Lower bound on the cosmological constant from the classicality of the early Universe

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We use the quantum unimodular theory of gravity to relate the value of the cosmological constant, Λ, and the energy scale for the emergence of cosmological classicality. The fact that Λ and unimodular time are complementary quantum variables implies a perennially quantum Universe should Λ be zero (or, indeed, fixed at any value). Likewise, the smallness of Λ puts an upper bound on its uncertainty, and thus a lower bound on the unimodular clock's uncertainty or the cosmic time for the emergence of classicality. Far from being the Planck scale, classicality arises at around 7×10^{11} GeV for the observed Λ, and taking the region of classicality to be our Hubble volume. We confirm this argument with a direct evaluation of the wave function of the Universe in the connection representation for unimodular theory. Our argument is robust, with the only leeway being in the comoving volume of our cosmological classical patch, which should be bigger than that of the observed last scattering surface. Should it be taken to be the whole of a closed Universe, then the constraint depends weakly on Ω_k: for $-\Omega_k < 10^{-3}$, classicality is reached at $> 4\times10^{12}$ GeV. If it is infinite, then this energy scale is infinite, and the Universe is always classical within the minisuperspace approximation. It is a remarkable coincidence that the only way to render the Universe classical just below the Planck scale is to define the size of the classical patch as the scale of nonlinearity for a red spectrum with the observed spectral index $n_s = 0.967(4)$ (about 10^{11} times the size of the current Hubble volume). In the context of holographic cosmology, we may interpret this size as the scale of confinement in the dual 3D quantum field theory, which may be probed (directly or indirectly) with future cosmological surveys.

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I. INTRODUCTION

It is usually asserted that the Universe becomes quantum at the Planck time, but the arguments behind this are often nothing more than flimsy dimensional analysis. A closer examination shows that the issue depends on the concrete quantum gravity theory, and even then it may hinge on nongeneric details (such as the choice of state or wave function). In this paper, we show that this is certainly the case in quantum unimodular theory [1–5], where the cosmological constant and unimodular (or four-volume) time appear as quantum complementaries, subject to an uncertainty relation. This implies a relation between the nonzero value of the cosmological constant, Λ, and the emergence of large-scale classicality in the early Universe.

Within such a theory, if Λ were zero (or any fixed value), then the clock uncertainty would be infinite, and the Universe would be perennially quantum. More generally, stating that Λ is small only makes sense if the uncertainty in Λ is smaller than its central value. This places a lower bound on the clock's uncertainty and on the time for the emergence of classicality in a unimodular theory. Thus, a lower bound in Λ translates into an upper bound on the temperature at the emergence of classicality, during the cosmological radiation-dominated era, which is parametrically smaller than the Planck temperature. In this paper, we will find that for the observed values of Λ and our

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comoving volume, the Universe becomes classical only for temperatures lower than about 10^{12} GeV.

The argument presented here is very generic and robust, as we show in progressively greater technical detail, starting in Secs. II and III (basic argument), and closing in Secs. IV and V (refinements). Indeed, the Λ used for defining a unimodular clock does not even need to be the observed Λ, should there be radiative corrections, as we show at the end of Sec. V. This decouples our argument from some formulations of the cosmological constant problem [6,7] (as well as from some of the corresponding solutions [8], which can be formulated as additions to the basic model used here [9]).

The only leeway is in the volume of the cosmological patch where we require classical behavior. It must be larger than the current observable Universe, but how close to this we do not know. If the Universe were closed or finite, its classical size would provide an upper bound on how large this patch is, but it could be much smaller. Conversely, in an infinite Universe, the energy scale of classicality would be infinite if classicality were required over an infinite patch (and there would be no quantum epoch, the Universe remaining classical within the minisuperspace approximation). This is because the commutation relations between Λ and its clock involve the inverse of the comoving volume of this patch.

Given the need for apparent fine-tuning for anything between the current Hubble scale and infinity (and so requiring an energy scale for classicality of 10^{12} GeV and infinity), in Sec. VI we make a surprising discovery: The only way to render the Universe classical at the Planck scale is to define the size of the classical patch as the scale of nonlinearity for a red spectrum coinciding with the observed spectral index $n_s = 0.967(4)$. This length scale is huge but not infinite: about 10^{11} times the size of the current Hubble volume. We further discuss the interpretation of this finding in the context of holographic cosmology.

Finally, Sec. VII summarizes our results and outlines future steps, including possible observational tests for the two very distinct quantum cosmologies that emerge from our analysis.

Throughout this paper, we use natural units $\hbar = c = 1$ (with some exceptions, where explicit $\hbar$ is noted for clarity) and the definition of reduced Planck length: $l_P \equiv \sqrt{8\pi G} \simeq (2.44 \times 10^{18}\ \mathrm{GeV})^{-1}$.

II. BACKGROUND

We work within the Henneaux and Teitelboim formulation of unimodular gravity [4], where full diffeomorphism invariance is preserved, but one adds to the base action S_0 (here standard general relativity) an additional term:

$$S_0 \to S = S_0 - \gamma \int d^4x \Lambda(\partial_\mu T^\mu) \tag{1}$$

(where γ is an arbitrary normalization factor inserted for later convenience). In this expression, T^μ is a density, and so the added term is diffeomorphism invariant while not requiring the use of the metric or the connection. Since T^μ does not appear in S_0, we have

$$\frac{\delta S}{\delta T^\mu} = 0 \Rightarrow \partial_\mu \Lambda = -\frac{1}{\gamma}\frac{\delta S_0}{\delta T^\mu} = 0, \tag{2}$$

i.e., on-shell constancy of Λ. The other equation of motion is

$$\frac{\delta S}{\delta \Lambda} = 0 \Rightarrow \partial_\mu T^\mu = \frac{1}{\gamma}\frac{\delta S_0}{\delta \Lambda} = -\frac{\sqrt{-g}}{8\pi G_N \gamma}, \tag{3}$$

(where G_N is Newton's constant), and so T^0 is proportional to a well-known candidate for relational time: four-volume time [4,10,11] (a 4D generalization of the earlier Misner's 3D volume time [12]). Since the metric and connection do not appear in the new term, the Einstein equations (and other field equations) are left unchanged. Thus, classically nothing changes, except that Λ becomes a constant of motion instead of a parameter in the Lagrangian.

However, the quantum theory is radically different. Performing a $3+1$ split of the new term, we find that Λ is now a variable conjugate to the relational time T. Upon quantization, they become duals satisfying commutation relations. If q^A represents the other degrees of freedom of matter and geometry (metric or connection), the Hamiltonian constraint can be written either in terms of Λ [resulting in the standard Wheeler-DeWitt equation for timeless $\psi_s(q^A, \Lambda)$] or in terms of its conjugate time T [leading to a Schrödinger-like equation for $\psi(q^A, T)$] [13–15]. The general solution takes the form

$$\psi(q^A, T) = \int d\Lambda \mathcal{A}(\Lambda) \exp\left[-\frac{i}{\mathfrak{h}}\Lambda T\right]\psi_s(q^A; \Lambda). \tag{4}$$

This is only a slight generalization of Eq. (70) in Smolin's groundbreaking paper [11], with q^A taken to be the Ashtekar connection, and ψ_s the Chern-Simons state. To the best of our knowledge, this is the earliest appearance of this solution in the literature.

In what follows, unless noted otherwise, we consider a cosmological minisuperspace reduction. Then the base action (before the addition of radiation and dust matter) becomes

$$S_0 = \frac{3V_c}{8\pi G}\int \mathrm{d}t\left(\dot{b}a^2 - Na\left(-(b^2 + k) + \frac{\Lambda a^2}{3}\right)\right), \tag{5}$$

where N is the lapse function, a is the expansion factor, b is the connection variable (on-shell $b = N\dot{a}$), k is the curvature (taken to be 1 later in the paper), and V_c is the comoving volume of the spatial region under study. It is convenient to choose $\gamma = 3/(8\pi G_N)$ in Eq. (1) so that the Poisson brackets of Λ and T mimic those of b and a^2:

$$\{b, a^2\} = \{\Lambda, T\} = \frac{8\pi G}{3V_c}. \qquad (6)$$

Then, classically (on-shell), we have

$$\dot{T} = \{T, H\} = -\frac{Na^3}{3} \Rightarrow T = -\frac{1}{3}\int dtNa^3. \qquad (7)$$

Quantum mechanically, we have commutation relations:

$$[\Lambda, T] = \mathrm{i}\mathfrak{h} \coloneqq \frac{8\pi \mathrm{i} G\hbar}{3V_c} = \mathrm{i}\frac{l_P^2}{3V_c}, \qquad (8)$$

and the general solutions in Eq. (4) have a reduced form in the connection representation:

$$\psi(b, T) = \int d\Lambda\, \mathcal{A}(\Lambda) \exp\left[-\frac{\mathrm{i}}{\mathfrak{h}}\Lambda T\right]\psi_s(b;\Lambda). \qquad (9)$$

An advantage of the unimodular extension is that it suggests a natural inner product [15,16]:

$$\langle\psi_1|\psi_2\rangle = \int d\Lambda\, \mathcal{A}_1^\star(\Lambda)\mathcal{A}_2(\Lambda). \qquad (10)$$

This is automatically conserved, so that the theory is unitary. It allows for the construction of normalizable wave packets, whereas the original fixed-Λ solutions, just like any plane wave, are non-normalizable. It also implies a definition of probability and a measure in b space (as we spell out in Sec. IV; see Refs. [15–17]).

In closing, we note that we could subject this construction to a canonical transformation $\Lambda \to \phi(\Lambda)$ and $T \to T_\phi = T/\phi'(\Lambda)$, for a generic function ϕ. All such theories are classically equivalent (and equivalent to GR), but their quantum mechanics is different. Their solutions [Eq. (4)] are different: a Gaussian in Λ is not a Gaussian in a generic $\phi(\Lambda)$; the frequency ΛT is not invariant under the canonical transformation. The natural unimodular inner product [Eq. (10)] is also not invariant [15,16]. Although all these quantum theories are different, for a generic ϕ chosen *within reason*, their border with the semiclassical limit is the same, as we will comment in more detail later.

III. GENERIC ARGUMENT

We first propose a generic argument that does not depend on the detailed dynamics (although it does rely on the inner product [Eq. (10)], and it may be argued that the inner product choice already prefigures knowledge of the dynamics [15,16]). The fact that Λ and T satisfy commutation relations [Eq. (8)], that they are Hermitian under Eq. (10), and that physical states are normalizable under this product, implies a Heisenberg uncertainty relation:

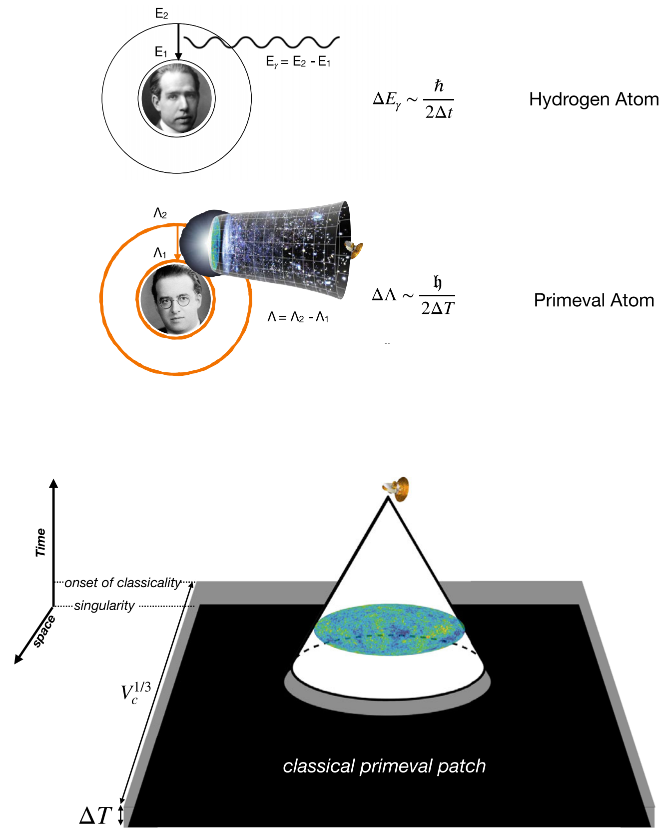

FIG. 1. Top: Analogy between the quantum creation of a photon (from Bohr's hydrogen atom) and that of the Universe (from Lemaître's primeval atom), implying uncertainties for the energy and cosmological constant, respectively. Bottom: The conformal diagram of the big bang spacetime, indicating the past light cone, and the classical primeval patch.

$$\sigma(\Lambda)\sigma(T) \geq \frac{\mathfrak{h}}{2} = \frac{l_P^2}{6V_c} \qquad (11)$$

(which can be intuitively depicted in the top panel of Fig. 1). The inequality is saturated when $\mathcal{A}(\Lambda)$ is a Gaussian:

$$\mathcal{A}(\Lambda) = \sqrt{\mathbf{N}(\Lambda_0, \sigma_\Lambda)} = \frac{\exp\left[-\frac{(\Lambda-\Lambda_0)^2}{4\sigma_\Lambda^2}\right]}{(2\pi\sigma_\Lambda^2)^{1/4}}, \qquad (12)$$

so that for these states a given σ_Λ translates into the minimal $\sigma(T) = \sigma_T = \mathfrak{h}/(2\sigma_\Lambda)$, which, we stress, is constant in time. This is enough to derive a generic lower bound on Λ from the fact that the early Universe should be (semi) classical for times $T > T_\star$, for a given time $T_\star$. In the next section, we will derive explicit solutions from the dynamics, showing that such a σ_T translates into a $\sigma(b)$ implying the same border between classical and quantum regimes, but the generic argument in this section may be enough for most tastes.

A physical analogue is the broadening of the atomic emission lines (Fig. 1, top panel), which is described by a Lorentzian profile:

$$\mathcal{A}(\Lambda) = \sqrt{\frac{\mathfrak{h}}{2\pi\sigma(T)}} \times \frac{1}{\Lambda - \Lambda_0 + \frac{i\mathfrak{h}}{2\sigma(T)}}, \tag{13}$$

$$P(\Lambda) = \frac{\mathfrak{h}}{2\pi\sigma(T)} \times \frac{1}{(\Lambda - \Lambda_0)^2 + \frac{\mathfrak{h}^2}{4\sigma(T)^2}}. \tag{14}$$

In this case, in contrast to the Gaussian wave function in Eq. (12), the variance of Λ is divergent, but the 68% confidence region is $\sigma^{68\%}(\Lambda) \simeq 0.9\mathfrak{h}/\sigma(T)$.

The main point is that in quantum unimodular theory, even when Λ is subdominant, it supplies a quantum clock for the Universe via its conjugate.[1] For the Universe to be classical at time T, the clock's uncertainty σ_T should be negligible for the relevant timing purposes: i.e., $\sigma_T \ll T$. Hence, the classicality of the early Universe imposes a lower bound on Λ. If Λ were zero, then $\sigma(\Lambda) = 0$, implying via Eq. (11) that $\sigma_T = \infty$, and indicating a permanently quantum gravitational Universe. Generally, stating that Λ is "small" only makes sense if $\sigma(\Lambda)$ is "smaller" than its central value Λ_0: $\sigma(\Lambda) \ll \Lambda_0$. This implies a lower bound on σ_T, and therefore also on the time $T_\star$ when $\sigma_T \sim T$ (so that classicality occurs for $T > T_\star$).

Since we do not know how much smaller than Λ_0 the $\sigma(\Lambda)$ actually is, we parametrize $\sigma(\Lambda) = \epsilon\Lambda_0$ with $\epsilon < 1$. Saturation of Eq. (11) then produces

$$\sigma(T) = \sigma_T = \frac{l_P^2}{6V_c\Lambda_0\epsilon} > \frac{l_P^2}{6V_c\Lambda_0}. \tag{15}$$

This implies a Universe in the realm of quantum cosmology at times T such that $\sigma(T)/T > 1$—that is, for $T < T_\star = \sigma_T$. Hence, we can only ignore quantum gravity at time T if

$$T > T_\star = \frac{l_P^2}{6V_c\Lambda_0\epsilon} > \frac{l_P^2}{6V_c\Lambda_0}. \tag{16}$$

As in any quantum cosmology argument based on minisuperspace, the question arises as to what V_c should be. We offer three possibilities:

(1) $V_c \simeq (4\pi/3)(\pi/H_0)^3$—that is, the comoving volume corresponding to the present observable Universe. The rationale behind this is that we do not know if the Universe is classical or quantum on a larger scale.

(2) The whole of a spherical Universe—i.e., $k = 1$, $V_c = 2\pi^2$ if we set $a = 1$ when the Universe has unit radius. If we set $a = 1$ today, then the 3D volume of the Universe is $V_3 = 2\pi^2 a^3 k^{-3/2} = 2\pi^2 a^3 H_0^{-3}(-\Omega_k)^{-3/2}$, so

$$V_c = 2\pi^2 H_0^{-3}(-\Omega_k)^{-3/2}. \tag{17}$$

This introduces a free parameter, Ω_k, in our prediction.

(3) There is an intrinsic infrared cutoff for the size of the classical primeval patch. We can parametrize

$$V_c = \frac{\alpha}{H_0^3}, \tag{18}$$

with $\alpha > 1$ varying from model to model.

We combine all of this into

$$\sigma(T) = \sigma_T = \frac{l_P^2}{6H_0^{-3}\Lambda_0\alpha\epsilon} = \frac{l_P^5\sqrt{\Lambda_0}}{18\alpha\epsilon\sqrt{3\Omega_\Lambda^3}}, \tag{19}$$

where $\alpha > 1$ and $\epsilon < 1$ pull in opposite directions and $\Omega_\Lambda \simeq 0.69$. We can also obtain Eq. (17) by setting $\alpha = 2\pi^2(-\Omega_k)^{-3/2}$.

We can now relate unimodular time and redshift via a change of variables,

$$-T(z) = \frac{1}{3}\int_z^\infty \frac{d\tilde{z}}{(1+\tilde{z})^4 H(\tilde{z})}, \tag{20}$$

and use the Friedmann equation to find

$$T(z) = -\frac{1}{15H_0\sqrt{\Omega_m}}\frac{z^5}{\sqrt{z_{eq}}}. \tag{21}$$

We can also use, under the assumption of adiabatic expansion,

$$H^2(z) = \frac{\pi^2}{90} l_P^2 g(z)\theta(z)^4, \tag{22}$$

$$g(z)\theta(z)^3(1+z)^{-3} = g_0\theta_{\rm CMB}^3, \tag{23}$$

$$g_0 = 3.91, \qquad \theta_{\rm CMB} = 2.73\ \mathrm{K} = 9.65 \times 10^{-32} l_P^{-1}, \tag{24}$$

to obtain

$$-T(z) \simeq \frac{\sqrt{10}g_0\theta_{\rm CMB}^3}{5\pi l_P g(z)^{3/2}\theta(z)^5}, \tag{25}$$

$$\frac{\sigma(T)}{|T|} = \frac{5\pi l_P g(z)^{3/2}\theta(z)^5\sigma_T}{\sqrt{10}g_0\theta_{\rm CMB}^3}, \tag{26}$$

[1] In standard unimodular theory, this is the only quantum clock. In other theories, one could consider multiple clocks at different epochs of the Universe [13–17], or even at the same epoch [18]. The constraints on each of these different theories are specific to each of them.

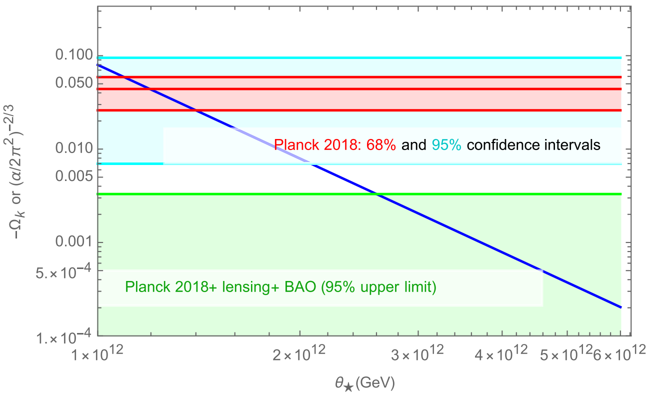

FIG. 2. The relationship between the classicality temperature $\theta_\star$ and the spatial curvature Ω_k [Eq. (31)], or the volume parameter of the classical primeval patch α [Eq. (30)]. Shaded areas show the 68% and 95% constraints on Ω_k from Planck 2018, and the Planck 2018 + lensing + baryonic acoustic oscillations (BAO) of the BOSS DR12 galaxy survey [19] (which are clearly in tension —e.g., Refs. [20,21]).

where we have used the fact that $\sigma(T) = \sigma_T$ does not change with time. Inserting Eq. (19), we arrive at

$$\frac{\sigma(T)}{|T|} = \frac{\sqrt{5}\pi g(z)^{3/2}}{18\alpha g_0\sqrt{6\Omega_\Lambda^3}} \times \frac{l_P^3\theta^5\sqrt{\Lambda_0}}{\theta_{\rm CMB}^3}, \tag{27}$$

so that the Universe can only be classical at the relevant large scales for

$$\theta < \theta_\star \simeq \left(\frac{18\alpha g_0\sqrt{6\Omega_\Lambda^3}}{\sqrt{5}\pi g_\star^{3/2}} \times \frac{\theta_{\rm CMB}^3}{l_P^3\sqrt{\Lambda_0}}\right)^{1/5}. \tag{28}$$

For a whole closed Universe, this becomes

$$\theta < \theta_\star \simeq \left(\frac{36\pi g_0\sqrt{6(-\Omega_\Lambda/\Omega_k)^3}}{\sqrt{5}g_\star^{3/2}} \times \frac{\theta_{\rm CMB}^3}{l_P^3\sqrt{\Lambda_0}}\right)^{1/5}. \tag{29}$$

For $g_\star = 107$ (for the ultrarelativistic Standard Model) and $\Lambda_0 \simeq 7.3 \times 10^{-121} l_P^{-2}$, these imply

$$\theta_\star \simeq \frac{2.8 \times 10^{-7}}{l_P}\left(\frac{\alpha}{4\pi^4/3}\right)^{1/5} = 6.8 \times 10^{11}\ \text{GeV}\left(\frac{\alpha}{4\pi^4/3}\right)^{1/5}, \tag{30}$$

or in a closed Universe,

$$\theta_\star \simeq \frac{7.7 \times 10^{-7}}{l_P}\left(\frac{-\Omega_k}{0.01}\right)^{-3/10} = 1.9 \times 10^{12}\ \text{GeV}\left(\frac{-\Omega_k}{0.01}\right)^{-3/10}, \tag{31}$$

which is shown in Fig. 2, and compared to the current observational bounds on Ω_k. As we see, rather than being the Planck scale, the most natural scale for the emergence of classicality in this theory is 10^{12} GeV.

This is the main conclusion in this paper, and the rest of the paper is devoted to refining the argument and checking how robust it might or might not be. This conclusion will be reinforced and vindicated, until at the very end of this paper, where a surprising discovery provides an interesting exception to the rule.

IV. GAUSSIAN STATES IN MINISUPERSPACE

It is possible to confirm the argument in the last section with explicit dynamical solutions. For a Universe with Λ, matter and radiation, the ψ_s in Eq. (4) must satisfy the Hamiltonian constraint for

$$\mathcal{H} = Na\left(-(b^2+k) + \frac{\Lambda a^2}{3} + \frac{m_m}{a} + \frac{m_r}{a^2}\right). \tag{32}$$

General solutions in the connection representation have been found in Refs. [15–17]. They can be written as

$$\psi_s = \frac{1}{\sqrt{2\pi\mathfrak{h}}} e^{\frac{i}{\mathfrak{h}}P(b,\phi)} \tag{33}$$

for functions P which take the simple asymptotic forms

$$P \approx \frac{3}{\Lambda}\mathcal{L}_{CS} \approx \frac{3}{\Lambda}\frac{b^3}{3} \tag{34}$$

$$\approx C_1 - \frac{2}{9}\frac{\Lambda m_m^4}{3}\frac{1}{b^9} \tag{35}$$

$$\approx C_2 - \frac{1}{5}\frac{\Lambda m_r^2}{3}\frac{1}{b^5} \tag{36}$$

in the Λ, matter, and radiation epochs, respectively (we have ignored the curvature k and highlighted the dependence in Λ via $3/\Lambda$). Here, C_1 and C_2 are constants which are irrelevant for our purposes. Assuming a sharp Gaussian for $\mathcal{A}(\Lambda)$, we can Taylor-expand P and explicitly evaluate Eq. (4) as

$$\psi(b,T) = \frac{\psi(b,T;\Lambda_0)}{(2\pi\sigma_T^2)^{\frac{1}{4}}}\exp\left[-\frac{(X^{\rm eff}-T)^2}{4\sigma_T^2}\right], \tag{37}$$

where

$$\psi(b,T;\Lambda_0) = e^{\frac{i}{\mathfrak{h}}(P(b;\Lambda_0)-\Lambda_0\cdot T)} \tag{38}$$

is the monochromatic partial wave for the central value $\Lambda = \Lambda_0$ (which is a pure phase), where

$$X^{\rm eff} = \frac{\partial P}{\partial \Lambda}\bigg|_{\Lambda_0}, \tag{39}$$

and where

$$\sigma_T = \frac{\mathfrak{h}}{2\sigma_\Lambda} = \frac{l_P^2}{6V_c\sigma_\Lambda} \tag{40}$$

indeed saturates the Heisenberg bound, and therefore is constant, as assumed in the argument of the previous section. For the three epochs of the Universe, we have

$$X^{\rm eff} \approx -\frac{b^3}{\Lambda^2} \tag{41}$$

$$\approx -\frac{2}{27}\frac{m_m^4}{b^9} \tag{42}$$

$$\approx -\frac{1}{15}\frac{m_r^2}{b^5}, \tag{43}$$

respectively, and it can be checked that $\dot{X}^{\rm eff} = \dot{T} = -Na^3/3$ represents the classical trajectory. This is followed by the peak of the Gaussian, so the absence of quantum behavior can be assessed from the induced

$$\frac{\sigma(b)}{b} \approx \frac{\sigma(X^{\rm eff})}{b|\frac{\partial X^{\rm eff}}{\partial b}|} = \frac{\sigma_T}{b|\frac{\partial X^{\rm eff}}{\partial b}|} \tag{44}$$

following from error propagation and $\sigma(X^{\rm eff}) = \sigma_T$. Thus,

$$\frac{\sigma(b)}{b} \approx \frac{\Lambda_0^2}{3}\frac{\sigma_T}{b^3} \tag{45}$$

$$\approx \frac{3}{2}\frac{\sigma_T}{m_m^4} b^9 \propto b^9 \propto z^{9/2} \tag{46}$$

$$\approx \frac{1}{3}\frac{\sigma_T}{m_r^2} b^5 \propto b^5 \propto z^5 \tag{47}$$

for the three epochs in the life of the Universe. Considering that we are just entering the Lambda epoch (so that up to factors of order 1, $H_0^2 = b_0^2 \sim \Lambda_0$), this implies that up to factors of order 1,

$$\frac{\sigma(b)}{b} \approx \frac{\sigma_T}{H_0}\frac{z^5}{\sqrt{z_{eq}}} \sim \frac{\sigma(T)}{|T|}, \tag{48}$$

where we have used Eq. (21) in the last step.

Hence, not only do the explicit minisuperspace solutions vindicate the essential assumption that $\sigma(T)$ is a constant and that its effects translate into uncertainties in the cosmic evolution (in terms of b), but they do not lead to significant numerical corrections.

V. ROBUSTNESS WITH REGARDS TO THE CHOICE OF FUNCTION $\phi(\Lambda)$

The choice of $\phi(\Lambda)$ (see the end of Sec. II) leads to different quantum theories; indeed, the most natural one to emerge from the dynamics is $\phi = 3/\Lambda$, the "wave number" appearing in the Chern-Simons state [cf. Eq. (34); see also Refs. [15,22,23]). However, unless ϕ is very contrived, this does not affect the discussion in this paper, modulo factors of order 1. For example, for any power-law $\phi \propto \Lambda^n$, if $\sigma(\Lambda)/\Lambda_0 = \epsilon \ll 1$, then, from small-error propagation,

$$\frac{\sigma(\phi)}{\phi_0} \approx |n|\frac{\sigma(\Lambda)}{\Lambda_0} = |n|\epsilon, \tag{49}$$

and nothing changes qualitatively in our arguments unless $|n| \gg 1$ or $|n| \ll 1$ (i.e., making the results applicable to the topical case $\phi = 3/\Lambda$). More generally, if a Gaussian is sufficiently sharply peaked, then the distribution of any function of its random variable is also *approximately* a sharply peaked Gaussian with variance obtained by small-error propagation $\sigma(\phi) \approx \phi'\sigma(\Lambda)$.

So all we need is for $\phi'\Lambda/\phi$ to be order 1 at Λ_0. The arguments in Secs. III and IV then follow. A coherent state in Λ is quasicoherent in any ϕ, and vice versa. The arguments in Sec. III follow through, because

$$\frac{\sigma(T_\phi)}{T_\phi} = \frac{\mathfrak{h}}{2\sigma(\phi)T_\phi} \approx \frac{\mathfrak{h}}{2\sigma(\Lambda)T} = \frac{\sigma(T)}{T}. \tag{50}$$

Likewise for the arguments in Sec. IV, since

$$X_\phi^{\rm eff} = \frac{X^{\rm eff}}{\phi_0'},$$

so that the extra factor in $T_\phi = T/\phi'$ cancels throughout (in the peak trajectory and in $\delta b/b$). Obviously, we can design functions for which the argument fails because the small-error propagation formula breaks down. Any power law with very large or small n would do this. However, one might argue that these are very contrived situations.

We can also consider $\Lambda_{\rm tot} = \Lambda + \Lambda_1$—that is, two cosmological constants entering the Hamiltonian constraint, but only one contributing to the unimodular clock. This could be a case of radiative corrections according to some authors [6,7]. In this case, our argument still *does* go through. Although Λ does not need to be small (Λ and Λ_1 have opposite signs), its variance must still be small because

$$\sigma(\Lambda) = \sigma(\Lambda_{\rm tot}) = \epsilon\Lambda_{{\rm tot}\,0} \ll \Lambda_{{\rm tot}\,0}. \tag{51}$$

In other words, the variance in Λ would have to be small with regards to the *total* Lambda for the cancellation to leave $\Lambda_{\rm tot}$ with $\sigma(\Lambda_{\rm tot}) \ll \Lambda_{{\rm tot}\,0}$. Hence, we obtain the same relation between the time of classicality $T_\star$ and the *total* cosmological constant, $\Lambda_{\rm tot}$.

The same is true in the sequester model [8,9], where one removes the space-time average of the trace of the Einstein equations, so that the unimodular Λ is not observable. Although this is true classically, one can only do this within $\sigma(\Lambda)$ in the semiclassical theory, so that $\sigma(\Lambda)$ propagates into the observable $\Lambda_{\rm obs}$ defined in Refs. [8,9] [that is, $\sigma(\Lambda) = \sigma(\Lambda_{\rm obs}) = \epsilon\Lambda_{\rm obs\,0} \ll \Lambda_{\rm obs\,0}$].[2] Again, the same relation is obtained between $T_\star$ and $\Lambda_{\rm obs}$. The situation, however, is complicated by the fact that in Ref. [9] there are two clocks (a Ricci clock as well as a unimodular clock), so that a new layer comes into the argument. We defer a full analysis to a future publication.

VI. IN SEARCH OF A SUPERHORIZON INFRARED SCALE

The only leeway we have in our results therefore relates to V_c, the comoving volume of the classical primeval patch, and obviously we would destroy our bound by letting $V_c \to \infty$, since then $\mathfrak{h} \to 0$ and all uncertainties would go to zero. There is, however, no reason for choosing this—quite the contrary. Indeed, in this case the energy scale for classicality would be infinite—that is, at least in the minisuperspace limit, the Universe would not be subjected to quantum gravity, even deep in the Planck epoch.

Nonetheless, we use this limit as inspiration for trying to find a physically motivated context in which our bound would emerge weakened, and instead seek a context within which the Planck scale could be the scale of classicality for these theories: $\theta_\star l_P \sim 1$. Given that (at least for now) we cannot see beyond the last scattering surface at 13 Gpc, the scale that sets the size of the classical primeval patch only has a firm lower limit. But how big can it really be?

Current observations indicate a red spectrum of primordial scalar perturbations, implying that fluctuations become more nonlinear on large superhorizon scales. Similar to quantum chromodynamics (QCD), this may lead to a strong coupling scale in the deep infrared. Following this hypothesis, we will use the solution to the one-loop renormalization group (RG) equation for a generic renormalizable theory (such as 4D Yang-Mills) to capture the running of the scalar power spectrum:

$$P_s(k) = \frac{A_s}{1 - (n_s - 1)\ln(k/k_0)}, \tag{52}$$

where n_s is the scalar spectral index, and $k_0 = 0.05\ {\rm Mpc}^{-1} \simeq 221 H_0$ is the conventional pivot scale. We can see that this power spectrum diverges at

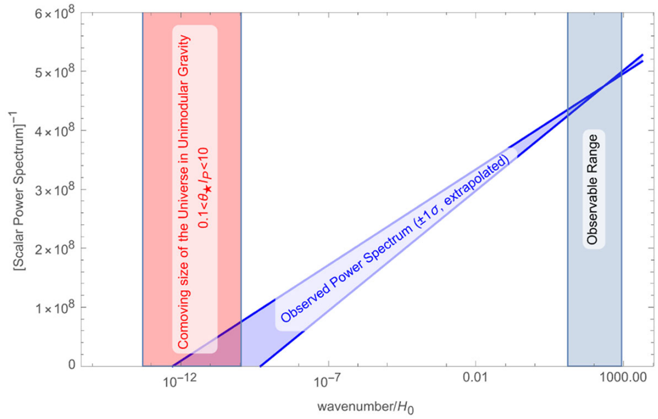

FIG. 3. Observed cosmological scalar power spectrum (blue region), extrapolated to large scales ($k \ll H_0$), from the observable range (gray range). Interestingly, the wave number range for which $P_s^{-1}(k)$ crosses zero [i.e., the scalar power spectrum diverges, Eq. (53)] coincides with the size of the comoving region that explains the observed cosmological constant in unimodular gravity [Eq. (30)], assuming that a classical cosmos emerges around Planck temperature, i.e., $0.1 \lesssim \theta_\star l_P \lesssim 10$.

$$k_\star = k_0 \exp\left(\frac{1}{n_s - 1}\right). \tag{53}$$

Notice that A_s does not appear in this expression because it concerns a scale for a divergence, therefore erasing any fine-tuning that might come from $A_s \sim 10^{-9}$. If we identify this scale with the (inverse of the) size of the classical primeval patch—i.e., $k_\star \sim V_c^{-1/3} = H_0/\alpha^{1/3}$ [with α defined in Eq. (18)]—combining Eqs. (28) and (53) yields

$$n_s = 1 - \frac{1}{32.17 + \ln(\theta_\star l_P)} \simeq 0.9689 + 1.6 \times 10^{-4} \ln(\theta_\star l_P). \tag{54}$$

Now, imagine we require classicality to emerge somewhere within $0.1 \lesssim \theta_\star l_P \lesssim 10$. This implies

$$0.965 < n_s < 0.972, \tag{55}$$

which is consistent with the current combined Planck 2018 bound: $n_s = 0.9665 \pm 0.0038$ [19].

This is a very surprising result, given that the two numbers Λ and n_s are *a priori* not related at all. The only way to get the Planck scale as the scale of classicality in this model is for the classical patch to be not infinite, but about 10^{11} times the linear size of the current horizon—i.e., precisely the scale that becomes nonlinear given the observed scalar spectral tilt of $n_s = 0.967(4)$. This coincidence is depicted in Fig. 3.

The qualitative connection between the RG flow of 3D quantum field theories and the cosmological power spectra can be made concrete in the context of

[2]Note that if $\sigma^2(\Lambda_1) > 0$, it would only tighten the constraints obtained here. We defer a full discussion to future work.

McFadden & Skenderis' Holographic Cosmology program [24]. In particular, our surprising discovery of the connection between Λ and n_s through the Planck scale points to renormalizable 3D quantum theories (which have logarithimc running), and it suggests that the cosmological correlations may be fully described by a finite number of couplings within these theories, which (broadly speaking) includes Chern-Simons 3-form coupling, as well as Φ^6 and Yukawa couplings.[3] To the best of our knowledge, determining the classes of these theories that lead to IR confinement (as suggested by our observed value of Λ) remains an open question.

VII. CONCLUSIONS

In summary, we have related the value of the cosmological constant and the energy scale for the emergence of classicality in a unimodular Universe, a lower bound in the former translating into an upper bound in the latter, and vice versa. The argument is robust with respect to many technicalities, with the exception of the choice of the comoving volume taken for the "classical primeval patch" of the Universe. If this is taken to be the current Hubble volume or even the whole of a closed Universe with a small but non-negligible Ω_k, then this scale is naturally around 10^{12} GeV, and not the Planck scale. It would be interesting to relate this number to the amplitude of the fluctuations in inflationary models operating at this energy scale.

A second surprise found in this paper is obtained if we allow ourselves latitude for a much larger patch of classicality, V_c, specifically setting it to be the scale where the primordial fluctuations become divergent for a red spectrum. While this is not entirely model independent, it suggests that a very large classical primeval patch can lift the energy scale of classicality to the Planck scale for values of n_s close to the observed ones. Note that if the scale V_c were to be infinite (i.e., if an infinite Universe were to be globally classical), then the energy scale for classicality would be infinite, and not the Planck energy either. Hence, it is a remarkable coincidence that the observed values of spectral tilt of $n_s = 0.967(4)$ and cosmological constant $\Lambda \simeq 7.0(2) \times 10^{-121} l_P^{-2}$ [19] point to a value of V_c that set the energy scale of classicality at the Planck energy. In the context of holographic cosmology [24], this coincidence suggests that the holographic 3D quantum field theory that describes our cosmological observations must be a renormalizable theory with an IR confinement scale of 10^{11} times the current comoving Hubble radius.

So, what next?

On the theoretical front, a clearer understanding of what may happen as we approach the scale of the classical primeval patch is required. In the context of our first scenario with $\theta_\star \sim 10^{12}$ GeV, one may be tempted to entertain the rich zoology of eternally inflating models, but intriguingly, a positive curvature with any appreciable $|\Omega_k| > 10^{-4}$ (is believed to) rule them out entirely [25]. Qualitatively, it will be hard to reconcile a near-scale-invariant spectrum (as observed) with a small scale of nonlinearity comparable to Hubble radius *in any model*. Nonetheless, interesting lessons may be learnt from 3D lattice simulations [26] in the context of super-renormalizble holographic dual theories (e.g., Refs. [24,27,28]). A less charted, but potentially more fertile territory may be a systematic study of the RG flows in renormalizable 3D quantum field theories that manifest confinement in the IR, and connects to our second scenario with $\theta_\star l_P \sim 1$. At a more foundational level, one may wonder whether there exists a holographic interpretation of the unimodular gravity.

On the observational front, there will be "dragons" (or other new physics) beyond the cosmological horizon. Indeed, in the first scenario, the quantum cosmology dragons should be in our face and right around the corner. Maybe an explanation for the infamous CMB anomalies [29] such as "The axis of evil" [30], "Planck Evidence for a closed Universe" [20,21] (Fig. 2), or rather more subtle "cosmological zero modes" [31] could be their tail? The fingerprints of the second scenario will be more subtle, but also more robust. For example, Eq. (52) predicts the running of the spectral index to be $\frac{dn_s}{d \ln k} = (n_s - 1)^2$, setting a clear target for the next generation of cosmological surveys (e.g., Ref. [32]). Furthermore, we expect the same IR strong coupling scale, $k_\star$ [Eq. (53)] for both scalar and tensor modes. Therefore, using the same functional form as Eq. (52) for tensors, we further can predict the tilt for tensors $n_t = n_s - 1$ (should they ever be detected).

The two scenarios are certainly distinguishable and falsifiable separately. For example, the observation of topological defects left over from a phase transition at an energy scale above 10^{12} GeV would kill the first scenario (but we are not holding our breath). Furthermore, given that such a low scale of classicality only allows for low-scale inflation, inflationary modes should be unobservable.[4] There are also other intriguing but very model-dependent possibilities for the first scenario, for example regarding the amplitude of scalar fluctuations. The ratio between $\theta_\star$ and the Planck scale is then 10^{-7}, not too different from 10^{-5}. Could the value of Λ and that of the amplitude of the primordial fluctuations be related?

We close with general comments regarding other work on the cosmological constant. We note that the argument we presented here gives a (minimal) width for the distribution of Λ, and not its central value, solving what Weinberg referred to as the "new cosmological constant problem" [33]. The "old cosmological problem" of why $\Lambda = 0$ is contained within (or at the center) of this distribution may find a solution through the nonperturbative structure of quantum

[3] K. Skenderis, private communication.

[4] We thank Tony Padilla for pointing this out.

gravity (e.g., Refs. [34–37]). We note also that the scale V_c could appear in the variance $\sigma(\Lambda)$ itself, as happens in causal set models [38–40], where Λ is a Poissonian process. This does not affect our argument, since the point made here is that $\sigma(\Lambda)$ must be smaller than the central value Λ_0 on whatever scale of classicality, V_c, we have defined.

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[1] W. G. Unruh, Unimodular theory of canonical quantum gravity, Phys. Rev. D **40**, 1048 (1989).
[2] L. Smolin, Quantization of unimodular gravity and the cosmological constant problems, Phys. Rev. D **80**, 084003 (2009).
[3] K. V. Kuchař, Does an unspecified cosmological constant solve the problem of time in quantum gravity?, Phys. Rev. D **43**, 3332 (1991).
[4] M. Henneaux and C. Teitelboim, The cosmological constant and general covariance, Phys. Lett. B **222**, 195 (1989).
[5] R. Carballo-Rubio, L. J. Garay, and G. García-Moreno, Unimodular Gravity vs General Relativity: A status report, arXiv:2207.08499.
[6] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).
[7] A. Padilla, Lectures on the Cosmological Constant Problem, arXiv:1502.05296.
[8] N. Kaloper and A. Padilla, Sequestering the Standard Model Vacuum Energy, Phys. Rev. Lett. **112**, 091304 (2014).
[9] N. Kaloper, A. Padilla, D. Stefanyszyn, and G. Zahariade, Manifestly Local Theory of Vacuum Energy Sequestering, Phys. Rev. Lett. **116**, 051302 (2016).
[10] L. Bombelli, W. E. Couch, and R. J. Torrence, Time as spacetime four-volume and the Ashtekar variables, Phys. Rev. D **44**, 2589 (1991).
[11] L. Smolin, Unimodular loop quantum gravity and the problems of time, Phys. Rev. D **84**, 044047 (2011).
[12] C. W. Misner, Absolute zero of time, Phys. Rev. **186**, 1328 (1969).
[13] S. Gielen and L. Menéndez-Pidal, Unitarity, clock dependence and quantum recollapse in quantum cosmolog, Classical Quantum Gravity **39**, 075011 (2022).
[14] J. Magueijo, Cosmological time and the constants of nature, Phys. Lett. B **820**, 136487 (2021).
[15] J. Magueijo, Connection between cosmological time and the constants of nature, Phys. Rev. D **106**, 084021 (2022).
[16] S. Gielen and J. Magueijo, Quantum analysis of the recent cosmological bounce in comoving Hubble length, arXiv:2201.03596.
[17] B. Alexandre and J. Magueijo, Possible quantum effects at the transition from cosmological deceleration to acceleration, Phys. Rev. D **106**, 063520 (2022).
[18] B. Alexandre and J. Magueijo, Semiclassical limit problems with concurrent use of several clocks in quantum cosmology, Phys. Rev. D **104**, 124069 (2021).
[19] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).
[20] W. Handley, Curvature tension: Evidence for a closed universe, Phys. Rev. D **103**, L041301 (2021).
[21] E. Di Valentino, A. Melchiorri, and J. Silk, Planck evidence for a closed Universe and a possible crisis for cosmology, Nat. Astron. **4**, 196 (2019).
[22] H. Kodama, Holomorphic wave function of the Universe, Phys. Rev. D **42**, 2548 (1990).
[23] J. Magueijo, Real Chern-Simons wave function, Phys. Rev. D **104**, 026002 (2021).
[24] P. McFadden and K. Skenderis, Holography for cosmology, Phys. Rev. D **81**, 021301 (2010).
[25] M. Kleban and M. Schillo, Spatial curvature falsifies eternal inflation, J. Cosmol. Astropart. Phys. 06 (2012) 029.
[26] G. Cossu, L. D. Debbio, A. Juttner, B. Kitching-Morley, J. K. L. Lee, A. Portelli, H. B. Rocha, and K. Skenderis, Nonperturbative Infrared Finiteness in a Superrenormalizable Scalar Quantum Field Theory, Phys. Rev. Lett. **126**, 221601 (2021).
[27] N. Afshordi, C. Coriano, L. D. Rose, E. Gould, and K. Skenderis, From Planck Data to Planck Era: Observational Tests of Holographic Cosmology, Phys. Rev. Lett. **118**, 041301 (2017).
[28] N. Afshordi, E. Gould, and K. Skenderis, Constraining holographic cosmology using Planck data, Phys. Rev. D **95**, 123505 (2017).
[29] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. VII. Isotropy and statistics of the CMB, Astron. Astrophys. **641**, A7 (2020).
[30] K. Land and J. Magueijo, Examination of Evidence for a Preferred Axis in the Cosmic Radiation Anisotropy, Phys. Rev. Lett. **95**, 071301 (2005).
[31] N. Afshordi and M. C. Johnson, Cosmological zero modes, Phys. Rev. D **98**, 023541 (2018).
[32] X. Li, N. Weaverdyck, S. Adhikari, D. Huterer, J. Muir, and H.-Y. Wu, The quest for the inflationary spectral runnings in

the presence of systematic errors, Astrophys. J. **862**, 137 (2018).
[33] S. Weinberg, The Cosmological Constant Problems (Talk given at Dark Matter 2000, February, 2000), arXiv:astro-ph/0005265.
[34] S. W. Hawking, The cosmological constant is probably zero, Phys. Lett. **134B**, 403 (1984).
[35] N. Afshordi, Gravitational Aether and the thermodynamic solution to the cosmological constant problem, arXiv:0807.2639.
[36] S. Aslanbeigi, G. Robbers, B. Z. Foster, K. Kohri, and N. Afshordi, Phenomenology of gravitational aether as a solution to the old cosmological constant problem, Phys. Rev. D **84**, 103522 (2011).
[37] N. Kaloper and A. Westphal, A Quantum-Mechanical Mechanism for Reducing the Cosmological Constant, arXiv:2204.13124.
[38] R. D. Sorkin, Causal sets: Discrete gravity (Notes for the Valdivia Summer School), in *School on Quantum Gravity* (2003), pp. 305–327, arXiv:gr-qc/0309009.
[39] M. Ahmed, S. Dodelson, P. B. Greene, and R. Sorkin, Everpresent Λ, Phys. Rev. D **69**, 103523 (2004).
[40] N. Zwane, N. Afshordi, and R. D. Sorkin, Cosmological tests of Everpresent Λ, Classical Quantum Gravity **35**, 194002 (2018).