

**Pseudo-Nambu-Goldstone dark matter from non-Abelian gauge symmetry**Hajime Otsuka<sup>1,\*</sup>, Takashi Shimomura<sup>1,2,†</sup>, Koji Tsumura<sup>1,‡</sup>, Yoshiki Uchida<sup>1,§</sup> and Naoki Yamatsu<sup>3,||</sup><sup>1</sup>*Department of Physics, Kyushu University, 744 Motoooka, Nishi-ku, Fukuoka 819-0395, Japan*<sup>2</sup>*Faculty of Education, University of Miyazaki, 1-1 Gakuen-Kibanadai-Nishi, Miyazaki 889-2192, Japan*<sup>3</sup>*Department of Physics, National Taiwan University, Taipei, Taiwan 10617, Republic of China*

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We propose a pseudo-Nambu-Goldstone boson (pNGB) dark matter (DM) model based on an additional non-Abelian gauge symmetry  $SU(2)_D$ . The gauge symmetry  $SU(2)_D$  is spontaneously broken to a global custodial symmetry  $U(1)_V$  via the nonvanishing vacuum expectation values of  $SU(2)_D$  doublet and triplet scalar fields. Due to the exact global symmetry  $U(1)_V$ , the lightest  $U(1)_V$  charged particle becomes stable. We assume that the lightest charged particle in the model is the charged complex pNGB, which we regard as DM. It avoids the strong constraints from current DM direct detection experiments due to the property of NGB. We find that the measured energy density of DM can be reproduced when the DM mass is larger than the half of the Higgs mass, where the lower limit generally comes from the constraint of DM invisible decay and the upper limit from DM direct detection experiments depends on the model parameters.

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The standard model (SM) in particle physics is able to explain the results of accelerator experiments with the exception of a few anomalies. However, some phenomena that cannot be explained within the scope of the SM have emerged. One of the important issues in modern particle physics and cosmology is the search for the identity of dark matter (DM). The existence of dark matter (DM) has been confirmed by several astronomical observations such as spiral galaxies [1,2], gravitational lensing [3], cosmic microwave background [4], and collision of bullet cluster [5].

There are a lot of DM candidates since the nature of DM is not yet understood. One such candidate is called the weakly interacting massive particle (WIMP). An attractive property of WIMP DM is that it can be generated thermally, which can be experimentally verified by introducing a nongravitational effect. In order to achieve the DM's relic abundance, the mass of WIMPs is expected to be in the range of  $\mathcal{O}(10)$  GeV to  $\mathcal{O}(100)$  TeV. Because of the nongravitational interactions of WIMPs, direct and indirect detections are expected. There is

still no clear signal for WIMPs, and hence direct detections yield strong constraints on WIMP masses and interactions.

Several mechanisms in WIMP DM models are proposed to avoid the severe constraints of the direct detection by considering, e.g., a fermion DM with pseudoscalar interactions [6–11] and a pseudo-Nambu-Goldstone boson (pNGB) DM with additional global  $U(1)$  group symmetry [12–27]. As pointed out in the original pNGB DM model [14], the DM has the property of Nambu-Goldstone (NG) mode, so the coupling of the DM with the SM Higgs boson is proportional to its momentum. As a result, the scattering cross sections of the DM with the SM particles via the Higgs bosons are strongly suppressed, while the annihilation cross sections of the DM to the SM particles are kept.

Recently, a pNGB DM model based on gauged  $G_{\text{SM}} \times U(1)_{B-L}$  symmetry, which extends the softly broken  $U(1)$  global symmetry to the gauged  $U(1)_{B-L}$  symmetry, was proposed [22,23], where  $G_{\text{SM}} := SU(3)_C \times SU(2)_W \times U(1)_Y$ . The DM direct detection cross section is naturally suppressed as the same as the original pNGB DM model. On the other hand, the pNGB DM decays into SM particles mediated by the  $U(1)_{B-L}$  gauge boson. As a result, the  $U(1)_{B-L}$  symmetry breaking scale is greater than  $10^{13}$  GeV for the DM mass  $< 1$  TeV to escape the constraint from DM stability, where the bound from gamma-ray observations is strong as roughly the DM lifetime  $\gtrsim 10^{27}$  s for two body decays [28]. In addition, the  $G_{\text{SM}} \times U(1)_{B-L}$  pNGB DM model has been extended to  $SO(10)$  grand unified theory (GUT) [29,30]. In this model, the vacuum expectation value (VEV) of the intermediate symmetry breaking scale is greater than  $10^{10}$  GeV and the DM mass is only allowed to be slightly below half the Higgs boson mass from the requirements of DM stability and grand unification and

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TABLE I. The field content in the pNGB DM model is shown in the  $G_{\text{SM}} \times SU(2)_D$  basis, where the fermions belong to  $(1/2, 0)$  under  $SL(2, \mathbb{C})$ .

|           | $Q$      | $u^c$              | $d^c$              | $L$      | $e^c$    | $H$      | $\Phi$   | $\Delta$ | $G_{\mu\nu}$ | $W_{\mu\nu}$ | $B_{\mu\nu}$ | $W'_{\mu\nu}$ |
|-----------|----------|--------------------|--------------------|----------|----------|----------|----------|----------|--------------|--------------|--------------|---------------|
| $SU(3)_C$ | <b>3</b> | $\bar{\mathbf{3}}$ | $\bar{\mathbf{3}}$ | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>8</b>     | <b>1</b>     | <b>1</b>     | <b>1</b>      |
| $SU(2)_W$ | <b>2</b> | <b>1</b>           | <b>1</b>           | <b>2</b> | <b>1</b> | <b>2</b> | <b>1</b> | <b>1</b> | <b>1</b>     | <b>3</b>     | <b>1</b>     | <b>1</b>      |
| $U(1)_Y$  | +1/6     | -2/3               | +1/3               | -1/2     | +1       | +1/2     | 0        | 0        | 0            | 0            | 0            | 0             |
| $SU(2)_D$ | <b>1</b> | <b>1</b>           | <b>1</b>           | <b>1</b> | <b>1</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>1</b>     | <b>1</b>     | <b>1</b>     | <b>3</b>      |

also the constraints of the Higgs invisible decay and the gamma-ray observations for DM annihilations.

The purpose of this paper is to propose a new pNGB DM model based on non-Abelian gauge symmetry  $SU(2)_D$ .<sup>1</sup> Unlike the  $G_{\text{SM}} \times U(1)_{B-L}$  and  $SO(10)$  pNGB DM model, we will confirm that the DM is stabilized due to the residual  $U(1)$  symmetry of the  $SU(2)$  custodial symmetry [32].<sup>2</sup> We will show that in our pNGB DM model the VEV of  $SU(2)_D$  breaking scale can be allowed to be roughly  $\mathcal{O}(1)$  TeV without introducing very high energy scale.

The paper is organized as follows. In Sec. II, we introduce an  $SU(2)$  pNGB DM model. In Sec. III, we analyze vacuum structures and symmetry breaking patterns of the model. In Sec. IV, we analyze the scalar potential of the system. In Sec. V, we investigate the mass spectra of scalar fields in this model. In Sec. VI, we examine the constraints from direct detection experiments and the thermal relic abundance of DM for our DM candidate. Section VII is devoted to summary and discussions. In Appendix, we show the detailed calculation of DM-quark scattering amplitude.

## II. THE MODEL

The model consists of the SM gauge fields, an  $SU(2)_D$  gauge field  $W_\mu^a$  ( $a = 1, 2, 3$ ), a complex scalar field in  $\mathbf{2}$  of  $SU(2)_D$   $\Phi$ , and a real scalar field in  $\mathbf{3}$  of  $SU(2)_D$   $\Delta$ . The matter content in the non-Abelian pNGB DM model is summarized in Table I.

The Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2} \text{tr}[G_{\mu\nu} G^{\mu\nu}] - \frac{1}{2} \text{tr}[W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
& - \frac{1}{2} \text{tr}[W'_{\mu\nu} W'^{\mu\nu}] + (D_\mu H)^\dagger (D^\mu H) + (D_\mu \Phi)^\dagger (D^\mu \Phi) \\
& + \frac{1}{2} \text{tr}[(D_\mu \Delta)(D^\mu \Delta)] - \mathcal{V}(H, \Phi, \Delta) \\
& + \bar{Q} i \not{D} Q + \bar{u}^c i \not{D} u^c + \bar{d}^c i \not{D} d^c + \bar{L} i \not{D} L + \bar{e}^c i \not{D} e^c \\
& - (y_u Q u^c H + y_d Q d^c H^\dagger + y_e L e^c H^\dagger + \text{H.c.}), \quad (2.1)
\end{aligned}$$

<sup>1</sup>A pNGB DM model based on non-Abelian global symmetry  $SU(2)_g$  and Abelian gauge symmetry  $U(1)_X$  has been proposed in Ref. [31]. This model can be regarded as a low-energy effective model that is realized in a special parameter region in our model.

<sup>2</sup>A DM model using custodial symmetry emerging from non-Abelian gauge symmetry  $SU(2)_D$  for DM stability has been proposed in Ref. [33], although it is not a pNGB DM model.

where  $D_\mu = \partial_\mu + i g_s G_\mu + i g_2 W_\mu + i g_1 B_\mu + i g'_2 W'_\mu$ ;  $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu + i g [F_\mu, F_\nu]$ , where  $F = G, W, B, W'$  and  $g_s, g_2, g_1, g'_1$  are gauge fields and gauge coupling constants of  $SU(3)_C, SU(2)_W, U(1)_Y, SU(2)_D$ , respectively. The scalar potential  $\mathcal{V}(H, \Phi, \Delta)$  contains quadratic, cubic, and quartic coupling terms,

$$\begin{aligned}
\mathcal{V}(H, \Phi, \Delta) = & -\mu_H^2 H^\dagger H - \mu_\Phi^2 \Phi^\dagger \Phi - \frac{1}{2} \mu_\Delta^2 \text{Tr}(\Delta^2) \\
& + \sqrt{2}((\kappa_1 + i\kappa_2)\tilde{\Phi}^\dagger \Delta \Phi + (\kappa_1 - i\kappa_2)\Phi^\dagger \Delta \tilde{\Phi}) \\
& + 2\sqrt{2}\kappa_3 \Phi^\dagger \Delta \Phi + \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\
& + \frac{1}{4} \lambda_\Delta \text{Tr}(\Delta^2)^2 + \lambda_{H\Phi} (H^\dagger H)(\Phi^\dagger \Phi) \\
& + \lambda_{H\Delta} (H^\dagger H) \text{Tr}(\Delta^2) + \lambda_{\Phi\Delta} (\Phi^\dagger \Phi) \text{Tr}(\Delta^2), \quad (2.2)
\end{aligned}$$

where  $\tilde{\Phi}(x) = i\sigma_2 \Phi(x)^*$ ;  $\mu_H^2, \mu_\Phi^2$ , and  $\mu_\Delta^2$  are real parameters with dimension 2,  $\kappa_a$  ( $a = 1, 2, 3$ ) are real parameters with dimension 1, and  $\lambda_H, \lambda_\Phi, \lambda_\Delta, \lambda_{H\Phi}, \lambda_{H\Delta}$ , and  $\lambda_{\Phi\Delta}$  are dimensionless real parameters. We use the following notation:

$$\begin{aligned}
\Delta &= \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & -\eta_3 \end{pmatrix}, \\
\Phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \quad (2.3)
\end{aligned}$$

Under the  $SU(2)_D$  transformation,  $\Phi(x)$  and  $\Delta(x)$  behave as

$$\Phi(x) \rightarrow U(x)\Phi(x), \quad \Delta(x) \rightarrow U(x)\Delta(x)U(x)^\dagger, \quad (2.4)$$

where  $U(x)$  is the  $SU(2)_D$  unitary transformation  $U(x) = \exp[i\theta_a(x)\frac{\sigma_a}{2}]$ ;  $\theta_a(x)$  ( $a = 1, 2, 3$ ) are the parameters of the  $SU(2)_D$  gauge transformation and  $\sigma_a$  stand for the Pauli matrices. Note that it is easy to check invariant terms under  $G_{\text{SM}} \times SU(2)_D$  by using GroupMath [34] and Sym2Int [35,36]. (For Lie groups, see, e.g., Ref. [37].)

We will analyze the relations between vacuum structures and symmetry breaking patterns in the next section. In the model, the invariant terms that contain only the scalar field  $\Phi(x)$  in  $\mathbf{2}$  of  $SU(2)_D^{\text{local}}$  are invariant under a larger global symmetry  $SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$ . To check this extended

$$SU(2)_{\Delta} \times SU(2)_{\Phi_L} \times SU(2)_{\Phi_R} \xrightarrow{\kappa\text{-term}} SU(2)_L \times U(1)_R$$

FIG. 1. The global symmetry  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$  breaking pattern is shown. The  $\kappa$  term stands for the soft symmetry breaking term. In the figure, the superscript, global, is omitted.

global symmetry, it is convenient to introduce a bidoublet or  $2 \times 2$  matrix notation for  $\Phi(x)$  as

$$\Sigma(x) := \begin{pmatrix} \tilde{\Phi}(x) & \Phi(x) \end{pmatrix}. \quad (2.5)$$

This notation is convenient to understand so-called  $SU(2)$  custodial symmetry [32]. We will find that the stability of the DM is realized by a “ $U(1)_V^{\text{global}}$  custodial symmetry,” which is a  $U(1)$  subgroup of the  $SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$  diagonal subgroup  $SU(2)_V^{\text{global}}$ .

By using the complex scalar field in  $\mathbf{2}$  of  $SU(2)_D$   $\Sigma(x)$  instead of  $\Phi(x)$ , the scalar potential in Eq. (2.2) can be written as

$$\begin{aligned} \mathcal{V}(H, \Sigma, \Delta) = & -\mu_H^2 H^\dagger H - \frac{\mu_\Sigma^2}{2} \text{Tr}(\Sigma^\dagger \Sigma) - \frac{1}{2} \mu_\Delta^2 \text{Tr}(\Delta^2) \\ & - \sqrt{2} \kappa_1 \text{Tr}(\sigma_1 \Sigma^\dagger \Delta \Sigma) - \sqrt{2} \kappa_2 \text{Tr}(\sigma_2 \Sigma^\dagger \Delta \Sigma) \\ & - \sqrt{2} \kappa_3 \text{Tr}(\sigma_3 \Sigma^\dagger \Delta \Sigma) + \lambda_H (H^\dagger H)^2 \\ & + \frac{\lambda_\Phi}{4} (\text{Tr}(\Sigma^\dagger \Sigma))^2 + \frac{1}{4} \lambda_\Delta \text{Tr}(\Delta^2)^2 \\ & + \frac{1}{2} \lambda_{H\Phi} (H^\dagger H) \text{Tr}(\Sigma^\dagger \Sigma) + \frac{1}{2} \lambda_{H\Delta} (H^\dagger H) \text{Tr}(\Delta^2) \\ & + \frac{1}{2} \lambda_{\Phi\Delta} \text{Tr}(\Sigma^\dagger \Sigma) \text{Tr}(\Delta^2), \end{aligned} \quad (2.6)$$

where we used a relation  $(-i\sigma_2)\Sigma^*(i\sigma_2) = \Sigma$ .

We verify what kind of global symmetry exists in the potential given in Eq. (2.6). First, the potential  $\mathcal{V}(0, \Sigma, 0)$  is invariant under  $SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$ .  $\Sigma$  is the bidoublet representation under  $SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$ :

$$\Sigma \rightarrow U_{\Phi_L} \Sigma U_{\Phi_R}^\dagger, \quad U_{\Phi_L} = e^{i\theta_{\Phi_L}^a \sigma_a}, \quad U_{\Phi_R} = e^{i\theta_{\Phi_R}^a \sigma_a}, \quad (2.7)$$

where  $\theta_{\Phi_L}^a$  and  $\theta_{\Phi_R}^a$  are parameters of  $SU(2)_{\Phi_L}^{\text{global}}$  and  $SU(2)_{\Phi_R}^{\text{global}}$  transformations, respectively. Second, the potential  $\mathcal{V}(0, 0, \Delta)$  is invariant under  $SU(2)_\Delta^{\text{global}}$  transformation.  $\Delta$  is the adjoint representation under  $SU(2)_\Delta^{\text{global}}$ :

$$\Delta \rightarrow U_\Delta \Delta U_\Delta^\dagger, \quad U_\Delta = e^{i\theta_\Delta^a \sigma_a}, \quad (2.8)$$

where  $\theta_\Delta^a$  is a parameter of  $SU(2)_\Delta^{\text{global}}$ , and the global transformation corresponds to a global subgroup transformation of the gauge group  $SU(2)_D^{\text{local}}$  transformation.

Third, under  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$ , each  $\kappa_a$  term transforms as

$$\text{Tr}(\sigma_a \Sigma^\dagger \Delta \Sigma) \rightarrow \text{Tr}(U_{\Phi_R}^\dagger \sigma_a U_{\Phi_R} \Sigma^\dagger U_{\Phi_L}^\dagger U_\Delta \Delta U_\Delta^\dagger U_{\Phi_L} \Sigma). \quad (2.9)$$

This term is invariant under  $SU(2)_L^{\text{global}} \times U(1)_{\Phi_{Ra}}^{\text{global}}$ , where  $SU(2)_L^{\text{global}}$  represents  $SU(2)_\Delta = SU(2)_{\Phi_L}$ , and  $U(1)_{\Phi_{Ra}}^{\text{global}}$  corresponds to the  $\sigma_a$  direction of  $SU(2)_{\Phi_R}^{\text{global}}$ . The combination of the  $\kappa_1$ ,  $\kappa_2$ , and  $\kappa_3$  terms is also invariant under  $SU(2)_L^{\text{global}} \times U(1)_{\Phi_R}^{\text{global}}$ :

$$\begin{aligned} \text{Tr}((\kappa_1 \sigma_1 + \kappa_2 \sigma_2 + \kappa_3 \sigma_3) \Sigma^\dagger \Delta \Sigma) \\ \rightarrow \text{Tr}(U_{\Phi_R}^\dagger (\kappa_1 \sigma_1 + \kappa_2 \sigma_2 + \kappa_3 \sigma_3) U_{\Phi_R} \Sigma^\dagger \Delta \Sigma), \end{aligned} \quad (2.10)$$

where the  $U(1)_{\Phi_R}^{\text{global}}$  transformation is associated with the  $(\kappa_1 \sigma_1 + \kappa_2 \sigma_2 + \kappa_3 \sigma_3)$  direction in  $SU(2)_{\Phi_R}^{\text{global}}$ . Therefore, the potential is invariant under  $SU(2)_L^{\text{global}} \times U(1)_{\Phi_R}^{\text{global}}$ . Without losing generality, we can choose the  $\kappa_a \sigma_a$  direction associated with the remaining  $U(1)_{\Phi_R}^{\text{global}}$  symmetry. Therefore, in the following we will take the  $\kappa_3 \sigma_3$  direction and denote  $\kappa_3$  as  $\kappa$  and  $U(1)_{\Phi_R^3}^{\text{global}}$  as  $U(1)_R^{\text{global}}$ . In other words, we remove  $\kappa_1$  and  $\kappa_2$  by using the  $SU(2)_{\Phi_R}^{\text{global}}$  transformation. Therefore, the potential in Eq. (2.6) is invariant under  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$ . The global symmetry  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi_L}^{\text{global}} \times SU(2)_{\Phi_R}^{\text{global}}$  breaking pattern associated with the explicit breaking terms is shown in Fig. 1.

### III. VACUUM STRUCTURE

We consider vacuum structures of  $\Sigma(x)$  and  $\Delta(x)$ . The system we are currently considering has  $SU(2)_D^{\text{local}}$  (or  $SU(2)_L^{\text{global}}$ ) and  $U(1)_R^{\text{global}}$  symmetry. By using a total of four degrees of freedom of  $SU(2)_D^{\text{local}}$  gauge and  $U(1)_R^{\text{global}}$  transformations, without loss of generality, we take the VEVs of  $\Sigma$  and  $\Delta$  as  $\langle \Delta \rangle = (v_{\eta_1} \sigma_1 + v_{\eta_3} \sigma_3) / \sqrt{2}$  and  $\langle \Sigma \rangle = v_\Phi I / \sqrt{2}$ , i.e.,

$$\langle \Delta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{\eta_3} & v_{\eta_1} \\ v_{\eta_1} & -v_{\eta_3} \end{pmatrix}, \quad \langle \Sigma \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Phi & 0 \\ 0 & v_\Phi \end{pmatrix}, \quad (3.1)$$

where we remove the VEVs of  $\eta_2$ ,  $\phi_1$ ,  $\phi_2$ , and  $\phi_4$ . The gauge symmetry  $SU(2)_D^{\text{local}}$  breaking patterns are shown in

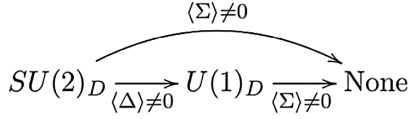


FIG. 2. The gauge symmetry  $SU(2)_D^{\text{local}}$  breaking patterns are shown.  $\langle \Sigma \rangle \neq 0$  and  $\langle \Delta \rangle \neq 0$  represent the spontaneous symmetry breaking (SSB) by the VEV of  $\Sigma$  and  $\Delta$  in **2** and **3** of  $SU(2)_D^{\text{local}}$ .  $U(1)_D$  is a local subgroup of  $SU(2)_D^{\text{local}}$ . In the figure, the superscript, local, is omitted.

Fig. 2. As is well known, the nonvanishing VEV of a complex scalar field in **2** of  $SU(2)$  breaks  $SU(2)$  symmetry completely, so a total of three Nambu-Goldstone (NG) or pseudo NG (pNG) modes appear. More specifically, when  $SU(2)$  global symmetry is exact, three NG modes appear; when  $SU(2)$  global symmetry is softly broken to  $U(1)$  global symmetry by explicit breaking terms, one NG and two pNG modes appear; when  $SU(2)$  global symmetry is completely softly broken by explicit breaking terms, three pNG modes appear. The nonvanishing VEV of a real scalar field in **3** of  $SU(2)$  breaks  $SU(2)$  symmetry to  $U(1)$  symmetry, so a total of two NG or pNG modes appear.

Furthermore, when  $\kappa = 0$ , the system has  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  symmetry. By using the degrees of freedom of  $SU(2)_\Delta^{\text{global}}$ ,  $SU(2)_{\Phi L}^{\text{global}}$ , and  $SU(2)_{\Phi R}^{\text{global}}$  transformations, we can take the VEVs of  $\Sigma$  and  $\Delta$  as

$$\langle \Delta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{\eta_3} & 0 \\ 0 & -v_{\eta_3} \end{pmatrix}, \quad \langle \Sigma \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Phi & 0 \\ 0 & v_\Phi \end{pmatrix}, \quad (3.2)$$

Here we check what kind of symmetry is preserved by the VEVs of  $\Sigma$  and  $\Delta$  in Eqs. (3.1) and (3.2). First, we consider the VEV of  $\Sigma$  as  $\langle \Sigma \rangle = v_\Phi / \sqrt{2} I$ . Under  $SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$ , the VEV transforms as

$$\begin{aligned} \langle \Sigma \rangle &\rightarrow U_{\Phi L} \langle \Sigma \rangle U_{\Phi R}^\dagger = e^{i\theta_{\Phi L a} \sigma_a} \frac{v_\Phi}{\sqrt{2}} I e^{-i\theta_{\Phi R a} \sigma_a} \\ &= \frac{v_\Phi}{\sqrt{2}} I e^{i(\theta_{\Phi L a} - \theta_{\Phi R a}) \sigma_a} = \langle \Sigma \rangle e^{i(\theta_{\Phi L a} - \theta_{\Phi R a}) \sigma_a}. \end{aligned} \quad (3.3)$$

Therefore, only for  $\theta_{\Phi L a} = \theta_{\Phi R a}$ , the VEV is invariant. That is, only  $SU(2)_{\Phi V}^{\text{global}}$  remains. In the case, a total of three NG or pNG modes appear.

Next, we consider the VEV of  $\Delta$  in Eqs. (3.1) and (3.2). When  $\kappa = 0$ , under  $SU(2)_\Delta^{\text{global}}$ , the VEV in Eq. (3.2) transforms as

$$\langle \Delta \rangle \rightarrow U_\Delta \langle \Delta \rangle U_\Delta^\dagger. \quad (3.4)$$

For  $\theta_{\Delta 1} = \theta_{\Delta 2} = 0$ , the VEV is invariant because  $[U_\Delta, \langle \Delta \rangle] = 0$ . That is, only  $U(1)_\Delta^{\text{global}}$  associated with

$\sigma_3$  remains. Under  $SU(2)_\Delta^{\text{global}}$ , the VEV in Eq. (3.1) transforms as

$$\langle \Delta \rangle \rightarrow U_\Delta \langle \Delta \rangle U_\Delta^\dagger. \quad (3.5)$$

When  $\kappa \neq 0$ , for  $\theta_{\Delta 3} = \frac{v_{\eta_3}}{v_{\eta_1}} \theta_{\Delta 1}$ ,  $\theta_{\Delta 2} = 0$ , the VEV is invariant because  $[U_\Delta, \langle \Delta \rangle] = 0$ . That is,  $U(1)_\Delta^{\text{global}}$  associated with a linear combination of  $\sigma_1$  and  $\sigma_3$  remains. From the above discussion, regardless of the configuration of the VEV of  $\Delta$ ,  $SU(2)_\Delta^{\text{global}}$  symmetry is broken to  $U(1)_\Delta^{\text{global}}$ . Therefore, a total of two NG or pNG modes appear.

We summarize NG or pNG modes in this model. When  $\Sigma(x)$  in **2** of  $SU(2)$  acquires a nonvanishing VEV, a total of three NG or pNG mode appear. When  $\Delta(x)$  in **3** of  $SU(2)$  acquires a nonvanishing VEV, a total of two NG or pNG mode appear. In the dark  $SU(2)_D$  sector, a total of up to five NG or pNG modes appear.

#### IV. ANALYZING THE POTENTIAL

We summarize how to find the vacuum that satisfies the global minimum of the potential for each set of model parameters below.

- (1) Write down the most general potential of fields such as  $\Sigma(x)$  and  $\Delta(x)$ . The potential  $\mathcal{V}$  depends on some degrees of freedoms  $v_X$  such as  $v_\Phi$  and  $v_{\eta_3}$ :

$$\mathcal{V}(\{v_X\}) = \mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3}). \quad (4.1)$$

- (2) Calculate the first derivatives of the potential  $\mathcal{V}(\{v_X\})$  with respect to all the variables  $\{v_X\}$ .

We find three stationary conditions as

$$\frac{\partial}{\partial v_X} \mathcal{V}(\{v_X\}) = 0. \quad (4.2)$$

- (3) Solve the simultaneous equations derived from the stationary conditions.

We find that the variables  $v_X$  are expressed as model parameters such as  $\mu_\Phi^2$  and  $\lambda_\Phi$ . Note that in some cases a VEV is related with another VEV, and some flat directions exist. This situation occurs when symmetry is unbroken.

- (4) Compare the values of the potential at all extrema and saddle points.

We find true vacua of the potential  $\mathcal{V}(\{v_X\})$  at each parameter region, where all the VEVs at the true vacuum must be real in our convention.

- (5) Check what kind of symmetry is realized at each vacuum for each parameter region.

(Note that the same procedure is commonly used, e.g., to analyze the vacuum structures of  $SU(N)$  symmetry breaking by elementary scalar fields [38,39] and



$E_6$ ,  $SU(N)$  and  $SO(N)$  symmetry breaking by composite scalar fields [40–44].)

To understand the vacuum structure of this system, we first consider the case  $\kappa = 0$ . After that, we will discuss the case  $\kappa \neq 0$ .

### A. Without soft symmetry breaking term ( $\kappa = 0$ )

We take the VEVs of  $\Sigma(x)$  and  $\Delta(x)$  given in Eq. (3.2). Substituting the VEVs into the potential of  $\Sigma(x)$  and  $\Delta(x)$  given in Eq. (2.6) with  $\kappa = 0$

$$\begin{aligned} \mathcal{V}(\Sigma, \Delta) = & -\frac{\mu_\Phi^2}{2} \text{Tr}(\Sigma^\dagger \Sigma) - \frac{1}{2} \mu_\Delta^2 \text{Tr}(\Delta^2) \\ & + \frac{\lambda_\Phi}{4} (\text{Tr}(\Sigma^\dagger \Sigma))^2 + \frac{1}{4} \lambda_\Delta \text{Tr}(\Delta^2)^2 \\ & + \frac{1}{2} \lambda_{\Phi\Delta} \text{Tr}(\Sigma^\dagger \Sigma) \text{Tr}(\Delta^2), \end{aligned} \quad (4.3)$$

the potential is given by

$$\begin{aligned} \mathcal{V}(v_\Phi, v_\Delta) = & -\frac{1}{2} \mu_\Phi^2 v_\Phi^2 - \frac{1}{2} \mu_\Delta^2 v_\Delta^2 + \frac{1}{4} \lambda_\Phi v_\Phi^4 \\ & + \frac{1}{4} \lambda_\Delta v_\Delta^4 + \frac{1}{2} \lambda_{\Phi\Delta} v_\Phi^2 v_\Delta^2, \end{aligned} \quad (4.4)$$

where we denote  $v_{\eta_3}$  as  $v_\Delta$ . This potential is invariant under  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  and  $SU(2)_D^{\text{local}}$  transformations shown in Figs. 1 and 2.

Next, we calculate the first derivatives of the potential  $\mathcal{V}(v_\Phi, v_\Delta)$  with respect to  $v_\Phi$  and  $v_\Delta$ .

$$\begin{aligned} \frac{\partial}{\partial v_\Phi} \mathcal{V}(v_\Phi, v_\Delta) = & v_\Phi (-\mu_\Phi^2 + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_\Delta^2), \\ \frac{\partial}{\partial v_\Delta} \mathcal{V}(v_\Phi, v_\Delta) = & v_\Delta (-\mu_\Delta^2 + \lambda_\Delta v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2). \end{aligned} \quad (4.5)$$

From the first derivatives, we find the following stationary conditions:

$$0 = v_\Phi (-\mu_\Phi^2 + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_\Delta^2), \quad (4.6)$$

$$0 = v_\Delta (-\mu_\Delta^2 + \lambda_\Delta v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2). \quad (4.7)$$

We analytically solve the simultaneous equations given in Eqs. (4.6) and (4.7) below.

(i) From Eq. (4.6), we find

$$v_\Phi = 0 \quad \text{or} \quad -\mu_\Phi^2 + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_\Delta^2 = 0. \quad (4.8)$$

(ii) First, for  $v_\Phi = 0$  case, from Eqs. (4.6) and (4.7), we find

$$v_\Delta = 0 \quad \text{or} \quad v_\Delta = \pm \sqrt{\frac{\mu_\Delta^2}{\lambda_\Delta}}. \quad (4.9)$$

For the first case,  $v_\Delta = 0$ , the VEVs are located at the origin

$$v_\Phi = v_\Delta = 0. \quad (4.10)$$

$SU(2)_D^{\text{local}}$  is unbroken, and  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  is also unbroken.

For the second case,  $v_\Delta = \pm \sqrt{\mu_\Delta^2/\lambda_\Delta}$ , the VEVs are given by

$$v_\Phi = 0, \quad v_\Delta = \pm \sqrt{\frac{\mu_\Delta^2}{\lambda_\Delta}}. \quad (4.11)$$

$SU(2)_D^{\text{local}}$  is broken to its subgroup  $U(1)_D^{\text{local}}$ , and  $U(1)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  remains.

(iii) Next we consider the second condition in Eq. (4.8).

From Eq. (4.7), we find

$$v_\Delta = 0 \quad \text{or} \quad -\mu_\Delta^2 + \lambda_\Delta v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2 = 0. \quad (4.12)$$

For the first case  $v_\Delta = 0$ , we find

$$v_\Phi = \pm \sqrt{\frac{\mu_\Phi^2}{\lambda_\Phi}}, \quad v_\Delta = 0. \quad (4.13)$$

$SU(2)_D^{\text{local}}$  is completely broken.  $SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  is broken to the  $SU(2)_{\Phi V}^{\text{global}}$  custodial symmetry that is the diagonal subgroup of  $SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$ , so  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  remains.

(iv) Finally, we consider the following simultaneous equations:

$$\begin{aligned} 0 = & -\mu_\Phi^2 + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_\Delta^2, \\ 0 = & -\mu_\Delta^2 + \lambda_\Delta v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2. \end{aligned} \quad (4.14)$$

Since the simultaneous equations can be decomposed into two quadratic equations of  $v_\Phi$  and  $v_\Delta$ , it can be solved as

$$v_\Phi = \pm \sqrt{\frac{\lambda_\Delta \mu_\Phi^2 - \lambda_{\Phi\Delta} \mu_\Delta^2}{\lambda_\Delta \lambda_\Phi - \lambda_{\Phi\Delta}^2}}, \quad v_\Delta = \pm \sqrt{\frac{\lambda_\Phi \mu_\Delta^2 - \lambda_{\Phi\Delta} \mu_\Phi^2}{\lambda_\Delta \lambda_\Phi - \lambda_{\Phi\Delta}^2}}, \quad (4.15)$$

where all sign combinations exist.  $SU(2)_D^{\text{local}}$  is completely broken.  $SU(2)_\Delta^{\text{global}}$  is broken to

TABLE II. The extrema and saddle points in the potential given in Eq. (4.4) for  $\kappa=0$  are shown. The potential energy at each extremum or saddle point and remaining gauge and global symmetry are also listed. # of NG represents the total number of NG modes. In the table, the superscript, local/global, is omitted.

| Name                            | $V_1$  | $V_2$   | $V_3$  | $V_4$   |
|---------------------------------|--|---|--|---|
| $(v_\Phi, v_\Delta)$            | (0,0)  | $(0, \pm\sqrt{\frac{\mu_\Delta^2}{\lambda_\Delta}})$      | $(\pm\sqrt{\frac{\mu_\Phi^2}{\lambda_\Phi}}, 0)$ | $(\pm\sqrt{\frac{\lambda_\Delta\mu_\Phi^2 - \lambda_{\Phi\Delta}\mu_\Delta^2}{\lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2}}, \pm(\Phi \leftrightarrow \Delta))$ |
| $\mathcal{V}(v_\Phi, v_\Delta)$ | 0  | $-\frac{\mu_\Delta^4}{4\lambda_\Delta}$                   | $-\frac{\mu_\Phi^4}{4\lambda_\Phi}$              | $-\frac{\lambda_\Phi\mu_\Delta^4 - 2\lambda_{\Phi\Delta}\mu_\Phi^2\mu_\Delta^2 + \lambda_\Delta\mu_\Phi^4}{4(\lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2)}$     |
| Gauge symmetry                  | $SU(2)_D$  | $U(1)_D$  | None   | None  |
| Global symmetry                 | $SU(2)_\Delta \times SU(2)_{\Phi L} \times SU(2)_{\Phi R}$ | $U(1)_\Delta \times SU(2)_{\Phi L} \times SU(2)_{\Phi R}$ | $SU(2)_\Delta \times SU(2)_{\Phi V}$             | $U(1)_\Delta \times SU(2)_{\Phi V}$   |
| # of NG                         | 0  | 2   | 3  | 5   |

$U(1)_\Delta^{\text{global}}$  and  $SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  is broken to the  $SU(2)_{\Phi V}^{\text{global}}$  custodial symmetry. Therefore  $U(1)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  remains.

We summarize the extrema and saddle points in the potential given in Eq. (4.4) in Table II. In the table, the potential energy at each extremum or saddle point, remaining gauge and global symmetry, and a total number of NG modes are also listed, where  $V_1$ ,  $V_2$ ,  $V_3$ , and  $V_4$  represent the names of the stationary points and the potential energies at  $SU(2)_D^{\text{local}}$ ,  $U(1)_D^{\text{local}}$ ,  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$ , and  $U(1)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  stationary points, respectively.

Next, we consider the correspondence between the parameter domain and the symmetry realized in the vacuum. First of all, the quartic coupling constants  $\lambda_\Delta$ ,  $\lambda_\Phi$ , and  $\lambda_{\Phi\Delta}$  must satisfy the following conditions to stabilize the potential  $\mathcal{V}(v_\Phi, v_\Delta)$  with finite values of the VEVs:

$$\lambda_\Delta > 0, \quad \lambda_\Phi > 0, \quad \lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2 > 0. \quad (4.16)$$

There are four stationary points  $V_{1,2,3,4}$  given in Table II. They are not always solutions in all parameter regions because the VEVs  $v_\Phi$  and  $v_\Delta$  are defined as real numbers. In fact,  $V_1$  is a solution in any  $\mu_\Phi^2$  and  $\mu_\Delta^2$  region;  $V_2$  is a solution for  $\mu_\Delta^2 > 0$ ;  $V_3$  is a solution for  $\mu_\Phi^2 > 0$ ; and  $V_4$  is a solution for  $\mu_\Delta^2 > 0$  and  $\mu_\Phi^2 > 0$ .

We will find the true vacuum by comparing the potential energies of stationary points. When  $\mu_\Delta^2 > 0$  and  $\mu_\Phi^2 > 0$ , the potential energy preserving  $U(1)_\Delta^{\text{global}} \times SU(2)_{\Phi V}$  is lower than the other potential energies preserving  $SU(2)_D^{\text{local}}$ ,  $U(1)_D^{\text{local}}$ , and  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  because

$$V_4 - V_1 = -\left\{ \frac{\lambda_\Phi}{4(\lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2)} \left( \mu_\Delta^2 - \frac{\lambda_{\Phi\Delta}}{\lambda_\Phi} \mu_\Phi^2 \right)^2 + \frac{\mu_\Phi^4}{4\lambda_\Phi} \right\} < 0, \quad (4.17)$$

$$V_4 - V_2 = -\frac{(\lambda_\Delta\mu_\Phi^2 - \lambda_{\Phi\Delta}\mu_\Delta^2)^2}{4\lambda_\Delta(\lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2)} < 0, \quad (4.18)$$

$$V_4 - V_3 = -\frac{(\lambda_\Phi\mu_\Delta^2 - \lambda_{\Phi\Delta}\mu_\Phi^2)^2}{4\lambda_\Phi(\lambda_\Delta\lambda_\Phi - \lambda_{\Phi\Delta}^2)} < 0 \quad (4.19)$$

from Eq. (4.16). For the other parameter spaces of  $(\mu_\Phi^2, \mu_\Delta^2)$ , it is easy to find that for  $\mu_\Phi^2 < 0$  and  $\mu_\Delta^2 < 0$ ,  $SU(2)_D^{\text{local}}$  is realized at the vacuum; for  $\mu_\Phi^2 < 0$  and  $\mu_\Delta^2 > 0$ ,  $U(1)_D^{\text{local}}$  is realized at the vacuum; for  $\mu_\Phi^2 > 0$  and  $\mu_\Delta^2 < 0$ ,  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  is realized at the vacuum. The global symmetry  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  breaking patterns are shown in Fig. 3.

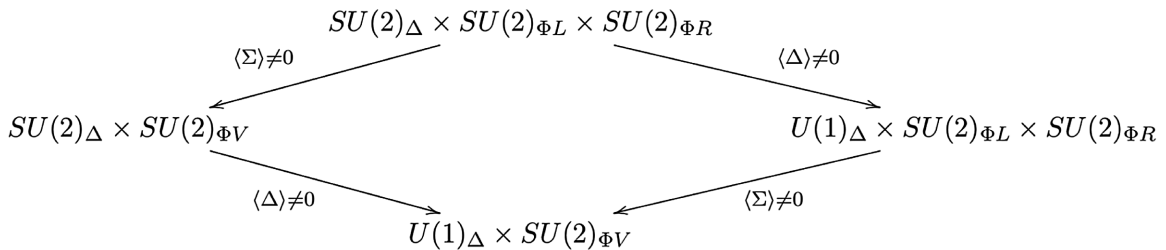


FIG. 3. The global symmetry  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  breaking patterns for  $\kappa=0$  are shown.  $\langle \Sigma \rangle \neq 0$  and  $\langle \Delta \rangle \neq 0$  represent the SSB by the VEV of  $\Sigma$  and  $\Delta$  in  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{3}, \mathbf{1}, \mathbf{1})$  of  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$ , respectively. In the figure, the superscript, global, is omitted.

### B. With soft symmetry breaking term ( $\kappa \neq 0$ )

We will now begin analyzing the potential for the case  $\kappa \neq 0$ . We take the VEVs of  $\Sigma(x)$  and  $\Delta(x)$  given in Eq. (3.1). Substituting the VEVs into the potential of  $\Sigma(x)$  and  $\Delta(x)$  given in Eq. (2.6)

$$\begin{aligned} \mathcal{V}(\Sigma, \Delta) = & -\frac{\mu_\Phi^2}{2} \text{Tr}(\Sigma^\dagger \Sigma) - \frac{1}{2} \mu_\Delta^2 \text{Tr}(\Delta^2) - \sqrt{2} \kappa \text{Tr}(\sigma_3 \Sigma^\dagger \Delta \Sigma) \\ & + \frac{\lambda_\Phi}{4} (\text{Tr}(\Sigma^\dagger \Sigma))^2 + \frac{1}{4} \lambda_\Delta \text{Tr}(\Delta^2)^2 \\ & + \frac{1}{2} \lambda_{\Phi\Delta} \text{Tr}(\Sigma^\dagger \Sigma) \text{Tr}(\Delta^2), \end{aligned} \quad (4.20)$$

the potential is given by

$$\begin{aligned} \mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3}) = & -\frac{1}{2} \mu_\Phi^2 v_\Phi^2 - \frac{1}{2} \mu_\Delta^2 (v_{\eta_1}^2 + v_{\eta_3}^2) - \kappa v_\Phi v_{\eta_3} \\ & + \frac{1}{4} \lambda_\Phi v_\Phi^4 + \frac{1}{4} \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2)^2 \\ & + \frac{1}{2} \lambda_{\Phi\Delta} v_\Phi^2 (v_{\eta_1}^2 + v_{\eta_3}^2). \end{aligned} \quad (4.21)$$

We recall that the  $\kappa$  term is invariant under  $SU(2)_D^{\text{local}}$ , but breaks  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  to  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$ . Therefore, this system is invariant under  $SU(2)_D^{\text{local}}$  gauge and  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$  global transformations. Note that  $\Sigma(x)$  and  $\Delta(x)$  belong to  $\mathbf{2}(\pm 1)$  and  $\mathbf{3}(0)$  of  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$ , respectively, where the numbers in boldface denote  $SU(2)_L^{\text{global}}$  representation and numbers in parentheses denote  $U(1)_R^{\text{global}}$  charges.

Next, we calculate the first derivatives of the potential  $\mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3})$  with respect to  $v_\Phi, v_{\eta_1}, v_{\eta_3}$ .

$$\begin{aligned} \frac{\partial}{\partial v_\Phi} \mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3}) = & v_\Phi (-\mu_\Phi^2 - 2\kappa v_{\eta_3} + \lambda_\Phi v_\Phi^2 \\ & + \lambda_{\Phi\Delta} (v_{\eta_1}^2 + v_{\eta_3}^2)), \\ \frac{\partial}{\partial v_{\eta_1}} \mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3}) = & v_{\eta_1} (-\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2) + \lambda_{\Phi\Delta} v_\Phi^2), \\ \frac{\partial}{\partial v_{\eta_3}} \mathcal{V}(v_\Phi, v_{\eta_1}, v_{\eta_3}) = & v_{\eta_3} (-\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2) \\ & + \lambda_{\Phi\Delta} v_\Phi^2) - \kappa v_\Phi^2. \end{aligned} \quad (4.22)$$

From the first derivatives, we find the following stationary conditions:

$$0 = v_\Phi (-\mu_\Phi^2 - 2\kappa v_{\eta_3} + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} (v_{\eta_1}^2 + v_{\eta_3}^2)), \quad (4.23)$$

$$0 = v_{\eta_1} (-\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2) + \lambda_{\Phi\Delta} v_\Phi^2), \quad (4.24)$$

$$0 = v_{\eta_3} (-\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2) + \lambda_{\Phi\Delta} v_\Phi^2) - \kappa v_\Phi^2. \quad (4.25)$$

We analytically solve the simultaneous equations given by Eqs. (4.23)–(4.25) below.

(i) From Eq. (4.23), we find

$$\begin{aligned} v_\Phi = 0 \quad \text{or} \\ -\mu_\Phi^2 - 2\kappa v_{\eta_3} + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} (v_{\eta_1}^2 + v_{\eta_3}^2) = 0. \end{aligned} \quad (4.26)$$

(ii) First, for  $v_\Phi = 0$  case, from Eqs. (4.23)–(4.25), we find

$$v_{\eta_1} = v_{\eta_3} = 0 \quad \text{or} \quad -\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2)^2 = 0. \quad (4.27)$$

For the first case, the VEVs are located at the origin

$$v_\Phi = v_{\eta_1} = v_{\eta_3} = 0. \quad (4.28)$$

$SU(2)_D^{\text{local}}$  is unbroken, and  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$  is also unbroken.

For the second case, we can take the following VEVs by using the  $SU(2)_L^{\text{global}}$  transformation:

$$v_\Phi = 0, \quad v_{\eta_1} = 0, \quad v_{\eta_3} = \pm \sqrt{\frac{\mu_\Delta^2}{\lambda_\Delta}}. \quad (4.29)$$

$SU(2)_D^{\text{local}}$  is broken to its subgroup  $U(1)_D^{\text{local}}$ , and  $U(1)_L^{\text{global}} \times U(1)_R^{\text{global}}$  remains.

(iii) Next we consider the second condition in Eq. (4.26). From Eq. (4.24), we find

$$v_{\eta_1} = 0 \quad \text{or} \quad -\mu_\Delta^2 + \lambda_\Delta (v_{\eta_1}^2 + v_{\eta_3}^2) + \lambda_{\Phi\Delta} v_\Phi^2 = 0. \quad (4.30)$$

From Eq. (4.25), the above second condition leads to  $\kappa = 0$ , but due to  $\kappa \neq 0$ ,  $v_{\eta_1} = 0$ . For  $v_\Phi \neq 0$  and  $v_{\eta_1} = 0$ , we need to solve the following simultaneous equations:

$$0 = -\mu_\Phi^2 - 2\kappa v_{\eta_3} + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_{\eta_3}^2, \quad (4.31)$$

$$0 = v_{\eta_3} (-\mu_\Delta^2 + \lambda_\Delta v_{\eta_3}^2 + \lambda_{\Phi\Delta} v_\Phi^2) - \kappa v_\Phi^2. \quad (4.32)$$

The solutions of the simultaneous equations lead to  $v_\Phi \neq 0$  and  $v_{\eta_3} \neq 0$ , so the vacuum of these solutions breaks  $SU(2)_L^{\text{global}} \times U(1)_R^{\text{global}}$  to  $U(1)_V^{\text{global}}$ .

The simultaneous equations in Eqs. (4.31) and (4.32) can be decomposed into a cubic equations for  $v_{\eta_3}$  and a quadratic equation for  $v_\Phi$ . From the vacuum solutions listed in Table II and the soft symmetry breaking  $\kappa$  term, the three solutions of the cubic equation correspond to one  $SU(2)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  and two  $U(1)_\Delta^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  global symmetry vacuum solutions in the  $\kappa = 0$  case. We can solve the exact solutions of the simultaneous

TABLE III. The extrema and saddle points in the potential given in Eq. (4.21) for  $\kappa \neq 0$  are shown. The potential energy at each extremum or saddle point and remaining gauge and global symmetry are also listed, where  $v_{\eta_1} = 0$ . # of NG represents the total number of NG and pNGB modes. In the table, the superscript, local/global, is omitted. For  $U(1)_{\Phi}^{\text{global}}$  case, we omit  $O(\kappa)$  if there is already a value greater than  $\kappa$ .

| Name                                | $V_I$                   | $V_{II}$   | $V_{III}$  | $V_{IV}$  |
|-------------------------------------|-------------------------|--|--|---|
| $(v_{\Phi}, v_{\eta_3})$            | (0,0)                   | $(0, \pm\sqrt{\frac{\mu_{\Delta}^2}{\lambda_{\Delta}}})$ | $(\pm\sqrt{\frac{\mu_{\Phi}^2}{\lambda_{\Phi}}}, O(\kappa))$ | $(\pm\sqrt{\frac{\lambda_{\Delta}\mu_{\Phi}^2 - \lambda_{\Phi}\mu_{\Delta}^2}{\lambda_{\Delta}\lambda_{\Phi} - \lambda_{\Phi}^2}}, \pm(\Phi \leftrightarrow \Delta))$   |
| $\mathcal{V}(v_{\Phi}, v_{\eta_3})$ | 0                       | $-\frac{\mu_{\Delta}^4}{4\lambda_{\Delta}}$              | $-\frac{\mu_{\Phi}^4}{4\lambda_{\Phi}}$                      | $-\frac{\lambda_{\Phi}\mu_{\Delta}^4 - 2\lambda_{\Phi}\mu_{\Phi}^2\mu_{\Delta}^2 + \lambda_{\Delta}\mu_{\Phi}^4}{4(\lambda_{\Delta}\lambda_{\Phi} - \lambda_{\Phi}^2)}$ |
| Gauge symmetry                      | $SU(2)_D$               | $U(1)_D$   |  | None  |
| Global symmetry                     | $SU(2)_L \times U(1)_R$ | $U(1)_L \times U(1)_R$                                   |  | $U(1)_V$  |
| # of NG                             | 0                       | 2  | 3  | 5   |

equations because of just cubic and quadratic equations, but they are too complicated to show here. Instead, we can find approximate solutions to the simultaneous equations by using the solutions around the  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  and  $U(1)_{\Delta}^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  vacuum solutions. The detailed values are not important for the discussion here, so we omit the particular form, but there are solutions around the  $\kappa = 0$  solution in Table II as follows. From the solution for the  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  vacuum in the  $\kappa = 0$  case,

$$v_{\Phi} = \pm\sqrt{\frac{\mu_{\Phi}^2}{\lambda_{\Phi}}} + O(\kappa), \quad v_{\eta_3} = O(\kappa). \quad (4.33)$$

From the solution for the  $U(1)_{\Delta}^{\text{global}} \times SU(2)_{\Phi V}^{\text{global}}$  vacuum in the  $\kappa = 0$  case,

$$v_{\Phi} = \pm\sqrt{\frac{\lambda_{\Delta}\mu_{\Phi}^2 - \lambda_{\Phi}\mu_{\Delta}^2}{\lambda_{\Delta}\lambda_{\Phi} - \lambda_{\Phi}^2}} + O(\kappa),$$

$$v_{\eta_3} = \pm\sqrt{\frac{\lambda_{\Phi}\mu_{\Delta}^2 - \lambda_{\Phi}\mu_{\Phi}^2}{\lambda_{\Delta}\lambda_{\Phi} - \lambda_{\Phi}^2}} + O(\kappa). \quad (4.34)$$

Next, we consider the correspondence between the parameter domain and the symmetry realized in the vacuum. Since the  $\kappa$  term does not affect the shape of the potential at infinity, the constraint of the parameter region from the stability condition to potential is the same for  $\kappa \neq 0$  as for  $\kappa = 0$ , which is given in Eq. (4.16). In the region where  $\kappa$  can be treated perturbatively, the true vacuum does not change, so the results for the case  $\kappa = 0$  are applicable. Therefore, for  $\mu_{\Phi}^2$  and  $\mu_{\Delta}^2 > 0$ ,  $U(1)_V^{\text{global}}$  is realized at the vacuum; for  $\mu_{\Phi}^2 < 0$  and  $\mu_{\Delta}^2 < 0$ ,  $SU(2)_D^{\text{local}}$  is realized at the vacuum; for  $\mu_{\Phi}^2 < 0$  and  $\mu_{\Delta}^2 > 0$ ,  $U(1)_D^{\text{local}}$  is realized at the vacuum. The extrema and saddle points in the potential are summarized in Table III. The global symmetry  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  breaking patterns are shown in Fig. 4.

Before investigating the mass spectra of the scalar sectors, we comment on the would-be NG and pNG modes. For  $V_I$  of Table III, there are no NG modes. For  $V_{II}$ , there are two NG modes, and they are absorbed by the  $SU(2)_D^{\text{local}}/U(1)_D^{\text{local}}$  gauge bosons. For  $V_{III}$ , there are three NG modes, and they are absorbed by the  $SU(2)_D^{\text{local}}$  gauge bosons. For  $V_{IV}$ , there are five NG modes. Three of the five NG modes are absorbed in the  $SU(2)_D^{\text{local}}$  gauge boson.

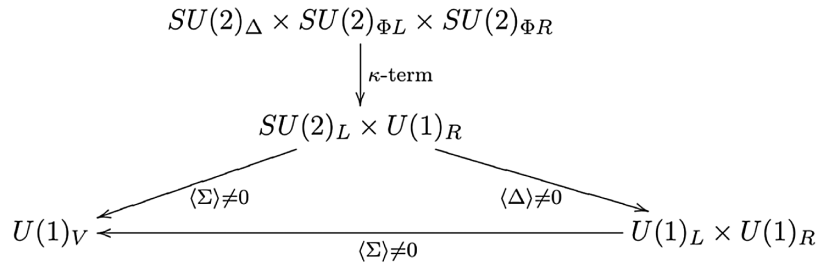


FIG. 4. The global symmetry  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  breaking patterns for  $\kappa \neq 0$  are shown. The  $\kappa$  term stands for a soft symmetry breaking term;  $\langle \Sigma \rangle \neq 0$  and  $\langle \Delta \rangle \neq 0$  represent the SSB by the VEVs of  $\Sigma$  and  $\Delta$  in  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{3}, \mathbf{1}, \mathbf{1})$  of  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$ .  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{3}, \mathbf{1}, \mathbf{1})$  of  $SU(2)_{\Delta}^{\text{global}} \times SU(2)_{\Phi L}^{\text{global}} \times SU(2)_{\Phi R}^{\text{global}}$  are decomposed into two  $\mathbf{2}$  and one  $\mathbf{3}$  of  $SU(2)_L$ , respectively. In the figure, the superscript, global, is omitted.



The remaining two NG modes are real scalar modes with  $U(1)_V^{\text{global}}$  charges and are identified as one complex scalar.

As can be seen from the above discussion, a charged pNGB that can be regarded as a DM appears only when  $\mu_\Phi^2 > 0$  and  $\mu_\Delta^2 > 0$ . In the following, we consider such a case.

## V. MASS SPECTRUM

Here we investigate the mass spectra of the scalar fields  $\Sigma(x)$  (or  $\Phi(x)$ ),  $\Delta(x)$ , and  $H(x)$  for the parameter region  $\mu_\Phi^2 > 0$  and  $\mu_\Delta^2 > 0$ , where  $U(1)_V^{\text{global}}$  symmetry is realized at a vacuum. In particular, we confirm that there is a  $U(1)_V^{\text{global}}$  charged complex scalar with a mass proportional to the  $\kappa$  parameter. It corresponds to the pNG mode, which will be regarded as a DM candidate.

First, we check the potential terms associated with  $\Sigma$  and  $\Delta$ . Here we consider the following field expression at the vacuum:

$$\begin{aligned} \Delta(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Delta + \eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & -v_\Delta - \eta_3 \end{pmatrix}, \\ \Sigma(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Phi + \phi_3 - i\phi_4 & \phi_1 + i\phi_2 \\ \phi_1 - i\phi_2 & v_\Phi + \phi_3 + i\phi_4 \end{pmatrix}, \end{aligned} \quad (5.1)$$

where we denote the VEV of  $\eta_3$  as  $v_\Delta$ , and the values of  $v_\Delta$  and  $v_\Phi$  for  $\kappa \rightarrow 0$  are given in Eq. (4.34). In our convention, the generator of the remaining  $U(1)_V^{\text{global}}$  corresponds to the  $\sigma_3$  direction. Therefore,  $\eta_1$ ,  $\eta_2$ ,  $\phi_1$ , and  $\phi_2$  have the same

$U(1)_V^{\text{global}}$  charge, while  $\eta_3$ ,  $\phi_3$ , and  $\phi_4$  have no  $U(1)_V^{\text{global}}$  charge. The  $U(1)_V^{\text{global}}$  charge of fields can be checked by using, e.g., the generator of the  $U(1)_V^{\text{global}}$  charge  $\sigma_3$ .

The stationary conditions in Eqs. (4.31) and (4.32) for  $v_\Phi, v_\Delta \neq 0$  can be written as

$$\mu_\Phi^2 = -2\kappa v_\Delta + \lambda_\Phi v_\Phi^2 + \lambda_{\Phi\Delta} v_{\eta_3}^2, \quad (5.2)$$

$$\mu_\Delta^2 = \lambda_\Delta v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2 - \kappa \frac{v_\Phi^2}{v_\Delta}, \quad (5.3)$$

where we replaced  $v_{\eta_3}$  as  $v_\Delta$ .

Substituting Eqs. (5.1)–(5.3) into the potential  $\mathcal{V}(\Sigma, \Delta)$  given in Eq. (4.20), the potential is given as

$$\mathcal{V}(\eta_1, \eta_2, \eta_3, \phi_1, \phi_2, \phi_3, \phi_4) = \mathcal{V}_0 + \mathcal{V}_2 + \mathcal{V}_3 + \mathcal{V}_4, \quad (5.4)$$

where  $\mathcal{V}$  stands for  $V_{\text{IV}}$  in Table III, and the subscript of  $\mathcal{V}_j$  ( $j = 0, 2, 3, 4$ ) denotes the mass dimension of the operator. Note that the tadpole term of the potential  $\mathcal{V}_1$  disappears from the stationary conditions. The constant terms of the potential of the real scalar fields  $(\eta_1, \eta_2, \eta_3, \phi_1, \phi_2, \phi_3, \phi_4)$  are given by

$$\mathcal{V}_0 = -\frac{1}{4}(\lambda_\Delta v_\Delta^4 + \lambda_\Phi v_\Phi^4 + 2\lambda_{\Phi\Delta} v_\Delta^2 v_\Phi^2 - 2\kappa v_\Delta v_\Phi^2). \quad (5.5)$$

The quadratic terms are given by

$$\begin{aligned} \mathcal{V}_2 &= \frac{1}{2} \begin{pmatrix} \phi_1 & \eta_1 \end{pmatrix} \begin{pmatrix} 4\kappa v_\Delta & 2\kappa v_\Phi \\ 2\kappa v_\Phi & \frac{v_\Phi^2}{v_\Delta} \kappa \end{pmatrix} \begin{pmatrix} \phi_1 \\ \eta_1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \phi_2 & \eta_2 \end{pmatrix} \begin{pmatrix} 4\kappa v_\Delta & -2\kappa v_\Phi \\ -2\kappa v_\Phi & \frac{v_\Phi^2}{v_\Delta} \kappa \end{pmatrix} \begin{pmatrix} \phi_2 \\ \eta_2 \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \phi_3 & \eta_3 \end{pmatrix} \begin{pmatrix} 2\lambda_\Phi v_\Phi^2 & 2\lambda_{\Phi\Delta} v_\Phi v_\Delta - 2v_\Phi \kappa \\ 2\lambda_{\Phi\Delta} v_\Phi v_\Delta - 2v_\Phi \kappa & 2\lambda_\Delta v_\Delta^2 + \frac{v_\Phi^2}{v_\Delta} \kappa \end{pmatrix} \begin{pmatrix} \phi_3 \\ \eta_3 \end{pmatrix} + 0 \times \phi_4^2. \end{aligned} \quad (5.6)$$

Since the determinant of  $(\eta_1, \phi_1)$  is zero, one of the eigenvalues is zero. Furthermore, if  $\kappa$  is set to zero, both eigenvalues are zero. The same is true for  $(\eta_2, \phi_2)$ . Since the determinant of  $(\eta_3, \phi_3)$  is non-zero, even for  $\kappa \rightarrow 0$  zero eigenvalues do not appear. Instead of it, a  $U(1)_V^{\text{global}}$  neutral scalar field  $\phi_4$  is always massless. Therefore, we find that one of the two linear combinations of  $\eta_1$  and  $\phi_1$  is an NG mode that is absorbed by an  $SU(2)_D^{\text{local}}$  gauge boson, and the other is a pNG mode. The same is true for  $\eta_2$  and  $\phi_2$ .  $\phi_4$  is an NG mode, and  $\eta_3$  and  $\phi_3$  are Higgs modes. Similarly, the cubic and quartic terms are given by

$$\begin{aligned} \mathcal{V}_3 &= 2\kappa(\eta_1 \phi_1 \phi_3 - \eta_2 \phi_2 \phi_3 + \eta_2 \phi_1 \phi_4 + \eta_1 \phi_2 \phi_4) + \kappa \eta_3 (\phi_1^2 + \phi_2^2 - \phi_3^2 - \phi_4^2) \\ &+ \lambda_\Delta v_\Delta \eta_3 (\eta_1^2 + \eta_2^2 + \eta_3^2) + \lambda_\Phi v_\Phi \phi_3 (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \\ &+ \lambda_{\Phi\Delta} v_\Delta \eta_3 (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \lambda_{\Phi\Delta} v_\Phi \phi_3 (\eta_1^2 + \eta_2^2 + \eta_3^2), \end{aligned} \quad (5.7)$$

$$\mathcal{V}_4 = \frac{1}{4} \lambda_\Delta (\eta_1^2 + \eta_2^2 + \eta_3^2)^2 + \frac{1}{4} \lambda_\Phi (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2 + \frac{1}{2} \lambda_{\Phi\Delta} (\eta_1^2 + \eta_2^2 + \eta_3^2) (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2). \quad (5.8)$$

For the calculation of the scattering amplitudes of SM particles and a charged pNGB in the next section, here we switch on the scalar field  $H$  in **2** of  $SU(2)_W^{\text{local}}$  listed in Table I and rewrite charged scalar fields as follows:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad \phi_{(\pm)} := \frac{1}{\sqrt{2}} (\phi_1 \pm i\phi_2),$$

$$\eta_{(\pm)} := \frac{1}{\sqrt{2}} (\eta_1 \mp i\eta_2), \quad (5.9)$$

where the subscripts (positive and negative signs) enclosed in parentheses indicate the sign of the  $U(1)_V$  charge, and  $v$  stands for the VEV of the SM Higgs boson, which breaks  $SU(2)_W^{\text{local}} \times U(1)_Y^{\text{local}}$  into  $U(1)_{\text{EM}}^{\text{local}}$ . In the rest of this section, we will examine the mass matrix when a

$U(1)_V^{\text{global}}$  neutral scalar field  $h$  is added to the mass matrix given in Eq. (5.6), where the field  $h$  is a main component of the SM Higgs boson.

First, we consider the mass eigenstates for the  $U(1)_V^{\text{global}}$  charged scalar fields  $\phi_{1,2}$  and  $\eta_{1,2}$ . When we rewrite  $\phi_{1,2}$  and  $\eta_{1,2}$  in terms of  $\phi_{(\pm)}$  and  $\eta_{(\pm)}$  as in Eq. (5.9), the mass term of  $\phi_{1,2}$  and  $\eta_{1,2}$  given in Eq. (5.6) can be rewritten as

$$\mathcal{V}_2 \ni \frac{1}{2} (\phi_{(\mp)} \quad \eta_{(\mp)}) M_C^2 \begin{pmatrix} \phi_{(\pm)} \\ \eta_{(\pm)} \end{pmatrix}, \quad M_C^2 := \begin{pmatrix} 4\kappa v_\Delta & 2\kappa v_\Phi \\ 2\kappa v_\Phi & \frac{v_\Phi^2}{v_\Delta} \kappa \end{pmatrix}. \quad (5.10)$$

The above mass matrix can be easily diagonalized, and the mass and mixing matrix are given by

$$m_{G_{(\pm)}} = 0, \quad m_\varphi^2 = \kappa \left( 4v_\Delta + \frac{v_\Phi^2}{v_\Delta} \right), \quad \begin{pmatrix} G_{(\pm)} \\ \varphi_{(\pm)} \end{pmatrix} := \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_{(\pm)} \\ \eta_{(\pm)} \end{pmatrix}, \quad (5.11)$$

where

$$\sin \beta = \frac{v_\Phi}{\sqrt{v_\Phi^2 + 4v_\Delta^2}}, \quad \cos \beta = \frac{2v_\Delta}{\sqrt{v_\Phi^2 + 4v_\Delta^2}}, \quad (5.12)$$

$G_{(\pm)}$  is the would-be NG modes of  $SU(2)_D^{\text{local}}$ , and  $\varphi_{(\pm)}$  is a  $U(1)_V^{\text{global}}$  charged scalar mode that is a pNG mode of  $SU(2)_V^{\text{global}}/U(1)_V^{\text{global}}$ , which will be identified as a DM.

Second, even when  $h$  exists, there is no mass mixing of  $\phi_4$  and  $h$ , so  $\phi_4$  remains massless. That is,  $\phi_4$  is a would-be NG mode of  $SU(2)_D^{\text{local}}$ . Further,  $\phi_4$  has no charge of  $U(1)_D^{\text{local}}$  and  $U(1)_V^{\text{global}}$ . Therefore,  $\phi_4$  can be identified with the neutral NG mode  $G_0$ .

Finally, the mass matrix of a  $U(1)_V^{\text{global}}$  neutral sector  $(\eta_3, \phi_3, h)$  is given by

$$M_H^2 := \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H\Phi} v v_\Phi & 2\lambda_{H\Delta} v v_\Delta \\ \lambda_{H\Phi} v v_\Phi & 2\lambda_\Phi v_\Phi^2 & 2\lambda_{\Phi\Delta} v_\Phi v_\Delta - 2v_\Phi \kappa \\ 2\lambda_{H\Delta} v v_\Delta & 2\lambda_{\Phi\Delta} v_\Phi v_\Delta - 2v_\Phi \kappa & 2\lambda_\Delta v_\Delta^2 + \frac{v_\Phi^2}{v_\Delta} \kappa - 2v_\Phi \kappa \end{pmatrix}. \quad (5.13)$$

Since this mass matrix is a real symmetric matrix, it can be diagonalized by a unitary matrix (orthogonal matrix)  $U_H$ , where  $U_H^\dagger U_H = I$  and  $U_H^\dagger = U_H^T$ . That is,  $U_H M_H^2 U_H^\dagger = (M_H^2)^{\text{diag}}$ , where  $(M_H^2)^{\text{diag}}$  is a  $3 \times 3$  diagonal matrix. The mass eigenstates can be expressed from the original basis as follows:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} := U_H \begin{pmatrix} h \\ \phi_3 \\ \eta_3 \end{pmatrix}, \quad (5.14)$$

where  $h_j$  ( $j = 1, 2, 3$ ) are mass eigenstates with no  $U(1)_V^{\text{global}}$  charge, and  $h_1$  is identified as the observed SM Higgs mode with a mass of about 125 GeV. The exact eigenvalues and eigenvectors are too complicated to show

here. Instead of it, we show the approximate mass eigenvalues and mass mixing matrix when  $v_\Delta$  is sufficiently larger than  $v$  and  $v_\Phi$ . For  $v_\Delta \gg v_\Phi, v$ , the mixing matrix  $U_H$  is given by

$$U_H = \begin{pmatrix} 1 & 0 & -\frac{\lambda_{H\Delta} v}{\lambda_\Delta v_\Delta} \\ 0 & 1 & -\frac{\lambda_{\Phi\Delta} v_\Phi}{\lambda_\Delta v_\Delta} \\ \frac{\lambda_{H\Delta} v}{\lambda_\Delta v_\Delta} & \frac{\lambda_{\Phi\Delta} v_\Phi}{\lambda_\Delta v_\Delta} & 1 \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} + O\left(\frac{v^2}{v_\Delta^2}, \frac{v_\Phi^2}{v_\Delta^2}\right),$$

$$\tan 2\alpha \simeq \frac{2v v_\Phi (\lambda_{H\Phi} \lambda_\Phi - \lambda_{H\Delta} \lambda_{\Phi\Delta})}{v^2 (\lambda_{H\Delta}^2 - \lambda_H \lambda_\Delta) - v_\Phi^2 (\lambda_{S\Phi}^2 - \lambda_\Phi \lambda_\Delta)}. \quad (5.15)$$

The mass eigenvalues for  $h_i$  are given by

$$\begin{aligned} m_{h_1}^2 &\simeq \lambda_H v^2 - \frac{\lambda_{H\Delta}^2 \lambda_\Phi - 2\lambda_{H\Phi} \lambda_{H\Delta} \lambda_{\Phi\Delta} + \lambda_\Delta \lambda_{H\Phi}^2}{\lambda_\Phi \lambda_\Delta - \lambda_{\Phi\Delta}^2} v_\Phi^2, \\ m_{h_2}^2 &\simeq \frac{\lambda_\Phi \lambda_\Delta - \lambda_{\Phi\Delta}^2}{\lambda_\Delta} v_\Phi^2 + \frac{(\lambda_\Delta \lambda_{H\Phi} - \lambda_{H\Delta} \lambda_{\Phi\Delta})^2}{\lambda_\Delta (\lambda_\Phi \lambda_\Delta) - \lambda_{\Phi\Delta}^2} v^2, \\ m_{h_3}^2 &\simeq \lambda_\Delta v_\Delta^2. \end{aligned} \quad (5.16)$$

Here we comment on the mass matrix in Eq. (5.13). The similar mass matrix has been analyzed in  $G_{\text{SM}} \times U(1)_{B-L}$  and  $SO(10)$  pNGB DM models Refs. [22,23,29,30], but only the case  $v_\Delta \gg v_\Phi, v$  such as  $v_\Delta > O(10^{10})$  GeV and  $v_\Phi, v = O(10^2)$  GeV is allowed in those models due to the DM stability problem. In this model, the stability of the DM is guaranteed by  $U(1)$ , but when we identify  $\varphi(\pm)$  as the DM, the direct detection leads to some constraints, which we will discuss in the next section.

## VI. DIRECT DETECTION AND RELIC ABUNDANCE

In this section, we will show how the model introduced in Sec. II is constrained by various DM experiments. Firstly, we study the scattering amplitudes of a DM candidate  $\varphi(\pm)$  and SM fermions via the SM Higgs and additional scalar fields shown in Fig. 5. In the original pNGB DM model (Ref. [14] for Abelian and Ref. [31] for non-Abelian case), the soft breaking term is a scalar bilinear term and gives only an origin of pNGB DM mass term. Such soft breaking terms preserve a nature of NGB for DM and gives derivative portal

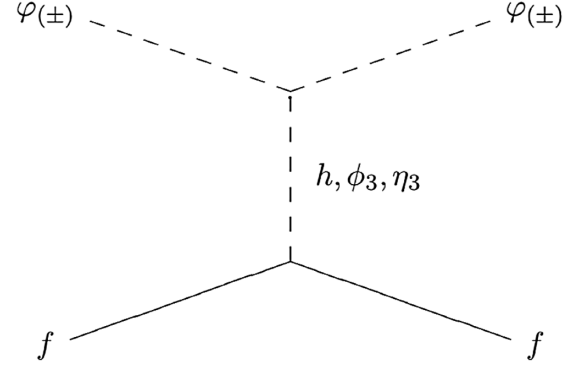


FIG. 5. Tree-level scattering of  $\varphi(\pm)$  and the SM fermions  $f$  via the scalar fields  $h, \phi_3, \eta_3$  is shown.

interactions, resulting in vanishing DM-nucleon scattering amplitudes in  $t \rightarrow 0$  limit. In the model we introduced in Sec. II, however, the soft breaking term found in Eq. (4.20) is a scalar trilinear term, and gives not only an origin of pNGB DM mass term but also additional  $h_i \varphi_{(+)} \varphi_{(-)}$  portal interactions proportional to soft breaking parameter  $\kappa$ , just like Refs. [22,29]. These portal interactions give rise to new contribution in addition to canceling diagrams, resulting in a nonzero DM-nucleon scattering process even in  $t \rightarrow 0$  limit. Therefore, we must look into parameter regions that escape direct detection constraints.

Apart from the vanishing part in  $t \rightarrow 0$  limit, the DM-nucleon scattering amplitude  $\mathcal{A}_{dd}$  in our model is proportional to soft breaking parameter  $\kappa$ , which is replaced with the DM mass  $m_\varphi$  by using Eq. (5.11) as

$$\mathcal{A}_{dd} \underset{\sim}{\propto} \frac{m_\varphi^2 m_f}{4v_\Delta^2 v} \left[ v_\Phi \sin 2\alpha_z \left( \frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2} \right) + 4v_\Delta \left( -\frac{1}{m_{h_1}^2} \alpha_y \cos \alpha_z + \frac{1}{m_{h_2}^2} \alpha_x \sin \alpha_z \right) \right], \quad (6.1)$$

where  $m_f$  denotes the mass of SM fermions  $f$ ;  $\alpha_x, \alpha_y$ , and  $\alpha_z$  stand for the mixing angles of  $\phi_3$ - $\eta_3$ ,  $h$ - $\eta_3$ , and  $h$ - $\phi_3$ , respectively;

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x & \sin \alpha_x \\ 0 & -\sin \alpha_x & \cos \alpha_x \end{pmatrix} \begin{pmatrix} \cos \alpha_y & 0 & \sin \alpha_y \\ 0 & 1 & 0 \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{pmatrix} \begin{pmatrix} \cos \alpha_z & \sin \alpha_z & 0 \\ -\sin \alpha_z & \cos \alpha_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ \phi_3 \\ \eta_3 \end{pmatrix}. \quad (6.2)$$

Note that the mixing angles  $\alpha_x$  and  $\alpha_y$  are expressed in terms of VEVs of the scalar fields and four point interaction coefficients as

$$\alpha_x \simeq -\frac{\lambda_{H\Delta} v}{\lambda_\Delta v_\Delta}, \quad \alpha_y \simeq -\frac{\lambda_{\Phi\Delta} v_\Phi}{\lambda_\Delta v_\Delta}. \quad (6.3)$$

Note also that we retain only the first order term for  $v/v_\Delta$  or  $v_\Phi/v_\Delta$  in Eq. (6.1). See Appendix for the detailed derivation.

As commented in the previous section, the previous models such as  $G_{\text{SM}} \times U(1)_{B-L}$  and  $SO(10)$  pNGB DM models required a very high  $v_\Delta$  due to DM longevity. The high  $v_\Delta$  also brings about a small DM-nucleon scattering amplitude, because it is suppressed by  $1/v_\Delta^2$ . In this model, on the other hand, the stability of the DM is guaranteed by  $U(1)_V$ , so  $v_\Delta$  is expected to be allowed to be a much smaller scale than  $O(10^{10})$  GeV, as we will see later.

In the remainder of this section, we compare the spin-independent (SI) DM-nucleon cross section  $\sigma_{\text{SI}}$  and show limitations on  $v_{\Delta}$  from recent DM experiments [45,46]. In the model, the SI cross section  $\sigma_{\text{SI}}$  is approximately given by

$$\begin{aligned} \sigma_{\text{SI}} \simeq & \frac{1}{16\pi} \left( \frac{m_{\phi}^2}{2v_{\Delta}^2} \frac{m_f}{v} \left[ v_{\Phi} \sin 2\alpha_z \left( \frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2} \right) \right. \right. \\ & \left. \left. + 4v_{\Delta} \left( -\frac{1}{m_{h_1}^2} \alpha_y \cos \alpha_z + \frac{1}{m_{h_2}^2} \alpha_x \sin \alpha_z \right) \right] \right)^2 \\ & \times \frac{m_p^4 f_N^2}{(m_{\phi} + m_p)^2}, \end{aligned} \quad (6.4)$$

where the proton mass  $m_p \simeq 0.938$  GeV, and  $f_N \simeq 0.3$ . Note that we retain only the first order term for  $v/v_{\Delta}$  or  $v_{\Phi}/v_{\Delta}$  in Eq. (6.4). For the conversion formula from DM-quark scattering to DM-nucleon scattering, see e.g., Ref. [47].

The thermal relic abundance of DM in a model that can be regarded as a low-energy effective description of this model has been calculated in Ref. [31], and it has been shown that the observed value can be reproduced when the DM mass satisfies the condition  $m_{\text{DM}} \gtrsim m_{h_1}/2$ . Furthermore, there is a constraint from Higgs invisible decay when the DM mass is less than half of the Higgs boson mass. Therefore, in the following, we mainly focus on regions where the DM mass is more than half the Higgs boson mass:  $m_{\phi} \gtrsim m_{h_1}/2$ .

As a benchmark parameter set, we fix mass parameters for the second and third neutral Higgs fields as  $m_{h_2} = 300$  GeV and  $m_{h_3} = 500$  GeV, respectively. We take a sample set  $(\sin \alpha_x, \sin \alpha_y, \sin \alpha_z) = (0.06, 0.05, 0.1)$ . We assume that  $SU(2)_D$  gauge bosons are heavy.

In Fig. 6, we show allowed parameter regions consistent with various experimental constraints, varying the ratio of the  $SU(2)_D$  doublet and triplet VEV as  $v_{\Phi}/v_{\Delta} = 1/4$  and  $1/10$ . Solid lines express parameter contours reproducing an observed DM energy density,  $\Omega h^2 = 0.12$ . Three dips in solid lines correspond to the resonance contributions from  $h_1$ ,  $h_2$ , and  $h_3$ . Therefore, they locate at the half of their masses,  $m_{h_1}/2 = 62.5$  GeV,  $m_{h_2}/2 = 150$  GeV, and  $m_{h_3}/2 = 250$  GeV, respectively. The small depression at  $m_{\phi} = 300$  GeV is due to the opening of a new annihilation channel,  $\varphi_{(+)}\varphi_{(-)} \rightarrow h_2 h_2$ . Dashed lines represent constraints from a direct detection LUX-ZEPLIN experiment [46] recasted to the upper limit for the VEV ratio  $v/v_{\Phi}$ . The direct detection constraints become tight at large DM mass region. That is because the DM-nucleon scattering amplitude is proportional to a soft-breaking parameter, namely, the DM mass square  $m_{\phi}^2$ , as we show in Eq. (6.1). The gray shaded region satisfies  $m_{Z'} < 2m_{\phi}$  with  $m_{Z'}$  being mass of the  $SU(2)_D$  neutral gauge boson. In this region, the  $SU(2)_D$  neutral gauge boson can also become DM candidate, which

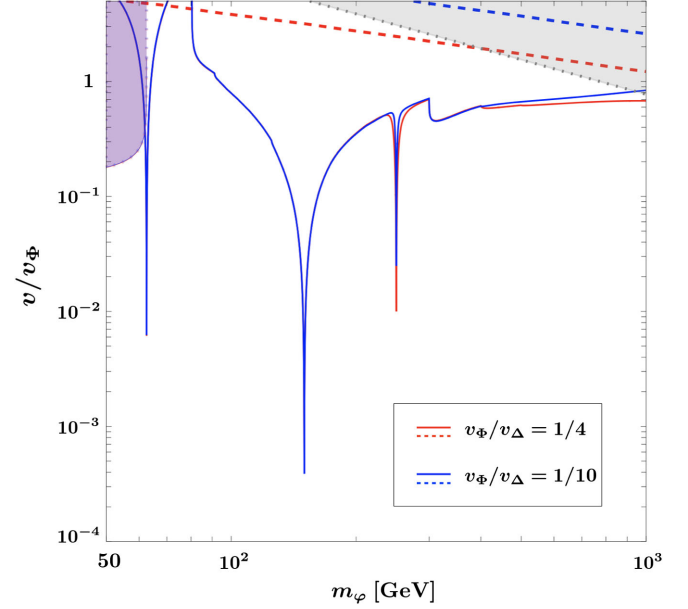


FIG. 6. Constraints from a direct detection experiment and a prediction for the relic abundance in our  $SU(2)$  pNGB DM model. Solid lines express a parameter contour corresponding to  $\Omega h^2 = 0.12$ , while dashed line represents a direct detection constraints from the LUX-ZEPLIN experiment [46]. The gray shaded region is the region where the  $SU(2)_D$  neutral gauge boson can also become DM candidate, which is not dealt with in this analysis for simplicity. The purple shaded region is excluded by Higgs invisible decay constraints [48].

is not dealt with in this analysis for simplicity. The purple shaded region is excluded by Higgs invisible decay constraints [48]. The Higgs invisible decay width in this model shows  $v_{\Phi}/v_{\Delta}$ -dependence only through subleading terms. Therefore, the excluded region colored in purple is common for VEV ratio  $v_{\Phi}/v_{\Delta} = 1/4$  and  $1/10$ .

We apply the same method in Refs. [47,49,50] to calculate the DM-nucleon scattering cross-sections and thermally averaged total annihilation cross sections. We find that the relic abundance does not change so much when we vary the VEV ratio  $v_{\Phi}/v_{\Delta}$ . We also find that there are plenty of allowed parameter regions which escape the direct detection constraints and reproduce a correct DM relic abundance at the same time.

## VII. SUMMARY AND DISCUSSIONS

We proposed a new pNGB DM model based on non-Abelian gauge symmetry  $SU(2)_D$ , in which scalars in **2** and **3** of  $SU(2)_D$  are introduced. We analyzed the structure of the symmetry and its breaking patterns in detail by analyzing the scalar potential. We found that when the mass parameters of the scalars  $\mu_{\Phi}^2$  and  $\mu_{\Delta}^2$  are positive in our convention, the  $SU(2)_D$  gauge symmetry is spontaneously broken to the exact  $U(1)_V$  global symmetry by the VEVs of the scalars in **2** and **3** of  $SU(2)_D$ . The charged pNGB under

the  $U(1)_V$  custodial symmetry appears, and is identified as DM. The stability of the DM is guaranteed by the exact  $U(1)_V$  custodial symmetry. From Fig. 6, we showed that the relic abundance is correctly reproduced while escaping the severe constraints from the direct detection experiments.

We comment on the additional  $SU(2)_D$  gauge symmetry breaking scale. In the  $G_{\text{SM}} \times U(1)_{B-L}$  and  $SO(10)$  pNGB DM models [22,23,29,30], the VEV of the additional  $U(1)_{B-L}$  gauge symmetry breaking scale must be higher than  $\mathcal{O}(10^{10})$  GeV to suppress DM decay rate, while in our new pNGB DM model the VEV of  $SU(2)_D$  breaking scale is allowed to be roughly  $\mathcal{O}(1)$  TeV or higher due to the stability of DM guaranteed by the  $U(1)_V$  custodial symmetry. Complementary verification by accelerator experiments may be possible in some parameter regions in our model.

In the  $SU(2)_g \times U(1)_X$  pNGB DM model [31], it has been pointed out that an additional gauge coupling of  $U(1)_X$  is not asymptotically free and there is a Landau pole in the high-energy region, so this problem can be tackled by extending to  $SU(2)$  gauge theory. However, it is not enough to extend the additional gauge sector part to non-Abelian gauge symmetry because the SM gauge group includes an Abelian gauge symmetry  $U(1)_Y$ . To address this issue, we have to discuss extensions to grand unified theory (GUT) [37,51,52]. The extension of this model to GUT models will be left as future work.

## ACKNOWLEDGMENTS

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## APPENDIX: DM-QUARK SCATTERING AMPLITUDE

We will derive DM-quark scattering amplitude given in Eq. (6.1). The scalar kinetic terms  $\mathcal{K}$  and scalar potential  $\mathcal{V}$  parts of the Lagrangian in Eq. (2.1) are given as

$$\mathcal{K} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2} \text{Tr}((D_\mu \Sigma)^\dagger D^\mu \Sigma) + \frac{1}{2} \text{Tr}(D_\mu \Delta D^\mu \Delta), \quad (\text{A1})$$

$$\begin{aligned} \mathcal{V} = & -\mu_H^2 H^\dagger H - \frac{1}{2} \mu_\Phi^2 \text{Tr}(\Sigma^\dagger \Sigma) - \frac{1}{2} \mu_\Delta^2 \text{Tr}(\Delta^2) \\ & - \sqrt{2} \kappa \text{Tr}(\sigma_3 \Sigma^\dagger \Delta \Sigma) + \lambda_H (H^\dagger H)^2 \\ & + \frac{\lambda_\Phi}{4} (\text{Tr}(\Sigma^\dagger \Sigma))^2 + \frac{\lambda_\Delta}{4} (\text{Tr}(\Delta^2))^2 \\ & + \frac{\lambda_{H\Phi}}{2} (H^\dagger H) \text{Tr}(\Sigma^\dagger \Sigma) + \frac{\lambda_{H\Delta}}{2} (H^\dagger H) \text{Tr}(\Delta^2) \\ & + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr}(\Sigma^\dagger \Sigma) \text{Tr}(\Delta^2), \end{aligned} \quad (\text{A2})$$

where  $\Sigma = (\tilde{\Phi}, \Phi)$  and  $\Delta$  are  $SU(2)_D^{\text{local}}$  bi-doublet and real triplet fields, respectively.

For deriving relevant interactions for DM-quark scattering amplitude, we adopt nonlinear basis, because in the nonlinear basis, the Higgs-portal interactions only come from kinetic terms in Eq. (A1) and the soft breaking term in the first line of Eq. (A2), which makes derivation of DM-quark scattering amplitude much easier than that in the linear basis. Note that the result Eq. (6.1) is the same regardless of whether we choose a linear or no-linear basis. The polar decomposition for bidoublet and real triplet fields are given as

$$\begin{aligned} \Sigma &= \frac{v_\Phi + \phi_3}{\sqrt{2}} \xi_\Sigma \quad \text{with} \\ \xi_\Sigma &= \exp\left(\frac{i}{v_\Phi} (\phi_{(+)} \sigma_{(+)} + \phi_{(-)} \sigma_{(-)} + \phi_4 \sigma_3)\right), \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \Delta &= \frac{v_\Delta + \eta_3}{\sqrt{2}} \xi_\Delta \sigma_3 \xi_\Delta \quad \text{with} \\ \xi_\Delta &= \exp\left(-\frac{i}{2v_\Delta} (\eta_{(+)} \sigma_{(+)} + \eta_{(-)} \sigma_{(-)})\right), \end{aligned} \quad (\text{A4})$$

where  $v_\Phi$ , and  $v_\Delta$  are VEVs for each scalar field.  $\sigma_{(\pm)}$  is expressed in terms of the first and second Pauli matrices as  $\sigma_{(\pm)} = (\sigma_1 \mp i\sigma_2)/\sqrt{2}$ . Note that  $SU(2)$  generators  $\sigma_{(\pm)}$ ,  $\sigma_3$  satisfy the normalization conditions  $\text{Tr}(\sigma_{(+)} \sigma_{(+)}) = \text{Tr}(\sigma_{(-)} \sigma_{(-)}) = 0$ ,  $\text{Tr}(\sigma_{(+)} \sigma_{(-)}) = \text{Tr}(\sigma_3 \sigma_3) = 2$ , and bidoublet and real triplet fields satisfy

$$\text{Tr}(\Sigma^\dagger \Sigma) = (v_\Phi + \phi_3)^2, \quad \text{Tr}(\Delta^2) = (v_\Delta + \eta_3)^2. \quad (\text{A5})$$

We evaluate scattering amplitudes of the DM  $\varphi_{(\pm)}$  and the SM fermions  $f$  shown in Fig. 5. Substituting polar decompositions Eqs. (A3) and (A4) into the potential  $\mathcal{V}$  and kinetic terms  $\mathcal{K}$  given in Eqs. (A1) and (A2), and extracting cubic scalar interactions relevant to DM-fermion scattering, we get



$$-\mathcal{V}_{\text{NL}} \supset -\frac{2}{v_{\Phi}} \phi_3(\phi_{(-)}, \eta_{(-)}) M_C^2 \begin{pmatrix} \phi_{(+)} \\ \eta_{(+)} \end{pmatrix} - \frac{1}{v_{\Delta}} \eta_3(\phi_{(-)}, \eta_{(-)}) M_C^2 \begin{pmatrix} \phi_{(+)} \\ \eta_{(+)} \end{pmatrix}, \quad (\text{A6})$$

$$\mathcal{K}_{\text{NL}} \supset \left(1 + \frac{\phi_3}{v_{\Phi}}\right)^2 \partial_{\mu} \phi_{(+)} \partial^{\mu} \phi_{(-)} + \left(1 + \frac{\eta_3}{v_{\Delta}}\right)^2 \partial_{\mu} \eta_{(+)} \partial^{\mu} \eta_{(-)}. \quad (\text{A7})$$

where  $M_C^2$  is given in Eq. (5.10). The cubic interactions in Eq. (A6) can be rewritten in terms of mass eigenstate  $\varphi_{(\pm)}$  as

$$-\mathcal{V}_{\text{NL}} \supset \left(-2 \frac{m_{\varphi}^2}{v_{\Phi}} \phi_3 - \frac{m_{\varphi}^2}{v_{\Delta}} \eta_3\right) \varphi_{(+)} \varphi_{(-)}. \quad (\text{A8})$$

The cubic interactions in Eq. (A7) can be rewritten in terms of  $\varphi_{(\pm)}$  as

$$\begin{aligned} \mathcal{K}_{\text{NL}} &\supset 2 \left( \frac{\sin^2 \beta}{v_{\Phi}} \phi_3 + \frac{\cos^2 \beta}{v_{\Delta}} \eta_3 \right) \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)} \\ &= \left( \frac{2}{v_{\Phi}} \phi_3 + \frac{1}{v_{\Delta}} \eta_3 \right) \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)} \\ &\quad - \left( 2 \frac{v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2} \phi_3 + \frac{1}{v_{\Delta}} \frac{4v_{\Delta}^2 - v_{\Phi}^2}{v_{\Phi}^2 + 4v_{\Delta}^2} \eta_3 \right) \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)}. \end{aligned} \quad (\text{A9})$$

Combining the first term in the last line of Eq. (A9) with Eq. (A8), we get<sup>3</sup>

<sup>3</sup>To obtain Eq. (A11), we used

$$\begin{aligned} \rho \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)} &= \frac{1}{2} \partial_{\mu} [\rho (\partial^{\mu} \varphi_{(+)} \varphi_{(-)} + \rho \varphi_{(+)} (\partial^{\mu} \varphi_{(-)} \\ &\quad - (\partial^{\mu} \rho) \varphi_{(+)} \varphi_{(-)})] + \frac{1}{2} (\partial^2 \rho) \varphi_{(+)} \varphi_{(-)} \\ &\quad - \frac{1}{2} \rho (\partial^2 \varphi_{(+)} \varphi_{(-)} - \frac{1}{2} \rho \varphi_{(+)} (\partial^2 \varphi_{(-)}) \end{aligned} \quad (\text{A10})$$

with  $\rho = \phi_3, \eta_3$ . Total derivative terms in the first line of Eq. (A10) are irrelevant, so we dropped them in Eq. (A11).

$$\begin{aligned} &\left( \frac{2}{v_{\Phi}} \phi_3 + \frac{1}{v_{\Delta}} \eta_3 \right) \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)} \\ &\quad - \left( 2 \frac{m_{\varphi}^2}{v_{\Phi}} \phi_3 + \frac{m_{\varphi}^2}{v_{\Delta}} \eta_3 \right) \varphi_{(+)} \varphi_{(-)} \\ &= \left( \frac{2}{v_{\Phi}} \phi_3 + \frac{1}{v_{\Delta}} \eta_3 \right) \left[ -\frac{1}{2} ([\partial^2 + m_{\varphi}^2] \varphi_{(+)} \varphi_{(-)} \right. \\ &\quad \left. - \frac{1}{2} \varphi_{(+)} ([\partial^2 + m_{\varphi}^2] \varphi_{(-)}) \right] \\ &\quad + \left( \frac{2}{v_{\Phi}} (\partial^2 \phi_3) + \frac{1}{v_{\Delta}} (\partial^2 \eta_3) \right) \varphi_{(+)} \varphi_{(-)}. \end{aligned} \quad (\text{A11})$$

We find that all the terms appearing in Eq. (A11) are irrelevant to the DM-fermion scattering in direct detection; the first line vanishes due to on-shell conditions for pNGB DM; the second line gives contributions proportional to momentum-transfer  $t = (p_2 - p_1)^2$  with  $p_{1,2}$  being incoming and out-going DM momentum. This will also vanish when we take  $t \rightarrow 0$  limit.

The remaining cubic interaction relevant to DM-fermion scattering shown in Fig. 5 is the second term of Eq. (A9):

$$\mathcal{K}_{\text{NL}} \supset - \left( 2 \frac{v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2} \phi_3 + \frac{1}{v_{\Delta}} \frac{4v_{\Delta}^2 - v_{\Phi}^2}{v_{\Phi}^2 + 4v_{\Delta}^2} \eta_3 \right) \partial_{\mu} \varphi_{(+)} \partial^{\mu} \varphi_{(-)}. \quad (\text{A12})$$

Note that this term decouples when we assume  $v_{\Delta} \gg v_{\Phi}$ . Further, by using the relation in Eq. (A10), we obtain

$$\begin{aligned} \mathcal{L}_{\text{NL}} &\supset -\frac{1}{2} \left( 2 \frac{v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2} \partial^2 \phi_3 + \frac{1}{v_{\Delta}} \frac{4v_{\Delta}^2 - v_{\Phi}^2}{v_{\Phi}^2 + 4v_{\Delta}^2} \partial^2 \eta_3 \right) \varphi_{(+)} \varphi_{(-)} \\ &\quad + \frac{1}{2} \left( 2 \frac{v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2} \phi_3 + \frac{1}{v_{\Delta}} \frac{4v_{\Delta}^2 - v_{\Phi}^2}{v_{\Phi}^2 + 4v_{\Delta}^2} \eta_3 \right) \\ &\quad \times ((\partial^2 \varphi_{(+)} \varphi_{(-)} + \varphi_{(+)} (\partial^2 \varphi_{(-)})), \end{aligned} \quad (\text{A13})$$

where we ignored the total derivative. The first term vanishes for  $t \rightarrow 0$  limit. By replacing  $\partial^2 \varphi_{(\pm)}$  to  $m_{\varphi}^2 \varphi_{(\pm)}$ , the effective DM-scalar interaction for the DM-fermion scattering becomes for  $t \rightarrow 0$

$$\begin{aligned} &\left( 2 \frac{v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2} \phi_3 + \frac{1}{v_{\Delta}} \frac{4v_{\Delta}^2 - v_{\Phi}^2}{v_{\Phi}^2 + 4v_{\Delta}^2} \eta_3 \right) m_{\varphi}^2 \varphi_{(+)} \varphi_{(-)} \\ &= (\kappa_{\varphi\varphi h} \quad \kappa_{\varphi\varphi\phi_3} \quad \kappa_{\varphi\varphi\eta_3}) \begin{pmatrix} h \\ \phi_3 \\ \eta_3 \end{pmatrix} \varphi_{(+)} \varphi_{(-)}, \end{aligned} \quad (\text{A14})$$

where

$$\begin{pmatrix} \kappa_{\varphi\varphi h} \\ \kappa_{\varphi\varphi\phi_3} \\ \kappa_{\varphi\varphi\eta_3} \end{pmatrix} = \frac{m_\varphi^2}{v_\Phi^2 + 4v_\Delta^2} \begin{pmatrix} 0 \\ 2v_\Phi \\ \frac{4v_\Delta^2 - v_\Phi^2}{v_\Delta} \end{pmatrix}. \quad (\text{A15})$$

Next, Yukawa interaction terms of the scalar fields and SM fermions  $f$  on the vacuum are given by

$$\frac{m_f}{v} h \bar{f} f = \frac{m_f}{v} (1 \ 0 \ 0) \begin{pmatrix} h \\ \phi_3 \\ \eta_3 \end{pmatrix} \bar{f} f, \quad (\text{A16})$$

where  $m_f$  stands for a mass parameter of the SM fermion  $f$ .

By using the DM-scalar and scalar-fermion interactions in Eqs. (A15) and (A16), we find that the scattering amplitude shown in Fig. 5 for  $t \rightarrow 0$  is given by

$$\mathcal{A}_{dd} \underset{\sim}{\propto} (\kappa_{\varphi\varphi h} \ \kappa_{\varphi\varphi\phi_3} \ \kappa_{\varphi\varphi\eta_3}) (M_H^2)^{-1} \frac{m_f}{v} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (\text{A17})$$

where  $(M_H^2)^{-1} = U_H^{-1} [(M_H^2)^{\text{diag}}]^{-1} U_H$ ; the approximate form of  $U_H$  for  $v_\Delta \gg v_\Phi, v$  is given in Eq. (5.15). Substituting Eq. (A15) into Eq. (A17), we get

$$\mathcal{A}_{dd} \underset{\sim}{\propto} \frac{m_\varphi^2}{4v_\Delta^2} \frac{m_f}{v} \left[ v_\Phi \sin 2\alpha_z \left( \frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2} \right) + 4v_\Delta \left( -\frac{1}{m_{h_1}^2} \alpha_y \cos \alpha_z + \frac{1}{m_{h_2}^2} \alpha_x \sin \alpha_z \right) \right], \quad (\text{A18})$$

which is identical to Eq. (6.1).

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