left(u \cdot \nabla \sigma^{\mu\nu} + \frac{\nabla \cdot u}{3} \sigma^{\mu\nu} \right) + \dots. \quad (1.24)$$

- (i) The shear relaxation time was computed for the $\mathcal{N} = 4$ $SU(N)$ SYM plasma in the planar limit and at infinitely large 't Hooft coupling constant in [9]

$$T\tau_\pi|_{\mathcal{N}=4} = \frac{2 - \ln 2}{2\pi}. \quad (1.25)$$

- (ii) The finite 't Hooft coupling corrections for the $\mathcal{N} = 4$ QGP τ_π were evaluated in [25]

$$T\tau_\pi|_{\mathcal{N}=4} = \frac{2 - \ln 2}{2\pi} + \frac{375}{32\pi} \zeta(3) (g_{\text{YM}}^2 N)^{-3/2} + \dots. \quad (1.26)$$

To report results obtained in this work we introduce

$$\mathcal{R}_{\tau_\pi} \equiv \frac{2\pi}{2 - \ln 2} T\tau_\pi, \quad (1.27)$$

where the normalization is chosen with (1.25) in mind.

- (i) Unlike the gravitation susceptibility, the shear relaxation time of a QGP is free from renormalization scheme ambiguities, see Sec. II for details.
(ii) In the supersymmetric $\mathcal{N} = 2^*$ theory with $m_b^2 = m_f^2$,

$$\mathcal{R}_{\tau_\pi}|_{\mathcal{N}=2^*, m_b=m_f} \in [1, 1.1(5)], \quad \text{as } \frac{m_b^2}{T^2} \in [0, +\infty), \quad (1.28)$$

while in the cascading gauge theory

$$\mathcal{R}_{\tau_\pi}|_{\text{cascading}} \in [1, 1.8(8)], \quad \text{as } T \in [T_{\chi\text{SB}}, +\infty), \quad (1.29)$$

where $T_{\chi\text{SB}}$ is given in (1.17).

- (iii) In the vicinity of the critical point (1.19), the relaxation time of the $\mathcal{N} = 2^*$ plasma diverges as (see Sec. IV)

$$\mathcal{R}_{\tau_\pi}|_{\mathcal{N}=2^*, m_f=0} \propto \mp \frac{1}{\sqrt{T - T_{\text{crit}}}}. \quad (1.30)$$

Note that the minus sign occurs on the thermodynamic branch with $c_s^2 > 0$; we find that

$$T\tau_\pi|_{\mathcal{N}=2^*, m_f=0} < 0 \quad \text{for } \frac{T}{T_{\text{crit}}} \in (1, 1.003(0)). \quad (1.31)$$

Given that the model discussed is a top-down holography, it would be extremely interesting to study whether a negative shear relaxation time implies some instabilities and/or causality violations. Typically, a combination of transport coefficients appears in physical observables. For example, the dispersion relation of the sound waves takes the form

$$\omega = \pm c_s q - i\Gamma q^2 \pm \frac{\Gamma}{c_s} \left(c_s^2 \tau_{\text{eff}} - \frac{\Gamma}{2} \right) q^3 + \mathcal{O}(q^4), \quad (1.32)$$

where

$$\Gamma = \frac{2\eta}{3sT} + \frac{\zeta}{2sT}, \quad \tau_{\text{eff}} = \frac{\tau_\pi + \frac{3}{4}\frac{\zeta}{\eta}\tau_\Pi}{1 + \frac{3}{4}\frac{\zeta}{\eta}}, \quad (1.33)$$

with τ_Π being the *bulk* relaxation time [10]. It was determined in [37] that $\tau_{\text{eff}} > 0$ in $\mathcal{N} = 2^*$ QGP in the phase with $c_s^2 > 0$, and diverges as $\tau_{\text{eff}} T \propto \pm (1 - T_{\text{crit}}/T)^{-1/2}$ in the critical region (1.19).

- (iv) The shear relaxation time of the cascading QGP is positive and finite, but is not analytic in the critical region (1.21) (see Sec. III)

$$\mathcal{R}_{\tau_\pi}|_{\text{cascading}} = \underbrace{\text{const}}_{>0} \mp \sqrt{T - T_u}. \quad (1.34)$$

The rest of the paper is organized as follows. In Sec. II we review the holographic framework used to compute κ . We explain why κ , but not τ_π , is renormalization scheme dependent in theories with $\Delta = \{2, 3\}$ relevant couplings. We discuss κ and τ_π of the cascading gauge theory and $\mathcal{N} = 2^*$ QGPs in Secs. III and IV correspondingly.

II. HOLOGRAPHIC COMPUTATION OF κ

Given the second-order formulation of the relativistic hydrodynamics reviewed in section I, the (xy, xy) -component of the retarded stress-energy tensor Green's function in the limit of the small frequency ω and the small momentum $q = |\vec{q}|$ takes the form [10]

$$G_R^{xy,xy}(\omega, q) = P - i\eta\omega + \underbrace{\left(\eta\tau_\pi - \frac{\kappa}{2} + \kappa^*\right)}_{\equiv T^2\hat{\Gamma}_\omega}\omega^2 + \underbrace{\left(-\frac{\kappa}{2}\right)}_{\equiv T^2\hat{\Gamma}_q}q^2 + \mathcal{O}(\omega q^2, \omega^3), \quad (2.1)$$

where we introduced dimensionless quantities $\hat{\Gamma}_\omega$ and $\hat{\Gamma}_q$. Notice that

$$T^2(\hat{\Gamma}_\omega + \hat{\Gamma}_q) = \eta\tau_\pi + \kappa^* - \kappa = \eta\tau_\pi - \frac{T}{2} \frac{d\kappa}{dT}, \quad (2.2)$$

where we used (1.7).

The computation of the Green's function (2.1) in holography was explained in [38]:

- (i) Consider the five-dimensional bulk gravitational action S_{bulk} , dual to some boundary QFT. The thermal equilibrium state of the boundary gauge theory is dual to a black brane geometry,

$$ds_5^2 = -c_1^2 dt^2 + c_2^2 \mathbf{x}^2 + c_3^2 d\rho^2, \quad (2.3)$$

where $c_i = c_i(\rho)$ are functions of the radial coordinate ρ . We assume that $\rho \rightarrow 0$ is the asymptotic boundary, while $\rho \rightarrow \rho_H$ is a regular Schwarzschild horizon,

$$\lim_{\rho \rightarrow 0} \frac{c_1}{c_2} = 1, \quad \lim_{\rho \rightarrow \rho_H} c_1 = 0. \quad (2.4)$$

- (ii) The retarded correlation function $G_R^{xy,xy}$ can be extracted from the quadratic boundary effective

action for the metric fluctuations $\varphi(t, z, \rho) \equiv \frac{1}{2} c_2^{-2} \delta g_{xy}(t, z, \rho)$,

$$\varphi^b(\omega, q) = \int d\omega dq e^{i\omega t - iqz} \varphi(t, z, \rho)|_{\rho \rightarrow 0}, \quad (2.5)$$

given by

$$S_{\text{boundary}}[\varphi^b] = \int \frac{d\omega dq}{(2\pi)^2} \varphi^b(-\omega, -q) \mathcal{F}(\omega, q) \varphi^b(\omega, q), \quad (2.6)$$

as

$$G_R^{xy,xy}(\omega, q) = -2\mathcal{F}(\omega, q). \quad (2.7)$$

- (iii) The boundary metric functional in (2.6) is defined as

$$S_{\text{boundary}}[\varphi^b] = \lim_{\rho \rightarrow 0} (S_{\text{bulk}}^\rho[\varphi] + S_{\text{GH}}[\varphi] + S^{\text{counter}}[\varphi]), \quad (2.8)$$

where S_{bulk}^ρ is the regularized bulk gravitational action, evaluated on-shell for the bulk metric fluctuation φ , subject to the following boundary conditions:

$$(a): \lim_{\rho \rightarrow 0} \varphi(t, z, \rho) = \varphi^b(t, z);$$

$$(b): \varphi(t, z, \rho) \text{ is an incoming wave at the horizon, i.e., as } \rho \rightarrow \rho_H. \quad (2.9)$$

Also, S_{GH} is the standard Gibbons-Hawking term over the regularized boundary. The purpose of the boundary counterterm S^{counter} is to remove divergences of the regularized boundary action $S_{\text{bulk}}^\rho + S_{\text{GH}}$ as $\rho \rightarrow 0$, rendering the renormalized boundary action (2.8) finite.

To evaluate $\hat{\Gamma}_\omega$ and $\hat{\Gamma}_q$ we need the boundary functional (2.8) to quadratic order in $\mathcal{O}(\omega^2, q^2)$ —thus we need the on-shell solution for φ to this order as well. It was shown in [39] that the equation for φ is simply that of the minimally coupled massless scalar in the background metric (2.3). Furthermore, the solution can be expanded as [39]

$$\varphi(t, z, \rho) = e^{-i\omega t + iqz} \underbrace{\varphi^b(\omega, q)}_{(a)} \underbrace{\left(\frac{c_1}{c_2}\right)^{-i\omega Q}}_{(b)} (1 + \omega^2 z_2(\rho) + q^2 z_3(\rho) + \mathcal{O}(\omega q^2, \omega^3)), \quad (2.10)$$

where

$$Q \equiv \frac{1}{2\pi T}. \quad (2.11)$$

with T being the Hawking temperature of the black brane (2.3). In (2.10) we highlighted components of the solution that take care of the boundary conditions (2.9). The radial functions $\{z_2, z_3\}$ satisfy

$$\begin{aligned}
0 &= z_2'' + \left(\ln \frac{c_1 c_2^3}{c_3} \right)' z_2' + \frac{c_2^2}{c_1^2} - \mathcal{Q}^2 \left[\left(\ln \frac{c_1}{c_2} \right)' \right]^2, \\
0 &= z_3'' + \left(\ln \frac{c_1 c_2^3}{c_3} \right)' z_3' - \frac{c_2^2}{c_2^2}, \quad (2.12)
\end{aligned}$$

should vanish at the boundary, i.e., as $\rho \rightarrow 0$, and remain regular at the horizon, i.e., as $\rho \rightarrow \rho_H$.

The on-shell regularized bulk action $S_{\text{bulk}}^\rho[\varphi]$ is a total derivative,⁷ however, we need to discard the contribution from the horizon [38].

The subtle piece in the boundary functional (2.8) comes from $S^{\text{counter}}[\varphi]$. This boundary counterterm action includes finite counterterms, that can lead to renormalization ambiguities in $\hat{\Gamma}_\omega$ and $\hat{\Gamma}_q$. Suppose that our holographic model has operators of conformal dimension $\Delta = \{2, 3\}$, the dual bulk gravitational scalars are correspondingly $\{\alpha, \chi\}$, with coupling constants λ_2 and λ_3 . Thus, close to the AdS₅ boundary we have⁸

$$\alpha = \lambda_2 \rho^2 \ln \rho + \mathcal{O}(\rho^2), \quad \chi = \lambda_3 \rho + \mathcal{O}(\rho^3 \ln \rho). \quad (2.13)$$

Holographic models with such operator content have finite counterterms such as [31]

$$S_{\text{finite}}^{\text{counter}} = \frac{1}{16\pi G_5} \int_{\partial \mathcal{M}_5} dx^4 \sqrt{-\gamma} R_4^\gamma \left(\delta_b \frac{\alpha}{\ln \rho} + \delta_f \chi^2 \right), \quad (2.14)$$

where $\gamma_{ij}(\rho)$ is a four dimensional metric on the regularized boundary $\partial \mathcal{M}_5$, and R_4^γ is the Ricci scalar constructed from this metric, with ρ being treated as an external parameter. δ_b and δ_f are arbitrary constants specifying the renormalization scheme. Evaluation of $S_{\text{finite}}^{\text{counter}}$ on the bulk fluctuation φ produces⁹ nonvanishing as $\rho \rightarrow 0$ terms coming from $R_4^\gamma[\varphi]$,

$$\begin{aligned}
R_4^\gamma[\varphi] &= \frac{1}{2c_2^2} \varphi \partial_{zz}^2 \varphi - \frac{1}{2c_1^2} \varphi \partial_{tt}^2 \varphi + \mathcal{O}(\varphi^4)|_{\rho \rightarrow 0} \\
&= \frac{\rho^2}{2} \int \frac{d\omega dq}{(2\pi)^2} \varphi^b(-\omega, -q) [\omega^2 - q^2] \varphi^b(\omega, q) \\
&\quad + \mathcal{O}((\varphi^b)^4), \quad (2.15)
\end{aligned}$$

resulting in the renormalization scheme dependence of the retarded Green's function (2.1)

$$G_{R,\text{finite}}^{\text{xy,xy}}(\omega, q) = \frac{\delta_b \lambda_2 + \delta_f \lambda_3^2}{16\pi G_5} (q^2 - \omega^2). \quad (2.16)$$

⁷See [30] for the $\mathcal{N} = 2^*$ gauge theory and [40] for the cascading gauge theory.

⁸In the $\mathcal{N} = 2^*$ model $\lambda_2 \sim m_b^2$ and $\lambda_3 \sim m_f$.

⁹We set the asymptotic AdS₅ radius $L = 1$.

The $(\delta_b \lambda_2 + \delta_f \lambda_3^2)$ factor immediately implies that the gravitational susceptibility κ is renormalization scheme dependent, as in (1.14). It is clear that evaluating the logarithmic derivative $\frac{d\kappa}{d \ln T}$ completely removes this scheme dependence. Furthermore, the $(q^2 - \omega^2)$ structure of the renormalization ambiguity in (2.16) implies that the sum $T^2(\hat{\Gamma}_\omega + \hat{\Gamma}_q)$ from (2.1) is always renormalization scheme independent. As a result, see (2.2), the shear relaxation time τ_π is renormalization scheme unambiguous as well.¹⁰

III. CASCADING QGP

For a recent review of the cascading gauge theory see [41]. In this section we will follow notations of the above reference. We omit the technical details and highlight the results only.

The effective five-dimensional gravitational action used to describe the chirally symmetric¹¹ cascading gauge theory plasma contains the Einstein-Hilbert term, and four scalars dual to operators of conformal dimensions $\Delta = \{4, 4, 6, 8\}$. There are no relevant operators, and thus, while the holographic renormalization of the theory is not unique [42], the gravitational susceptibility κ of the theory is renormalization scheme independent.

The black brane dual to the chirally symmetric phase of the cascading QGP is characterized by 13 parameters (see Eqs. (A.55) and (A.58) of [41]):

$$\begin{aligned}
\text{UV: } &\{K_0, f_{a,1,0}, f_{4,0}, f_{c,4,0}, g_{4,0}, f_{a,6,0}, f_{c,8,0}\}, \\
\text{IR: } &\{f_{a,0}^h, f_{c,0}^h, h_0^h, K_{1,0}^h, g_0^h, f_1^h\}, \quad (3.1)
\end{aligned}$$

where K_0 sets the strong coupling scale of the cascading gauge theory¹² (see Eq. (2.48) of [41]):

$$\Lambda^2 = \sqrt{2} e^{-K_0}. \quad (3.2)$$

Parameters (3.1) determine the thermodynamics of the theory (see Eqs. (A.59), (A.86), (A.92) of [41]):

$$\begin{aligned}
8\pi G_5 \epsilon &= -\frac{3}{2} f_{4,0} + \frac{3}{2} f_{c,4,0}, & 8\pi G_5 P &= -\frac{1}{2} f_{4,0} - \frac{3}{2} f_{c,4,0}, \\
4G_5 s &= (f_{a,0}^h)^2 \sqrt{f_{c,0}^h h_0^h}, & T &= \frac{f_1^h}{4\pi \sqrt{h_0^h}}. \quad (3.3)
\end{aligned}$$

For a given black brane geometry, a solution of (2.12) is further characterized by four parameters:

¹⁰The shear viscosity η is universal $\frac{\eta}{s} = \frac{1}{4\pi}$ [22], and is renormalization scheme independent [39].

¹¹The cascading QGP with spontaneously broken chiral symmetry is unstable [34].

¹²We work in the computation scheme with $P = g_s = 1$.

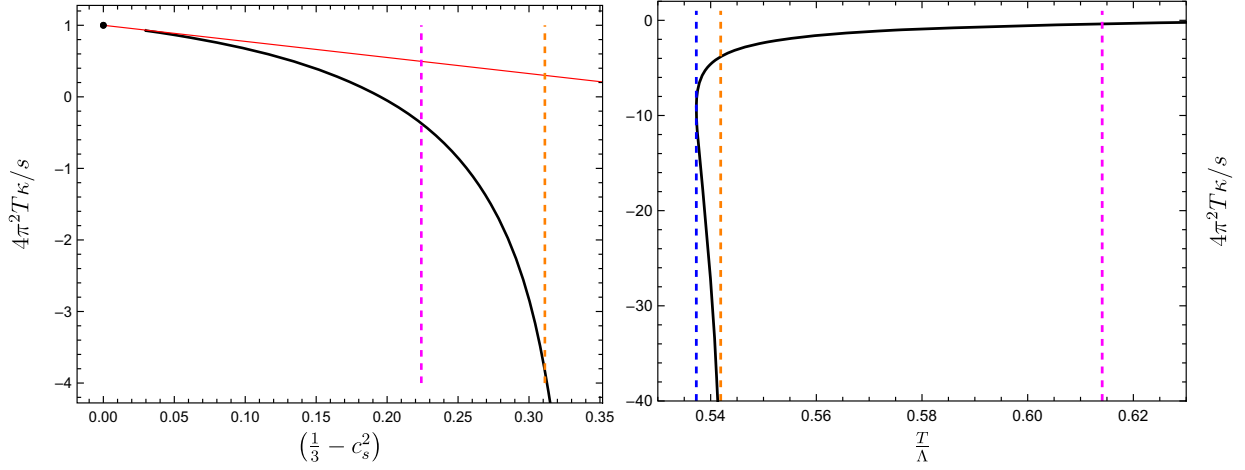


FIG. 1. The normalized gravitational susceptibility κ of the cascading QGP as a function of the nonconformal deformation parameter $(\frac{1}{3} - c_s^2)$ (the left panel), and the ratio $\frac{T}{\Lambda}$ (the right panel). The red line is the leading near-conformal approximation, and the vertical lines represent various phase transitions in this QGP.

$$\begin{aligned} \text{UV: } & \{z_{2,4,0}, z_{3,4,0}\}, \\ \text{IR: } & \{z_{2,0}^h, z_{3,0}^h\}. \end{aligned} \quad (3.4)$$

Implementing the holographic framework of Sec. II, we confirm the general structure of the retarded Green's function (2.1), and identify

$$\begin{aligned} 16\pi G_5 \hat{\Gamma}_w &= \frac{\pi^2 h_0^h}{2(f_1^h)^2} (f_{a,1,0}^2 (6K_0 - 7) - 128z_{2,4,0}), \\ 16\pi G_5 \hat{\Gamma}_q &= -\frac{\pi^2 h_0^h}{2(f_1^h)^2} (f_{a,1,0}^2 (6K_0 - 7) + 128z_{3,4,0}). \end{aligned} \quad (3.5)$$

In Fig. 1 we present the cascading QGP normalized gravitational susceptibility $\frac{4\pi^2 T \kappa}{s}$ as a function of the *universal* nonconformal deformation¹³ $(\frac{1}{3} - c_s^2)$ (the left panel) and as a function of model-specific $\frac{T}{\Lambda}$. The black dot indicates holographic $\mathcal{N} = 4$ SYM result (1.11). The red line is the independently computed¹⁴ leading perturbative near-conformal approximation. It agrees with an accuracy of $\sim 10^{-6}$ with the analytic result of [44],

$$\frac{4\pi^2 T \kappa}{s} = 1 - \frac{9}{4} \left(\frac{1}{3} - c_s^2 \right) + \mathcal{O}((1 - 3c_s^2)^2). \quad (3.6)$$

The vertical dashed magenta line indicates the first-order confinement/deconfinement phase transition at $T = T_c$, and the vertical dashed orange line indicates the second-order

¹³This parameter is useful in comparing different holographic models among themselves, and with the lattice QCD data (when available). Its use was originally advocated for in [43].

¹⁴The thermal state of the cascading QGP is constructed perturbatively in the limit $\ln \frac{T}{\Lambda} \gg 1$, see Appendix D of [41], followed by the corresponding perturbative solution of (2.12).

chiral symmetry breaking phase transition at $T = T_{\chi\text{SB}}$. Finally, the vertical dashed blue line indicates the terminal temperature of the chirally symmetric phase of the cascading gauge theory plasma, see (1.21).

In Fig. 2 we present the results for the normalized shear relaxation time \mathcal{R}_{τ_π} (1.27) of the cascading gauge theory plasma. Here, the agreement with the leading near-conformal analytic result of [44] is $\sim 3 \times 10^{-6}$,

$$\mathcal{R}_{\tau_\pi} = 1 + \frac{9(16 - \pi^2)}{32(2 - \ln 2)} \left(\frac{1}{3} - c_s^2 \right) + \mathcal{O}((1 - 3c_s^2)^2). \quad (3.7)$$

In Fig. 3 we focus on the behavior of the cascading gauge theory susceptibility

$$\hat{\kappa} \equiv 16\pi G_5 \kappa \quad (3.8)$$

close to criticality, see (1.21): the left panel presents dimensionless quantity $\frac{\hat{\kappa}}{T^2}$, and the right panel shows its temperature derivative. From the plots it is clear that the near-critical susceptibility of the cascading QGP is given by (1.23); given (2.1), the latter implies (1.34).

IV. $\mathcal{N} = 2^*$ QGP

In this section we follow notations of [35]. We omit the technical details and highlight the results only.

The effective five-dimensional gravitational action used to describe $\mathcal{N} = 2^*$ gauge theory plasma contains the Einstein-Hilbert term, and two scalars dual to operators of conformal dimensions $\Delta = \{2, 3\}$. Following the general discussion in Sec. II, we expect two-parameter family of the renormalization scheme dependence of its gravitational susceptibility.

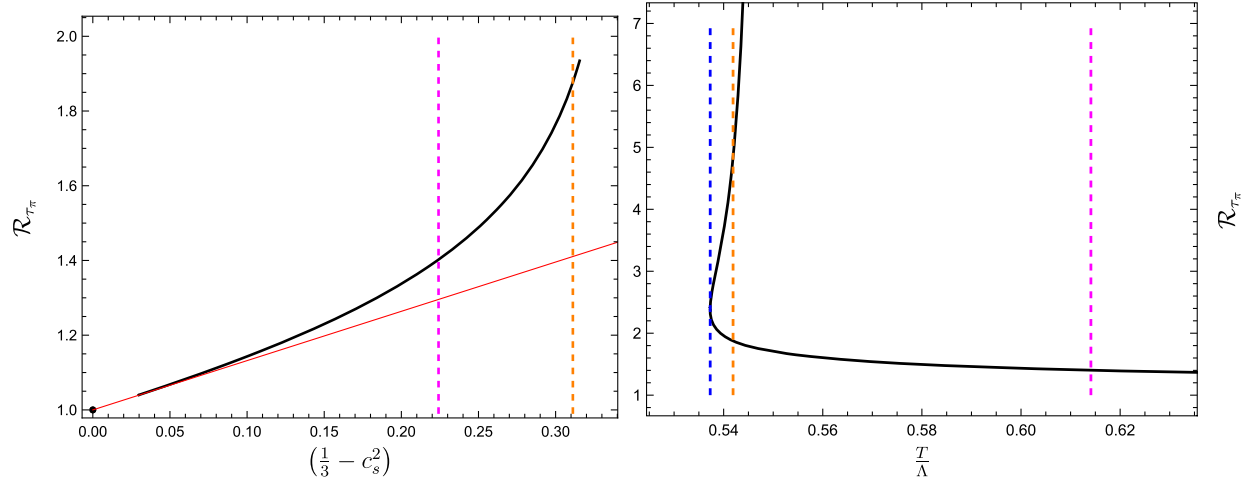


FIG. 2. The normalized shear relaxation time \mathcal{R}_{τ_π} (1.27) of the cascading QGP as a function of the nonconformal deformation parameter $(\frac{1}{3} - c_s^2)$ (the left panel), and the ratio $\frac{T}{\Lambda}$ (the right panel). The red line is the leading near-conformal approximation, and the vertical lines represent various phase transitions in this QGP.

The black brane dual to $\mathcal{N} = 2^*$ QGP is characterized by 8 parameters (see Eqs. (2.19) and (2.31) of [35]):

$$\begin{aligned} \text{UV: } & \{\hat{\delta}_3, \rho_{11}, \rho_{10}, \chi_0, \chi_{10}\}, \\ \text{IR: } & \{a_h, r_0, c_0\}, \end{aligned} \quad (4.1)$$

where $\{\rho_{11}, \chi_0\}$ are the mass parameters of $\mathcal{N} = 2^*$ gauge theory (see Eq. (3.12) of [35]):

$$\rho_{11} = \frac{\sqrt{2}}{24\pi^2} e^{-6a_h} \left(\frac{m_b}{T}\right)^2, \quad \chi_0 = \frac{1}{2^{3/4}\pi} e^{-3a_h} \left(\frac{m_f}{T}\right). \quad (4.2)$$

Parameters (4.1) determine the thermodynamics of the theory (see Eqs. (2.36)–(2.39) of [35]):

$$\begin{aligned} 16\pi G_5 P &= \frac{1}{2} \hat{\delta}_3^4 \left(1 + \rho_{11}^2 (24 \ln 2 - 96 \ln \hat{\delta}_3 + 16\delta_2 + 24) \right. \\ &\quad + 2\chi_{10}\chi_0^2 - 24\rho_{10}\rho_{11} \\ &\quad \left. + \chi_0^4 \left(-\frac{2}{3} \ln 2 + \frac{8}{3} \ln \hat{\delta}_3 + \delta_1 + \frac{10}{9} \right) \right), \\ 16\pi G_5 \epsilon &= 2\hat{\delta}_3^4 - 16\pi G_5 P, \quad 4G_5 s = \hat{\delta}_3^3 e^{3a_h}, \\ T &= \frac{\hat{\delta}_3}{2\pi} e^{-3a_h}. \end{aligned} \quad (4.3)$$

In (4.3) the arbitrary constants δ_1 and δ_2 introduce the scheme dependence to one-point correlation function of the $\mathcal{N} = 2^*$ boundary stress-energy tensor. With Minkowski space supersymmetry, i.e., when $m_b^2 = m_f^2$,

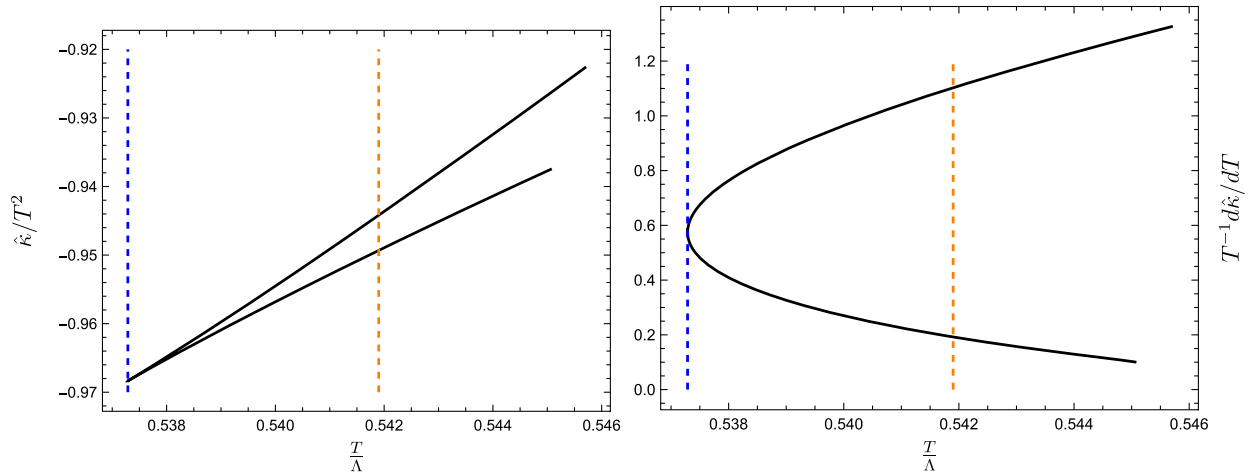


FIG. 3. Behavior of $\hat{\kappa} \equiv 16\pi G_5 \kappa$ close to criticality, i.e., as $T \rightarrow T_u + 0$, see (1.21). T_u is represented by the vertical dashed blue line. The leading nonanalytic term of the cascading gauge theory gravitational susceptibility close to criticality is $\kappa_{\text{nonanalytic}} \propto \pm (T - T_u)^{3/2}$.

and correspondingly $\chi_0^2 = 6\rho_{11}$, the supersymmetry preserving renormalization requires

$$0 = 9\delta_1 + 6 + 4\delta_2. \quad (4.4)$$

For a given black brane geometry, a solution of (2.12) is further characterized by four parameters:

$$\begin{aligned} \text{UV: } & \{z_{2,2,0}, z_{3,2,0}\}, \\ \text{IR: } & \{z_{2,0}^h, z_{3,0}^h\}. \end{aligned} \quad (4.5)$$

Implementing the holographic framework of Sec. II, we confirm the general structure of the retarded Green's function (2.1), and identify

$$\begin{aligned} 16\pi G_5 \hat{\Gamma}_w &= \pi^2 e^{6a_h} \left(-\frac{2\sqrt{2}}{9} (12 \ln \hat{\delta}_3 + 9\delta_3 + 5 - 3 \ln 2) \chi_0^2 - 8\sqrt{2} \delta_4 \rho_{11} - 4z_{2,2,0} \right), \\ 16\pi G_5 \hat{\Gamma}_q &= -\pi^2 e^{6a_h} \left(-\frac{2\sqrt{2}}{9} (12 \ln \hat{\delta}_3 + 9\delta_3 + 5 - 3 \ln 2) \chi_0^2 - 8\sqrt{2} \delta_4 \rho_{11} + 4z_{3,2,0} \right). \end{aligned} \quad (4.6)$$

Arbitrary constants $\delta_3 \leftrightarrow \delta_f$ and $\delta_4 \leftrightarrow \delta_b$ introduce the renormalization scheme dependence in accordance with the general discussion in Sec. II.

An interesting feature of the $\mathcal{N} = 2^*$ QGP with $m_b^2 = m_f^2$ is that the limit $\frac{T}{m_b} \rightarrow 0$ is given by the conformal thermodynamics of a certain five-dimensional theory, compactified on S^1 [45]. The local properties of plasma, such as the transport coefficients, are unaffected by the compactification. The precise matching of the $\mathcal{N} = 2^*$ thermodynamics in the $\frac{T}{m_b} \rightarrow 0$ limit with that of the CFT_5 thermodynamics was explained in [46]; it can be easily extended to the matching of the Green's functions, with the (perhaps the obvious) result:

$$\begin{aligned} \lim_{T/m_b \rightarrow 0} \mathcal{R}_\kappa|_{\mathcal{N}=2^*, m_b=m_f} &= \mathcal{R}_\kappa|_{\text{CFT}_5}, \\ \lim_{T/m_b \rightarrow 0} \mathcal{R}_{\tau_\pi}|_{\mathcal{N}=2^*, m_b=m_f} &= \mathcal{R}_{\tau_\pi}|_{\text{CFT}_5}, \end{aligned} \quad (4.7)$$

where in view of the renormalization scheme dependence of the gravitational susceptibility of the $\mathcal{N} = 2^*$ QGP we use (1.15). The relaxation time τ_π of the CFT_5 plasma was computed in [47]

$$\begin{aligned} \tau_\pi T|_{\text{CFT}_5} &= \frac{5}{8\pi} \left(2 - \frac{\pi}{5} \sqrt{1 - \frac{2}{\sqrt{5}}} + \frac{1}{\sqrt{5}} \coth^{-1} \sqrt{5} - \frac{1}{2} \ln 5 \right) \\ &\Rightarrow \mathcal{R}_{\tau_\pi}|_{\text{CFT}_5} = 1.15(4). \end{aligned} \quad (4.8)$$

We reproduce (4.8), and additionally find

$$\mathcal{R}_{\tau_\pi}|_{\text{CFT}_5} = \frac{5}{4}. \quad (4.9)$$

In Fig. 4 we collect the gravitational susceptibility parameter \mathcal{R}_κ , see (1.15), for all the models we study: the $\mathcal{N} = 2^*$ plasma with $m_b^2 = m_f^2$ (the gray curve), the $\mathcal{N} = 2^*$ plasma with $m_b \neq 0$ and $m_f = 0$ (the green curve), and the cascading QGP (the black curve). The vertical dashed blue line identifies the critical behavior as $c_s^2 \rightarrow 0$. Notice that while \mathcal{R}_κ diverges for the $\mathcal{N} = 2^*$ QGP with $m_f = 0$, it remains finite for the cascading gauge theory plasma.

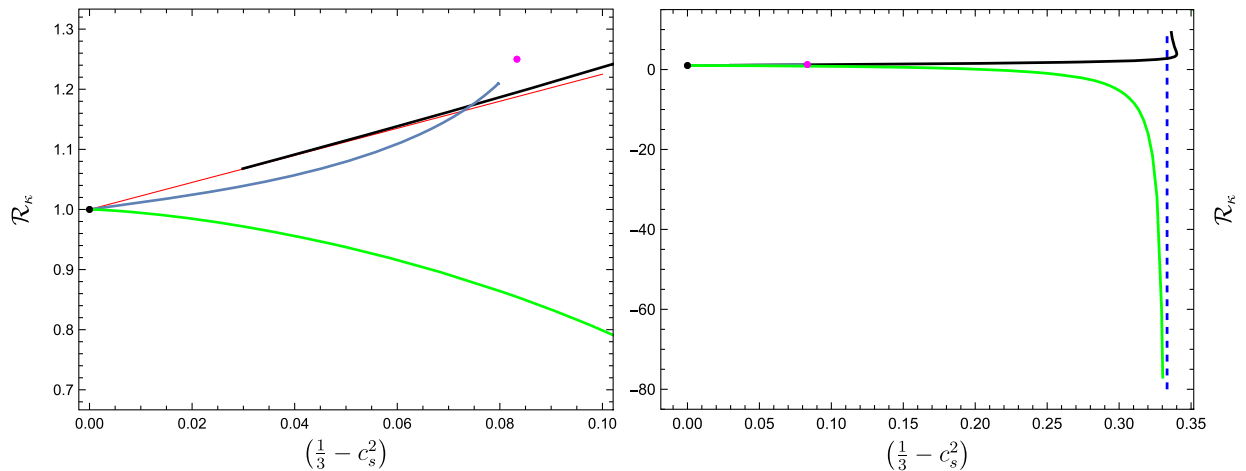


FIG. 4. The gravitational susceptibility parameter \mathcal{R}_κ , see (1.15), for the $\mathcal{N} = 2^*$ plasma with $m_b^2 = m_f^2$ (the gray curve), the $\mathcal{N} = 2^*$ plasma with $m_b \neq 0$ and $m_f = 0$ (the green curve), and the cascading QGP (the black curve). The vertical dashed blue line identifies the critical behavior as $c_s^2 \rightarrow 0$. Notice that while \mathcal{R}_κ diverges for the $\mathcal{N} = 2^*$ QGP with $m_f = 0$, it remains finite for the cascading gauge theory plasma.

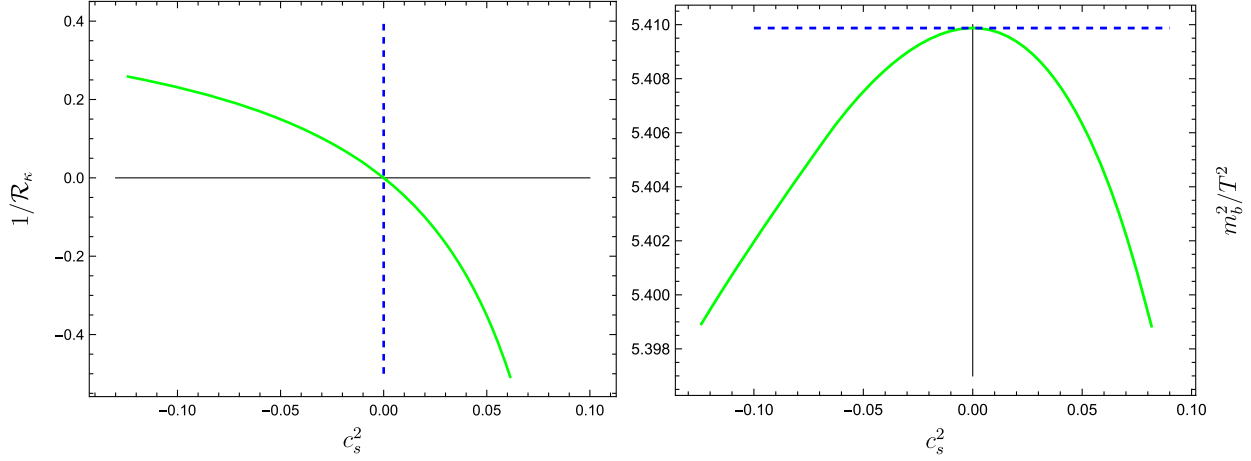


FIG. 5. The $\mathcal{N} = 2^*$ QGP with $m_f = 0$ has a terminal temperature T_{crit} (1.19), represented by the horizontal dashed blue line. Close to criticality, the gravitational susceptibility parameter \mathcal{R}_κ of the model diverges as $\mathcal{R}_\kappa \propto -\frac{1}{c_s^2} \propto \mp \frac{1}{\sqrt{T-T_{\text{crit}}}}$.

$\mathcal{N} = 2^*$ plasma with $m_b \neq 0$ and $m_f = 0$ (the green curve), and the cascading QGP (the black curve). The black dot indicates the $\mathcal{N} = 4$ SYM result (1.11), and the magenta dot indicates the CFT₅ result (4.9). The red line is the near-conformal approximation to \mathcal{R}_κ for the cascading gauge theory plasma. The vertical dashed blue line identifies the critical behavior with the vanishing speed of the sound waves, i.e., $T \rightarrow T_{\text{crit}}$ (1.19) for $\mathcal{N} = 2^*$ plasma with $m_f = 0$, and $T \rightarrow T_u$ (1.21) for the cascading gauge theory plasma.

In Fig. 5 we highlight the critical behavior of \mathcal{R}_κ in the $\mathcal{N} = 2^*$ QGP with $m_f = 0$. Since in the critical region, see the right panel,

$$T - T_{\text{crit}} \propto (c_s^2)^2, \quad (4.10)$$

and, see the left panel,

$$\mathcal{R}_\kappa \propto -\frac{1}{c_s^2}, \quad (4.11)$$

we extract the divergent critical behavior of \mathcal{R}_κ as in (1.20).

In Fig. 6 we collect the shear relaxation time parameter \mathcal{R}_{τ_π} , see (1.27), for all the models we study (the left panel): the $\mathcal{N} = 2^*$ plasma with $m_b^2 = m_f^2$ (the gray curve), the $\mathcal{N} = 2^*$ plasma with $m_b \neq 0$ and $m_f = 0$ (the green curve), and the cascading QGP (the black curve). The black dot

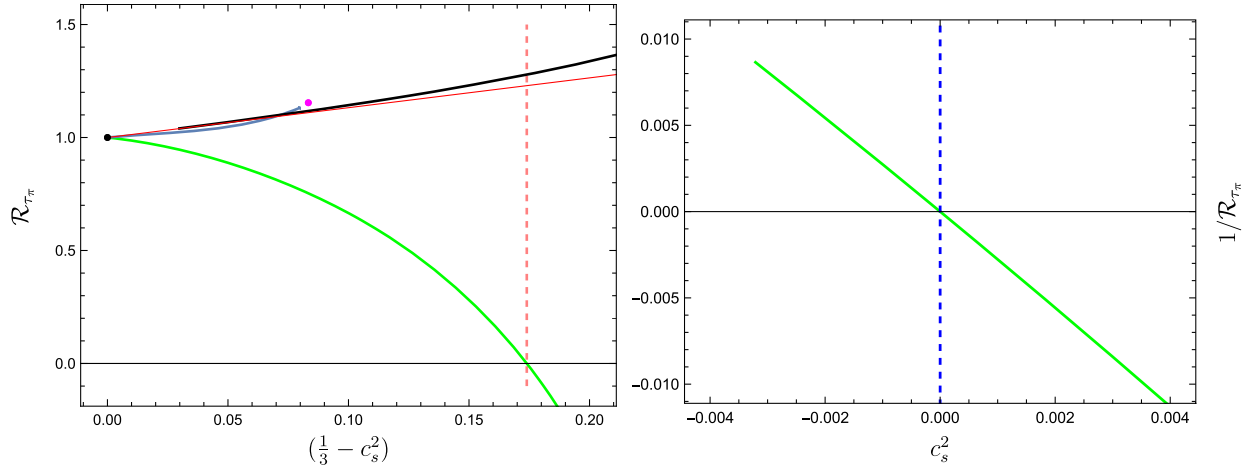


FIG. 6. The left panel: the shear relaxation time parameter \mathcal{R}_{τ_π} , see (1.27), for all the models we study: the $\mathcal{N} = 2^*$ plasma with $m_b^2 = m_f^2$ (the gray curve), the $\mathcal{N} = 2^*$ plasma with $m_b \neq 0$ and $m_f = 0$ (the green curve), and the cascading QGP (the black curve). Notice that below some temperature (the vertical pink dashed line), the shear relaxation time of the $\mathcal{N} = 2^*$ QGP with $m_f = 0$ becomes negative. The right panel: the shear relaxation time of the $\mathcal{N} = 2^*$ QGP with $m_f = 0$ diverges as in (1.30) as $c_s^2 \rightarrow 0$, represented by the vertical dashed blue line.

indicates the $\mathcal{N} = 4$ SYM result (1.25), and the magenta dot indicates the CFT₅ result (4.8). The red line is the near-conformal approximation to \mathcal{R}_{τ_π} for the cascading gauge theory plasma. Notice that the shear relaxation time of the $\mathcal{N} = 2^*$ plasma with $m_f = 0$ becomes negative for $T < 1.003(0)T_{\text{crit}}$, represented by the vertical dashed pink line. In the right panel we show the critical behavior of the shear relaxation time of the $\mathcal{N} = 2^*$ QGP with $m_f = 0$ as $T \rightarrow T_{\text{crit}}$, $\mathcal{R}_{\tau_\pi} \propto -\frac{1}{c_s^2}$, leading to (1.30).

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