Alleviating H_0 tension in Horndeski gravity

Maria Petronikolou,[1,*] Spyros Basilakos,[2,3,4,†] and Emmanuel N. Saridakis[3,5,6,‡]

[1]*Department of Physics, National Technical University of Athens, Zografou Campus GR 157 73, Athens, Greece*
[2]*Academy of Athens, Research Center for Astronomy and Applied Mathematics, Soranou Efessiou 4, 11-527 Athens, Greece*
[3]*National Observatory of Athens, Lofos Nymfon, 11852 Athens, Greece*
[4]*School of Sciences, European University Cyprus, Diogenes Street, Engomi, 1516 Nicosia, Cyprus*
[5]*CAS Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*
[6]*Departamento de Matemáticas, Universidad Católica del Norte, Avda. Angamos 0610, Casilla 1280 Antofagasta, Chile*

(Received 10 October 2021; accepted 7 December 2022; published 30 December 2022)

We show that the H_0 tension can be alleviated in the framework of Horndeski/generalized Galileon gravity. In particular, since the terms depending on G_5 control the friction in the Friedmann equation, we construct specific subclasses in which it depends only on the field's kinetic energy. Since the latter is small at high redshifts, namely at redshifts which affected the CMB structure, the deviations from ΛCDM cosmology are negligible, however as time passes it increases and thus at low redshifts the Hubble function acquires increased values in a controlled way. We consider two models; one with quadratic and one with quartic dependence on the field's kinetic energy. In both cases we show the alleviation of the tension, resulting to $H_0 \approx 74$ km/s/Mpc for particular parameter choices. Finally, we examine the behavior of scalar metric perturbations, showing that the conditions for absence of ghost and Laplacian instabilities are fulfilled throughout the evolution, and we confront the models with Supernovae type Ia and cosmic chronometer data.

DOI: 10.1103/PhysRevD.106.124051

I. INTRODUCTION

The Standard Model of cosmology, namely Λ cold dark matter (ΛCDM) plus inflation in the framework of general relativity, proves to be very efficient in describing the Universe evolution, both at the background and perturbation levels [1,2]. However, theoretical issues such as the cosmological constant problem and the nonrenormalizability of general relativity, as well as the possibility of a dynamical nature for the late-time acceleration, led to the appearance of various extensions and modifications. In general these belong to two classes. In the first class one maintains general relativity as the underlying gravitational theory but adds extra components, such as the dark energy sectors [3,4]. In the second class one constructs modified theories of gravity, which possess general relativity as a particular limit but which in general provide the necessary extra degree(s) of freedom that can drive the Universe acceleration [5–7].

Recently, an additional motivation in favor of extensions/modifications of the concordance cosmology has appeared, namely, the need to incorporate tensions such as the H_0 and σ_8 ones. The former arises from the fact that the Planck Collaboration estimation for the present day cosmic expansion rate is $H_0 = (67.27 \pm 0.60)$ km/s/Mpc [8], which is in tension at about 4.4σ with the 2019 SH0ES Collaboration (R19) direct measurement, i.e., $H_0 = (74.03 \pm 1.42)$ km/s/Mpc, obtained using the Hubble Space Telescope observations of 70 long-period Cepheids in the Large Magellanic Cloud [9] (note that combination with gravitational lensing and time-delay data increases the deviation at 5.3σ [10]). Additionally, the σ_8 tension is related to the parameter which quantifies the matter clustering within spheres of $8h^{-1}$ Mpc radius, and the possible deviation between the cosmic microwave background (CMB) estimation [8] and the SDSS/BOSS measurement [11–13]. If these tensions are not a result of unknown systematics, which at least concerning the H_0 one seems progressively less possible to be the case, then one should indeed seek for alleviation in extensions of the standard lore of cosmology.

In principle one has two main directions to alleviate the H_0 tension. On one hand we could alter the Universe content and interactions while maintaining general

*petronikoloumaria@mail.ntua.gr
†svasil@academyofathens.gr
‡msaridak@phys.uoa.gr

relativity as the gravitational theory [14–50], and on the other hand we could seek for a solution in modified gravity. Since the second direction maintains the advantages that modified gravities bring related to renormalizability and early- and late-time acceleration, it might be preferable. Furthermore, since the H_0 tension implies that the Universe expands faster than what ΛCDM cosmology predicts, in order to alleviate it one should seek for a modified gravity that qualitatively leads to "less gravitational power" at intermediate and late times. Hence, during the last five years many models of modified gravity have been proposed as candidates for the potential alleviation of the H_0 tension [51–76].

In this work we are interested in alleviating the H_0 tension in the framework of Horndeski gravity. Horndeski gravity [77], which is equivalent to generalized Galileon theory [78–80], is the most general four-dimensional scalar-tensor theory with one propagating scalar degree of freedom, that has second-order field equations and thus is free from Ostrogradski instabilities [81]. Hence, by choosing suitable subclasses of the theory we can obtain a cosmological behavior that is almost identical with that of ΛCDM at early times, but which at intermediate times deviates from it due to the weakening of the gravitational interaction, thus alleviating the tension (see also [82,83] for a different approach on the problem using cubic covariant Galileon formulation).

The plan of the work is the following: In Sec. II we present Horndeski gravity, providing the background cosmological equations as well as the conditions for pathologies absent at the perturbation level. In Sec. III we construct specific subclasses of Horndeski gravity that can alleviate the H_0 tension, we compare them to ΛCDM behavior, and we confront them with Supernovae type Ia (SNIa) and cosmic chronometer (CC) data. Finally, in Sec. IV we give a summary of the results and we conclude.

II. HORNDESKI GRAVITY

In this section we briefly review Horndeski gravity, or equivalently generalized Galileon theory. We first give the corresponding general action, and by applying it in a cosmological framework we extract the background Friedmann equations. Additionally, we give the perturbation equations around such a background, and we provide the conditions for the absence of instabilities.

The most general Lagrangian with one scalar degree of freedom coupled to curvature terms, with second-order field equations is [77,84,85]

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i, \tag{1}$$

with

$$\mathcal{L}_2 = K(\phi, X), \tag{2}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \tag{3}$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)], \tag{4}$$

$$\begin{aligned}\mathcal{L}_5 = {}& G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) \\ & -\frac{1}{6}G_{5,X}[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) \\ & + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)],\end{aligned} \tag{5}$$

where ∇_μ is the covariant derivative and $\Box$ the d'Alembertian. In the above expressions R is the Ricci scalar and $G_{\mu\nu}$ the Einstein tensor, while the functions K and G_i ($i = 3, 4, 5$) depend on the scalar field ϕ and its kinetic energy $X = -\partial^\mu\phi\partial_\mu\phi/2$. Moreover, $G_{i,X}$ and $G_{i,\phi}$ ($i = 3, 4, 5$) denote the partial derivatives of G_i in terms of X and ϕ, i.e., $G_{i,X} \equiv \partial G_i/\partial X$ and $G_{i,\phi} \equiv \partial G_i/\partial\phi$. Hence, the total action of the theory will be

$$S = \int d^4x\sqrt{-g}(\mathcal{L} + \mathcal{L}_m), \tag{6}$$

where g is the metric determinant, and $\mathcal{L}_m$ accounts for the matter content of the Universe, which corresponds to a perfect fluid with energy density ρ_m and pressure p_m.

We consider an expanding universe described by a flat homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry with metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{7}$$

with $a(t)$ the scale factor. Varying the action (6) with respect to the metric, and imposing the above FRW form we obtain the two generalized Friedmann equations,

$$\begin{aligned}&2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} \\ &\quad - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m,\end{aligned} \tag{8}$$

$$\begin{aligned}&K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 8\dot{H}XG_{4,X} - 12H^2XG_{4,X} - 4H\dot{X}G_{4,X} - 8HX\dot{X}G_{4,XX} \\ &\quad + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 4H^2X^2\ddot{\phi}G_{5,XX} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} \\ &\quad + 4HX(\dot{X} - HX)G_{5,\phi X} + 4HX\dot{\phi}G_{5,\phi\phi} + 2[2(\dot{H}X + H\dot{X}) + 3H^2X]G_{5,\phi} = -p_m,\end{aligned} \tag{9}$$

where dots mark derivatives with respect to t, and where we have defined the Hubble parameter $H \equiv \dot{a}/a$. Additionally, varying (6) with respect to $\phi(t)$ leads to its equation of motion, namely

$$\frac{1}{a^3}\frac{d}{dt}(a^3 J) = P_\phi, \tag{10}$$

where

$$\begin{aligned} J \equiv\ & \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} - 12HXG_{4,\phi X} \\ & + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) \\ & + 2H^3X(3G_{5,X} + 2XG_{5,XX}) \\ & + 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X}), \end{aligned} \tag{11}$$

$$\begin{aligned} P_\phi \equiv\ & K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) \\ & + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} \\ & - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}. \end{aligned} \tag{12}$$

Finally, the system of equations closes by considering the matter conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{13}$$

Having obtained the background equations of motion, one can proceed to the investigation of perturbations [85–87]. In this work we are interested in the scalar perturbations, and specifically on the conditions of absence of ghosts and Laplacian instabilities, in order to ensure that our solutions are cosmologically viable. In particular, in order for Horndeski/generalized Galileon theory to be free from Laplacian instabilities associated with the scalar field propagation speed one should have [85]

$$c_S^2 \equiv \frac{3(2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2 \dot{w}_2)}{w_1(4w_1 w_3 + 9w_2^2)} \geq 0. \tag{14}$$

Similarly, for the absence of perturbative ghosts one should have [85]

$$Q_S \equiv \frac{w_1(4w_1 w_3 + 9w_2^2)}{3w_2^2} > 0. \tag{15}$$

In the above expressions we have set

$$w_1 \equiv 2(G_4 - 2XG_{4,X}) - 2X(G_{5,X}\dot{\phi}H - G_{5,\phi}), \tag{16}$$

$$\begin{aligned} w_2 \equiv\ & -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H \\ & + 4(\dot{\phi}G_{4,\phi X} - 4HG_{4,X})X + 2G_{4,\phi}\dot{\phi} \\ & + 8X^2HG_{5,\phi X} + 2HX(6G_{5,\phi} - 5G_{5,X}\dot{\phi}H) \\ & - 4G_{5,XX}\dot{\phi}X^2H^2, \end{aligned} \tag{17}$$

$$\begin{aligned} w_3 \equiv\ & 3X(K_{,X} + 2XK_{,XX}) \\ & + 6X(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6H\dot{\phi}G_{3,X}) \\ & + 18H(4HX^3G_{4,XXX} - 5X\dot{\phi}G_{4,\phi X} + 7HG_{4,X}X \\ & -HG_4 - G_{4,\phi}\dot{\phi} + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,\phi XX}) \\ & + 6H^2X(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} - 18G_{5,\phi} \\ & +13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X}), \end{aligned} \tag{18}$$

$$w_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}. \tag{19}$$

We mention here that a negative sound speed square should be definitely avoided, however a sound speed square larger than one does not necessarily imply pathologies and acausal behavior [88,89].

Lastly, we mention here that in Horndeski theories the gravitational-wave speed is in general different than 1, namely than the light speed. In particular, we have [85]

$$c_T^2 \equiv \frac{w_4}{w_1} \geq 0, \tag{20}$$

and as we can see from (16) and (19) the G_5 terms may have an effect according to the cosmological evolution.

III. ALLEVIATING THE H_0 TENSION

In the previous section we presented the cosmological equations in the framework of Horndeski/generalized Galileon gravity. In this section we desire to use particular subclasses of the theory in order to obtain an alleviating of the H_0 tension. Our strategy is the following: since the simplest model in Horndeski cosmology is ΛCDM one, arising from $G_4 = 1/(16\pi G)$, $K = -2\Lambda = \text{const}$, and $G_3 = G_5 = 0$, we want to introduce deviations which will be negligible at high redshifts, in which CMB structure is formed, but that will play a role at low redshifts, in which direct Hubble measurements take place. In particular, since it is known that the terms depending on G_5 affect the friction term on the scalar field [90–97], we could consider G_5 functions depending only on the kinetic energy X in a way that their effect is negligible at high redshifts while being gradually important in a controlled way at low redshifts.

Having these in mind, in the following we will consider $G_4 = 1/(16\pi G)$ and $G_3 = 0$, which are also the case in ΛCDM cosmology, and we will impose a simple scalar field potential and standard kinetic term. Hence $K = -V(\phi) + X$, and we will consider the G_5 term to depend only on X, namely $G_5(\phi, X) = G_5(X)$. In this case, the Friedmann equations (8) and (9) become

$$H^2 = \frac{8\pi G}{3}(\rho_{\mathrm{DE}} + \rho_m), \tag{21}$$

$$\dot{H} = -4\pi G(\rho_{\mathrm{DE}} + p_{\mathrm{DE}} + \rho_m + p_m). \tag{22}$$

In these equations we have defined an effective dark energy sector with energy density and pressure respectively,

$$\rho_{\mathrm{DE}} = 2X - K + 2H^3 X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}), \tag{23}$$

$$p_{\mathrm{DE}} = K - 2XG_{5,X}(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi}) - 4H^2X^2\ddot{\phi}G_{5,XX}, \tag{24}$$

and thus the dark-energy equation of state parameter becomes

$$w_{\mathrm{DE}} \equiv \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}}}. \tag{25}$$

Note that the scalar-field conservation equation (10) becomes simply

$$\dot{\rho}_{\mathrm{DE}} + 3H(\rho_{\mathrm{DE}} + p_{\mathrm{DE}}) = 0. \tag{26}$$

As we mentioned above we want to make our model to coincide with ΛCDM cosmology at high redshifts. Thus, it proves convenient to use the redshift $z = -1 + a_0/a$ as the independent variable, fixing the current scale factor $a_0 = 1$ [therefore $\dot{H} = -(1+z)H(z)H'(z)$ where primes denote derivatives with respect to z]. Introducing, as usual, the matter density parameter through $\Omega_m \equiv \frac{8\pi G\rho_m}{3H^2}$, we can express the Hubble function in the case of ΛCDM cosmology as

$$H_{\Lambda\mathrm{CDM}}(z) \equiv H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}, \tag{27}$$

with H_0 the Hubble parameter at present and Ω_{m0} the present value of the matter density parameter.

Hence, we want to suitably choose $G_5(X)$ forms in order for the $H(z)$ obtained from (21), (23) to coincide with $H_{\Lambda\mathrm{CDM}}(z)$ of (27) at $z = z_{\mathrm{CMB}} \approx 1100$, namely $H(z \to z_{\mathrm{CMB}}) \approx H_{\Lambda\mathrm{CDM}}(z \to z_{\mathrm{CMB}})$, but give $H(z \to 0) > H_{\Lambda\mathrm{CDM}}(z \to 0)$. In the following subsections we will consider two subcases of the $G_5(X)$ term separately. For simplicity, from now on we focus on the dust matter case, i.e., we impose $p_m = 0$, while for the scalar-potential without loss of generality we choose $K = -V_0\phi + X$.

A. Model I: $G_5(X) = \xi X^2$

The first model we consider is the one with $G_5(X) = \xi X^2$, i.e., G_5 has a quadratic dependence on the field's kinetic energy. In this case (23) and (24) respectively become

$$\rho_{\mathrm{DE}} = \frac{\dot{\phi}^2}{2} + V_0\phi + 7\xi H^3\dot{\phi}^5, \tag{28}$$

$$p_{\mathrm{DE}} = \frac{\dot{\phi}^2}{2} - V_0\phi - \xi\dot{\phi}^4(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 5H^2\ddot{\phi}). \tag{29}$$

As described above we chose the model parameter V_0 and the initial conditions for the scalar field in order to obtain $H(z_{\mathrm{CMB}}) = H_{\Lambda\mathrm{CDM}}(z_{\mathrm{CMB}})$ and $\Omega_{m0} = 0.31$ in agreement with [98], and we leave ξ as the parameter that determines the late-time deviation from ΛCDM cosmology.

In Fig. 1 we depict the normalized $H(z)/(1+z)^{3/2}$ as a function of the redshift, for ΛCDM cosmology and for our model with various choices of ξ. As we can see, indeed our model coincides with ΛCDM cosmology at high and intermediate redshifts, while at small redshifts the proposed Model I gives higher values. In particular, the present-day value H_0 depends on the model parameter ξ, and it can be around $H_0 \approx 74$ km/s/Mpc for $\xi = 1.3$ (in H_0 units, i.e., where the ΛCDM H_0 is 1). Specifically, the tension can be alleviated at 3σ if $1.2 < \xi < 1.7$. Hence, we can see that this particular subclass of Horndeski/generalized Galileon gravity can alleviate the H_0 tension due to the effect of the kinetic energy dependent G_5 term. Specifically, at early times the field's kinetic term is negligible and hence the $G_5(X)$ terms do not introduce any deviation from ΛCDM scenario; however, as time passes they increase in a controlled and suitable way in order to make the Hubble function, and thus H_0 too, increase. Note that, since $H_0 \approx 10^{-61}$ in Planck units, the fact that $V_0 = 0.08$ and $\xi = 1.3$ in H_0 units implies that $V_0 \approx 0.5 \times 10^{-61}$ and $\xi \approx 8 \times 10^{365}$ in Planck units (in Planck units we obtain characteristic values of $\dot{\phi}$ and ϕ around 10^{-60}), which is the expected scale for the quantities of a scenario that describes the Universe acceleration (ξ has dimensions of $[M]^{-9}$ i.e., $\xi^{1/9} \sim 10^{40}$ GeV^{-1}).

Let us make a comment here on the specific mechanism behind the tension alleviation. In general, the alleviation of the H_0 tension or/and the σ_8 tension, is a complex issue, and it usually arises as a collective result of many effects. If one remains in the class of late-time modification (without

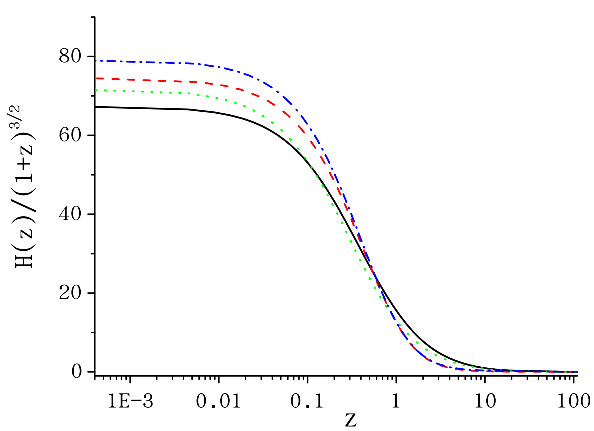

FIG. 1. The normalized $H(z)/(1+z)^{3/2}$ in units of km/s/Mpc as a function of the redshift, for ΛCDM cosmology (black solid) and for Model I with $V_0 = 0.08$ and with $G_5(X) = \xi X^2$, for $\xi = 1.5$ (green dotted), $\xi = 1.3$ (red—dashed) and $\xi = 1$ (blue dashed-dotted), in H_0 units. We have imposed $\Omega_{m0} \approx 0.31$.

examining possible early-time solutions, as it is the aim of this work) then one efficient mechanism is to have $w_{\rm DE} < -1$ at some recent redshift, since a "phantom" dark energy implies "faster" expansion. Nevertheless, this requirement is efficient but not necessary, since the decrease of the effective Newton's constant at intermediate redshifts is also an efficient mechanism [71,99] (see also the discussion in the recent review [100]), since "weaker" gravity implies "faster" expansion. In the models proposed in the present work, although many terms are involved, the alleviation of the tension arises from such a decreased effective Newton's constant, brought about in turn by the friction term. In particular, in Horndeski theories we have [101,102]

$$\frac{G_{\rm eff}}{G} = \frac{1}{2}(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}HXG_{5,X})^{-1}, \quad (30)$$

and for the present Model I this exhibits a decrease as can be seen in Fig. 2. Hence, although $w_{\rm DE}$ in this scenario does not evolve into the phantom regime, as we show in Fig. 3 (note that contrary to tracking dark energy models [103], $w_{\rm DE}$ remains strictly negative for $z \gg 1$, i.e., for $100 \le z < 1000$ the $w_{\rm DE}$ is around -0.13), the aforementioned decrease in the effective Newton's constant is adequate to alleviate the tension.

Finally, we examine the stability of the obtained solution, by investigating the sound speed square c_S^2 given in (14) and the quantity Q_S given in (15). In Fig. 4 we depict their evolution as a function of the redshift for the background solution given above. As we can see, the stability conditions are always satisfied, and hence the obtained solutions are free from ghost and Laplacian instabilities (we mention that for a general $G_5 \neq 0$ the c_S^2 is not identically 1; however, for the chosen $G_5(X)$ with the chosen ξ and the imposed initial conditions we can bring it to be almost 1 during the whole evolution). Additionally, the value of Q_S for large z is close to zero. However, one can verify that it always remains positive, and for $z \gg 1$ (i.e., for $z \to 1000$) it is around 0.005.

Lastly, in Fig. 5 we depict the corresponding gravitational wave speed square c_T^2 given in (20). As we mentioned after (20), in Horndeski theories the G_5 terms in general lead to a gravitational-wave speed different than 1 (namely, than the light speed). In the present model, the gravitational wave speed is not identically one; however, for the chosen parameter value, since the G_5 terms are small compared to G_4, and moreover since $\ddot{\phi}$ is of the same order of $\dot{\phi}H$, the gravitational wave speed is very close to 1. Specifically, the numerical difference between c_T and unity for small z ($z < 0.5$) is less than $\lesssim 10^{-15}$, while for $z > 1$ it becomes larger, namely around 5×10^{-10}. However, this is not in contradiction with LIGO-VIRGO bounds [104], since both GW170817 and GW190425 neutron-star—neutron-star merger events are at very close distances,

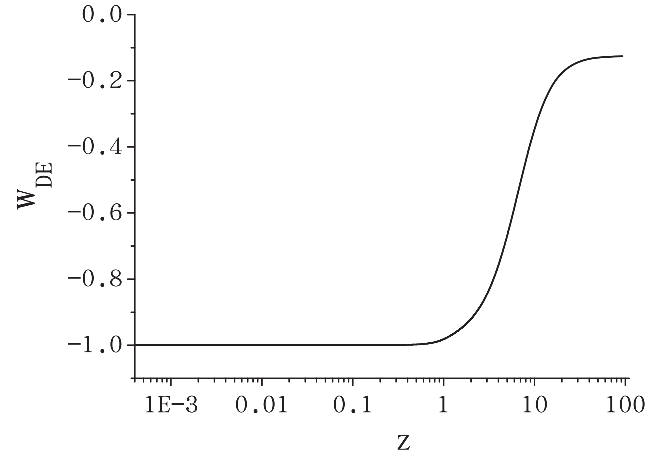

FIG. 3. The effective dark energy equation-of-state parameter $w_{\rm DE}$ given in (25) as a function of the redshift, for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ in H_0 units.

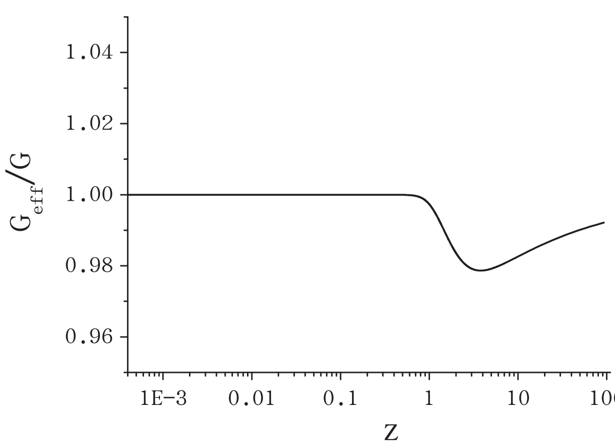

FIG. 2. The normalized effective Newton's constant $\frac{G_{\rm eff}}{G}$ given in (30) as a function of the redshift, for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ in H_0 units.

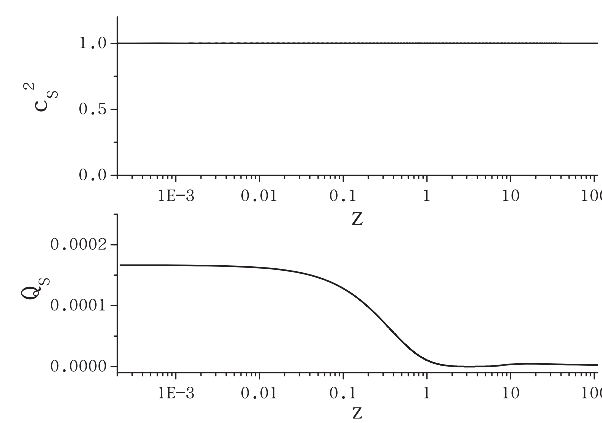

FIG. 4. The sound speed square c_S^2 given in (14) and the quantity Q_S given in (15), as functions of the redshift for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ in H_0 units. The model is free from ghost and Laplacian instabilities.

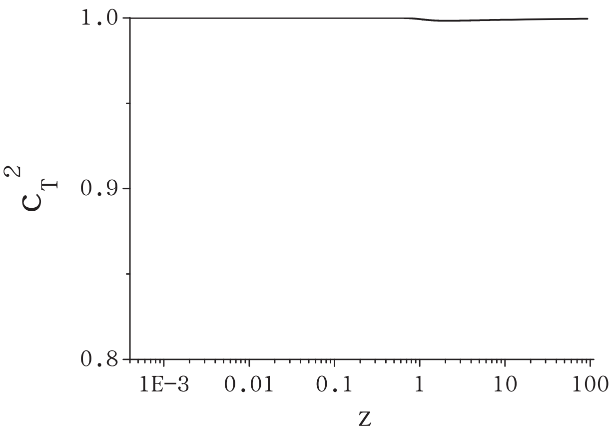

FIG. 5. The gravitational wave speed square c_T^2 given in (20) as a function of the redshift, for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ in H_0 units.

namely at redshifts around 0.01, and thus strictly speaking the observational verification of the gravitational wave speed is only for very low redshifts (and also for the specific frequency range of LIGO-VIRGO). In summary, the present model is able to pass the LIGO-VIRGO bounds, however one could still try to construct Horndeski models that can alleviate the tension but have c_T even closer to 1.

B. Model II: $G_5(X) = \lambda X^4$

The second model we consider is the one with $G_5(X) = \lambda X^4$, i.e., G_5 has a quartic dependence on the field's kinetic energy. In this case (23) and (24) respectively become

$$\rho_{\mathrm{DE}} = \frac{\dot{\phi}^2}{2} + V_0\phi + \frac{11}{2}\lambda H^3\dot{\phi}^9, \tag{31}$$

$$p_{\mathrm{DE}} = \frac{\dot{\phi}^2}{2} - V_0\phi - \frac{\lambda\dot{\phi}^8}{2}(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 9H^2\ddot{\phi}). \tag{32}$$

In Fig. 6 we present $H(z)/(1+z)^{3/2}$ as a function of the redshift for ΛCDM scenario and for our model with various choices of λ. The two models coincide at high and intermediate redshifts, but at small redshifts Model II gives a higher value, while H_0 depends on the model parameter λ. Specifically, it can be around $H_0 \approx 74$ km/s/Mpc for $\lambda = 1$ in H_0 units, and in general the tension can be alleviated at 3σ if $0.5 < \lambda < 1.2$ (in Planck units we have $V_0 \sim 10^{-61}$ and $\lambda \sim 10^{510}$, and since λ has dimensions of $[M]^{-17}$ we acquire $\lambda^{1/17} \sim 10^{30}$ GeV^{-1}). Thus, we observe that this kinetic dependent subclass of Horndeski/generalized Galileon gravity can also alleviate the H_0 tension, since the $G_5(X)$ term that controls the friction term in the Friedmann equation is negligible at high redshifts while it increases and plays a role at low redshifts. Lastly, we mention that the behavior of c_S^2 and Q_S is similar to the one of Fig. 4, i.e., the scenario at hand is free from ghost and Laplacian instabilities.

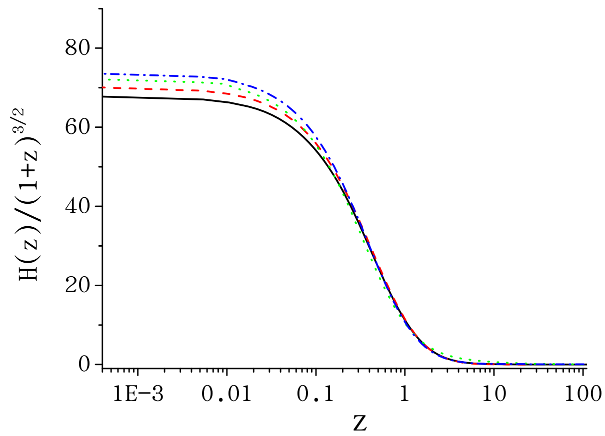

FIG. 6. The normalized $H(z)/(1+z)^{3/2}$ as a function of the redshift, for ΛCDM cosmology (black—solid) and for Model II with $V_0 = 1$ and with $G_5(X) = \lambda X^4$, for $\lambda = 0.5$ (red dashed), $\lambda = 0.9$ (green dotted) and $\lambda = 1$ (blue dashed-dotted), in H_0 units. We have imposed $\Omega_{m0} \approx 0.31$.

We close this section by confronting the models at hand with Supernovae type Ia (SNIa) and cosmic chronometer cosmological data. In particular, concerning SNIa it is known that

$$2.5\log\left[\frac{L}{l(z)}\right] = \mu \equiv m(z) - M = 5\,\log\left[\frac{d_L(z)_{\mathrm{obs}}}{\mathrm{Mpc}}\right] + 25, \tag{33}$$

with $l(z)$ and $m(z)$ the apparent luminosity and apparent magnitude, and L and M the absolute luminosity and magnitude, respectively, while $d_L(z)_{\mathrm{obs}}$ is the luminosity distance. On the other hand, the theoretical value of the luminosity distance is

$$d_L\,(z)_{\mathrm{th}} \equiv (1+z)\int_0^z \frac{dz'}{H(z')}. \tag{34}$$

Since we know the evolution of $H(z)$ in our models, as well as $H_{\Lambda\mathrm{CDM}}(z)$, in Fig. 7 we depict the apparent minus absolute magnitude predicted theoretically for our models as well as ΛCDM cosmology, on top of the binned Pantheon sample SNIa data points from [105]. As we can see, the agreement is very good, and the proposed models have a slightly higher accelerating behavior, as expected.

Additionally, the CC dataset is based on the measurements of $H(z)$ using the relative ages of massive and passively evolving galaxies and the corresponding estimation of dz/dt [106]. In Fig. 8 we confront the theoretically predicted $H(z)$ behavior, as well as the one of ΛCDM cosmology, with the $H(z)$ CC data from [107] at 3σ confidence level. The agreement is very good, and the

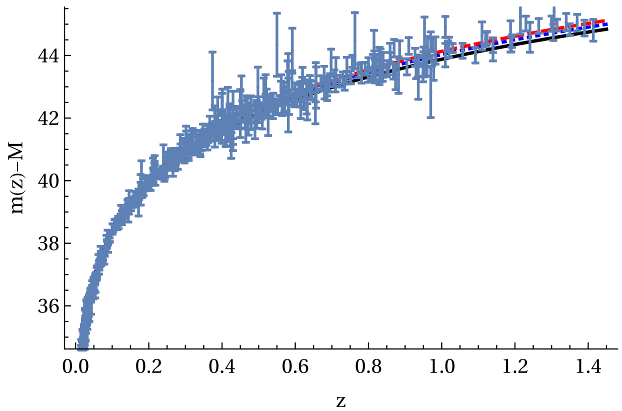

FIG. 7. The apparent minus absolute magnitude predicted theoretically for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ (red—dashed) and for Model II with $V_0 = 1$ and with $\lambda = 1$ (blue dotted) in H_0 units, on top of the Pantheon SNIa data points from [105]. For comparison we depict the ΛCDM curve (black solid) too.

$H(z)$ evolution of the proposed models lies within the prediction of the direct measurements of the $H(z)$ from the CC data, having again a slightly higher accelerating behavior at low redshifts, for the parameter sets $\{\Omega_{m0}, V_0, \xi\} = \{0.31, 0.08, 1.3\}$ and $\{\Omega_{m0}, V_0, \lambda\} = \{0.31, 1, 1\}$.

In summary, there exist regions of the free parameters that are able to reproduce the observed Hubble function evolution and at late times potentially alleviate the H_0 tension, implying also the viability of the examined models. Definitely, in order to conclude on whether a specific model can alleviate the cosmological tensions, a full confrontation with all observational datasets is required. The present work is just an initial approach on the subject, in order to reveal the mechanism that is able to lift the present Hubble parameter value comparing to ΛCDM scenario (following the general requirements of [99,100]). The detailed verification of viability for the proposed models and their results, applying likelihood analysis and model selection criteria on full cosmological datasets, lies beyond the scope of the current work and will be presented in a forthcoming project.

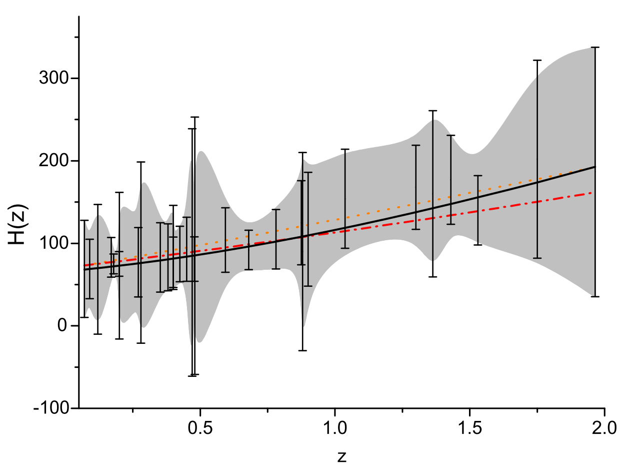

FIG. 8. The $H(z)$ in units of Km/s/Mpc as a function of the redshift, for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ (red dashed-dotted), and for Model II with $V_0 = 1$ and with $\lambda = 1$ (orange dashed), in H_0 units, on top of the CC data points from [107] at 3σ confidence level. For comparison we depict the ΛCDM curve (black solid) too. We have imposed $\Omega_{m0} \approx 0.31$.

IV. CONCLUSIONS

The H_0 tension, unless it is caused by some unknown systematics or is related with some basic data-handling error, may provide a strong indication towards the modification of Standard Model of cosmology. In the present work we investigated the possibility for its alleviation through Horndeski/generalized Galileon gravity.

In particular, knowing that the terms depending on G_5 control the friction term in the Friedmann equation, we constructed specific subclasses depending only on the field's kinetic energy X. Since the kinetic energy is small at high redshifts, namely at redshifts which affected the CMB structure, the deviations from ΛCDM cosmology are negligible. However as time passes X increases in a controlled way and it leads to a decrease in the effective Newton's constant, and thus at low redshifts $H(z)$ acquires increased values.

We considered two models, one with quadratic and one with quartic dependence on the field's kinetic energy. In both cases we showed that at high and intermediate redshifts the Hubble function behaves identically to that of ΛCDM scenario, however at low redshifts it acquires increased values, resulting to $H_0 \approx 74$ km/s/Mpc for particular parameter choices. Hence, these subclasses of Horndeski/generalized Galileon gravity can alleviate the H_0 tension. We mention that the above behavior is obtained without a tuning in the initial conditions of ϕ and $\dot{\phi}$ (we do not have much freedom since we set $\Omega_{\mathrm{DE}0} \approx 0.7$ and moreover we desire to have $w_{\mathrm{DE}}(z = 0)$ around -1). However, the amount of tuning comes mainly in the selection of the functions G_i's and $K(\phi, X)$, since only a small subclass of them can fulfill the above requirements.

As a self-consistency test we examined the behavior of scalar metric perturbations, showing that the conditions for absence of ghost and Laplacian instabilities are fulfilled throughout the evolution, and hence that the proposed solutions are stable and free from pathologies. Finally, for completeness we confronted the proposed models with SNIa and CC data, as a first evidence that they are viable and in agreement with observations.

In summary, in this pilot project we showed that the H_0 tension can be alleviated in the modified gravity framework of Horndeski/generalized Galileon theory, due to the weakening of gravity at low redhifts by the terms depending on the scalar field's kinetic energy. Definitely, in order to obtain a more concrete verification of the above result one should perform a full observational confrontation, using the full datasets, namely data from SNIa, baryonic acoustic oscillations (BAO), cosmic chronometers, redshift space distortion (RSD), CMB shift temperature and polarization, and $f\sigma_8$ observations, performing also the comparison to ΛCDM concordance scenario using various information criteria. Such a full and detailed analysis, lies beyond the scope of this first work, and it is left for a future project.

ACKNOWLEDGMENTS

The authors would like to acknowledge the contribution of the COST Action CA18108 "Quantum Gravity Phenomenology in the multimessenger approach".

[1] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **09**, 373 (2000).
[2] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[3] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[4] Y.-F. Cai, E. N. Saridakis, M. R. Setare, and J.-Q. Xia, Phys. Rep. **493**, 1 (2010).
[5] E. N. Saridakis *et al.* (CANTATA Collaboration), arXiv: 2105.12582.
[6] S. Capozziello and M. De Laurentis, Phys. Rep. **509**, 167 (2011).
[7] Y. F. Cai, S. Capozziello, M. De Laurentis, and E. N. Saridakis, Rep. Prog. Phys. **79**, 106901 (2016).
[8] N. Aghanim *et al.* (Planck Collaboration), arXiv: 1807.06209.
[9] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri, and D. Scolnic, Astrophys. J. **876**, 85 (2019).
[10] K. C. Wong *et al.*, Mon. Not. R. Astron. Soc. **498**, 1420 (2020).
[11] S. Alam *et al.* (BOSS Collaboration), Mon. Not. R. Astron. Soc. **470**, 2617 (2017).
[12] M. Ata *et al.*, Mon. Not. R. Astron. Soc. **473**, 4773 (2018).
[13] P. Zarrouk *et al.*, Mon. Not. R. Astron. Soc. **477**, 1639 (2018).
[14] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess, and J. Silk, Classical Quantum Gravity **38**, 153001 (2021).
[15] E. Di Valentino, L. A. Anchordoqui, O. Akarsu, Y. Ali-Haimoud, L. Amendola, N. Arendse, M. Asgari, M. Ballardini, S. Basilakos, E. Battistelli *et al.*, Astropart. Phys. **131**, 102605 (2021).
[16] E. Di Valentino, A. Melchiorri, and J. Silk, Phys. Rev. D **92**, 121302 (2015).
[17] J. L. Bernal, L. Verde, and A. G. Riess, J. Cosmol. Astropart. Phys. 10 (2016) 019.
[18] S. Pan, W. Yang, E. Di Valentino, E. N. Saridakis, and S. Chakraborty, Phys. Rev. D **100**, 103520 (2019).
[19] S. Pan, W. Yang, C. Singha, and E. N. Saridakis, Phys. Rev. D **100**, 083539 (2019).
[20] W. Yang, S. Pan, E. Di Valentino, and E. N. Saridakis, Universe **5**, 219 (2019).
[21] W. Yang, S. Pan, E. Di Valentino, E. N. Saridakis, and S. Chakraborty, Phys. Rev. D **99**, 043543 (2019).
[22] S. Kumar and R. C. Nunes, Phys. Rev. D **94**, 123511 (2016).
[23] E. Di Valentino, A. Melchiorri, and O. Mena, Phys. Rev. D **96**, 043503 (2017).
[24] E. Di Valentino, C. Bøehm, E. Hivon, and F. R. Bouchet, Phys. Rev. D **97**, 043513 (2018).
[25] T. Binder, M. Gustafsson, A. Kamada, S. M. R. Sandner, and M. Wiesner, Phys. Rev. D **97**, 123004 (2018).
[26] E. Di Valentino, A. Melchiorri, E. V. Linder, and J. Silk, Phys. Rev. D **96**, 023523 (2017).
[27] J. Solà, A. Gómez-Valent, and J. de Cruz Pérez, Phys. Lett. B **774**, 317 (2017).
[28] W. Yang, S. Pan, E. Di Valentino, R. C. Nunes, S. Vagnozzi, and D. F. Mota, J. Cosmol. Astropart. Phys. 09 (2018) 019.
[29] F. D'Eramo, R. Z. Ferreira, A. Notari, and J. L. Bernal, J. Cosmol. Astropart. Phys. 11 (2018) 014.
[30] V. Poulin, T. L. Smith, T. Karwal, and M. Kamionkowski, Phys. Rev. Lett. **122**, 221301 (2019).
[31] A. Shafieloo, D. K. Hazra, V. Sahni, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. **473**, 2760 (2018).
[32] L. Lancaster, F. Y. Cyr-Racine, L. Knox, and Z. Pan, J. Cosmol. Astropart. Phys. 07 (2017) 033.
[33] K. V. Berghaus and T. Karwal, Phys. Rev. D **101**, 083537 (2020).
[34] K. L. Pandey, T. Karwal, and S. Das, J. Cosmol. Astropart. Phys. 07 (2020) 026.
[35] S. Adhikari and D. Huterer, Phys. Dark Universe **28**, 100539 (2020).
[36] D. Benisty, Proc. Sci., CORFU2021 (**2022**) 338 [arXiv: 1912.11124].
[37] A. Perez, D. Sudarsky, and E. Wilson-Ewing, Gen. Relativ. Gravit. **53**, 7 (2021).
[38] S. Pan, W. Yang, and A. Paliathanasis, Mon. Not. R. Astron. Soc. **493**, 3114 (2020).
[39] G. Benevento, W. Hu, and M. Raveri, Phys. Rev. D **101**, 103517 (2020).
[40] A. Banerjee, H. Cai, L. Heisenberg, E. Ó. Colgáin, M. M. Sheikh-Jabbari, and T. Yang, Phys. Rev. D **103**, L081305 (2021).
[41] E. Elizalde, M. Khurshudyan, S. D. Odintsov, and R. Myrzakulov, Phys. Rev. D **102**, 123501 (2020).
[42] P. D. Alvarez, B. Koch, C. Laporte, and Á. Rincón, J. Cosmol. Astropart. Phys. 06 (2021) 019.

[43] A. De Felice, S. Mukohyama, and M. C. Pookkillath, Phys. Lett. B **816**, 136201 (2021).
[44] B. S. Haridasu, M. Viel, and N. Vittorio, Phys. Rev. D **103**, 063539 (2021).
[45] O. Seto and Y. Toda, Phys. Rev. D **103**, 123501 (2021).
[46] J. L. Bernal, L. Verde, R. Jimenez, M. Kamionkowski, D. Valcin, and B. D. Wandelt, Phys. Rev. D **103**, 103533 (2021).
[47] G. Alestas and L. Perivolaropoulos, Mon. Not. R. Astron. Soc. **504**, 3956 (2021).
[48] E. Elizalde, J. Gluza, and M. Khurshudyan, arXiv: 2104.01077.
[49] C. Krishnan, R. Mohayaee, E. Ó. Colgáin, M. M. Sheikh-Jabbari, and L. Yin, Classical Quantum Gravity **38**, 184001 (2021).
[50] A. Theodoropoulos and L. Perivolaropoulos, Universe **7**, 300 (2021).
[51] F. K. Anagnostopoulos, S. Basilakos, and E. N. Saridakis, Phys. Rev. D **100**, 083517 (2019).
[52] A. El-Zant, W. El Hanafy, and S. Elgammal, Astrophys. J. **871**, 210 (2019).
[53] M. Braglia, M. Ballardini, F. Finelli, and K. Koyama, Phys. Rev. D **103**, 043528 (2021).
[54] T. Abadi and E. D. Kovetz, Phys. Rev. D **103**, 023530 (2021).
[55] Y. F. Cai, M. Khurshudyan, and E. N. Saridakis, Astrophys. J. **888**, 62 (2020).
[56] C. Escamilla-Rivera and J. Levi Said, Classical Quantum Gravity **37**, 165002 (2020).
[57] W. E. V. Barker, A. N. Lasenby, M. P. Hobson, and W. J. Handley, Phys. Rev. D **102**, 024048 (2020).
[58] D. Wang and D. Mota, Phys. Rev. D **102**, 063530 (2020).
[59] M. Ballardini, M. Braglia, F. Finelli, D. Paoletti, A. A. Starobinsky, and C. Umiltà, J. Cosmol. Astropart. Phys. 10 (2020) 044.
[60] F. X. Linares Cedeño and U. Nucamendi, Phys. Dark Universe **32**, 100807 (2021).
[61] S. Basilakos, S. Nesseris, F. K. Anagnostopoulos, and E. N. Saridakis, J. Cosmol. Astropart. Phys. 08 (2018) 008.
[62] S. A. Adil, M. R. Gangopadhyay, M. Sami, and M. K. Sharma, Phys. Rev. D **104**, 103534 (2021).
[63] S. D. Odintsov, D. S. C. Gómez, and G. S. Sharov, Nucl. Phys. **B966**, 115377 (2021).
[64] E. Di Valentino, A. Melchiorri, O. Mena, and S. Vagnozzi, Phys. Rev. D **101**, 063502 (2020).
[65] S. Vagnozzi, Phys. Rev. D **102**, 023518 (2020).
[66] B. Hu and M. Raveri, Phys. Rev. D **91**, 123515 (2015).
[67] N. Khosravi, S. Baghram, N. Afshordi, and N. Altamirano, Phys. Rev. D **99**, 103526 (2019).
[68] E. Belgacem, Y. Dirian, S. Foffa, and M. Maggiore, J. Cosmol. Astropart. Phys. 03 (2018) 002.
[69] R. C. Nunes, J. Cosmol. Astropart. Phys. 05 (2018) 052.
[70] M. X. Lin, M. Raveri, and W. Hu, Phys. Rev. D **99**, 043514 (2019).
[71] S. F. Yan, P. Zhang, J. W. Chen, X. Z. Zhang, Y. F. Cai, and E. N. Saridakis, Phys. Rev. D **101**, 121301 (2020).
[72] R. D'Agostino and R. C. Nunes, Phys. Rev. D **101**, 103505 (2020).
[73] F. K. Anagnostopoulos, S. Basilakos, and E. N. Saridakis, Phys. Rev. D **103**, 104013 (2021).
[74] S. Capozziello, M. Benetti, and A. D. A. M. Spallicci, Found. Phys. **50**, 893 (2020).
[75] E. N. Saridakis, S. Myrzakul, K. Myrzakulov, and K. Yerzhanov, Phys. Rev. D **102**, 023525 (2020).
[76] W. J. C. da Silva and R. Silva, Eur. Phys. J. Plus **136**, 543 (2021).
[77] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).
[78] A. De Felice and S. Tsujikawa, Phys. Rev. D **84**, 124029 (2011).
[79] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, Phys. Rev. D **84**, 064039 (2011).
[80] J. Renk, M. Zumalacárregui, F. Montanari, and A. Barreira, J. Cosmol. Astropart. Phys. 10 (2017) 020.
[81] M. Ostrogradsky, Mem. Acad. St. Petersbourg **VI 4**, 385 (1850).
[82] S. Peirone, G. Benevento, N. Frusciante, and S. Tsujikawa, Phys. Rev. D **100**, 063540 (2019).
[83] N. Frusciante, S. Peirone, L. Atayde, and A. De Felice, Phys. Rev. D **101**, 064001 (2020).
[84] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011).
[85] A. De Felice and S. Tsujikawa, J. Cosmol. Astropart. Phys. 02 (2012) 007.
[86] A. De Felice and S. Tsujikawa, Phys. Rev. Lett. **105**, 111301 (2010).
[87] S. Appleby and E. V. Linder, J. Cosmol. Astropart. Phys. 03 (2012) 043.
[88] E. Babichev, V. Mukhanov, and A. Vikman, J. High Energy Phys. 02 (2008) 101.
[89] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, J. Cosmol. Astropart. Phys. 10 (2010) 026.
[90] E. N. Saridakis and S. V. Sushkov, Phys. Rev. D **81**, 083510 (2010).
[91] S. Capozziello and G. Lambiase, Gen. Relativ. Gravit. **31**, 1005 (1999).
[92] G. Koutsoumbas, K. Ntrekis, and E. Papantonopoulos, J. Cosmol. Astropart. Phys. 08 (2013) 027.
[93] K. Feng, T. Qiu, and Y. S. Piao, Phys. Lett. B **729**, 99 (2014).
[94] G. Koutsoumbas, K. Ntrekis, E. Papantonopoulos, and E. N. Saridakis, J. Cosmol. Astropart. Phys. 02 (2018) 003.
[95] H. Mohseni Sadjadi and P. Goodarzi, Phys. Lett. B **732**, 278 (2014).
[96] I. Dalianis, G. Koutsoumbas, K. Ntrekis, and E. Papantonopoulos, J. Cosmol. Astropart. Phys. 02 (2017) 027.
[97] S. Karydas, E. Papantonopoulos, and E. N. Saridakis, Phys. Rev. D **104**, 023530 (2021).
[98] N. Aghanim *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).
[99] L. Heisenberg, H. Villarrubia-Rojo, and J. Zosso, arXiv: 2201.11623.
[100] E. Abdalla, G. F. Abellán, A. Aboubrahim, A. Agnello, O. Akarsu, Y. Akrami, G. Alestas, D. Aloni, L. Amendola, L. A. Anchordoqui *et al.*, J. High Energy Astrophys. **34**, 49 (2022).
[101] E. Bellini and I. Sawicki, J. Cosmol. Astropart. Phys. 07 (2014) 050.

[102] S. Peirone, K. Koyama, L. Pogosian, M. Raveri, and A. Silvestri, Phys. Rev. D **97**, 043519 (2018).
[103] P. J. Steinhardt, L. M. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).
[104] J. M. Ezquiaga and M. Zumalacárregui, Phys. Rev. Lett. **119**, 251304 (2017).
[105] D. M. Scolnic, D. O. Jones, A. Rest, Y. C. Pan, R. Chornock, R. J. Foley, M. E. Huber, R. Kessler, G. Narayan, and A. G. Riess *et al.*, Astrophys. J. **859**, 101 (2018).
[106] R. Jimenez and A. Loeb, Astrophys. J. **573**, 37 (2002).
[107] H. Yu, B. Ratra, and F. Y. Wang, Astrophys. J. **856**, 3 (2018).