Erratum: Theoretical analysis of the leptonic decays $B \to \ell\ell\ell'\bar{\nu}_{\ell'}$ [Phys. Rev. D 105, 014028 (2022)]

Mikhail A. Ivanov and Dmitri Melikhov

(Received 9 November 2022; published 16 December 2022)

DOI: 10.1103/PhysRevD.106.119901

We correct formulas from [1] for the Bremsstrahlung part of the $B \to l^+l^-l'\nu'$ amplitude for the case of a massive lepton $m_l' \neq 0$ and discuss its impact on the behavior of the distribution at small q^2. All results for massless leptons remain unchanged.

I. THE BREMSSTRAHLUNG CONTRIBUTION

The Bremsstrahlung contribution to the $B \to \gamma^* l'\nu'$ amplitude, $Q_l = Q_B$ (see Fig. 1, the momenta are defined such that $p = q + q'$, $q' = k + k'$) for a massive lepton l' ($m_{l'} \equiv m \neq 0$) and a massless neutrino is proportional to $\varepsilon^*_\alpha(q)A^L_\alpha$ with

$$\begin{aligned} A^L_\alpha &= if_B Q_B p_\nu \left\{ \bar{u}(k)\gamma_\alpha \frac{\hat{q}+\hat{k}+m}{m^2-(k+q)^2}\gamma_\nu(1-\gamma_5)v(k')\right\} \\ &= if_B Q_B \left\{ \bar{u}(k)\gamma_\alpha \frac{\hat{q}+\hat{k}+m}{m^2-(k+q)^2}(\hat{k}+\hat{q}-m+m)(1-\gamma_5)v(k')\right\} \\ &= if_B Q_B(-g_{\alpha\nu})\bar{u}(k)\gamma_\nu(1-\gamma_5)v(k') + if_B Q_B m\bar{u}(k)\gamma_\alpha \frac{\hat{q}+\hat{k}+m}{m^2-(k+q)^2}(1-\gamma_5)v(k') \\ &= A^{L,0}_\alpha + A^{L,1}_\alpha, \end{aligned} \tag{1}$$

where we have isolated two structures $A^{L,0}_\alpha$ and $A^{L,1}_\alpha$,

$$A^{L,0}_\alpha \equiv if_B Q_B\left(-g_{\alpha\nu} - \frac{q'_\alpha q'_\nu}{q'q}\right)\bar{u}(k)\gamma_\nu(1-\gamma_5)v(k'), \tag{2}$$

$$\begin{aligned} A^{L,1}_\alpha &\equiv if_B Q_B\left[m\bar{u}(k)\gamma_\alpha\frac{\hat{q}+\hat{k}+m}{m^2-(k+q)^2}(1-\gamma_5)v(k') + \frac{q'_\alpha q'_\nu}{q'q}\bar{u}(k)\gamma_\nu(1-\gamma_5)v(k')\right] \\ &= if_B Q_B m\left[\bar{u}(k)\gamma_\alpha\frac{\hat{q}+\hat{k}+m}{m^2-(k+q)^2}(1-\gamma_5)v(k') + \frac{q'_\alpha}{q'q}\bar{u}(k)(1-\gamma_5)v(k')\right]. \end{aligned} \tag{3}$$

These structures are defined such that they contain no singularity at $q^2 \to 0$ (except for $q'^2 = M_B^2$) and satisfy the following useful relations

$$q_\alpha A^{L,0}_\alpha = -iQ_B f_B p_\nu\, \bar{u}(k)\gamma_\nu(1-\gamma_5)v(k'), \tag{4}$$

$$q_\alpha A^{L,1}_\alpha = 0. \tag{5}$$

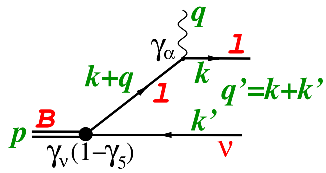

FIG. 1. Feynman diagram describing the Bremsstrahlung amplitude.

II. COMBINING $A_\alpha^{L,0}$ WITH THE HADRONIC CONTRIBUTION

Recall that the hadronic tensor $T^5_{\alpha\nu}$ given by Eq. (2.3) of [1] satisfies the following Eq. (2.5) of [1]

$$q_\alpha T^5_{\alpha\nu} = iQ_B f_B p_\nu.$$

Therefore, it is convenient to combine the hadronic contribution to the amplitude $A(B \to ll l'\nu')$ with $A_\alpha^{L,0}$ and write

$$A_{\rm axial}(B\to\gamma^* l'\nu') = A^{(0)}_{\rm axial}(B\to\gamma^* l'\nu') + A^{(1)}_{\rm axial}(B\to\gamma^* l'\nu'), \tag{6}$$

where $[\bar l' \equiv \bar u(k),\ \nu' \equiv v(k')]$

$$A^{(0)}_{\rm axial}(B\to\gamma^* l'\nu') = ie\frac{G_F}{\sqrt{2}}V_{ub}\varepsilon^*_\alpha(q)\bar l'\gamma_\nu(1-\gamma_5)\nu'\left\{\left(g_{\alpha\nu}-\frac{q_\alpha q_\nu}{q^2}\right)q'qf_{1A}+\left(q'_\alpha-\frac{q'q}{q^2}q_\alpha\right)[p_\nu f_{2A}+q_\nu f_{3A}]+Q_Bf_B\frac{q_\alpha p_\nu}{q^2}\right\}$$
$$+\,ie\frac{G_F}{\sqrt{2}}V_{ub}\varepsilon^*_\alpha(q),\bar l'\gamma_\nu(1-\gamma_5)\nu'\left\{Q_Bf_B\left(-g_{\alpha\nu}-\frac{q'_\alpha q'_\nu}{q'q}\right)\right\}, \tag{7}$$

and

$$A^{(1)}_{\rm axial}(B\to\gamma^* l'\nu') = e\frac{G_F}{\sqrt{2}}V_{ub}\varepsilon^*_\alpha(q)A^{(L,1)}_\alpha. \tag{8}$$

$A^{(L,1)}_\alpha$ contains both axial and vector parts; here only the axial part contributes. The explicit form of $A^{(L,1)}_\alpha$ is not important for our argument—only its property (4) and the absence of a singularity at $q^2 = 0$ are essential.

III. $A^{(0)}_{\rm axial}$

Using Eq. (2.14) from [1], one obtains

$$A^{(0)}_{\rm axial}(B\to\gamma^* l'\nu') = ie\frac{G_F}{\sqrt{2}}V_{ub}\bar l'\gamma_\nu(1-\gamma_5)\nu'\varepsilon^*_\alpha(q)\times\left\{(g_{\alpha\nu}q'q-q'_\alpha q_\nu)\frac{F_{1A}}{M_B}+q'_\alpha q_\nu\frac{F_{2A}}{M_B}+q'_\alpha q'_\nu\left(\frac{F'_{2A}}{M_B}-\frac{2Q_Bf_B}{M_B^2-q^2-q'^2}\right)\right\}. \tag{9}$$

Notice the new term accompanying the form factor F'_{2A} compared to Eq. (2.15) of [1]. By virtue of (2.17) from [1], the term proportional to $q'_\alpha q'_\nu$ vanishes at $q^2 = 0$ in agreement with Ref. [2].

IV. $A^{(1)}_{\rm axial}$

It is important to check that $A^{(1)}_{\rm axial}$ which contains factor m does not generate contributions to $|A|^2$ finite at q^2. To show this we consider the following quantity (the sum runs over polarizations of l' and ν')

$$L_{\alpha\beta}(q,q') = \sum_{\text{leptons}} A_\alpha^{L,1}(q,q')A_\beta^{L,1}(q,q') = (q^2 g_{\alpha\beta} - q_\alpha q_\beta)C_1(q^2,q'^2) + (q^2 q'_\alpha - q^q q_\alpha)(q^2 q'_\beta - q^q q_\beta)C_2(q^2,q'^2). \quad (10)$$

The Lorentz structure of $L_{\alpha\beta}$ is dictated by its symmetry, $L_{\alpha\beta} = L_{\beta\alpha}$, and the transversity condition $q_\alpha A_\alpha^{L,1}(q,q') = 0$. Moreover, the invariant functions $C_{1,2}(q^2,q'^2)$ have no poles at $q^2 = 0$ since the amplitude $A_\alpha^{L,1}(q,q')$ was defined such that it is finite at $q^2 = 0$. After the convolution with the leptonic tensor corresponding to l^+l^-, we see that the amplitude $A^{L,1}$ generates a contribution that has zero at $q^2 = 0$. The same conclusion may be drawn for the interference term $A^0_{\text{Brems}}A^{L,1}$.

V. CONCLUSION

So we conclude that Eq (4.4) of [1] for $A|^2_{m_l'}$ should be modified by replacing $F'_{2A} \to F'_{2A} - 2Q_B f_B/(M_B^2 - q^2 - q'^2)$. By virtue of (2.17) from [1], the latter combination vanishes at $q^2 \to 0$ thus providing no enhancement of $|A|^2_{m_l'}$ at small q^2. We therefore conclude that the lepton mass effect plays no role also in $B \to e^+e^-\mu\nu_\mu$ decay.

The first line of Table III of [1] should read

Mode	$q^2 = [4m_e^2, 4m_\mu^2]$	$q^2 = [4m_\mu^2, 0.4\ \text{GeV}^2]$	$q^2 = [0.4\ \text{GeV}^2,\ 1\ \text{GeV}^2]$	$q^2 = [1\ \text{GeV}^2, q^2_{\text{max}}]$	Total
$e^+e^-\mu\nu_\mu$	1.96×10^{-8}	6.79×10^{-9}	2.51×10^{-8}	4.14×10^{-10}	5.19×10^{-8}

Equation (6.3) from [1] takes the form

$$\text{Br}(B \to e^+e^-\mu\nu_\mu) = (5.241^{+2.6}_{-1.05}|_{\lambda_b} \pm 0.70|_{\text{weak ffs}})10^{-8}.$$

ACKNOWLEDGMENTS

We would like to thank D. van Dyk for useful discussions.

[1] M. A. Ivanov and D. Melikhov, Phys. Rev. D **105**, 014028 (2022).
[2] S. Kürten, M. Zanke, B. Kubis, and D. van Dyk, arXiv:2210.09832.