Alternative gauged $U(1)_R$ symmetric model in light of the CDF II W-boson mass anomaly

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We consider an explanation of the CDF II W boson mass anomaly by $Z-Z'$ mixing with $U(1)_R$ gauge symmetry under which right-handed fermions are charged. It is found that $U(1)_R$ is preferred to be leptophobic to accommodate the anomaly while avoiding other experimental constraints. In such a case we require extra charged leptons to cancel quantum anomalies and the SM charged leptons get masses via interactions with the extra ones. These interactions also induce muon $g-2$ and lepton flavor violations. We discuss muon $g-2$, possible flavor constraints, neutrino mass generation via inverse seesaw mechanism, and collider physics regarding Z' production for parameter space explaining the W boson mass anomaly.

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I. INTRODUCTION

Precision measurements of electroweak observable are a good test of the standard model (SM) and would provide a hint of beyond the SM. CDFII collaboration recently reported updated result of the SM charged-gauge boson (W-boson) mass [1]

$$m_W = (80.433 \pm 0.0064_{\mathrm{stat}} \pm 0.0069_{\mathrm{syst}})\ \mathrm{GeV}, \quad (1)$$

which deviates from the SM prediction by 7σ, where the SM prediction indicates $m_W = (80.357 \pm 0.006)$ GeV. The disagreement also appears to the previous global combination of data from LEP, CDF, D0, and ATLAS where they give the mass range of $m_W = (80.379 \pm 0.012)$ GeV [2]. This anomaly suggests new physics (NP) beyond the SM [3–101], and can be interpreted as the deviation of ΔT oblique parameter [102,103], where the oblique parameters are zero in the SM.

One of the straightforward explanations of the anomaly can be realized by introducing extra $U(1)$ gauge symmetry where the SM Higgs field is charged under it. Then ΔT is shifted by effect of mass mixing between the SM Z and a new neutral gauge boson Z'. In particular one of the minimal scenarios is the explanation by Z' with right-handed $U(1)_R$ symmetry [104]. The symmetry is originally proposed by chiral anomaly cancellations with three right-handed neutrinos, and it is well testable at the International Linear Collider (ILC) or the Large Hadron Collider (LHC) due to observing the difference of chirality [105–110]. However the W-boson anomaly cannot be explained if charged-leptons have nonzero $U(1)_R$ charge because of the constraints from W and Y oblique parameters [7]. Then, only the possibility to explain the W-boson anomaly along this idea is that only the SM Higgs field and quarks have nonzero charge under $U(1)_R$ symmetry. Note that another possibility is $U(1)_H$ case where only Higgs doublet is charged under it. In this case we need another Higgs doublet to induce the SM fermion masses and two Higgs doublets can also contribute to ΔT parameter at loop level. Remarkably in our setting W boson anomaly is explained by purely Z–Z' mixing effect.

In this letter, we extend the original $U(1)_R$ model to make the Z' to be leptophobic in order to explain this anomaly avoiding other electroweak precision tests. Then extra $SU(2)$ singlet charged leptons are required to cancel quantum anomalies. As a result the masses of the SM charged leptons are obtained via interactions between the SM lepton and the extra charged leptons. Such interactions also induce lepton anomalous magnetic moments and lepton flavor violations (LFVs) at loop level [111,112].

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TABLE I. Charge assignments of the our fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$, where its upper index a is the number of family that runs over 1–3.

	Q_L^a	u_R^a	d_R^a	L_L^a	e_R^a	N_R^a	N_L^a	E_L^a	E_R^a	H	φ_1	φ_2
$SU(3)_C$	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_L$	**2**	**1**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**2**	**1**	**1**
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	0	-1	-1	$\frac{1}{2}$	0	0
$U(1)_R$	0	1	-1	0	0	1	0	0	-1	1	-1	$-\frac{1}{2}$

In addition, we include discussion of active Majorana neutrino mass matrix via inverse seesaw scenario [107].

This paper is organized as follows. In Sec. II, we introduce our model and show relevant formulas for phenomenology. In Sec. III, we show our phenomenological analysis of muon $g-2$, LFVs and collider physics. Finally we devote the summary of our results and the conclusion.

II. MODEL SETUP AND CONSTRAINTS

Here, we review our model. We introduce three vector-like neutral fermions $N_{L,R}$ and singly charged fermions $E_{L,R}$, where only N_R and E_R has nonzero $U(1)_R$ charge with 1 and -1, respectively. It suggests that masses of charged-lepton arise not via the SM type but via mixings among singly charged fermions. One can straightforwardly confirm the typical four patterns of chiral anomaly cancellations per one generation; $[U(1)_R] = [U(1)_R]^3 = [U(1)_R]^2[U(1)_Y] = [U(1)_R][U(1)_Y]^2 = 0$.[1] As for scalar sector, we introduce an isospin singlet fields φ_1 and φ_2 with $U(1)_R$ charge -1 and $-\frac{1}{2}$ to spontaneous $U(1)_R$ symmetry breaking where SM-like Higgs H also has nonzero charge under $U(1)_R$ in order to induce Yukawa Lagrangian. Each of VEVs is denoted by $\langle H\rangle \equiv [0, v/\sqrt{2}]^T$ and $\langle\varphi_{1(2)}\rangle \equiv v'_{1(2)}/\sqrt{2}$. All the field contents and their assignments are summarized in Table I.

A. Lagrangian and scalar masses

The relevant lepton Yukawa Lagrangian under these symmetries is given by

$$-\mathcal{L}_Y = (y_\ell)_{ab}\bar{L}_L^a H E_R^b + (y_E)_{aa}\varphi_1 \bar{E}_L^a E_R^a + (m_{Ee})_{ab}\bar{E}_L^a e_R^b + (y_D)_{ab}\bar{L}_L^a \tilde{H} N_R^b + (y_N)_{aa}\varphi_1 \bar{N}_L^a N_R^a + (M_{N_L})_{ab}\bar{N}_L^a N_L^{cb} + \text{H.c.}, \tag{2}$$

where $\tilde{H} \equiv i\sigma_2 H$, and upper(lower) indices $(a,b) = 1$–3 for fields(Yukawa or mass matrix) are the number of families, and y_E, y_N can be diagonal matrix without loss of generality.

[1] In our paper, quark sector is the same as the SM except for the fact that right-handed ones interact with Z' boson.

The scalar potential is given by

$$\mathcal{V} = \mu_1^2|\varphi_1|^2 - \mu_2^2|\varphi_2|^2 - \mu_H^2|H|^2 + \lambda_H|H|^4 + \lambda_1|\varphi_1|^4 + \lambda_2|\varphi_2|^4 - \mu_3(\varphi_1^*\varphi_2\varphi_2 + \text{H.c.}) + \lambda_3|\varphi_1|^2|H|^2 + \lambda_4|\varphi_2|^2|H|^2 + \lambda_5|\varphi_1|^2|\varphi_2|^2. \tag{3}$$

The scalar fields are parametrized as

$$H = \begin{bmatrix} w^+ \\ \frac{v+r+iz}{\sqrt{2}} \end{bmatrix}, \qquad \varphi_{1,2} = \frac{v'_{1,2} + r'_{1,2} + iz'_{1,2}}{\sqrt{2}}, \tag{4}$$

where w^+ and z are massless Nambu-Goldstone(NG) bosons which are absorbed by the SM gauge bosons W^+ and Z, and one linear combination of $z'_{1,2}$ corresponds to NG boson abosorbed by an extra Z' boson from $U(1)_R$. The VEVs are obtained from tadpole conditions $\frac{\partial\mathcal{V}}{\partial v} = \frac{\partial\mathcal{V}}{\partial v'_1} = \frac{\partial\mathcal{V}}{\partial v'_2} = 0$. We obtain the condition for v'_1 from $\frac{\partial\mathcal{V}}{\partial v'_1} = 0$ such that

$$\mu_1^2 v'_1 + \lambda_1 v_1'^3 - \frac{1}{2\sqrt{2}}\mu_3 v_2'^2 + \lambda_5 v'_1 v_2'^2 = 0. \tag{5}$$

Here the VEV of φ_1 is approximately given by

$$v'_1 \simeq \frac{1}{2\sqrt{2}}\frac{\mu_3 v_2'^2}{\mu_1^2 + \lambda_5 v_2'^2}, \tag{6}$$

where we assume v'_1 is much smaller than v'_2 and $v_1'^3$ term is ignored. The hierarchy of VEVs is consistently achieved by choosing $\mu_3 v'_2 \ll \mu_1^2$ and/or $\mu_3 v'_2 \ll \lambda_5 v_2'^2$. This VEV hierarchy is necessary to explain W-boson mass anomaly and to obtain sizable muon $g-2$ at the same time, as we discuss below. In this case, z'_2 corresponds to the NG boson to be absorbed by Z' boson.

In our analysis, we assume λ_4 and λ_5 to be negligibly small for simplicity, and only r and r'_1 mixes. Then, we obtain the mass matrix for CP even scalar, m_R^2, in the basis of (r, r'_1), where the mass eigenstates $\{h, H\}$ is found to be $(r, r'_1)^T = O_R(h, H)^T$, and mass eigenvalues are given by $m_{h,H}^2 = O_R^T m_R^2 O_R$. m_R^2 and O_R are obtained as

$$m_R^2 \simeq \begin{bmatrix} 2v^2\lambda_H & vv_1'\lambda_3 \\ vv_1'\lambda_3 & \mu_1^2 \end{bmatrix}, \qquad O_R = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}, \quad (7)$$

where $c_\theta(s_\theta)$ stands for $\cos\theta(\sin\theta)$ with $s_{2\theta} = \frac{2vv_1'\lambda_3}{m_h^2 - m_H^2}$. The mass eigenvalues are also calculated such that

$$m_{h,H}^2 \simeq (v^2\lambda_H + \mu_1^2) \mp \sqrt{(v^2\lambda_H - \mu_1^2)^2 + v^2 v_1'^2 \lambda_3^2}. \quad (8)$$

Here $h \equiv h_{\rm SM}$ is the SM Higgs, therefore, $m_h = 125$ GeV. The mixing effect for CP-even scalar is constrained by the measurements of Higgs production cross section and its decay branching ratio at the LHC, and $s_a \lesssim 0.3$ is provided by the current data [113]. The mass of r_2' is approximately given by $m_{r_2'} \simeq \sqrt{2\lambda_2} v_2'$ which is supposed to be much heavier than m_H.

B. Oblique parameters

Oblique parameters come from $Z_{\rm SM} - Z'$ *mixing*. Thus, we first discuss this effect. Since H has nonzero $U(1)_R$ charge, there is mixing between $Z_{\rm SM}$ and Z'. The resulting mass matrix in the basis of $(Z_{\rm SM}, Z')$ is given by

$$\begin{aligned} m_{Z_{\rm SM}Z'}^2 &\simeq \frac{1}{4}\begin{bmatrix} (g_1^2+g_2^2)v^2 & -2\sqrt{g_1^2+g_2^2}g'v^2 \\ -2\sqrt{g_1^2+g_2^2}g'v^2 & 4g'^2(v^2+v_2'^2) \end{bmatrix} \\ &= m_{Z'}^2 \begin{bmatrix} \epsilon_1^2 & -\epsilon_1\epsilon_2 \\ -\epsilon_1\epsilon_2 & 1+\epsilon_2^2 \end{bmatrix}, \end{aligned} \quad (9)$$

where $m_{Z_{\rm SM}} \equiv \frac{\sqrt{g_1^2+g_2^2}v}{2}$, $m_{Z'} \equiv g' v_2'$, $\epsilon_1 \equiv \frac{m_{Z_{\rm SM}}}{m_{Z'}}$, $\epsilon_2 \equiv \frac{v}{v_2'}$, g_1, g_2, and g' are gauge coupling of $U(1)_Y$, $SU(2)_L$, and $U(1)_R$, respectively. Note that we ignored v_1' in $m_{Z'}$ formula due to the relation $v_1' \ll v_2'$. Then its mass matrix is diagonalized by the two by two mixing matrix V as $V m_{Z_{\rm SM}Z'}^2 V^T \equiv {\rm Diag}(m_Z^2, m_{Z_R}^2)$, where we work under $\epsilon_2^2 \ll 1$ and

$$m_Z^2 \approx m_{Z_{\rm SM}}^2(1-\epsilon_2^2), \qquad m_{Z_R}^2 \approx m_{Z'}^2(1+\epsilon_1^2\epsilon_2^2), \quad (10)$$

$$V \approx \begin{bmatrix} c_Z & s_Z \\ -s_Z & c_Z \end{bmatrix}, \qquad \theta_Z = \frac{1}{2}\tan^{-1}\left[\frac{2\epsilon_1\epsilon_2}{1+\epsilon_2^2-\epsilon_1^2}\right]. \quad (11)$$

The Z_R mass can be approximated as $m_{Z_R} \simeq m_{Z'} = g' v_2'$ since $\epsilon_{1,2}$ is small. Thus gauge coupling g' is almost fixed if we choose values of m_{Z_R} and v'.

In our case, only ΔT is nonzero induced via $Z - Z'$ mixing thanks to zero $U(1)_R$ charges of L_L, e_R and defined by

$$\Delta T = \frac{1}{\alpha_{\rm em}} \frac{m_{Z_{\rm SM}}^2 - m_Z^2}{m_{Z_{\rm SM}}^2} \simeq \frac{\epsilon_2^2}{\alpha_{\rm em}}, \quad (12)$$

where we have used Eq. (10) in the last part of the above equation. Thus, ΔT is straightforwardly given by inserting $\epsilon_2 \equiv v/v_2'$, $v_2' = m_{Z'}/g'$, $v = 2m_Z \cos\theta_W/g_2$ with θ_W being the Weinberg angle and simply given by [7][2]

$$\Delta T \simeq \frac{v^2}{\alpha_{\rm em}} \frac{g'^2}{m_{Z'}^2} = \frac{4m_Z^2 \cos^2\theta_W}{g_2^2 \alpha_{\rm em}} \frac{g'^2}{m_{Z'}^2}. \quad (13)$$

Note also that the contribution to ΔT at loop level is negligible since our new particles are $SU(2)$ singlet. Although mixing between the SM charged lepton and new charged lepton discussed below can affect the ΔT at loop level it is highly suppressed by small mixing angle. Thus our relevant parameters for ΔT are only new gauge coupling g' and Z' mass $m_{Z'}$. From global fit including CDF II W-boson mass with $S \sim 0$ we obtain 1σ range of T as

$$0.09 \leq \Delta T \leq 0.14. \quad (14)$$

Then it is found from Eq. (13):

$$20 \text{ TeV} \lesssim \frac{m_{Z'}}{g'} (= v_2') \lesssim 31 \text{ TeV}, \quad (15)$$

where we used central value of Eq. (1) for m_W. This range is allowed by LEP constraints [114] and dijet searches at the LHC [115–117]. Here, we emphasize that our model only provides a sizable contribution to ΔT via Z–Z' mixing. Thus modification of W-boson mass is characterized by ΔT that is estimated by Z' mass, new gauge coupling and other electroweak observables $\{m_Z, \theta_W, \alpha_{\rm em}\}$. It is the advantage of our model that other oblique parameters are not modified.

Here we comment on the situation in the case of original $U(1)_R$ model. In this case, we have contributions to other oblique parameters W and Y due to Z' interactions with charged leptons [118]. The oblique parameters, including W and Y are defined from the effective Lagrangian

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2} W_\mu^3 \Pi_{33}(p^2) W^{3\mu} - \frac{1}{2} B_\mu \Pi_{00}(p^2) B^\mu \\ &- W_\mu^3 \Pi_{30}(p^2) B^\mu - W_\mu^+ \Pi_{WW}(p^2) W^{-\mu}, \end{aligned} \quad (16)$$

where W_μ^3, B^μ and $W^\pm$ are gauge fields corresponding to the third component of $SU(2)_L$, $U(1)_Y$ and W boson respectively, and p^2 is the momentum square carried by the gauge field. The oblique parameters W and Y are then given by

$$W = \frac{m_W^2}{2} \frac{d^2}{d(p^2)^2} \Pi_{33}|_{p^2=0}, \qquad Y = \frac{m_W^2}{2} \frac{d^2}{d(p^2)^2} \Pi_{00}|_{p^2=0}. \quad (17)$$

It is shown in Ref. [118] that $W = Y = 0$ is obtained when leptons are not charged under new $U(1)$ symmetry, which is

[2]Notice here that the other valid oblique parameters $\Delta S, \Delta W, \Delta Y$ are zero.

our case. Otherwise, the values of W and Y have similar order as $\hat{T} \equiv \alpha_{\rm em} T$ and the LHC constraints are $|W| \lesssim 1.8 \times 10^{-4}$ and $|Y| \lesssim 2.0 \times 10^{-4}$ at 1σ confidence level [7,119,120]. Thus the parameter region realizing ΔT in Eq. (14) is excluded by the constraints. Therefore we need leptophobic $U(1)_R$ to explain the W-boson anomaly.

C. Charged-lepton sector

The charged-lepton mass matrix is obtained via mixing among the singly charged fermions, and the form is give by [121]

$$\begin{pmatrix} \bar{e}_L \\ \bar{E}_L \end{pmatrix}^T \mathcal{M}_E \begin{pmatrix} e_R \\ E_R \end{pmatrix} = \begin{pmatrix} \bar{e}_L \\ \bar{E}_L \end{pmatrix}^T \begin{bmatrix} 0 & m_{eE} \\ m_{Ee} & M_E \end{bmatrix} \begin{pmatrix} e_R \\ E_R \end{pmatrix}, \tag{18}$$

$$\mathcal{M}_E {\mathcal{M}_E}^\dagger = \begin{bmatrix} m_{eE} m_{eE}^\dagger & m_{eE} M_E \\ M_E m_{eE}^\dagger & M_E^2 + m_{Ee} M_{Ee}^\dagger \end{bmatrix},$$
$${\mathcal{M}_E}^\dagger \mathcal{M}_E = \begin{bmatrix} m_{Ee}^\dagger m_{Ee} & m_{Ee}^\dagger M_E \\ M_E m_{Ee} & M_E^2 + m_{eE}^\dagger m_{eE} \end{bmatrix}, \tag{19}$$

where $m_{eE} \equiv y_\ell v/\sqrt{2}$, $M_E \equiv y_E v'/\sqrt{2}$. The mass matrix is diagonalized by the transformation $(e_{L(R)}, E_{L(R)}) \to V_{L(R)} \ell_{L(R)}$. Thus we can obtain diagonalization matrices V_L and V_R which respectively diagonalize $M_E M_E^\dagger$ and $M_E^\dagger M_E$ as $V_L^\dagger \mathcal{M}_E {\mathcal{M}_E}^\dagger V_L = V_R^\dagger {\mathcal{M}_E}^\dagger \mathcal{M}_E V_R = {\rm diag}|D_{E_a}|^2$ ($a = 1$–6), where the first three mass eigenstates correspond to the SM charged-leptons. We then write ${\rm diag} D_{E_a} = {\rm diag}(m_e, m_\mu, m_\tau, M_{E_1}, M_{E_2}, M_{E_3})$.

D. New contribution to $Z \to \ell\bar{\ell}$

Due to the mixing between the exotic singly charged fermions and the SM leptons, we have a new contribution to $Z \to \ell_i \bar{\ell}_j$. Their kinetic Lagrangian in terms of mass eigenvectors is given by

$$\frac{g_2}{c_W}\left[\left(-\frac{1}{2}\sum_{a=1}^3 V_{L_{ia}}^\dagger V_{L_{aj}} + s_W^2 \delta_{ij}\right)\bar{\ell}_{L_i}\gamma^\mu \ell_{L_j} + s_W^2 \delta_{ij} \bar{\ell}_{R_i}\gamma^\mu \ell_{R_j}\right] Z_\mu, \tag{20}$$

where s_W, c_W are short-hand notations of Weinberg angles; $\sin\theta_W, \cos\theta_W$, which are rewritten in terms of g_1, g_2. Then, the decay rate of $Z \to \ell_i \bar{\ell}_j$ is given by

$$\Gamma(Z \to \ell_i \bar{\ell}_j) \simeq \frac{g_2^2}{24\pi c_W^2} m_Z \left[\left|-\frac{1}{2}\sum_{a=1}^3 V_{L_{ia}}^\dagger V_{L_{aj}} + s_W^2 \delta_{ij}\right|^2 + s_W^4 \delta_{ij}\right], \tag{21}$$

where we assume $m_Z \gg m_\ell$. Here, one confirms that the SM contribution is derived when $\sum_{a=1}^3 V_{L_{ia}}^\dagger V_{L_{aj}} = 1$;

$$\Gamma(Z \to \ell_i \bar{\ell}_j)_{\rm SM} \simeq \frac{g_2^2}{24\pi c_W^2} m_Z \left[\frac{1}{4} - s_W^2 + 2 s_W^4\right]\delta_{ij}. \tag{22}$$

Thus, the new contribution of branching ratios are given by

$$\Delta{\rm BR}(Z \to \ell_i \bar{\ell}_j) = \frac{\Gamma(Z \to \ell_i \bar{\ell}_j) - \Gamma(Z \to \ell_i \bar{\ell}_j)_{\rm SM}}{\Gamma_Z^{\rm tot}} \quad \text{for } i = j, \tag{23}$$

$${\rm BR}(Z \to \ell_i \bar{\ell}_j) = \frac{\Gamma(Z \to \ell_i \bar{\ell}_j)}{\Gamma_Z^{\rm tot}} \text{ for } i \neq j, \tag{24}$$

where the total Z decay width $\Gamma_Z^{\rm tot} = 2.4952 \pm 0.0023$ GeV [113].

The current bounds on the lepton-flavor-(conserving) changing Z boson decay branching ratios(BRs) at 95% confidence level (CL) are given by [113]:

$$\begin{aligned} &\Delta{\rm BR}(Z \to e^\pm e^\mp) < \pm 4.2 \times 10^{-5}, \\ &\Delta{\rm BR}(Z \to \mu^\pm \mu^\mp) < \pm 6.6 \times 10^{-5}, \\ &\Delta{\rm BR}(Z \to \tau^\pm \tau^\mp) < \pm 8.3 \times 10^{-5}, \\ &{\rm BR}(Z \to e^\pm \mu^\mp) < 7.5 \times 10^{-7}, \\ &{\rm BR}(Z \to e^\pm \tau^\mp) < 9.8 \times 10^{-6}, \\ &{\rm BR}(Z \to \mu^\pm \tau^\mp) < 1.2 \times 10^{-5}. \end{aligned} \tag{25}$$

E. Lepton flavor violations and muon $g-2$

Due to mixing between the SM charged-lepton and heavier leptons, we have nonzero LFVs and muon $g-2$ via y_ℓ. The current experimental upper bounds on LFVs are given by [122,123]

$$\begin{aligned} &{\rm BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}, \\ &{\rm BR}(\tau \to \mu\gamma) \le 4.4 \times 10^{-8}, \\ &{\rm BR}(\tau \to e\gamma) \le 3.3 \times 10^{-8}. \end{aligned} \tag{26}$$

On the other hand, new results on the muon $(g-2)$ were recently published by the E989 collaboration at Fermilab [124]:

$$a_\mu^{\rm FNAL} = 116592040(54) \times 10^{-11}. \tag{27}$$

Combined with the previous BNL result, this means that the muon $(g-2)$ deviates from the SM prediction by 4.2σ level [124–145]:

$$\Delta a_\mu^{\rm new} = (25.1 \pm 5.9) \times 10^{-10}, \tag{28}$$

and it could be a verifiable signature of the physics beyond the SM.

In our scenario, the relevant Lagrangian to induce LFVs and muon $g-2$ in terms of mass eigenstate is given by[3]

$$\mathcal{L}_{LFV} = (Y_{\alpha\beta}c_\theta - \tilde{Y}_{\alpha\beta}s_\theta)h\bar{\ell}_\alpha P_R \ell_\beta + (Y_{\alpha\beta}s_\theta + \tilde{Y}_{\alpha\beta}c_\theta)H\bar{\ell}_\alpha P_R \ell_\beta + \text{H.c.}, \quad (29)$$

$$Y_{\alpha\beta} \equiv \frac{1}{\sqrt{2}} \sum_{a=1,2,3} \sum_{b=1,2,3} (V_L^\dagger)_{\alpha,a} (y_\ell)_{ab} (V_R)_{b+3,\beta}, \quad (30)$$

$$\tilde{Y}_{\alpha\beta} \equiv \frac{1}{\sqrt{2}} \sum_{a=1,2,3} \sum_{b=1,2,3} (V_L^\dagger)_{\alpha,a+3} (y_E)_{aa} (V_R)_{a+3,\beta}. \quad (31)$$

Then, the dominant contributions to LFVs and muon $g-2$ at one-loop level are given by

$$\text{BR}(\ell_\beta \to \ell_\alpha \gamma) \simeq \frac{12\pi^2 C_{\beta\alpha}}{(4\pi)^4 m_{\ell_\beta}^2 G_F^2} (|a_{L_{\alpha\beta}}|^2 + |a_{R_{\alpha\beta}}|^2), \quad (32)$$

$$\Delta a_\mu \simeq -\frac{m_\mu}{(4\pi)^2} (a_{L_{22}} + a_{R_{22}}), \quad (33)$$

$$a_{L_{\alpha\beta}} \approx -\frac{1}{4}(Y^\dagger_{\alpha\rho}c_\theta - \tilde{Y}^\dagger_{\alpha\rho}s_\theta) D_{E_\rho} (Y^\dagger_{\rho\beta}c_\theta - \tilde{Y}^\dagger_{\rho\beta}s_\theta) F(h_1, D_{E_\rho}) - \frac{1}{4}(Y^\dagger_{\alpha\rho}s_\theta + \tilde{Y}^\dagger_{\alpha\rho}c_\theta) D_{E_\rho} (Y^\dagger_{\rho\beta}s_\theta + \tilde{Y}^\dagger_{\rho\beta}c_\theta) F(h_2, D_{E_\rho}), \quad (34)$$

$$a_{R_{\alpha\beta}} \approx -\frac{1}{4}(Y_{\alpha\rho}c_\theta - \tilde{Y}_{\alpha\rho}s_\theta) D_{E_\rho} (Y_{\rho\beta}c_\theta - \tilde{Y}_{\rho\beta}s_\theta) F(h_1, D_{E_\rho}) - \frac{1}{4}(Y_{\alpha\rho}s_\theta + \tilde{Y}_{\alpha\rho}c_\theta) D_{E_\rho} (Y_{\rho\beta}s_\theta + \tilde{Y}_{\rho\beta}c_\theta) F(h_2, D_{E_\rho}), \quad (35)$$

$$F(m_{h_i}, D_{E_\rho}) \approx \frac{m_{h_i}^4 - 4m_{h_i}^2 D_{E_\rho}^2 + 3D_{E_\rho}^4 - 2m_{h_i}^2 (m_{h_i}^2 - 2D_{E_\rho}^2) \ln\left[\frac{m_{h_i}^2}{D_{E_\rho}^2}\right]}{(m_{h_i}^2 - D_{E_\rho}^2)^3}, \quad (36)$$

where $C_{21} \approx 1$, $C_{31} \approx 0.1784$, $C_{32} \approx 0.1736$, $h_1 \equiv h$, $h_2 \equiv H$. The dominant contribution is obtained from the diagram in Fig. 1 that does not have chiral suppression.

[3]Even though we have a contribution from a new gauge boson Z' to these phenomenologies, these are subdominant for the value of $g'/m_{Z'}$ in the range of Eq. (15). We have checked it numerically.

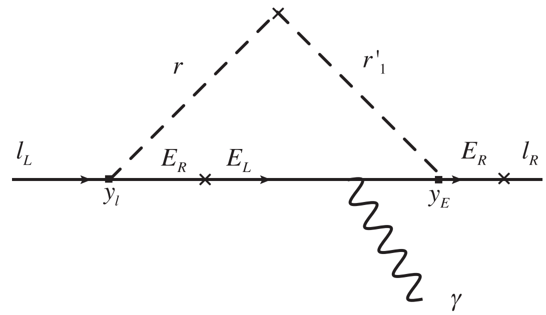

FIG. 1. The diagram that gives dominant contributions to $\ell \to \ell'\gamma$ and muon $g-2$ in flavor basis. The × mark indicates mass (mixing) insertion.

F. Neutrino mass via inverse seesaw mechanism

After spontaneous gauge symmetry breaking, we obtain neutral fermion mass matrix in the basis of (ν_L^c, N_R, N_L^c) as follows

$$M_N = \begin{bmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M & M_{N_L} \end{bmatrix}, \quad (37)$$

where $m_D \equiv v y_D/\sqrt{2}$ and $M \equiv y_N v_1'/\sqrt{2}$. When mass parameters satisfy $M_{N_L} \ll m_D \lesssim M$ active neutrino mass can be approximately written by

$$m_\nu \simeq m_D M^{-1} M_{N_L} (M^T)^{-1} m_D^T. \quad (38)$$

The neutrino mass matrix is diagonalized by a unitary matrix U_ν; $D_\nu = U_\nu^T m_\nu U_\nu$ with $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$. Including charged lepton mixing matrix, the PMNS matrix is defined by $U_{\text{PMNS}} \equiv V_L^\dagger U_\nu$.

We discuss constraint from nonunitarity which is described by a matrix U'_{PMNS}. This matrix is typically parametrized by the form of

$$U'_{\text{PMNS}} \equiv \left(1 - \frac{1}{2} F F^\dagger\right) U_{\text{PMNS}}, \quad (39)$$

where $F \equiv (M^T)^{-1} m_D$ is a Hermitian matrix. The global constraints on elements of $|FF^\dagger|$ are found combining several experimental results such as the SM W boson mass M_W, the effective Weinberg angle θ_W, several ratios of Z boson fermionic decays, invisible decay of Z, electroweak universality, measured Cabbibo-Kobayashi-Maskawa, and lepton flavor violations [146]. The result is then given by [147]

$$|FF^\dagger| \leq \begin{bmatrix} 2.5\times10^{-3} & 2.4\times10^{-5} & 2.7\times10^{-3} \\ 2.4\times10^{-5} & 4.0\times10^{-4} & 1.2\times10^{-3} \\ 2.7\times10^{-3} & 1.2\times10^{-3} & 5.6\times10^{-3} \end{bmatrix}. \quad (40)$$

If we require $|F| \sim 10^{-5}$ conservatively, $M_{N_L} \sim 1$–10 GeV can reproduce active neutrino mass scale. Also we require $y_D \sim 10^{-4}$ for $M = \mathcal{O}(1)$ TeV. Observed neutrino mixing can be easily obtained since we have sufficient number of free parameters $\{y_D, y_N, M_{N_L}\}$.

III. NUMERICAL ANALYSIS AND PHENOMENOLOGICAL CONSEQUENCES

In this section we carry out numerical study to estimate muon $g-2$ and LFVs. We also calculate Z' production cross section for parameter space explaining W-boson mass anomaly and show phenomenological implications of our scenario at hadron collider experiments.

A. Parameter scan for muon $g-2$ and LFVs

Here we discuss muon $g-2$ taking into account charged lepton masses and LFV constraints in our model performing numerical analysis. The dominant contribution to muon $g-2$ comes from the diagram in Fig. 1 since it has chirality change inside loop giving heavy charged lepton mass factor. The relevant free parameters are

$$\{(y_\ell)_{ab}, (y_E)_{aa}, (m_{Ee})_{ab}, \sin\theta, m_H\}, \tag{41}$$

where $a, b = 1$–3. We then scan these free parameters globally to search for best fit value of muon $g-2$. In Table II, we show our benchmark point(BP) found by numerical analysis. We find that large value of Yukawa coupling y_E is preferred to obtain sizable muon $g-2$ while exotic heavy charged lepton masses are around 700 GeV. Then values of v_1' and m_H are preferred to be around electroweak scale. This is the reason why we need two singlet scalars φ_1 and φ_2 to explain both muon $g-2$ and W-boson mass anomaly as the required scale of these VEVs are different; $v_1' \ll v_2'$. We also show $\mathrm{BR}(\ell \to \ell' \gamma)$ and find that those of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are close to the current upper limit. Thus it could be tested in future measurements. Furthermore we show $(\Delta)\mathrm{BR}(Z \to \ell\ell')$ values that might be also tested in future precision measurements for Z boson decay.

TABLE II. Benchmark point that explains muon $g-2$.

Input	
v_1'/GeV	284
m_H/GeV	245
$\sin\theta$	0.250
$[(y_\ell)_{11}, (y_\ell)_{12}, (y_\ell)_{13}]$	$[-0.000512, 0.00520, 0.00236]$
$[(y_\ell)_{21}, (y_\ell)_{22}, (y_\ell)_{23}]$	$[-0.000105, 0.000624, 0.0643]$
$[(y_\ell)_{31}, (y_\ell)_{32}, (y_\ell)_{33}]$	$[0.000148, 0.0145, 0.0647]$
$[(m_{eE})_{11}, (m_{eE})_{12}, (m_{eE})_{13}]/\mathrm{GeV}$	$[0.674, 16.8, 5.92]$
$[(m_{eE})_{21}, (m_{eE})_{22}, (m_{eE})_{23}]/\mathrm{GeV}$	$[-16.2, -46.3, 22.3]$
$[(m_{eE})_{31}, (m_{eE})_{32}, (m_{eE})_{33}]/\mathrm{GeV}$	$[-12.2, 69.5, 19.8]$
$[(y_E)_{11}, (y_E)_{22}, (y_E)_{33}]$	$[-3.26, 3.43, -3.41]$
Output	
$[m_{E_1}, m_{E_2}, m_{E_3}]/\mathrm{GeV}$	[655, 690, 694]
Δa_μ	2.11×10^{-9}
$\mathrm{BR}(\mu \to e\gamma)$	3.59×10^{-13}
$\mathrm{BR}(\tau \to e\gamma)$	9.91×10^{-11}
$\mathrm{BR}(\tau \to \mu\gamma)$	2.22×10^{-8}
$\Delta\mathrm{BR}(Z \to e^\pm e^\mp)$	-1.55×10^{-9}
$\Delta\mathrm{BR}(Z \to \mu^\pm \mu^\mp)$	-6.11×10^{-7}
$\Delta\mathrm{BR}(Z \to \tau^\pm \tau^\mp)$	-3.82×10^{-5}
$\Delta\mathrm{BR}(Z \to e^\pm \mu^\mp)$	6.27×10^{-17}
$\Delta\mathrm{BR}(Z \to e^\pm \tau^\mp)$	3.83×10^{-14}
$\Delta\mathrm{BR}(Z \to \mu^\pm \tau^\mp)$	3.64×10^{-11}

B. Signature of Z' production at collider

We consider signature of Z' production at hadron collider experiments focusing on parameter space that can explain W-boson mass anomaly. In Fig. 2, we show the 1σ region to explain W-boson mass anomaly on $\{m_{Z'}, g'\}$ plane where the red colored line indicates the value providing central value of ΔT in Eq. (14). For the parameter region we estimate cross section of $pp \to Z'$ process using *CalcHEP* [148] implementing relevant gauge interactions.

Our Z' dominantly decays into $\{q\bar{q}, N_R\bar{N}_R, E_R\bar{E}_R\}$ modes. The decay width is estimated by

$$\Gamma(Z' \to f\bar{f}) = \frac{g'^2 N_c}{12\pi} m_{Z'} \sqrt{1 - \frac{4m_f^2}{m_{Z'}^2}} \left(1 - \frac{m_f^2}{m_{Z'}^2}\right), \tag{42}$$

where N_c is color degrees of freedom. BRs of Z' decay can be estimated by the width. In Fig. 3, we show products of cross section and the BRs for jj and $E\bar{E}(N\bar{N})$ modes where we chose $m_{E(N)} = 700$ GeV for three generations. We thus obtain sizable cross section that can be tested at future LHC experiments. Extra fermions E_a and N_a dominantly decay into the SM lepton with boson as

$$E_a \to \ell_i h, \tag{43}$$

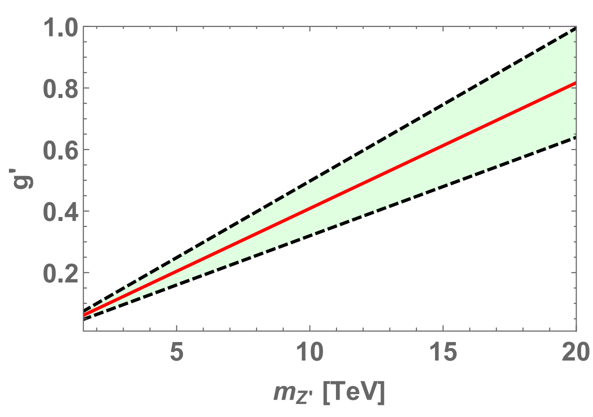

FIG. 2. The 1σ region to explain W-boson mass anomaly on $\{m_{Z'}, g'\}$ plane where the red colored line indicates the value providing central value of ΔT in Eq. (14).

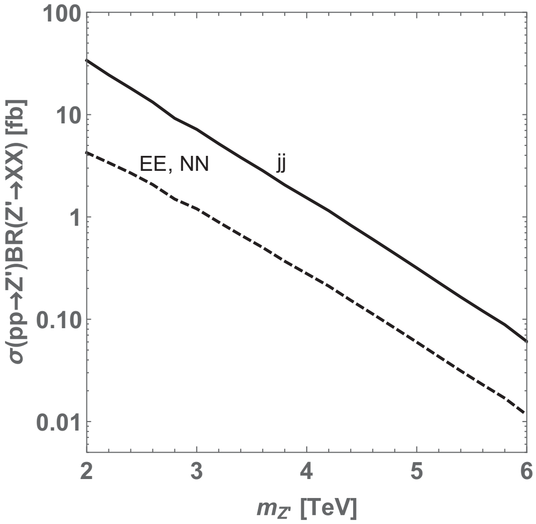

FIG. 3. The products of Z' production cross section and the BRs for jj and $E\bar{E}(N\bar{N})$ modes at the LHC 14 TeV where we chose $m_{E(N)} = 700$ GeV for three generations.

$$N_a \to \nu_i h, \nu_i Z, \ell_i W, \tag{44}$$

where lepton flavor dependence is determined by structure of Yukawa couplings. Therefore signatures of our Z' are dijet, top quark pair, and SM leptons with Higgs or gauge bosons. More detailed numerical analysis is beyond the scope of this paper and left as future work.

IV. SUMMARY AND CONCLUSIONS

We have proposed a model with leptophobic $U(1)_R$ gauge symmetry to explain CDF W-boson mass anomaly. The shift of W-boson mass is realized via neutral gauge boson mass which induces nonzero oblique ΔT parameter. Also we have shown that $U(1)_R$ should be leptophobic since parameter region to explain W-boson mass anomaly is excluded by the LHC constraints on W and Y oblique parameters if Z' is coupled to the SM charged leptons. It is then found that Z' mass and new gauge coupling should satisfy 20 TeV $\lesssim m_{Z'}/g' \lesssim 31$ TeV to accommodate CDF W-boson mass.

In our model charged lepton masses are induced via interactions between the SM leptons and extra charged leptons. These interactions also induce muon $g-2$ and LFVs at loop level. We have investigated muon $g-2$ and LFVs taking into current experimental constraints. It is found that some LFV BRs, $\mu \to e\gamma$ and $\tau \to \mu\gamma$, tends to be close to current upper limits when we explain muon $g-2$ and the BRs can be tested in future experiments. In addition we have discussed realization of active neutrino mass and mixings via inverse seesaw mechanism. Moreover we have discussed collider physics focusing on Z' production and its decay at the LHC. We have found that sizable cross section can be expected considering parameter region explaining the W-boson mass anomaly.

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