Generalized parton distributions in spin-3/2 particles

Dongyan Fu,[1,2,*] Bao-Dong Sun,[3,4,5,†] and Yubing Dong[1,2,‡]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[2]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 101408, China*
[3]*Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China*
[4]*Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China*
[5]*Helmholtz Institut für Strahlen-und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

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Generalized parton distribution functions (GPDs) of spin-3/2 particles are defined for the first time in this paper. Eight unpolarized and eight polarized GPDs are found. In the forward limit of GPDs, the structure functions and parton distribution functions are obtained. Then, the sum rules that connect the GPDs with the electromagnetic and gravitational form factors are explicitly displayed. Finally, the relations between GPDs and the helicity amplitudes of the system are derived.

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I. INTRODUCTION

The electromagnetic form factors (EMFFs) of Delta isobar (spin-3/2) have been explored extensively both experimentally and theoretically for a long history [1–6]. They show richer information on charge and magnetic structures carried by the spin-3/2 particles with respect to the lower spin hadrons like nucleon [7–10], deuteron [11–16], and rho meson [17–21] etc. In parallel, the gravitational form factors (GFFs), which are defined by factorizing the matrix element of the energy momentum tensor (EMT), characterize the mechanical properties such as the mass, the spin density, as well as the internal force distributions inside the particles [22–27]. Both EMFFs and GFFs can be related through the generalized parton distribution functions (GPDs), which is firstly introduced in describing the deeply virtual Compton scattering process for spin-1/2 particles [28–30] and then for spin-1 particles [31]. GPDs can also produce parton distribution functions (PDFs) and structure functions in the forward limit. Moreover, through the crossing symmetry, GPDs are also connected with generalized distribution amplitudes (GDAs) [32]. It makes the concept of GPDs very important to understand the abundant experimental and theoretical [by model-(in)dependent studies, lattice QCD calculations etc.] information of hadron structures in a unified theoretical framework [33–36]. Thus, GPDs and the related quantities have been receiving lots of interest for particles of spin-0 (e.g., pion and kaon mesons [37–42]), spin-1/2 (e.g., nucleon [43–46], and for nuclei, like ^{3}He [47,48]), and spin-1 (e.g., rho meson [49–52] and deuteron [47,48,53–55]), etc.

In contrast to the increasing interest in the EMFFs and GFFs of the delta isobar (spin-3/2), the definitions of GPDs for spin-3/2 hadrons are still missing. One could expect that the spin-3/2 GPDs would expand and deepen our understanding of the quantities such as PDFs [56], structure functions, EMFFs, and GFFs, etc. for higher spin ($\geq 3/2$) particles in a similar method as lower spin cases. Therefore it is of great interest to give the definitions and properties of spin-3/2 GPDs and it is the main purpose of this work. Now the most promising "measurement" is the lattice QCD calculation, e.g., the recent lattice calculation on spin-3/2 gluonic GFFs [57]. In the future electron-ion collision experiments, EIC [58] (under construction) and EicC [59] (planned), the candidate targets may possess higher spins, including spin-3/2 nuclei like $^7_3$Li, $^9_4$Be, and $^{11}_4$B, and even spin-5/2 ones like $^{17}_8$O, $^{25}_{12}$Mg, and $^{27}_{13}$Al [59]. Therefore, it would be possible to access the spin-3/2 GPDs and helicity amplitudes experimentally [53] in the near future. Nevertheless, the spin-3/2 GPDs by themselves are important in the theoretical aspect.

This work is organized as follows: In Sec. II, we give the definitions and properties of GPDs of the spin-3/2 system.

*fudongyan@ihep.ac.cn
†bao-dong.sun@m.scnu.edu.cn
‡dongyb@ihep.ac.cn

Section III displays the connections of structure functions and PDFs to GPDs, and two sum rules of the structure functions are derived for the spin-3/2 system. Moreover, the sum rules connecting GPDs with EMFFs and GFFs and the helicity amplitudes are explicitly given. Finally, Sec. IV will be devoted to a summary and discussion.

II. GENERALIZED PARTON DISTRIBUTIONS OF A SPIN-3/2 PARTICLE

The convention of the four-vector v in light-cone coordinates is given as

$$v = (v^+, v^-, \mathbf{v}_\perp), \text{ with } v^\pm = v^0 \pm v^3 \text{ and } \mathbf{v}_\perp = (v^1, v^2), \tag{1}$$

and the lightlike four-vector $n = (0, 2, \mathbf{0})$ with $n^2 = 0$. The scalar product of two four-vectors is $u \cdot v = \frac{1}{2}u^+ v^- + \frac{1}{2}u^- v^+ - \mathbf{u}_\perp \cdot \mathbf{v}_\perp$. The convention for the momenta is

$$P = \frac{p + p'}{2}, \qquad q = p' - p, \qquad t = q^2, \tag{2}$$

where p and p' are the initial and final momenta, respectively. The conventions of variables, skewness ξ and x in the representations of GPDs, are

$$\xi = -\frac{q \cdot n}{2P \cdot n} = -\frac{q^+}{2P^+} \quad (|\xi| \leq 1)$$
$$\text{and} \quad x = \frac{k \cdot n}{P \cdot n} = \frac{k^+}{P^+} \quad (-1 \leq x \leq 1), \tag{3}$$

where k is the loop momentum in Fig. 1. It shows that the four-momentum of the parton emitted from the initial particle is $k - q/2$ and the one absorbed in the final particle is $k + q/2$. The corresponding fraction of the momentum carried by the parton over that of the total system in the light cone direction is $x_q = (x + \xi)/(1 + \xi)$ for the initial one and $x'_q = (x - \xi)/(1 - \xi)$ for the final one [60].

The GPDs are defined through the nondiagonal matrix elements of quark and gluon nonlocal current operators at a lightlike separation [61]. The general decompositions for quarks can be written as

$$V_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \langle p', \lambda' | \bar{\psi}(-z/2) \not{n} \psi(z/2) | p, \lambda \rangle |_{z^+=0, \mathbf{z}=\mathbf{0}},$$
$$= -\bar{u}_{\alpha'}(p', \lambda') \mathcal{H}^{\alpha'\alpha}(x, \xi, t) u_\alpha(p, \lambda) \tag{4}$$

for the unpolarized case, and

$$A_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \langle p', \lambda' | \bar{\psi}(-z/2) \not{n} \gamma_5 \psi(z/2) | p, \lambda \rangle |_{z^+=0, \mathbf{z}=\mathbf{0}},$$
$$= -\bar{u}_{\alpha'}(p', \lambda') \tilde{\mathcal{H}}^{\alpha'\alpha}(x, \xi, t) u_\alpha(p, \lambda) \tag{5}$$

for the polarized case, where $\lambda(\lambda')$ are the helicities of the incoming (outgoing) spin-3/2 particles and $\alpha(\alpha')$ are reserved indices for the initial (final) states in this work. The tensors $\mathcal{H}^{\alpha'\alpha}$ and $\tilde{\mathcal{H}}^{\alpha'\alpha}$ define the GPDs as will be shown in the later context. The Rarita-Schwinger spinor $u_\alpha(p, \lambda)$ shown in A is normalized to $\bar{u}_\alpha(p, \lambda') u^\alpha(p, \lambda) = -2M\delta_{\lambda'\lambda}$.

To count the number of independent GPDs, one can define the helicity amplitudes for the scattering of a quark in a spin-3/2 particle as [33]

$$\mathcal{A}_{\lambda'\mu', \lambda\mu} = \int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \langle p', \lambda' | \mathcal{O}_{\mu'\mu}(z) | p, \lambda \rangle |_{z^+=0, \mathbf{z}=\mathbf{0}}. \tag{6}$$

The operators $\mathcal{O}_{\mu'\mu}$ describe a quark transferring from helicity μ to μ'. The two helicity-conserved operators are

$$\mathcal{O}_{++}(z) = \frac{1}{4}\bar{\psi}(-z/2)\gamma^+(1 + \gamma_5)\psi(z/2),$$
$$\mathcal{O}_{--}(z) = \frac{1}{4}\bar{\psi}(-z/2)\gamma^+(1 - \gamma_5)\psi(z/2). \tag{7}$$

With those definitions, one has

$$\mathcal{A}_{\lambda'\pm, \lambda\pm} = \frac{1}{2}(V_{\lambda'\lambda} \pm A_{\lambda'\lambda}), \tag{8}$$

where $\pm$ represents the quark helicities. The constraints

$$\mathcal{A}_{-\lambda'-\mu, -\lambda-\mu} = (-1)^{\lambda'-\lambda} \mathcal{A}^*_{\lambda'\mu, \lambda\mu},$$
$$\mathcal{A}(x, \xi, t)_{\lambda'\mu, \lambda\mu} = (-1)^{\lambda'-\lambda} \mathcal{A}^*(x, -\xi, t)_{\lambda\mu, \lambda'\mu} \tag{9}$$

can be obtained from the parity and time reversal invariances. And the analogous forms have been found for spin

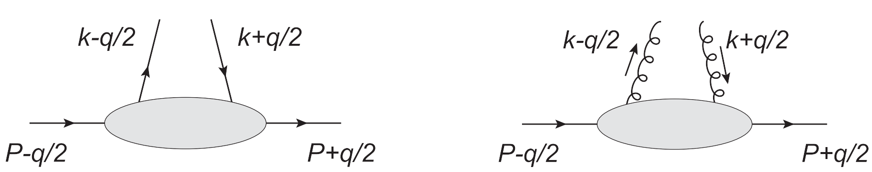

FIG. 1. Diagrams describe the GPDs for quarks (left) and gluons (right).

1/2 and 1 cases in Refs. [31,33]. With these constraints, we finally obtain up to eight independent unpolarized GPDs and eight independent polarized GPDs for the spin-3/2 system.

The building blocks to construct the Lorentz structures accompanying the spin-3/2 GPDs are P^μ, q^μ, n^μ, γ^μ, γ^5 (or Levi-Civita tensors $\epsilon^{\mu\nu\alpha\beta}$), $g^{\mu\nu}$, and $\sigma^{\mu\nu}$. With help of the on-shell identities given in Refs. [6,23] and the properties of the Rarita-Schwinger spinor (see A), one can obtain eight independent tensors for the unpolarized and eight for the polarized cases, which agrees with the number counting from the helicity amplitudes. Equivalently, one can build the tensors accompanying the GPDs of the spin-3/2 particle through the direct product between the unpolarized spin-1/2 and unpolarized spin-1 structures or the direct product between the polarized spin-1/2 and polarized spin-1 structures. It can be proven that the "polarized-polarized" pairs of tensors are equivalent to "unpolarized-unpolarized" ones. It is known that the tensors accompanying the unpolarized GPDs of a spin-1/2 particle [62,63] have two independent Lorentz structures,

$$1, \not{n}, \tag{10}$$

and the tensors for the spin-1 case are [31,49]

$$g^{\alpha'\alpha}, \quad P^{\alpha'}P^{\alpha}, \quad n^{[\alpha'}P^{\alpha]}, \quad n^{\{\alpha'}P^{\alpha\}}, \quad n^{\alpha'}n^{\alpha}, \tag{11}$$

with $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$ and $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$. The direct product gives

$$\begin{gathered} g^{\alpha'\alpha}, \quad P^{\alpha'}P^{\alpha}, \quad n^{[\alpha'}P^{\alpha]}, \quad n^{\{\alpha'}P^{\alpha\}}, \quad n^{\alpha'}n^{\alpha}, \\ g^{\alpha'\alpha}\not{n}, \quad P^{\alpha'}P^{\alpha}\not{n}, \quad n^{[\alpha'}P^{\alpha]}\not{n}, \quad n^{\{\alpha'}P^{\alpha\}}\not{n}, \quad n^{\alpha'}n^{\alpha}\not{n}. \end{gathered} \tag{12}$$

There are two on-shell identities, Eqs. (B1) and (B2) (see B), that can be employed to reduce two tensors $n^{\{\alpha'}P^{\alpha\}}$ and $n^{\{\alpha'}P^{\alpha\}}\not{n}$ in terms of the rest eight independent ones. Hence, there are eight independent unpolarized GPDs for the spin-3/2 case, which can be defined as

$$\begin{aligned} \mathcal{H}^{\alpha'\alpha} = {} & H_1\frac{g^{\alpha'\alpha}}{M} + H_2\frac{P^{\alpha'}P^{\alpha}}{M^3} + H_3\frac{n^{[\alpha'}P^{\alpha]}}{MP\cdot n} \\ & + H_4\left[\frac{3Mn^{\alpha'}n^{\alpha}}{(P\cdot n)^2} + \frac{g^{\alpha'\alpha}}{M}\right] + H_5\left[\frac{g^{\alpha'\alpha}\not{n}}{P\cdot n} - \frac{g^{\alpha'\alpha}}{M}\right] \\ & + H_6\frac{P^{\alpha'}P^{\alpha}\not{n}}{M^2P\cdot n} + H_7\frac{n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} \\ & + H_8\left[\frac{3M^2n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3} - \frac{3Mn^{\alpha'}n^{\alpha}}{(P\cdot n)^2}\right], \end{aligned} \tag{13}$$

where $\mathcal{H}^{\alpha'\alpha} \equiv \mathcal{H}^{\alpha'\alpha}(x,\xi,t)$ and $H_i \equiv H_i(x,\xi,t)$, and similar for the polarized case afterwards.

Analogously, the two independent Lorentz structures of the polarized spin-1/2 GPDs are [61]

$$\gamma^5, \quad \not{n}\gamma^5, \tag{14}$$

and the four independent tensor structures of the spin-1 polarized GPDs are [31,49]

$$i\epsilon^{nP\alpha'\alpha}, \quad i\epsilon^{nPq\{\alpha}P^{\alpha'\}}, \quad i\epsilon^{nPq[\alpha}P^{\alpha']}, \quad i\epsilon^{nPq\{\alpha}n^{\alpha'\}}. \tag{15}$$

Analogous to the unpolarized case, one expects that the Lorentz structures of the polarized GPDs in the spin-3/2 case can also be expressed as the direct product between the polarized and unpolarized structures for the spin-1/2 and spin-1 cases. The Lorentz structures of the polarized GPDs can be written in two equivalent ways: with polarization comes from either spin-1/2 part (γ^5 terms) or spin-1 part (the Levi-Civita tensors). With the constraints by the on-shell identities [Eqs. (B3) and (B4)], there are eight independent polarized GPDs in the spin-3/2 case:

$$\begin{aligned} \tilde{\mathcal{H}}^{\alpha'\alpha} = {} & \tilde{H}_1\frac{g^{\alpha'\alpha}}{M}\gamma^5 + \tilde{H}_2\frac{P^{\alpha'}P^{\alpha}}{M^3}\gamma^5 + \tilde{H}_3\frac{n^{\{\alpha'}P^{\alpha\}}}{MP\cdot n}\gamma^5 \\ & + \tilde{H}_4\frac{Mn^{\alpha'}n^{\alpha}}{(P\cdot n)^2}\gamma^5 + \tilde{H}_5\frac{3g^{\alpha'\alpha}}{\sqrt{5}P\cdot n}\not{n}\gamma^5 \\ & + \tilde{H}_6\frac{3P^{\alpha'}P^{\alpha}}{\sqrt{5}M^2(P\cdot n)}\not{n}\gamma^5 + \tilde{H}_7\frac{n^{\{\alpha'}P^{\alpha\}}}{(P\cdot n)^2}\not{n}\gamma^5 \\ & + \tilde{H}_8\left[\frac{\sqrt{5}M^2n^{\alpha'}n^{\alpha}}{(P\cdot n)^3} + \frac{g^{\alpha'\alpha}}{\sqrt{5}P\cdot n}\right]\not{n}\gamma^5. \end{aligned} \tag{16}$$

The time reversal does not provide further limits on the number of GPDs but determines their behavior under the sign change of the skewness parameter ξ,

$$H_i(x,\xi,t) = H_i(x,-\xi,t) \;\text{ with }\; i = 1,2,4,5,6,8, \tag{17a}$$

$$H_i(x,\xi,t) = -H_i(x,-\xi,t) \;\text{ with }\; i = 3,7, \tag{17b}$$

$$\tilde{H}_j(x,\xi,t) = -\tilde{H}_j(x,-\xi,t) \;\text{ with }\; j = 1,2,3,4, \tag{17c}$$

$$\tilde{H}_j(x,\xi,t) = \tilde{H}_j(x,-\xi,t) \;\text{ with }\; j = 5,6,7,8. \tag{17d}$$

$H_{3,7}$ and $\tilde{H}_{1,2,3,4}$ are T-odd GPDs and others are T-even GPDs. When $\xi = 0$, $H_{3,7}(x,0,t) = 0$ and $\tilde{H}_{1,2,3,4}(x,0,t) = 0$. It should be mentioned that the spin-0 and spin-1/2 GPDs are all T even, and the T-odd GPDs start to appear from the spin-1 cases, which are $H_4^{(S=1)}$ and $\tilde{H}_3^{(S=1)}$ [31].

In addition, for the gluon distributions in the spin-3/2 system, instead of the matrix elements in (4) and (5), we have

$$\begin{aligned} & \frac{n_{\beta'}n_\beta}{P\cdot n}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\langle p',\lambda'|F^{\beta'\mu}(-z/2)F_\mu^{\beta}(z/2)|p,\lambda\rangle|_{z^+=0,\mathbf{z}=\mathbf{0}} \\ & = -\bar{u}_{\alpha'}(p',\lambda')\mathcal{H}_g^{\alpha'\alpha}(x,\xi,t)u_\alpha(p,\lambda), \end{aligned} \tag{18a}$$

$$-i\frac{n_{\beta'}n_\beta}{P\cdot n}\int\frac{\mathrm{d}z^-}{2\pi}e^{ix(P\cdot z)}\langle p',\lambda'|F^{\beta'\mu}(-z/2)\tilde{F}_\mu{}^\beta(z/2)|p,\lambda\rangle|_{z^+=0,\mathbf{z}=\mathbf{0}}=-\bar{u}_{\alpha'}(p',\lambda')\tilde{\mathcal{H}}_g^{\alpha'\alpha}(x,\xi,t)u_\alpha(p,\lambda), \tag{18b}$$

with $\tilde{F}^{\alpha\beta}=\frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$. The tensors $\mathcal{H}_g^{\alpha'\alpha}$, $\tilde{\mathcal{H}}_g^{\alpha'\alpha}$ have the same structures as those for quark distributions given in Eqs. (13) and (16). It should be stressed that Diehl's convention [33] is used here, and the definitions of gluon GPDs under Ji's convention [64] would differ by a factor $2x$, i.e., $H_g=2xH_g^{(\mathrm{Ji})}$ and $\tilde{H}_g=2x\tilde{H}_g^{(\mathrm{Ji})}$.

III. PDFS, SUM RULES, AND HELICITY AMPLITUDES

A. The forward limit

It is known that the GPDs in the forward limit give the usual parton distribution functions. In the parton model for the spin-$3/2$ sector, there are four independent structure functions in deep inelastic scattering at leading twist and leading order in α_s. They are F_1, b_1, g_1, g_2 whose probabilistic interpretations in terms of quark densities read [65]

$$F_1(x)=\frac{1}{2}\sum_q e_q^2\frac{q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x)+q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x)}{2}+\{q\to\bar{q}\}, \tag{19a}$$

$$b_1(x)=\frac{1}{2}\sum_q e_q^2\frac{(q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x))-(q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x))}{2}+\{q\to\bar{q}\}, \tag{19b}$$

$$g_1(x)=\frac{1}{2}\sum_q e_q^2\frac{3(q_\uparrow^{\frac{3}{2}}(x)-q_\uparrow^{-\frac{3}{2}}(x))+(q_\uparrow^{\frac{1}{2}}(x)-q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}}+\{q\to\bar{q}\}, \tag{19c}$$

$$g_2(x)=\frac{1}{2}\sum_q e_q^2\frac{(q_\uparrow^{\frac{3}{2}}(x)-q_\uparrow^{-\frac{3}{2}}(x))-3(q_\uparrow^{\frac{1}{2}}(x)-q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}}+\{q\to\bar{q}\}, \tag{19d}$$

where $q_\uparrow^\lambda(x)$ stands for the probability to find a quark with momentum fraction x and positive helicity in the spin-$3/2$ particle with helicity λ. In addition, one has $q_\uparrow^\lambda(x)=q_\downarrow^{-\lambda}(x)$ from parity invariance. Here, g_2 is the new structure function as the spin goes from 1 up to $3/2$. In the forward limit, there are $\bar{u}_{\alpha'}P^{\alpha'}=P^\alpha u_\alpha=0$, so the only structures in Eqs. (13) and (16) that survive are those proportional to H_i and $\tilde{H}_i$ with $i=1,4,5,8$. Moreover, in the forward limit, $\tilde{H}_{1,4}$ vanish because of the time reversal relation (17) and $H_{5,8}$ vanish as well because of $\bar{u}_{\alpha'}(M\not{n}-P\cdot n)u_\alpha=0$. According to the results for the helicity amplitudes shown below, one gets

$$2H_1(x,0,0)=\frac{q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x)+q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x)}{2}, \tag{20a}$$

$$2H_4(x,0,0)=\frac{(q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x))-(q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x))}{2}, \tag{20b}$$

$$2\tilde{H}_5(x,0,0)=\frac{3(q_\uparrow^{\frac{3}{2}}(x)-q_\uparrow^{-\frac{3}{2}}(x))+(q_\uparrow^{\frac{1}{2}}(x)-q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}}, \tag{20c}$$

$$2\tilde{H}_8(x,0,0)=\frac{(q_\uparrow^{\frac{3}{2}}(x)-q_\uparrow^{-\frac{3}{2}}(x))-3(q_\uparrow^{\frac{1}{2}}(x)-q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}}, \tag{20d}$$

for $x>0$. Similar to the deuteron case [31], the corresponding relations for $x<0$ involve the antiquark distributions at $-x$, with an overall minus sign in the expressions for H_1 and H_4. With the sum rules (28) given in the next subsection for the GPDs, the structure functions have the following sum rules:

$$\int_0^1\mathrm{d}x b_1(x)=0,\qquad\int_0^1\mathrm{d}x g_2(x)=0, \tag{21}$$

if the quark sea $q-\bar{q}$ does not contribute to the integral. These two equalities in Eq. (21) are consistent with the sum rules derived from the rotation properties of the structure functions [65].

B. Sum rules

As shown in Ref. [33], the $(a+1)$th Mellin moments (in x) of the operator defining the quark GPDs of the system (4) lead to derivative operators between the two fields,

$$(P\cdot n)^{a+1}\int\mathrm{d}x x^a\int\frac{\mathrm{d}z^-}{2\pi}e^{ixP^+z^-}[\bar{\psi}(-z/2)\not{n}\psi(z/2)]|_{z^+=0,\mathbf{z}=0}=\left(i\frac{\mathrm{d}}{\mathrm{d}z^-}\right)^a[\bar{\psi}(-z/2)\not{n}\psi(z/2)]|_{z=0}=\bar{\psi}(0)\not{n}(i\overleftrightarrow{\partial}^+)^a\psi(0). \tag{22}$$

This relation at the operator level connects the quark GPDs with EMFFs ($a=0$), GFFs ($a=1$), and other FFs from higher rank current operators. For gluon GPDs, there exist similar relations as shown explicitly by Ref. [33].

The decompositions of the matrix elements of the vector [3] and axial vector [66,67] currents of the spin-3/2 case are

$$\langle p',\lambda'|\bar{\psi}(0)\gamma^\mu\psi(0)|p,\lambda\rangle = -2\bar{u}_{\alpha'}(p',\lambda')\left[g^{\alpha'\alpha}\left(G_1(t)\frac{P^\mu}{M}+G_5(t)\gamma^\mu\right)+\frac{P^{\alpha'}P^\alpha}{M^2}\left(G_2(t)\frac{P^\mu}{M}+G_6(t)\gamma^\mu\right)\right]u_\alpha(p,\lambda), \tag{23}$$

$$\langle p',\lambda'|\bar{\psi}(0)\gamma^\mu\gamma^5\psi(0)|p,\lambda\rangle = -2\bar{u}_{\alpha'}(p',\lambda')\left[g^{\alpha'\alpha}\left(-\tilde{G}_1(t)\frac{q^\mu}{2M}+\tilde{G}_5(t)\gamma^\mu\right)+\frac{P^{\alpha'}P^\alpha}{M^2}\left(-\tilde{G}_2(t)\frac{q^\mu}{2M}+\tilde{G}_6(t)\gamma^\mu\right)\right]\gamma^5u_\alpha(p,\lambda), \tag{24}$$

where a different set of notations is adopted for later convenience to exhibit their relations with the quark GPDs. And the relations between the different notations are $2(G_1,G_2,G_5,G_6)=(-a_2,c_2,-a_1,c_1)$ [3] and $2(\tilde{G}_1,\tilde{G}_2,\tilde{G}_5,\tilde{G}_6)=(-g_3,h_3,g_1,-h_1)$ [67]. Note that the matrix elements (as well as tensors $\mathcal{H}^{\alpha'\alpha}$, $\tilde{\mathcal{H}}^{\alpha'\alpha}$ and GPDs) are defined flavor by flavor, so one should multiply the electric or weak charges and sum over flavors to get the conventional form factors. The isospin symmetry is not specified in this work.

The GFFs for the spin-3/2 particle are defined as [23,25]

$$\begin{aligned}\langle p',\lambda'|\hat{T}^{\mu\nu}(0)|p,\lambda\rangle = -\bar{u}_{\alpha'}(p',\lambda')\Bigg[&\frac{P^\mu P^\nu}{M}\left(g^{\alpha'\alpha}F^T_{1,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{1,1}(t)\right)+\frac{(q^\mu q^\nu-g^{\mu\nu}q^2)}{4M}\left(g^{\alpha'\alpha}F^T_{2,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{2,1}(t)\right)\\&+Mg^{\mu\nu}\left(g^{\alpha'\alpha}F^T_{3,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{3,1}(t)\right)+\frac{P^{\{\mu}i\sigma^{\nu\}q}}{2M}\left(g^{\alpha'\alpha}F^T_{4,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{4,1}(t)\right)\\&-\frac{1}{M}(2q^{\{\mu}g^{\nu\}[\alpha'}P^{\alpha]}+8g^{\mu\nu}P^{\alpha'}P^\alpha-g^{\alpha'\{\mu}g^{\nu\}\alpha}q^2)F^T_{5,0}(t)+Mg^{\alpha'\{\mu}g^{\nu\}\alpha}F^T_{6,0}(t)\Bigg]u_\alpha(p,\lambda).\end{aligned} \tag{25}$$

The individual tensors in Eq. (12) make it more convenient for establishing the polynomiality sum rules. It is similar for the polarized case. Therefore, we introduce another set of coefficient functions that accompany the individual tensors,

$$E_1 = H_1+H_4-H_5,\quad E_4=3H_4-3H_8,\quad E_8=3H_8,$$
$$E_i=H_i \text{ with } i=2,3,5,6,7, \tag{26}$$

and

$$\tilde{E}_5=\frac{3}{\sqrt{5}}\tilde{H}_5+\frac{1}{\sqrt{5}}\tilde{H}_8,\quad \tilde{E}_6=\frac{3}{\sqrt{5}}\tilde{H}_6,\quad \tilde{E}_8=\sqrt{5}\tilde{H}_8,$$
$$\tilde{E}_j=\tilde{H}_j \text{ with } j=1,2,3,4,7. \tag{27}$$

Clearly, E_is ($\tilde{E}_i$s) are linear combinations of H_is ($\tilde{H}_i$s) that possess the same symmetry in Eq. (17).

Taking $a=0$ in Eq. (22) it gives the sum rules connecting quark GPDs with EMFFs,

$$\int_{-1}^1 \mathrm{d}x\, E_i(x,\xi,t)=G_i(t)\quad\text{with}\quad i=1,2,5,6, \tag{28a}$$

$$\int_{-1}^1 \mathrm{d}x\, \tilde{E}_i(x,\xi,t)=\xi\tilde{G}_i(t)\quad\text{with}\quad i=1,2, \tag{28b}$$

$$\int_{-1}^1 \mathrm{d}x\, \tilde{E}_i(x,\xi,t)=\tilde{G}_i(t)\quad\text{with}\quad i=5,6, \tag{28c}$$

$$\int_{-1}^1 \mathrm{d}x\, E_j(x,\xi,t)=\int_{-1}^1 \mathrm{d}x\, \tilde{E}_j(x,\xi,t)=0\quad\text{with}\quad j=3,4,7,8. \tag{28d}$$

The first moments of E_3 and $\tilde{E}_3$ vanish because of the time reversal as shown in Eq. (17). Moreover, the first moments of E_i ($i=4,7,8$) and $\tilde{E}_i$ ($i=4,7,8$) also disappear due to that the tensor structures $n^\mu n^\nu/(P\cdot n)^2$ and $n^\mu n^\nu n^\rho/(P\cdot n)^3$ have no correspondences in the factorization of the matrix elements of local currents.

A similar procedure can be done for the case of GFFs. Taking $a=1$ in Eq. (22) gives the sum rules connecting quark GPDs with GFFs,

$$\int_{-1}^1 \mathrm{d}x\, xE_1(x,\xi,t)=F^T_{1,0}(t)+\xi^2F^T_{2,0}(t)-2F^T_{4,0}(t), \tag{29a}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_2(x,\xi,t) = 2F^T_{1,1}(t) + 2\xi^2 F^T_{2,1}(t) - 4F^T_{4,1}(t), \tag{29b}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_3(x,\xi,t) = 8\xi F^T_{5,0}(t), \tag{29c}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_4(x,\xi,t) = \frac{2t}{M^2} F^T_{5,0}(t) + 2F^T_{6,0}(t), \tag{29d}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_5(x,\xi,t) = 2F^T_{4,0}(t), \tag{29e}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_6(x,\xi,t) = 4F^T_{4,1}(t), \tag{29f}$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_i(x,\xi,t) = 0 \quad \text{with} \quad i = 7, 8. \tag{29g}$$

The second Mellin moment of E_7 vanishes because of the time reversal invariance and that of E_8 disappears as well because the tensor structure $n^\mu n^\nu n^\rho/(P\cdot n)^3$ does not have the correspondence in the parametrization of GFFs in Eq. (25). For the gluon GPDs, the factor x should be absent and the integral should only go from 0 to 1.

C. Helicity amplitudes

The helicity amplitudes given in Eq. (6) can be expressed in terms of the obtained GPDs. To show the symmetry properties carried by the individual tensor structures, we again express helicity amplitudes in terms of the coefficient functions Es in Eq. (26) and $\tilde{E}$s in (27) instead of GPDs directly. We introduce the notations $|\mathbf{p}_\perp| e^{\pm i\phi} \equiv p^1 \pm i p^2$ and $|\mathbf{p}'_\perp| e^{\pm i\phi'} \equiv p'^1 \pm i p'^2$, and

$$\begin{aligned}
&C \equiv \sqrt{\frac{1-\xi}{1+\xi}} \frac{|\mathbf{p}_\perp|}{M} e^{-i\phi} - \sqrt{\frac{1+\xi}{1-\xi}} \frac{|\mathbf{p}'_\perp|}{M} e^{-i\phi'},\\
&D \equiv -\frac{t}{4M^2} - \frac{\xi^2}{1-\xi^2},\\
&K_{\pm i} \equiv A_1 E_i \pm \xi A_1 \tilde{E}_i + A_2 \tilde{K}_{\pm(i+4)} \quad \text{with} \quad i = 1\sim 4,\\
&\tilde{K}_{\pm j} \equiv E_j \pm \tilde{E}_j \quad \text{with} \quad j = 1\sim 8,
\end{aligned} \tag{30}$$

where

$$A_1 \equiv \frac{2}{\sqrt{1-\xi^2}}, \qquad A_2 \equiv 2\sqrt{1-\xi^2}. \tag{31}$$

Then the helicity amplitudes have the following forms:

$$2\mathcal{A}_{\frac{3}{2}+,\frac{3}{2}+} = K_{+1} + \frac{|C|^2}{8} K_{+2}, \tag{32}$$

$$\begin{aligned}
2\mathcal{A}_{\frac{3}{2}+,\frac{1}{2}+} = &-\sqrt{\frac{1+\xi}{1-\xi}} \frac{C}{\sqrt{3}} \left(K_{+1} - \frac{1+\xi}{2} K_{-3}\right)\\
&- \frac{C}{\sqrt{3}} \left(\tilde{K}_{-1} + \frac{|C|^2}{8} \tilde{K}_{-2}\right)\\
&- \sqrt{\frac{1+\xi}{1-\xi}} \frac{[D(1-\xi^2)+\xi]C}{2\sqrt{3}(1-\xi^2)} K_{+2},
\end{aligned} \tag{33}$$

$$\begin{aligned}
2\mathcal{A}_{\frac{3}{2}+,(-\frac{1}{2})+} = &\sqrt{\frac{1+\xi}{1-\xi}} \frac{C^2}{\sqrt{3}} \left(\tilde{K}_{-1} - \frac{1+\xi}{2} \tilde{K}_{+3}\right) - \frac{C^2}{8\sqrt{3}} K_{+2}\\
&+ \sqrt{\frac{1+\xi}{1-\xi}} \frac{[D(1-\xi^2)+\xi]C^2}{2\sqrt{3}(1-\xi^2)} \tilde{K}_{-2},
\end{aligned} \tag{34}$$

$$2\mathcal{A}_{\frac{3}{2}+,(-\frac{3}{2})+} = \frac{C^3}{8} \tilde{K}_{-2}, \tag{35}$$

$$\begin{aligned}
2\mathcal{A}_{\frac{1}{2}+,\frac{3}{2}+} = &\sqrt{\frac{1-\xi}{1+\xi}} \frac{C^*}{\sqrt{3}} \left(K_{+1} + \frac{1-\xi}{2} K_{+3}\right)\\
&+ \frac{C^*}{\sqrt{3}} \left(\tilde{K}_{+1} + \frac{|C|^2}{8M^2} \tilde{K}_{+2}\right)\\
&+ \sqrt{\frac{1-\xi}{1+\xi}} \frac{[D(1-\xi^2)-\xi]C^*}{2\sqrt{3}(1-\xi^2)} K_{+2},
\end{aligned} \tag{36}$$

$$\begin{aligned}
2\mathcal{A}_{\frac{1}{2}+,\frac{1}{2}+} = &-\frac{2}{3}\left[K_{+1} - \frac{1}{2}\left(K_{-1} + \frac{|C|^2}{8} K_{-2}\right) + K_{+3} - K_{-3} + (1-\xi^2) K_{+4}\right] - \frac{|C|^2}{3}\left[\sqrt{\frac{1+\xi}{1-\xi}} \tilde{K}_{+1} + \sqrt{\frac{1-\xi}{1+\xi}} \tilde{K}_{-1}\right]\\
&+ \frac{|C|^2}{6\sqrt{1-\xi^2}} (\tilde{K}_{+2} + \tilde{K}_{-2} + 4\xi \tilde{K}_{-3}) + \frac{2[D(\xi^2-1)+1]}{3} \left(\frac{2K_{+1}}{(1-\xi^2)} + \frac{K_{+3}}{1+\xi} - \frac{K_{-3}}{1-\xi}\right)\\
&- \frac{2[D^2(1-\xi^2)^2 - \xi^2]}{3(1-\xi^2)^2} K_{+2} + \frac{|C|^2[D(\xi^2-1)-1]}{6(1-\xi^2)} \left(\sqrt{\frac{1+\xi}{1-\xi}} \tilde{K}_{+2} + \sqrt{\frac{1-\xi}{1+\xi}} \tilde{K}_{-2}\right),
\end{aligned} \tag{37}$$

$$2\mathcal{A}_{\frac{1}{2}+,(-\frac{1}{2})+} = \frac{2C}{3}\left[\tilde{K}_{-1} - \frac{|C|^2}{16}\tilde{K}_{+2} - \tilde{K}_{+3} + \tilde{K}_{-3} + (1-\xi^2)\tilde{K}_{-4}\right] - \frac{C}{3}\left(\sqrt{\frac{1+\xi}{1-\xi}}K_{-1} + \sqrt{\frac{1-\xi}{1+\xi}}K_{+1}\right)$$
$$+ \frac{C}{6\sqrt{1-\xi^2}}(K_{+2} + K_{-2} + 4\xi K_{+3}) - \frac{2[D(\xi^2-1)+1]C}{3}\left(\frac{2\tilde{K}_{-1}}{(1-\xi^2)} - \frac{\tilde{K}_{+3}}{1-\xi} + \frac{\tilde{K}_{-3}}{1+\xi}\right)$$
$$+ \frac{2[D^2(\xi^2-1)^2-\xi^2]C}{3(1-\xi^2)^2}\tilde{K}_{-2} + \frac{[D(\xi^2-1)-1]C}{6(1-\xi^2)}\left(\sqrt{\frac{1+\xi}{1-\xi}}K_{-2} + \sqrt{\frac{1-\xi}{1+\xi}}K_{+2}\right), \quad (38)$$

$$2\mathcal{A}_{\frac{1}{2}+,(-\frac{3}{2})+} = \sqrt{\frac{1-\xi}{1+\xi}}\frac{C^2}{\sqrt{3}}\left(\tilde{K}_{-1} + \frac{1-\xi}{2}\tilde{K}_{-3}\right) - \frac{C^2}{8\sqrt{3}}K_{-2}$$
$$- \sqrt{\frac{1-\xi}{1+\xi}}\frac{[D(\xi^2-1)+\xi]C^2}{2\sqrt{3}(1-\xi^2)}\tilde{K}_{-2}. \quad (39)$$

The rest helicity amplitudes can be obtained through the relations in Eqs. (8) and (9). Obviously, $\mathcal{A}_{\frac{3}{2}+,\frac{3}{2}+}$ and $\mathcal{A}_{\frac{1}{2}+,\frac{1}{2}+}$ in Eqs. (32) and (37) are unchanged after the time reversal. Noted that C is a dimensionless complex number and depends on the transverse momenta of both initial and final states. Thus, the time reversal could turn C into its complex conjugate. It should be addressed that a common factor $C^{\lambda'-\lambda}\theta(\lambda'-\lambda) + C^{*\lambda-\lambda'}\theta(\lambda-\lambda')$, where $\theta(x-y)$ is the step function, can be extracted out of the helicity amplitude $A_{\lambda'+,\lambda+}$ when $\lambda' \neq \lambda$. In the forward limit, the factor C vanishes and so for $A_{\lambda'+,\lambda+}$s when $\lambda' \neq \lambda$. Further, we have $\mathcal{A}(x,0,0)_{\lambda+,\lambda+} = q^{\lambda}_{\uparrow}(x)$ and $\mathcal{A}(x,0,0)_{\lambda-,\lambda-} = q^{\lambda}_{\downarrow}(x)$; using this we find the relations in Eq. (20).

IV. SUMMARY AND DISCUSSIONS

In this work, the GPDs of the spin-3/2 particle are given for the first time. There are eight independent unpolarized GPDs and eight polarized ones. The independent tensors that accompany the distributions can be constructed through the direct product between the tensors accompanying the spin-1/2 unpolarized or polarized GPDs and those of the spin-1. Moreover, the structure functions can be expressed as the GPDs in the forward limit, and the sum rules connecting GPDs with EMFFs and GFFs are obtained through the Mellin moments. In the last subsection, the helicity amplitudes of the spin-3/2 particle are derived and expressed in terms of the coefficient functions which are linear combinations of the GPDs and they depend on the transverse momenta of the initial and final states. The parity and time reversal invariances are satisfied throughout. It is expected that the relations given in this work could be tested in the electron-ion collision experiments at future EIC and EicC with spin-3/2 targets, such as $^7_3$Li. Other measurements about the GPDs of the spin-3/2 particle, like the Ω baryon, may also be possible in the heavy ion collision, where the Ω baryon is rich in the final state. Finally, the numerical results for the unpolarized and polarized GPDs of a spin-3/2 particle, taking the Δ isobar as an example, will be given in our forthcoming work.

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APPENDIX A: THE RARITA-SCHWINGER SPINOR

The explicit form of the Rarita-Schwinger spinor of a spin-3/2 particle employed in our work is [68]

$$u^{\alpha}(p,\lambda) = \sum_{\rho,\sigma} C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma}\epsilon^{\alpha}(p,\rho)u(p,\sigma), \quad (A1)$$

where the coefficient in (A1) is the Clebsch-Gordan coefficient. The explicit light-front form expressions of the polarization vectors are derived from Ref. [69] by boosting from the rest momentum frame to the moving frame,

$$\boldsymbol{\epsilon}^{\alpha}(p,0) = \frac{1}{M}\left(p^+, p^- - \frac{2M^2}{p^+}, \boldsymbol{\epsilon}_{\perp}(p,0)\right)^{\mathrm{T}} \text{ with}$$
$$\boldsymbol{\epsilon}_{\perp}(p,0) = (p_1, p_2), \quad (A2a)$$

$$\epsilon^{\alpha}(p,+1) = -\left(0, \frac{\sqrt{2}(p_1 + ip_2)}{p^+}, \epsilon_{\perp}(p,+1)\right)^{\mathrm{T}} \text{ with}$$
$$\epsilon_{\perp}(p,+1) = \left(\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right), \tag{A2b}$$

$$\epsilon^{\alpha}(p,-1) = \left(0, \frac{\sqrt{2}(p_1 - ip_2)}{p^+}, \epsilon_{\perp}(p,-1)\right)^{\mathrm{T}} \text{ with}$$
$$\epsilon_{\perp}(p,-1) = \left(\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right). \tag{A2c}$$

The massive positive energy Dirac spinor can be written as [70]

$$u(p,\sigma) = \frac{(\not{p} + M)}{\sqrt{2p\cdot n}}\not{n}\chi_{\sigma}, \tag{A3}$$

where χ_σ is the rest frame spinor. Note that the Rarita-Schwinger spinor in Eq. (A1) satisfies the Rarita-Schwinger equation, as well as the subsidiary constraint equations,

$$(\not{p} - M)u^{\alpha}(p,\lambda) = 0, \quad \gamma_{\alpha}u^{\alpha}(p,\lambda) = 0, \quad \partial_{\alpha}u^{\alpha}(p,\lambda) = 0. \tag{A4}$$

APPENDIX B: ON-SHELL IDENTITIES

Some useful on-shell identities have been given in Refs. [6]:

$$n^{\{\alpha'}P^{\alpha\}}i\sigma^{nq} \doteq P\cdot n g^{\alpha'\alpha}i\sigma^{nq} + 2P\cdot nP^{\{\alpha'}n^{\alpha\}} + \frac{1}{2}(q\cdot n)^2 g^{\alpha'\alpha} + 2q\cdot nP^{[\alpha'}n^{\alpha]} + tn^{\alpha'}n^{\alpha}, \tag{B1}$$

$$n^{\{\alpha'}P^{\alpha\}} \doteq -M\left(1 - \frac{t}{4M^2}\right)g^{\alpha'\alpha}\left(\frac{P\cdot n}{M} + \frac{i\sigma^{nq}}{2M}\right) + g^{\alpha'\alpha}P\cdot n + \frac{2}{M}P^{\alpha'}P^{\alpha}\left(\frac{P\cdot n}{M} + \frac{i\sigma^{nq}}{2M}\right), \tag{B2}$$

in which $\doteq$ represents the relation in a similar manner as Gordon identity. Analogously, for the case of polarized GPDs, one can derive these on-shell identities:

$$4Mn^{[\alpha'}P^{\alpha]}\not{n}\gamma^5 \doteq [4(P\cdot n)^2 - (q\cdot n)^2]g^{\alpha'\alpha}\gamma^5 - 8(P\cdot n)n^{\{\alpha'}P^{\alpha\}}\gamma^5 + 4(q\cdot n)n^{[\alpha'}P^{\alpha]}\gamma^5 + 8P^2 n^{\alpha'}n^{\alpha}\gamma^5 + 2M(q\cdot n)g^{\alpha'\alpha}\not{n}\gamma^5, \tag{B3}$$

$$n^{[\alpha'}P^{\alpha]}\gamma^5 \doteq \frac{1}{2}g^{\alpha'\alpha}(q\cdot n)\gamma^5 - Mg^{\alpha'\alpha}\not{n}\gamma^5 + \frac{P^2}{M}g^{\alpha'\alpha}\not{n}\gamma^5 - \frac{2}{M}P^{\alpha'}P^{\alpha}\not{n}\gamma^5. \tag{B4}$$

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