Possible molecular states of $\bar{D}^*K^*$ (D^*K^*) and new exotic states $X_0(2900)$, $X_1(2900)$, $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$

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Two isosinglet hadron states $X_0(2900)$ and $X_1(2900)$ with $J = 0$ and 1, respectively, discovered by the LHCb collaboration in 2020, are identified as molecular bound states of $\bar{D}^*K^*$. Recently two structures $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ have been observed at the hadron spectra, and one would suspect if they also are molecular states of D^* and K^*. As long as they are of the molecular structures of D^*K^*, the hadron states must be in an isovector, namely $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ are $I_3 = -1$, 1 components of the isovector. If it is the case, then the corresponding $T^a_{cs0}(2900)^+$ of $I = 1, I_3 = 0$, and $T'^a_{cs0}(2900)^+$ of $I = 0, I_3 = 0$ so far have evaded experimental observation, but should be found by the future experiments. To testify this ansatz, in this paper we study the possible molecular structures of $\bar{D}^*K^*$ and D^*K^* within the Bethe-Salpeter framework. With reasonable input parameters it is found that $\bar{D}^*K^*$ isoscalar systems with $J^P = 0^+$ and 1^+ have solutions. The result supports the ansatz of $X_0(2900)$ and $X_1(2900)$ being molecular states of $\bar{D}^*K^*$. Whereas for the system of D^*K^* with $I = 1$ the corresponding Bethe-Salpeter equation has no solution. Thus we can draw a clear conclusion that $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ should not be bound states of D^* and K^*. The two structures observed by the LHCb collaboration may be caused by dynamics, such as the well-recognized triangle anomalies or other mechanisms.

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I. INTRODUCTION

In 2020 the LHCb collaboration reported two exotic states $X_0(2900)$ and $X_1(2900)$ in the D^-K^+ invariant mass spectrum [1]. Naturally the two states possess a $\bar{c}\bar{s}ud$ flavor structure. Very recently the LHCb collaboration claimed that two new hadronic states $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ in the mass distributions of $D_s^+\pi^-$ and $D_s^+\pi^+$ had been observed [2]. It strongly implies that the two states have $c\bar{s}\bar{u}d$ or $c\bar{s}u\bar{d}$ flavor structures.

The masses and widthes of the $X_0(2900)$, $X_1(2900)$, and $T^a_{cs0}(2900)$ are

$$m_{X_0(2900)} = (2866 \pm 7)\ \mathrm{MeV},$$
$$\Gamma_{X_0(2900)} = (57.2 \pm 12.9)\ \mathrm{MeV},$$
$$m_{X_1(2900)} = (2904 \pm 5)\ \mathrm{MeV},$$
$$\Gamma_{X_1(2900)} = (110.3 \pm 11.5)\ \mathrm{MeV},$$
$$m_{T^a_{cs0}(2900)} = (2908 \pm 11 \pm 2)\ \mathrm{MeV},$$
$$\Gamma_{T^a_{cs0}(2900)} = (136 \pm 23 \pm 11)\ \mathrm{MeV}.$$

Those states are composed with four different flavor quarks (or antiquarks), thus they are distinct from the previously confirmed exotic states such as $X(3872)$, $X(3940)$, $Y(3940)$, $Z(4430)^{\pm}$, $Z_{cs}(4000)$, $Z_{cs}(4220)$, and Z_b, Z'_b, T^+_{cc} [3–10].

Since $X_0(2900)$, $T^a_{cs0}(2900)^0$, and $T^a_{cs0}(2900)^{++}$ are made of $\bar{c}\bar{s}ud$, $c\bar{s}\bar{u}d$, and $c\bar{s}u\bar{d}$, respectively, they cannot be identified as the traditional mesons that contain a quark and an antiquark pair. Instead, they are suggested to be multiquark exotic states that were predicted at the early stage when the $SU(3)$ quark model [11] was raised. In fact, they might be molecular states, compact tetraquarks, mixing of both structures or nonhadronic resonances occurring due to dynamical effects [12,13]. To confirm their structures, one needs to invoke a synthesis of theoretical analysis and experimental observation.

Since the masses of $X_0(2900)$ and $X_1(2900)$ are near the threshold of $\bar{D}^*$ and K^*, they are identified as $\bar{D}^*K^*$ molecules. Similarly, as the masses of $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ are close to that of D^* and K^*, one is tempted

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to expect these new states may be molecular states of D^*K^*. In some works the possible $\bar{D}^*K^*$ or D^*K^* molecular states were proposed and studied [14–18].

Our starting point is that as long as $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ are molecular states composed of $c\bar{s}d\bar{u}$ and $c\bar{s}u\bar{d}$, respectively, they would be the $(I=1, I_3=-1)$ and $(I=1, I_3=1)$ components of an isovector. If it is the case, $T^a_{cs0}(2900)^+$ of $(I=1, I_3=0)$ corresponding to $c\bar{s}(u\bar{u}-d\bar{d})$ and $T'^a_{cs0}(2900)^+$ of $(I=0, I_3=0)$ for $c\bar{s}(u\bar{u}+d\bar{d})$ should exist and so far evade from experimental observation yet. We will probe this ansatz in this work.

Concretely, we study the possible bound state of $\bar{D}^*K^*$ and D^*K^* systems within the Bethe-Salpeter (B-S) framework where the relativistic corrections are automatically included [19–33].

In this work we employ the one-boson-exchange model to obtain the interaction kernels where the effective vertices (D^*D^*M and $\bar{D}^*\bar{D}^*M$) are taken from the heavy meson chiral perturbation theory [34–39] and the effective vertices (K^*K^*M) are adopted from Ref. [17]. The exchanged particles are some light mesons such as π, η, ρ, and ω. In Ref. [34] the authors indicated that the σ exchange makes an insignificant contribution, thus we omit it. The B-S equation is solved in momentum space so the kernel we obtain by calculating the corresponding Feynman diagrams can be used directly rather than converting it into a potential form in coordinate space.

In this work we are only concerned about the ground states where the orbital angular momentum between two constituent mesons is zero (i.e., $l=0$). For the molecular states which consist of two vector mesons their J^P may be 0^+, 1^+, and 2^+. For $\bar{D}^*K^*$ system there are four isospin states: $\frac{1}{\sqrt{2}}(K^{*+}D^{*-}-K^{*0}\bar{D}^{*0})(I=0)$ and $\frac{1}{\sqrt{2}}(K^{*+}D^{*-}+K^{*0}\bar{D}^{*0})(I=1, I_3=0)$, $K^{*0}D^{*-}(I=1, I_3=-1)$, $K^{*+}\bar{D}^{*0}(I=1, I_3=1)$. For D^*K^* system there are also four isospin states: $K^{*+}D^{*+}(I=1, I_3=1)$, $\frac{1}{\sqrt{2}}(K^{*+}D^{*0}-K^{*0}D^{*+})(I=1, I_3=0)$, $K^{*0}D^{*0}(I=1, I_3=-1)$, and $\frac{1}{\sqrt{2}}(K^{*+}D^{*0}+K^{*0}D^{*+})(I=0)$.

With the input parameters, these B-S equations are solved numerically. In some cases there no solutions satisfying the equation as long as the input parameters are set within a reasonable range, it implies that the proposed bound state should not appear in the nature. On the contrary, a solution of the B-S equation with reasonable parameters implies that the attraction between two constituents is sufficient to build up a bound state, thus a hadronic state (molecular) should be formed and emerge in the nature.

After this introduction we deduce the B-S equations and the corresponding kernels for the two vector systems with different quantum numbers. Then in Sec. III we present our numerical results along with explicitly displaying all input parameters. Section IV is devoted to a brief summary.

II. THE BETHE-SALPETER FORMALISM

In our early papers [22–24] we deduced the B-S equations for the systems containing two vectors. It is noted that the structures of the effective vertices are not the same as that given in [22–24], so the new scenario needs to be reformulated.

The quantum number J^P of the bound state composed of two vectors can be 0^+, 1^+, or 2^+.

A. The B-S equation of the 0^+ system

The B-S wave function of 0^+ state $|S\rangle$ composed of two vectors is defined as the following:

$$\langle 0|\mathrm{T}\phi_1^\mu(x_1)\phi_2^\nu(x_2)|S\rangle = \chi_S(x_1, x_2)g^{\mu\nu}, \tag{1}$$

where $\phi_1^\mu(x_1)$ and $\phi_2^\nu(x_2)$ are the field operators of two mesons, respectively. The relative coordinate x and the center of mass coordinate X are

$$X = \eta_1 x_1 + \eta_2 x_2, \qquad x = x_1 - x_2, \tag{2}$$

where $\eta_i = m_i/(m_1+m_2)$ and m_i $(i=1,2)$ is the mass of the ith constituent meson.

After some manipulations we obtain the B-S equation in the momentum space

$$\chi_S(p) = \frac{1}{4}\Delta_{1\mu\lambda}\int\frac{d^4p'}{(2\pi)^4}K_0^{\alpha\alpha'\mu\mu'}(p,p')\chi_S(p')\Delta_{2\mu'\lambda'}g_{\alpha\alpha'}g^{\lambda\lambda'}, \tag{3}$$

where $\Delta_{j\mu\lambda} = \frac{i}{p_j^2-m_j^2}(\frac{p_{j\mu}p_{j\lambda}}{m_j^2}-g_{\mu\lambda})$.

The relative momenta and the total momentum of the bound state in the equation are defined as

$$\begin{aligned} &p = \eta_2 p_1 - \eta_1 p_2, \qquad p' = \eta_2 p_1' - \eta_1 p_2', \\ &P = p_1 + p_2 = p_1' + p_2', \end{aligned} \tag{4}$$

where P denotes the total momentum of the bound state.

In this paper the exchanged mesons we consider are π, η, ρ, and ω [30,31], and the Feynman diagrams corresponding to these effective interactions are depicted in Fig. 1.

With the Feynman diagrams and the effective interaction we obtain

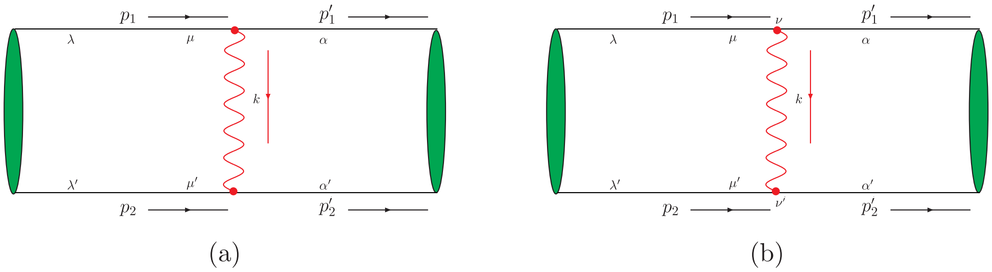

FIG. 1. A bound state composed of two vectors. (a) π (η) is exchanged. (b) ρ (ω) is exchanged.

$$
\begin{aligned}
K_0^{\alpha\alpha'\mu\mu'}(p,p') &= K_{0V}^{\alpha\alpha'\mu\mu'}(p,p',m_\rho) + K_{0V}^{\alpha\alpha'\mu\mu'}(p,p',m_\omega) + K_{0P}^{\alpha\alpha'\mu\mu'}(p,p',m_\pi) + K_{0P}^{\alpha\alpha'\mu\mu'}(p,p',m_\eta),\\
K_{0V}^{\alpha\alpha'\mu\mu'}(p,p',m_V) &= iC_V\frac{q_\nu q_{\nu'}/{m_V}^2 - g_{\nu\nu'}}{q^2-m_V^2}[g_{\bar{D}^*\bar{D}^*V}g^{\alpha\mu}(p_1+p_1')^\nu + 2g'_{\bar{D}^*\bar{D}^*V}(q^\alpha g^{\mu\nu} - q^\mu g^{\alpha\nu})]\\
&\quad\times g_{K^*K^*V}[-q^{\alpha'}g^{\nu'\mu'} + q^{\mu'}g^{\nu'\alpha'} - {p_2'}^{\nu'}g^{\mu'\alpha'} + {p_2'}^{\mu'}g^{\nu'\alpha'} - {p_2}^{\nu'}g^{\mu'\alpha'} + {p_2}^{\alpha'}g^{\nu'\mu'}]F(q)^2,\\
K_{0P}^{\alpha\alpha'\mu\mu'}(p,p',m_P) &= C_P[g_{\bar{D}^*\bar{D}^*P}\varepsilon^{\alpha\beta\mu\nu}q_\nu(p_1+p_1')_\beta][-g_{K^*K^*P}\varepsilon^{\alpha'\beta'\mu'\nu'}q_{\nu'}(p_2+p_2')_{\beta'}]\frac{-i}{q^2-m_P^2}F(q)^2,
\end{aligned}\tag{5}
$$

where P and V represent pseudoscalar and vector mesons. The contributions from vector exchanges are included in $K_{0V}^{\alpha\alpha'\mu\mu'}(p,p',m_V)$ and those for exchanging pseudoscalars are included in $K_{0P}^{\alpha\alpha'\mu\mu'}(p,p',m_P)$. The isospin coefficients C_V and C_P are collected in Tables I and II.

Since the constituent meson is not a point particle, a form factor at each interaction vertex among hadrons must be introduced to reflect the finite-size effects of these hadrons. The form factor is assumed to be in the following form:

$$
F(k) = \frac{\Lambda^2 - M_V^2}{\Lambda^2 - k^2},\tag{6}
$$

where Λ is a cutoff parameter.

It is not an easy task to solve Eq. (3). Defining $K_0(p,p') = \frac{1}{4}K_0^{\alpha\alpha'\mu\mu'}(p,p')(\frac{p_{2\mu'}p_{2\lambda'}}{m_2^2} - g_{\mu'\lambda'})(\frac{p_{1\mu}p_{1\lambda}}{m_1^2} - g_{\mu\lambda})g_{\alpha\alpha'}g^{\lambda\lambda'}$ and using the so-called instantaneous approximation: $p_0' = p_0 = 0$ for $K_0(p,p')$ the B-S equation can be reduced to

TABLE I. The isospin factor of the $\bar{D}^*K^*$ system.

	C_π	C_η	C_ρ	C_ω
$I=0$	$-\frac{3\sqrt{2}}{2}$	$\frac{\sqrt{6}}{6}$	$-\frac{3\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$I=1$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

TABLE II. The isospin factor of the D^*K^* system.

	C_π	C_η	C_ρ	C_ω
$I=0$	$\frac{3\sqrt{2}}{2}$	$\frac{\sqrt{6}}{6}$	$\frac{3\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$I=1$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

$$
\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_S(\mathbf{p}) = \frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_0(\mathbf{p},\mathbf{p}')\psi_S(\mathbf{p}'),\tag{7}
$$

where $E_i \equiv \sqrt{\mathbf{p}^2+m_i^2}$, $E = P^0$, the equal-time wave function is defined as $\psi_S(\mathbf{p}) = \int dp^0\chi_S(p)$ and

$$
\begin{aligned}
K_0(\mathbf{p},\mathbf{p}') &= K_{0V}(\mathbf{p},\mathbf{p}',m_\rho) + K_{0V}(\mathbf{p},\mathbf{p}',m_\omega)\\
&\quad + K_{0P}(\mathbf{p},\mathbf{p}',m_\pi) + K_{0P}(\mathbf{p},\mathbf{p}',m_\eta).
\end{aligned}\tag{8}
$$

The expressions of $K_{0V}(\mathbf{p},\mathbf{p}',m_V)$ and $K_{0P}(\mathbf{p},\mathbf{p}',m_P)$ can be found in Appendix B.

B. The B-S equation of the 1^+ system

The B-S wave function of 1^+ state $|V\rangle$ composed of two vectors is defined as

$$
\langle 0|T\phi_\alpha(x_1)\phi_{\alpha'}(x_2)|V\rangle = \frac{\epsilon_{\alpha\alpha'\tau\tau'}}{\sqrt{6}M}\chi_V(x_1,x_2)\varepsilon^\tau P^{\tau'},\tag{9}
$$

where ε^τ is the polarization vector of 1^+ state.

The corresponding B-S equation is

$$
\begin{aligned}
\chi_V(p) &= \frac{1}{6M^2}\epsilon^{\lambda\lambda'\omega\sigma}\varepsilon_\sigma P_\omega\Delta_{1\mu\lambda}\int\frac{d^4p'}{(2\pi)^4}K_1^{\alpha\alpha'\mu\mu'}(p,p')\\
&\quad\times\epsilon_{\alpha\alpha'\omega'\sigma'}\chi_V(p')\varepsilon^{\sigma'}P^{\omega'}\Delta_{2\mu'\lambda'},
\end{aligned}\tag{10}
$$

where $K_1^{\alpha\alpha'\mu\mu'}(p,p')$ is the same as $K_0^{\alpha\alpha'\mu\mu'}(p,p')$ in Eq. (5).

Defining $K_1(p,p') = \frac{K_1^{\alpha\alpha'\mu\mu'}(p,p')}{6M^2}\epsilon^{\lambda\lambda'\omega\sigma}\varepsilon_\sigma P_\omega(\frac{p_{2\mu'}p_{2\lambda'}}{m_2^2} - g_{\mu'\lambda'})(\frac{p_{1\mu}p_{1\lambda}}{m_1^2} - g_{\mu\lambda})\epsilon_{\alpha\alpha'\omega'\sigma'}\varepsilon^{\sigma'}P^{\omega'}$ the B-S equation is reduced to

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_V(\mathbf{p})=\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_1(\mathbf{p},\mathbf{p}')\psi_V(\mathbf{p}'),\quad(11)$$

with

$$K_1(\mathbf{p},\mathbf{p}')=K_{1V}(\mathbf{p},\mathbf{p}',m_\rho)+K_{1V}(\mathbf{p},\mathbf{p}',m_\omega)+K_{1P}(\mathbf{p},\mathbf{p}',m_\pi)+K_{1P}(\mathbf{p},\mathbf{p}',m_\eta).\quad(12)$$

The expressions of $K_{1V}(\mathbf{p},\mathbf{p}',m_V)$ and $K_{1P}(\mathbf{p},\mathbf{p}',m_P)$ can be found in Appendix B.

C. The B-S equation of the 2^+ system

The B-S wave function of the 2^+ state composed of two vectors is written as

$$\langle 0|T\phi^\alpha(x_1)\phi^{\alpha'}(x_2)|\mathcal{T}\rangle=\frac{1}{\sqrt{5}}\chi_{\mathcal{T}}(x_1,x_2)\varepsilon^{\alpha\alpha'},\quad(13)$$

where $\varepsilon^{\alpha\alpha'}$ is the polarization tensor of the 2^+ state.

The B-S equation can be expressed as

$$\chi_{\mathcal{T}}(p)=\frac{1}{5}\varepsilon^{\lambda\lambda'}\Delta_{1\mu\lambda}\int\frac{d^4p'}{(2\pi)^4}K_2^{\alpha\alpha'\mu\mu'}(p,p')\varepsilon_{\alpha\alpha'}\chi_{\mathcal{T}}(p')\Delta_{2\mu'\lambda'},\quad(14)$$

where $K_2^{\alpha\alpha'\mu\mu'}(p,p')$ is the same as $K_0^{\alpha\alpha'\mu\mu'}(p,p')$ in Eq. (5).

Defining $K_2(p,p')=\frac{K_2^{\alpha\alpha'\mu\mu'}(p,p')}{5}\varepsilon^{\lambda\lambda'}(\frac{p_{2\mu'}p_{2\lambda'}}{m_2^2}-g_{\mu'\lambda'})\times(\frac{p_{1\mu}p_{1\lambda}}{m_1^2}-g_{\mu\lambda})\varepsilon_{\alpha\alpha'}$ the B-S equation can be reduced to

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_{\mathcal{T}}(\mathbf{p})=\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_2(\mathbf{p},\mathbf{p}')\psi_{\mathcal{T}}(\mathbf{p}'),\quad(15)$$

where

$$K_2(\mathbf{p},\mathbf{p}')=K_{2V}(\mathbf{p},\mathbf{p}',m_\rho)+K_{2V}(\mathbf{p},\mathbf{p}',m_\omega)+K_{2P}(\mathbf{p},\mathbf{p}',m_\pi)+K_{2P}(\mathbf{p},\mathbf{p}',m_\eta).\quad(16)$$

The expressions of $K_{2V}(\mathbf{p},\mathbf{p}',m_V)$ and $K_{2P}(\mathbf{p},\mathbf{p}',m_P)$ can be found in Appendix B.

III. NUMERICAL RESULTS

Now let us solve the B-S equations (7), (11), and (15). Since we are interested in the ground state of a bound state, the function $\psi_J(\mathbf{p})$ (J stands for 0,1, or 2) only depends on the norm of the three-momentum and we may first integrate over the azimuthal angle of the functions in (7), (11), or (15)

$$\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_J(\mathbf{p},\mathbf{p}'),$$

to obtain a potential form $U_J(|\mathbf{p}|,|\mathbf{p}'|)$, then the B-S equation turns into a one-dimensional integral equation

$$\psi_J(|\mathbf{p}|)=\frac{(E_1+E_2)/E_1E_2}{E^2-(E_1+E_2)^2}\int d|\mathbf{p}'|U_J(|\mathbf{p}|,|\mathbf{p}'|)\psi_J(|\mathbf{p}'|).\quad(17)$$

As long as the potential $U_J(|\mathbf{p}|,|\mathbf{p}'|)$ is attractive and strong enough, the corresponding B-S equation has a solution(s) and we can obtain the mass of the possible bound state. We explained how to solve the one-dimensional integral equation in Refs. [22–26]

A. The results of the $\bar{D}^*K^*$ system

In our calculation the values of the parameters $g_{\bar{D}^*\bar{D}^*P}$, $g_{K^*K^*P}$, $g_{\bar{D}^*\bar{D}^*V}$, $g'_{\bar{D}^*\bar{D}^*V}$, and $g_{K^*K^*V}$ are presented in Appendix A. In Refs. [40,41] the authors suggested a relation: $\Lambda=m+\lambda\Lambda_{\rm QCD}$ where m is the mass of the exchanged meson, λ is a number of order $O(1)$, and $\Lambda_{\rm QCD}=220$ MeV. The masses of the concerned constituent mesons, $m_{\bar{D}^*}$ and m_{K^*} are directly taken from the databook [42].

Now let us try to calculate the eigenvalues of these systems of $\bar{D}^*K^*(J^P=0^+,I=0)$, $\bar{D}^*K^*(J^P=1^+,I=0)$, $\bar{D}^*K^*(J^P=2^+,I=0)$, $\bar{D}^*K^*(J^P=0^+,I=1)$, $\bar{D}^*K^*(J^P=1^+,I=1)$, and $\bar{D}^*K^*(J^P=2^+,I=1)$, respectively. We find that no value of λ can satisfy the equations of the systems with $I=1$, i.e., these B-S equations are unsolvable. It implies that the effective interaction between the two constituents is repulsive. For the system with $I=0$, $J=0$, or $J=1$ the B-S equation has a solution. It implies that $\bar{D}^*K^*$ can form an isospin scalar bound state with $J=0$ or $J=1$ by exchanging light mesons. However the B-S equation of the system with $I=0$, $J=2$ still has no solution even if we vary the value of Λ within a larger range. It seems that the attractive interaction between the two constituents is sufficiently strong for a $J=0$ or $J=1$ system but very weak for a $J=2$ one. In Ref. [17] the authors obtained the same results with a similar approach. In Table III several places are symbolized by the mark "–," which means that such bound states cannot exist due to the fact that the corresponding B-S equation has no solution. In Table IV the values of λ with different binding energy ΔE are presented. The corresponding wave functions with different binding energy ΔE are depicted in Fig. 2. In 2020 LHCb collaboration found two exotic states $X_0(2900)$ and $X_1(2900)$ and

TABLE III. The value of λ for the $\bar{D}^*K^*$ system ($\Delta E=33$ MeV).

	0^+	1^+	2^+
$I=0$	3.144	4.424	$\cdots$
$I=1$	$\cdots$	$\cdots$	$\cdots$

TABLE IV. The value of λ for the $I=0$ $\bar{D}^*K^*$ system with different ΔE (in unit of MeV).

	5	10	15	20	25
$J^P=0^+$	2.765	2.867	2.947	3.008	3.066
$J^P=1^+$	3.669	3.868	4.021	4.151	4.265

some theoretical works suggested that they should be $\bar{D}^*K^*$ molecular states. Our calculation supports the proposal that the two states should be $\bar{D}^*K^*$ molecular states with $I=0$. In Ref. [43] the authors suggested that $X_0(2900)$ is probably a compact $ud\bar{s}\bar{c}$ state with the quantum number $J^P=0^+$, but they could not explain the structure of $X_1(2900)$ simultaneously.

B. The results for the D^*K^* system

The signs of the isospin factors C_π and C_ρ for the D^*K^* system in Table II are opposite to those for $\bar{D}^*K^*$ shown in Table I. It seems that the effective interactions for the D^*K^* system with $I=0$ would be repulsive. However the coupling constants $g_{D^*D^*V}=-g_{\bar{D}^*\bar{D}^*V}$ and $g'_{D^*D^*V}=-g'_{\bar{D}^*\bar{D}^*V}$ make the interactions for D^*K^* systems with $I=0$ still being attractive. Indeed numerical results in Table V support the inference, i.e., the B-S equations have solution(s) except the $J=2$ system.

For the system with $I=1$ the contributions from exchanging ρ and ω nearly cancel each other and the contribution from exchanging π is canceled partly by that of the η exchange so that the interaction between the two constituents may be very weak whether it is attractive or repulsive. As we expected, in this case these equations have no solution. Numerical results indicate that these bound states $[K^{*+}D^{*+}(I=1,I_3=1)$, $\frac{1}{\sqrt{2}}(K^{*+}D^{*0}-K^{*0}D^{*+})(I=1,I_3=0)$, and $K^{*0}D^{*0}(I=1,I_3=-1)]$ should not exist but that $\frac{1}{\sqrt{2}}(K^{*+}D^{*0}+K^{*0}D^{*+})(I=0)$ can form a molecular state. Our calculation indicates that the two new structures $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$, recently reported by the LHCb collaboration, should not be explained as D^*K^* molecular states. Instead, in Ref. [44] it is suggested that the two peaks were resonancelike structures induced by the threshold effects.

Moreover we find there may exist an isoscalar D^*K^* state possessing a positive charge.

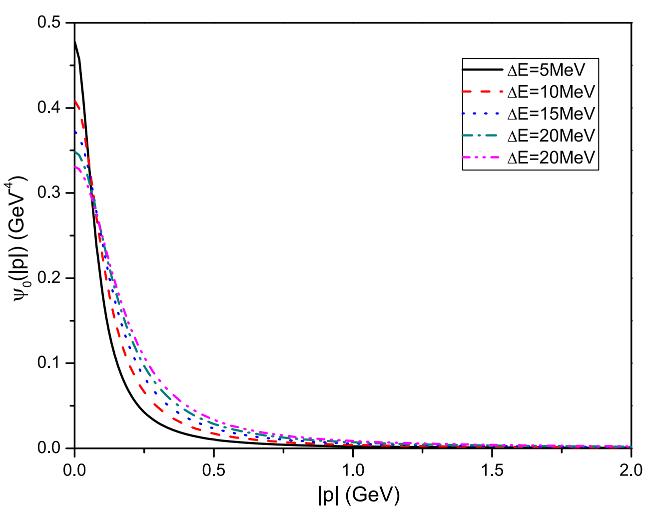

FIG. 2. The unnormalized wave functions of the bound state $\bar{D}^*K^*$ ($J^P=0^+$) with different binding energy.

TABLE V. The value of λ for the D^*K^* system ($\Delta E=33$ MeV).

	0^+	1^+	2^+
$I=0$	2.118	2.766	…
$I=1$	…	…	…

IV. A BRIEF SUMMARY

In this work we study whether $\bar{D}^*K^*$ and D^*K^* can form hadronic molecular states. We extend our previous works, where the B-S framework was applied to the systems of one vector and one pseudoscalar or two vectors [22–26], to study possible bound states with the concerned isospin-spin structures.

In order to obtain the interaction kernels for B-S equations by using the heavy meson chiral perturbation theory and the effective interaction to obtain the effective vertices, we eventually are able to calculate the corresponding Feynman diagrams where π, η, ρ, or ω are exchanged.

Our final results indicate that $\bar{D}^*$ and K^* can form two $I=0$ molecular states with $J=0$ and $J=1$. Naturally, the two states are identified as $X_0(2900)$ and $X_1(2900)$. As for the D^*K^* system, $I=1$ bound states cannot exist, thus we draw a definite conclusion that $T^a_{cs0}(2900)^0$ and $T^a_{cs0}(2900)^{++}$ cannot be explained as a D^*K^* molecular state. Moreover, it is also noted that there should be an $I=0$ D^*K^* state with a positive charge.

Since the parameters are fixed from data that span a relatively large range we cannot expect all the numerical results to be very accurate. The goal of this work is to study whether $\bar{D}^*K^*$ or D^*K^* can form a molecular state. Our results, even if not accurate, have obvious qualitative significance. Definitely, further theoretical and experimental works are badly needed for gaining better understanding of these exotic hadrons.

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APPENDIX A: THE EFFECTIVE INTERACTIONS

The effective interactions can be found in [34–36]

$$\mathcal{L}_{D^*D^*P} = g_{D^*D^*P}(D_b^{*\mu}\overleftrightarrow{\partial}^{\beta}D_a^{*\alpha\dagger})(\partial^{\nu}\mathcal{M})_{ba}\varepsilon_{\nu\mu\alpha\beta} + g_{\bar{D}^*\bar{D}^*P}(\bar{D}_b^{*\mu}\overleftrightarrow{\partial}^{\beta}\bar{D}_a^{*\alpha\dagger})(\partial^{\nu}\mathcal{M})_{ba}\varepsilon_{\nu\mu\alpha\beta}, \tag{A1}$$

$$\begin{aligned}\mathcal{L}_{D^*D^*V} &= ig_{D^*D^*V}(D_b^{*\nu}\overleftrightarrow{\partial}_{\mu}D_{a\nu}^{*\dagger})(\mathcal{V})^{\mu}_{ba} + ig'_{D^*D^*V}(D_b^{*\mu}D_a^{*\nu\dagger} - D_b^{*\mu\dagger}D_a^{*\nu})(\partial_{\mu}\mathcal{V}_{\nu} - \partial_{\nu}\mathcal{V}_{\mu})_{ba}\\ &\quad + ig_{\bar{D}^*\bar{D}^*V}(\bar{D}_b^{*\nu}\overleftrightarrow{\partial}_{\mu}\bar{D}_{a\nu}^{*\dagger})(\mathcal{V})^{\mu}_{ba} + ig'_{\bar{D}^*\bar{D}^*V}(\bar{D}_b^{*\mu}\bar{D}_a^{*\nu\dagger} - \bar{D}_b^{*\mu\dagger}\bar{D}_a^{*\nu})(\partial_{\mu}\mathcal{V}_{\nu} - \partial_{\nu}\mathcal{V}_{\mu})_{ba},\end{aligned} \tag{A2}$$

where a and b represent the flavors of light quarks. In Ref. [34] $\mathcal{M}$ and $\mathcal{V}$ are 3×3 Hermitian matrices

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

and,

$$\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K^{*0}} & \phi \end{pmatrix}$$

respectively.

In the chiral and heavy quark limit, the above coupling constants are $g_{D^*D^*P} = g_{\bar{D}^*\bar{D}^*P} = \frac{g}{f_\pi}$, $g_{D^*D^*V} = -g_{\bar{D}^*\bar{D}^*V} = -\frac{\beta g_V}{\sqrt{2}}$, $g'_{D^*D^*V} = -g'_{\bar{D}^*\bar{D}^*V} = -\sqrt{2}\lambda g_V M_{D^*}$ with $f_\pi = 132$ MeV [35], $g = 0.64$ [36], $\kappa = g$, $\beta = 0.9$, $g_V = 5.9$ [45], and $\lambda = 0.56\ \mathrm{GeV}^{-1}$ [46].

The effective interactions

$$\mathcal{L}_{K^*K^*P} = g_{K^*K^*P}(\partial^{\nu}K^{*\mu\dagger}\partial^{\alpha}PK^{*\beta})\varepsilon_{\nu\mu\alpha\beta} + \mathrm{H.c.}, \tag{A3}$$

$$\begin{aligned}\mathcal{L}_{K^*K^*V} &= i\frac{g_{K^*K^*V}}{2}(K^{*\mu\dagger}V_{\mu\nu}K^{*\nu} + K^{*\mu\nu\dagger}V_{\mu}K^*_{\nu}\\ &\quad + K^{*\mu\dagger}V^{\nu}K^*_{\mu\nu}),\end{aligned} \tag{A4}$$

with $K^*_{\mu\nu} = \partial_\mu K^*_\nu - \partial_\nu K^*_\mu$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

APPENDIX B: KERNEL

$$K_{0P} = \frac{-2iC_P g_{\bar{D}^*\bar{D}^*P}g_{K^*K^*P}F(\mathbf{q})^2}{(\mathbf{p}-\mathbf{p}')^2 + m_P^2}[-\mathbf{p}\cdot\mathbf{p}'^2 + \eta_1\eta_2 M^2(\mathbf{p}-\mathbf{p}')^2 + \mathbf{p}^2\mathbf{p}'^2], \tag{B1}$$

$$\begin{aligned}K_{0V} &= \frac{iC_V g'_{\bar{D}^*\bar{D}^*V}g_{K^*K^*V}F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]}\left\{\frac{9(\mathbf{p}-\mathbf{p}')^2}{2} + [6\mathbf{p}^4 + \mathbf{p}^2(2\mathbf{p}'^2 - 9\mathbf{p}\cdot\mathbf{p}')\right.\\
&\quad - M^2{\eta_1}^2(\mathbf{p}^2 + 2\mathbf{p}'^2 - 3\mathbf{p}\cdot\mathbf{p}') + \mathbf{p}\cdot\mathbf{p}'^2 - 4M^2\eta_1\eta_2(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(2{m_1}^2)\\
&\quad + (M^2\eta_1\eta_2 + \mathbf{p}^2)[6\mathbf{p}^4 + \mathbf{p}^2(2\mathbf{p}'^2 - 9\mathbf{p}\cdot\mathbf{p}') + M^2\eta_1\eta_2(3\mathbf{p}^2 + 2\mathbf{p}'^2 - 5\mathbf{p}\cdot\mathbf{p}')\\
&\quad + \mathbf{p}\cdot\mathbf{p}'^2 + 2M^2{\eta_2}^2(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(2{m_1}^2{m_2}^2)\\
&\quad \left. + [6\mathbf{p}^4 + \mathbf{p}^2(2\mathbf{p}'^2 - 9\mathbf{p}\cdot\mathbf{p}') + \mathbf{p}\cdot\mathbf{p}'^2 + M^2{\eta_2}^2(-5\mathbf{p}^2 - 2\mathbf{p}'^2 + 7\mathbf{p}\cdot\mathbf{p}')]/(2{m_2}^2)\right\}\\
&\quad + \frac{iC_V g_{\bar{D}^*\bar{D}^*V}g_{K^*K^*P}F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]}\{3(\mathbf{p}^2 - \mathbf{p}'^2)^2/(4{m_V}^2) + [(12M^2\eta_1\eta_2 + 3(\mathbf{p}+\mathbf{p}')^2)]/4\\
&\quad + [\mathbf{p}^2\mathbf{p}'^2 + M^2{\eta_1}^2(\mathbf{p}^2 - \mathbf{p}'^2) + 2M^2\eta_1\eta_2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2]/(4{m_1}^2)\\
&\quad - (\mathbf{p}^2 - \mathbf{p}'^2)[\mathbf{p}^2\mathbf{p}'^2 + M^2{\eta_1}^2(\mathbf{p}^2 - \mathbf{p}'^2) + 2M^2\eta_1\eta_2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2]/(4{m_1}^2{m_V}^2)\\
&\quad + [\mathbf{p}^2\mathbf{p}'^2 + M^2{\eta_2}^2(\mathbf{p}^2 - \mathbf{p}'^2) + 2M^2\eta_1\eta_2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2]/(4{m_2}^2)\\
&\quad + (\mathbf{p}^2 - \mathbf{p}'^2)[M^2{\eta_2}^2(\mathbf{p}-\mathbf{p}')^2 - \mathbf{p}^2\mathbf{p}'^2 + \mathbf{p}\cdot\mathbf{p}'^2]/(4{m_2}^2{m_V}^2)\\
&\quad + (M^2\eta_1\eta_2 + \mathbf{p}^2)[\mathbf{p}^2\mathbf{p}'^2 + M^2\eta_1\eta_2(6\mathbf{p}^2 + \mathbf{p}'^2 - 3\mathbf{p}\cdot\mathbf{p}') + M^2{\eta_1}^2(4\mathbf{p}^2 - 2\mathbf{p}\cdot\mathbf{p}')\\
&\quad - \mathbf{p}\cdot\mathbf{p}'^2 + M^2{\eta_2}^2(\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(4{m_1}^2{m_2}^2) - (M^2\eta_1\eta_2 + \mathbf{p}^2)(\mathbf{p}^2 - \mathbf{p}'^2)[\mathbf{p}^2\mathbf{p}'^2\\
&\quad + M^2\eta_1\eta_2(\mathbf{p}'^2 - \mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2 + {\eta_2}^2(M^2)(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(4{m_1}^2{m_2}^2{m_V}^2)\},\end{aligned} \tag{B2}$$

$$K_{1P} = \frac{-4iC_P g^2_{D^*D^*P} F(\mathbf{q})^2}{3[(\mathbf{p}-\mathbf{p}')^2 + m_P^2]} \eta_1\eta_2 M^2 (\mathbf{p}-\mathbf{p}')^2, \tag{B3}$$

$$\begin{aligned}
K_{1V} = {} & \frac{iC_V g_{\bar{D}^*\bar{D}^*V} g_{K^*K^*V} F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]} \{2(\mathbf{p}^2 - \mathbf{p}'^2)^2/(3m_V{}^2) + 2[6M^2\eta_1\eta_2 + (\mathbf{p}+\mathbf{p}')^2]/3 \\
& + (\mathbf{p}^2 - \mathbf{p}'^2)(2\mathbf{p}^4 - \mathbf{p}^2\mathbf{p}'^2 - \mathbf{p}\cdot\mathbf{p}'^2)/(6m_1{}^2 m_V{}^2) \\
& + (\mathbf{p}^2 - \mathbf{p}'^2)[-(\mathbf{p}^2\mathbf{p}'^2) + M^2\eta_2{}^2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') + \mathbf{p}\cdot\mathbf{p}'^2]/(3m_2{}^2 m_V{}^2) \\
& + [\mathbf{p}^2\mathbf{p}'^2 + M^2\eta_1\eta_2(6\mathbf{p}^2 - 4\mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2 + M^2\eta_2{}^2(\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(3m_2{}^2) \\
& + [8M^2\eta_1\eta_2\mathbf{p}^2 + 2\mathbf{p}^4 + \mathbf{p}\cdot\mathbf{p}'^2 + \mathbf{p}^2(\mathbf{p}'^2 + 4\mathbf{p}\cdot\mathbf{p}')]/(6m_1{}^2)\} \\
& + \frac{iC_V g'_{\bar{D}^*\bar{D}^*V} g_{K^*K^*V} F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]} \{2(\mathbf{p}-\mathbf{p}')^2 + [2\mathbf{p}^4 + \mathbf{p}^2(\mathbf{p}'^2 - 6\mathbf{p}\cdot\mathbf{p}') + 4M^2\eta_1\eta_2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') \\
& + 3\mathbf{p}\cdot\mathbf{p}'^2]/(3m_1{}^2) + 2[\mathbf{p}^2(2\mathbf{p}^2 + \mathbf{p}'^2 - 3\mathbf{p}\cdot\mathbf{p}') + M^2\eta_2{}^2(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(3m_2{}^2\}),
\end{aligned} \tag{B4}$$

$$\begin{aligned}
K_{2V} = {} & \frac{iC_V g_{\bar{D}^*\bar{D}^*V} g_{K^*K^*V} F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]} \{2(\mathbf{p}^2 - \mathbf{p}'^2)^2/(3m_V{}^2) + 2[6M^2\eta_1\eta_2 + (\mathbf{p}+\mathbf{p}')^2]/3 \\
& + (\mathbf{p}^2 - \mathbf{p}'^2)(6\mathbf{p}^4 - 7\mathbf{p}^2\mathbf{p}'^2 + \mathbf{p}\cdot\mathbf{p}'^2)/(30m_1{}^2 m_V{}^2) \\
& + (\mathbf{p}^2 - \mathbf{p}'^2)[-(\mathbf{p}^2\mathbf{p}'^2) + M^2\eta_2{}^2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') + \mathbf{p}\cdot\mathbf{p}'^2]/(3m_2{}^2 m_V{}^2) \\
& + 2\mathbf{p}^2(\mathbf{p}^2 - \mathbf{p}'^2)[-(\mathbf{p}^2\mathbf{p}'^2) + M^2\eta_2{}^2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') + \mathbf{p}\cdot\mathbf{p}'^2]/(15m_1{}^2 m_2{}^2 m_V{}^2) \\
& + [\mathbf{p}^2\mathbf{p}'^2 + M^2\eta_1\eta_2(6\mathbf{p}^2 - 4\mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2 + M^2\eta_2{}^2(\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/3m_2{}^2 \\
& + 2\mathbf{p}^2[\mathbf{p}^2\mathbf{p}'^2 + M^2\eta_1\eta_2(6\mathbf{p}^2 - 4\mathbf{p}\cdot\mathbf{p}') - \mathbf{p}\cdot\mathbf{p}'^2 + M^2\eta_2{}^2(\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(15m_1{}^2 m_2{}^2) \\
& + [40M^2\eta_1\eta_2\mathbf{p}^2 + 6\mathbf{p}^4 - \mathbf{p}\cdot\mathbf{p}'^2 + \mathbf{p}^2(7\mathbf{p}'^2 + 12\mathbf{p}\cdot\mathbf{p}')]/(30m_1{}^2)\} \\
& + \frac{iC_V g'_{\bar{D}^*\bar{D}^*V} g_{K^*K^*V} F(\mathbf{q})^2}{[-(\mathbf{p}-\mathbf{p}')^2 - m_V^2]} \{-2(\mathbf{p}-\mathbf{p}')^2 \\
& - 2[\mathbf{p}^2(2\mathbf{p}^2 + \mathbf{p}'^2 - 3\mathbf{p}\cdot\mathbf{p}') + M^2\eta_2{}^2(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}')]/(3m_2{}^2) \\
& + [-14\mathbf{p}^4 + 20M^2\eta_1\eta_2(\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{p}') - 9\mathbf{p}\cdot\mathbf{p}'^2 + \mathbf{p}^2(-7\mathbf{p}'^2 + 30\mathbf{p}\cdot\mathbf{p}')]/(15m_1{}^2) \\
& - 4[\mathbf{p}^4(2\mathbf{p}^2 + \mathbf{p}'^2 - 3\mathbf{p}\cdot\mathbf{p}') + M^2\eta_2{}^2\mathbf{p}^2(-\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{p}') \\
& + M^2\eta_1\eta_2(-3\mathbf{p}^4 - 3\mathbf{p}\cdot\mathbf{p}'^2 + \mathbf{p}^2\mathbf{p}'^2 + 5\mathbf{p}^2\mathbf{p}\cdot\mathbf{p}')]/(15m_1{}^2 m_2{}^2)\},
\end{aligned} \tag{B5}$$

$$K_{2P} = \frac{4iC_P g^2_{D^*D^*P} F(\mathbf{q})^2}{3[(\mathbf{p}-\mathbf{p}')^2 + m_P^2]} \eta_1\eta_2 M^2 (\mathbf{p}-\mathbf{p}')^2. \tag{B6}$$

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