# Polarization phenomena in the reaction $e^+ + e^- \to N + \bar{N} + \pi^0$ in the framework of the nonresonant mechanism

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The dependence of the nucleon polarization in the reaction $e^+ + e^- \to N + \bar{N} + \pi^0$ over different invariant variables is derived for the nonresonant mechanism. The nucleon polarization is expressed in terms of six invariant complex amplitudes, assuming the conservation of the hadron electromagnetic currents and the P invariance of the hadron electromagnetic interaction. An inclusive experimental setup when the proton (or the antiproton) and the pion are detected in coincidence is considered. Numerical estimations are performed for the so-called normal polarization in the energy range from threshold up to $s = 16\ \mathrm{GeV}^2$, using selected parametrizations of the nucleon electromagnetic form factors in frame of a formalism derived in a previous work for the calculation of the unpolarized differential cross section.

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## I. INTRODUCTION

The interaction between electrons and nucleons is considered the cleanest probe to investigate the nonperturbative aspects of quantum chromo dynamics (QCD), the theory of the strong interaction. In a previous work [1], we considered the reaction

$$e^+(k_1) + e^-(k_2) \to N(p_1) + \bar{N}(p_2) + \pi^0(k), \quad N = p, n \quad (1)$$

that is currently accessible at BESIII [2]. This reaction is the most simple "inelastic annihilation" reaction and is very sensitive to the electromagnetic structure of the hadron current. At electron accelerators, such as JLAB, MAMI, and ELSA, huge experimental programs are based on the reaction $e^- + N \to e^- + N + \pi$ to determine the properties of baryon resonances and transition electromagnetic form factors. All these experiments, including the crossing symmetry related reactions induced by antiproton (that will be investigated at PANDA (FAIR) [3]) and by pions (that is the object of study at HADES [4]) bring strong constraints on nucleon models.

The general formalism for the analysis of the annihilation reactions (1) was derived in Ref. [1], assuming the conservation of the hadron electromagnetic currents and the P invariance of the hadron electromagnetic interaction. Under these assumptions, the matrix element, which is the convolution of lepton and hadron currents, can be expressed by six independent complex invariant amplitudes. This statement remains true for different possible charge states of the pion, nucleon, and antinucleon. The differential cross sections and the different polarization observables including single-spin beam asymmetry, the nucleon (antinucleon) polarization, and the correlation between electron and nucleon polarization states can be expressed, in the general case, in terms of the bilinear combinations of these invariant amplitudes. In Ref. [1], we thoroughly investigated the contribution of the continuum (nonresonant mechanism; see Fig. 1) to double and single differential distributions over selected invariant variables in the case of unpolarized particles. An important ingredient of these calculations is the knowledge of nucleon form factors.

In the present work, this formalism is extended to polarization observables. The earlier study of the phase space in terms of invariant variables and the derived analytical expressions for the corresponding invariant amplitudes allow us to calculate also different polarization observables. The importance of polarization observables

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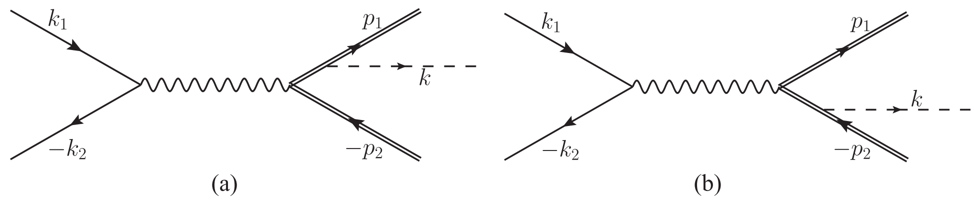



FIG. 1. The simplest Feynman diagrams describing the continuum (nonresonant) contribution to the process (1): (a) with intermediate nucleon and (b) with intermediate antinucleon.

cannot be overestimated, as shown by the proton form factor measurements in the spacelike region. In the case of elastic electron proton scattering the recoil polarization method could be systematically applied only recently, with the advent of high duty cycle, highly polarized electron beams at Jefferson Laboratory [5]. This method, suggested in Refs. [6,7] requires a longitudinally polarized electron beam and the measurement of the transverse polarization of a few GeV recoil proton (or of the spin asymmetry with a transversely polarized target). The precise determination of the electric to magnetic form factor ratio, achieved by the JLab-GEP Collaboration showed that the electric and magnetic distributions in the proton are different, contrary to what was previously assumed and gave rise to a deep revision of nucleon models. In the timelike region, form factors are of complex nature, and the study of polarization phenomena is necessary for their full determination.

Much progress was recently done in this field: the near threshold cross sections for the annihilation of $e^+e^-$ into proton-antiproton and neutron-antineutron were measured at Novosibirsk [8,9]. Following the results obtained by the *BABAR* Collaboration at SLAC [10,11] and the discovery of periodic oscillations in the proton form factor in Ref. [12], the BESIII Collaboration in Beijing enormously contributed to several works, using initial-state radiation [13,14] and the beam scan method [15]. Precise data in a large $q^2$ domain confirmed and completed the *BABAR* data, providing also the first separation between electric and magnetic form factors (in moduli) [15]. Precise data on the neutron effective form factors have also been published [16,17], highlighting similar oscillating behavior, but shifted by a phase, in the neutron effective form factors.

The magnitude of these oscillations is of the order of 10% above a smooth background that follows roughly a dipole $q^2$ dependence, as predicted by perturbative QCD. The smallness of the oscillations and their regularity points to an interference mechanism. Note, however, that two-and-a-half damped oscillations are visible on the proton, while the neutron data are less exhaustive. The nature of these oscillations is not clarified yet: rescattering in the final state [18], mixing of intermediate channels of different isotopic spin [19], resonances [20], or interfering amplitudes [21]. Depending on their origin, it is not straightforward to state if these oscillations play a visible role in the inelastic channel of interest here.

In the present paper, we consider polarization phenomena in the approximation of the nonresonant mechanism (see Fig. 1), widely using the results of Ref. [1] concerning the phase space of the final particles.

The hadron tensor for a longitudinally polarized electron beam and polarized nucleon is explicitly derived. In these conditions, we can investigate the single-spin effects due to the polarization of the electron beam (single-spin beam asymmetry) or to the polarization of the nucleon as well the double-spin observables, i.e., the correlation between electron and nucleon polarizations.

## II. FORMALISM

In our analysis, we consider three different possible independent polarization states of the nucleon which are defined in Ref. [1] as longitudinal $S^L$, transverse $S^T$, and normal $S^N$. Our calculation follows different steps. First, we obtain the full differential cross section over four invariant variables

$$s_1 = (k + p_1)^2, \qquad s_2 = (k + p_2)^2,$$
$$t_1 = (k_1 - p_1)^2, \qquad t_2 = (k_2 - p_2)^2,$$

accounting also for the terms depending on the nucleon spin states. Then, we perform the analytical integration over two variables using the sets of limits defined in Ref. [1] and find the spin-dependent double differential distributions over the pairs $(s_1, s_2), (t_2, s_1)$ and $(s_{12}, s_1), s_{12} = (p_1 + p_2)^2$. These distributions are derived taking explicitly into account all particle masses (even the electron one).

The corresponding polarization of the nucleon (longitudinal, transverse, or normal) is defined by the ratio of the spin-dependent part of the cross section to the spin-independent one. Performing one more integration, we obtain the differential cross sections over one invariant variable $(s_1, s_{12}, t_2)$ and then the corresponding single- and double-spin distributions for the nucleon polarizations.

The full differential cross section of the process (1) can be written in terms of the convolution of the leptonic and hadronic tensors and of the final particle phase space as

$$d\sigma = \frac{\alpha^2}{8\pi^3 q^6} L^{\mu\nu} H_{\mu\nu} dR_3,$$
$$dR_3 = \frac{d^3 p_1}{2E_1}\frac{d^3 p_2}{2E_2}\frac{d^3 k}{2E}\delta(k_1 + k_2 - p_1 - p_2 - k), \quad (2)$$

where $E_1(E_2)$ is the nucleon (antinucleon) energy and $E$ is the pion one. It is possible to choose the coordinate system in such a way that one of the final 3-momenta belongs to a definite plane, for example, to the $zx$ one. Such a choice, in fact, corresponds to the integration over one azimuthal angle. In this case, we can write the phase space $dR_3$ in terms of the invariant variables, namely, [22]

$$dR_3 = \frac{\pi}{16(s - 2m_e^2)} \frac{dt_1 dt_2 ds_1 ds_2}{\sqrt{-\Delta}}, \quad (3)$$

where $\Delta$ is the Gramian determinant (see Ref. [1] for the details). Both the convolution of the tensors and the Gramian determinant can be expressed through the variable $q^2 = s$ and the other invariants chosen for description of the phase space. Moreover, the condition $-\Delta > 0$ alone sets the limiting range for these invariant variables.

The spin-dependent part of the hadronic tensor, in the general case, has been obtained in Ref. [1] in terms of the invariant amplitudes for the nonresonant mechanism. We need to calculate its convolution with the leptonic tensor, which contributes to single-spin effects. Using the connection between the Dirac and Pauli form factors, $F_1$ and $F_2$, and the corresponding invariant amplitudes (see Eqs. (26) in Ref. [1]), we have

$$L^{\mu\nu} H_{\mu\nu}^{(s)}(S) = \frac{4 g^2_{\pi^0 N\bar{N}} \mathrm{Im}[F_1 F_2^*]}{M(k\cdot q - p\cdot q - q^2)[(k\cdot q)^2 - (p\cdot q)^2]^2}[4k\cdot q(k\cdot q k_1\cdot p - k\cdot k_1 p\cdot q)(S_1(k_1 k q S) + S_2(k_1 p q S)) + S_3(k p q S)], \qquad p = p_1 - p_2, \qquad (abcd) = \varepsilon^{\mu\nu\lambda\rho} a_\mu b_\nu c_\lambda d_\rho, \quad (4)$$

where $S$ is the nucleon spin 4-vector, $M(m)$ is the nucleon (neutral pion) mass, and

$$\begin{aligned} S_1 &= q^2(q^2 - p^2 - 4M^2) - 2k\cdot q(q^2 - 2M^2) + (k\cdot q)^2 - (p\cdot q)^2 - 4M^2 p\cdot q, \\ S_2 &= (p\cdot q - k\cdot q)^2 + m^2 q^2, \\ S_3 &= (k\cdot q - p\cdot q)\{p\cdot q(q^2 + p\cdot q)[4(k_1\cdot k)^2 + m^2 q^2] + (k\cdot q)^2[4(k_1\cdot p)^2 + 4k_1\cdot p k_1\cdot q + p^2 q^2] \\ &\quad - k\cdot q[q^2 k\cdot p(q^2 + 2p\cdot q) + 4k_1\cdot k[k_1\cdot q p\cdot q + k_1\cdot p(q^2 + 2p\cdot q)]]\} \\ &\quad - q^2 k\cdot q\{(k\cdot q)^3 + (p\cdot q)^3 + (k\cdot q)^2(4M^2 - 2q^2 - p\cdot q) + m^2 q^2 p\cdot q \\ &\quad + k\cdot q[q^2(q^2 + p^2 - 4M^2) - (p\cdot q)^2 - 4M^2 p\cdot q]\}. \end{aligned} \quad (5)$$

All the scalar products in Eq. (4) can be expressed via the invariant variables. Note also that

$$\mathrm{Im}[F_1 F_2^*] = \frac{\mathrm{Im}[G_E G_M^*]}{\tau - 1}, \qquad \tau = \frac{q^2}{4M^2}. \quad (6)$$

Equation (6) shows that the nucleon polarization due to the nonresonant mechanism gives information about the phase difference of the electric and magnetic form factors ($G_E$ and $G_M$ are the commonly used Sachs form factors [23], linearly related to $F_1$ and $F_2$).

Let us consider the effect originated by the longitudinal polarization of the nucleon when the direction of its 3-vector polarization (in the nucleon rest frame) is along $\mathbf{n} = -\mathbf{q}/|\mathbf{q}|$. In this case [1],

$$S_\mu = S_\mu^L = \frac{p_1\cdot q p_{1\mu} - M^2 q_\mu}{MK}, \qquad K = \sqrt{(p_1\cdot q)^2 - M^2 q^2}, \quad (7)$$

and the rhs of Eq. (4) simplifies as

$$L^{\mu\nu} H_{\mu\nu}^{(s)}(S^L) = -\frac{4 g^2_{\pi^0 N\bar{N}} \mathrm{Im}[F_1 F_2^*](s_1 + s_2 - 2M^2)(k_1 k_2 p_1 p_2)}{K(s_1 - M^2)^2(s_2 - M^2)} I(s_1, s_2, t_1, t_2), \quad (8)$$

with

$$I(s_1, s_2, t_1, t_2) = 2M^4 + (s - 2m_e^2)(s_1 + s_2) - 2s_1 s_2 + 2(s_1 t_1 + s_2 t_2) - 2M^2(s + t_1 + t_2 - 2m_e^2).$$

If the nucleon is polarized in such a way that the direction of the 3-vector polarization (in the nucleon rest frame) is along $[\mathbf{q}\times[\mathbf{k}\times\mathbf{q}]]/|[\mathbf{q}\times[\mathbf{k}\times\mathbf{q}]]|$, we have

$$S_\mu = S_\mu^T = \frac{(q^2 k\cdot p_1 - q\cdot p_1 k\cdot q)\tilde{p}_1^\mu + [(q\cdot p_1)^2 - q^2M^2]\tilde{k}^\mu}{KN}, \qquad \tilde{a}^\mu = a^\mu - \frac{a\cdot q q^\mu}{q^2}, \tag{9}$$

with

$$N = \sqrt{-(\mu k p_1 q)(\mu k p_1 q)}, \qquad N^2 = 2k\cdot q k\cdot p_1 q\cdot p_1 - q^2(k\cdot p_1)^2 - M^2(k\cdot q)^2 - m^2(q\cdot p_1)^2 + q^2M^2m^2.$$

In this case,

$$L^{\mu\nu}H^{(s)}_{\mu\nu}(S^T) = \frac{g^2_{\pi^0 N\bar{N}}\mathrm{Im}[F_1F_2^*](s_1+s_2-2M^2)(k_1k_2p_1p_2)}{MKN(s_1-M^2)^2(s_2-M^2)^2}J(s_1,s_2)I(s_1,s_2,t_1,t_2), \tag{10}$$

and

$$J(s_1,s_2) = 3M^6 - M^4(s+s_1+4s_2) + M^2[s(s_1+s_2)+s_2^2-3m^2s] + (s-s_2)(m^2s-s_1s_2).$$

In terms of invariant variables, the following relations hold:

$$\begin{aligned} K^2 &= \frac{1}{4}[(s-s_2)^2 - 2M^2(s+s_2) + M^4],\\ N^2 &= \frac{1}{4}\{-2M^6 + M^4(s+s_1+s_2+m^2) - M^2[s(s_1+s_2) - 2s_1s_2 - m^2(2s-s_1-s_2)]\\ &\quad + m^2[s(s_1+s_2)+s_1s_2-s^2] - m^4s + s_1s_2(s-s_1-s_2)\}. \end{aligned} \tag{11}$$

In both cases, the convolution of the leptonic and hadronic tensors contains the product $(k_1k_2p_1p_2)I(s_1,s_2,t_1,t_2)$. Therefore, all the dependence on the variables $t_1$ and $t_2$ of the spin-dependent part of the full differential cross section is contained in the factor $I(s_1,s_2,t_1,t_2)$ because the factor $(k_1k_2p_1p_2)$ just cancels the Gramian determinant in the phase space.

To calculate the corresponding double differential $(s_1,s_2)$ distribution, one needs to integrate with respect $t_1$ and $t_2$. This results in

$$\int_{t_{1-}}^{t_{1+}} dt_1 \int_{t_{2-}}^{t_{2+}} dt_2 I(s_1,s_2,t_1,t_2) = 0. \tag{12}$$

For $t_{1\pm}$ and $t_{2\pm}$, see Eqs. (19) and (20) in Ref. [1]. The $(s_1,s_2)$ distribution for the longitudinal and transverse nucleon polarizations vanishes, but this is not the case for the double $(t_2,s_1)$ distribution. Of course, after the integration over $t_2$, this last distribution also vanishes.

Here, we need to note that the factor $(k_1k_2p_1p_2)$ may be expressed in terms of the considered variables up to the sign only. To understand this problem, let us consider the center of mass system (c.m.s.) of the initial particles with the $z$ axis along the direction $\mathbf{k}_1$ and $\mathbf{p}_1$ in the plane $(x,z)$. In this system,

$$(k_1k_2p_1p_2) = \frac{s}{2}|\mathbf{p}_1|\cdot|\mathbf{p}_2|\sin\theta_1\sin\theta_2\sin\phi,$$

where $\theta_1(\theta_2)$ is the polar angle of the nucleon (antinucleon) and $\phi$ is the azimuthal angle of the antinucleon. We can express explicitly $\cos\phi$ in terms of invariant variables, which gives the quantity $\sin\phi$ up to the sign only.

Let us consider the normal nucleon polarization

$$S_\mu = S_\mu^N = \frac{(\mu k p_1 q)}{N}. \tag{13}$$

In this case, the convolution of the tensors is more complicated, and we report its expression in the limit $m_e \to 0$,

$$\begin{aligned} L^{\mu\nu}H^{(s)}_{\mu\nu}(S^N) &= \frac{g^2_{\pi^0 N\bar{N}}\mathrm{Im}[F_1F_2^*]}{Z}\{(2M^2-s_1-s_2)I(s_1,s_2,t_1,t_2)[(3M^4+M^2(s_1-3s_2)-s_1s_2+m^2s)C_1\\ &\quad + (m^2s - s_1s_2 + M^2(s_1+s_2) - M^4)C_2] + 4(s_2-M^2)C_3C_4\}, \end{aligned} \tag{14}$$

with

$$
\begin{aligned}
Z &= 4MN(s_1 - M^2)^2(s_2 - M^2)^2(s + M^2 - s_2),\\
C_1 &= (s_1 + s_2)[s_1(s_2 - t_1) - s_2 t_2] + s[s_1(t_1 - t_2 - 2s_2) + s_2(t_2 - t_1)]\\
&\quad + m^2 s(2s - s_1 - s_2 + 2t_1 + 2t_2) + 2M^6 - M^4(2s + s_1 + s_2 + 2t_1 + 2t_2)\\
&\quad + M^2[-2m^2 s - 2s_1 s_2 + 2s(s_1 + s_2) + 3s_1 t_1 + s_2 t_1 + s_1 t_2 + 3s_2 t_2],\\
C_2 &= s_1^2(t_1 - s_2) + (2s - s_2)[s(t_1 - t_2) + s_2 t_2] + s_1[s_2^2 + s(t_2 - 3t_1) + s_2(t_2 - t_1)]\\
&\quad + m^2 s(s_1 - s_2 + 2t_1 - 2t_2) + M^2[-2s(s_1 - s_2 + 2t_1 - 2t_2) - (s_1 - s_2)(t_1 + t_2)] + M^4(s_1 - s_2),\\
C_3 &= s_1 s_2(s_1 + s_2 - s) + m^2[m^2 s - M^4 + M^2(s_1 + s_2 - 2s) + s^2 - s_1 s_2 - s(s_1 + s_2)]\\
&\quad + 2M^6 - M^4(s + s_1 + s_2) + M^2[s(s_1 + s_2) - 2s_1 s_2],\\
C_4 &= m^2[2s(s_1 - M^2)(M^2 + s - s_2)] + 2M^8 - 2M^6[3s + 2(t_1 + t_2)]\\
&\quad + 2M^4[-2s_1 s_2 + 2s_1 t_1 + t_1^2 + 2s_2 t_2 + 2t_1 t_2 + t_2^2 + s(3s_1 + 2s_2 + 3t_1 + t_2)]\\
&\quad - M^2[(2s^2(t_1 - t_2) + 4(t_1 + t_2)(s_1(t_1 - s_2) + s_2 t_2) + s(s_1^2 + s_2^2 + 7s_1 t_1 + s_2 t_1 + 2t_1^2 + 3s_1 t_2 + 5s_2 t_2 - 2t_2^2)]\\
&\quad + s^2(s_1 + s_2)(t_1 - t_2) + 2[s_1(t_1 - s_2) + s_2 t_2]^2\\
&\quad + s[s_1^2(3t_1 - 2s_2) - s_1 s_2(t_1 - 5t_2) + 2s_1 t_1(t_1 - t_2) + s_2 t_2(s_2 + 2t_1 - 2t_2)]. \qquad (15)
\end{aligned}
$$

Note that in this case one has no problem with the ambiguity, so further we focus on the normal polarization.

## III. SINGLE-SPIN ASYMMETRY

The single-spin beam asymmetry is defined by the convolution of the spin-dependent antisymmetrical part of the leptonic tensor,

$$
L_{\mu\nu} = L_{\mu\nu}^{(s)} + L_{\mu\nu}^{(a)}, \quad L_{\mu\nu}^{(s)} = -q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}),
$$
$$
L_{\mu\nu}^{(a)} = 2im_e(\mu\nu\eta q), \qquad (16)
$$

where $m_e$ is the electron mass and $\eta$ is the 4-vector of its longitudinal polarization,

$$
q = k_1 + k_2, \quad (\mu\nu\eta q) = \varepsilon^{\mu\nu\lambda\rho}\eta_\lambda q_\rho, \quad \varepsilon^{0123} = +1,
$$

and the antisymmetrical spin-independent part of the hadronic tensor,

$$
H_{\mu\nu} = \frac{1}{2}(H_{\mu\nu}^{(s)}(0) + H_{\mu\nu}^{(a)}(0)) + H_{\mu\nu}^{(s)}(S) + H_{\mu\nu}^{(a)}(S). \qquad (17)
$$

where we use the same notations as in Ref. [1].

Both spin-independent and spin-dependent parts of the hadronic tensor are defined in Ref. [1] in the general case in terms of bilinear combinations of invariant amplitudes and corresponding independent tensor structures. Note, that for the nonresonant mechanism the spin-independent antisymmetrical part vanishes: $H_{\mu\nu}^{(a)}(0) = 0$. That is why in the present paper we do not consider the single-spin beam asymmetry. However, this situation is not general. For example, the decay of the virtual photon into a real $\pi^0$ and a virtual vector meson $V^0$, which interacts then with the real nucleon-antinucleon pair (see Fig. 2), leads to a nonzero value of $H_{\mu\nu}^{(a)}(0)$ and, consequently, to nonzero single-spin beam asymmetry.

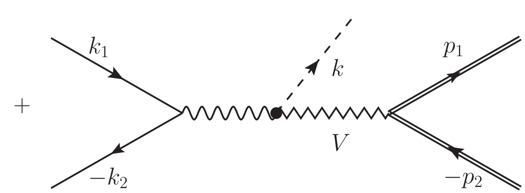
Thus, the considered single-spin effect arises due to the nucleon polarization and is defined by convolution of the symmetrical spin-independent part of the leptonic tensor and symmetrical spin-dependent part of the hadronic one:

$$
L^{(s)\mu\nu}H_{\mu\nu}^{(s)}(S).
$$

The double-spin effect is defined by

$$
L^{(a)\mu\nu}H_{\mu\nu}^{(a)}(S).
$$

In this paper, we study only the single-spin effects due to the nucleon polarization.



(c)

FIG. 2. The Feynman diagrams which describe the decay $\gamma^* \to \pi^0 + V^0$ with the subsequent transition $V^0 \to N + \bar{N}$.

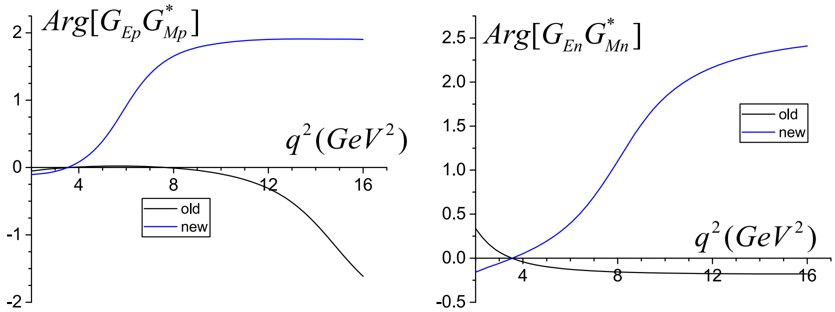



FIG. 3. Phase difference $\mathrm{Arg}[G_E G_M^*]$ (in radians) for two different parameterizations of the electric and magnetic nucleon form factors.

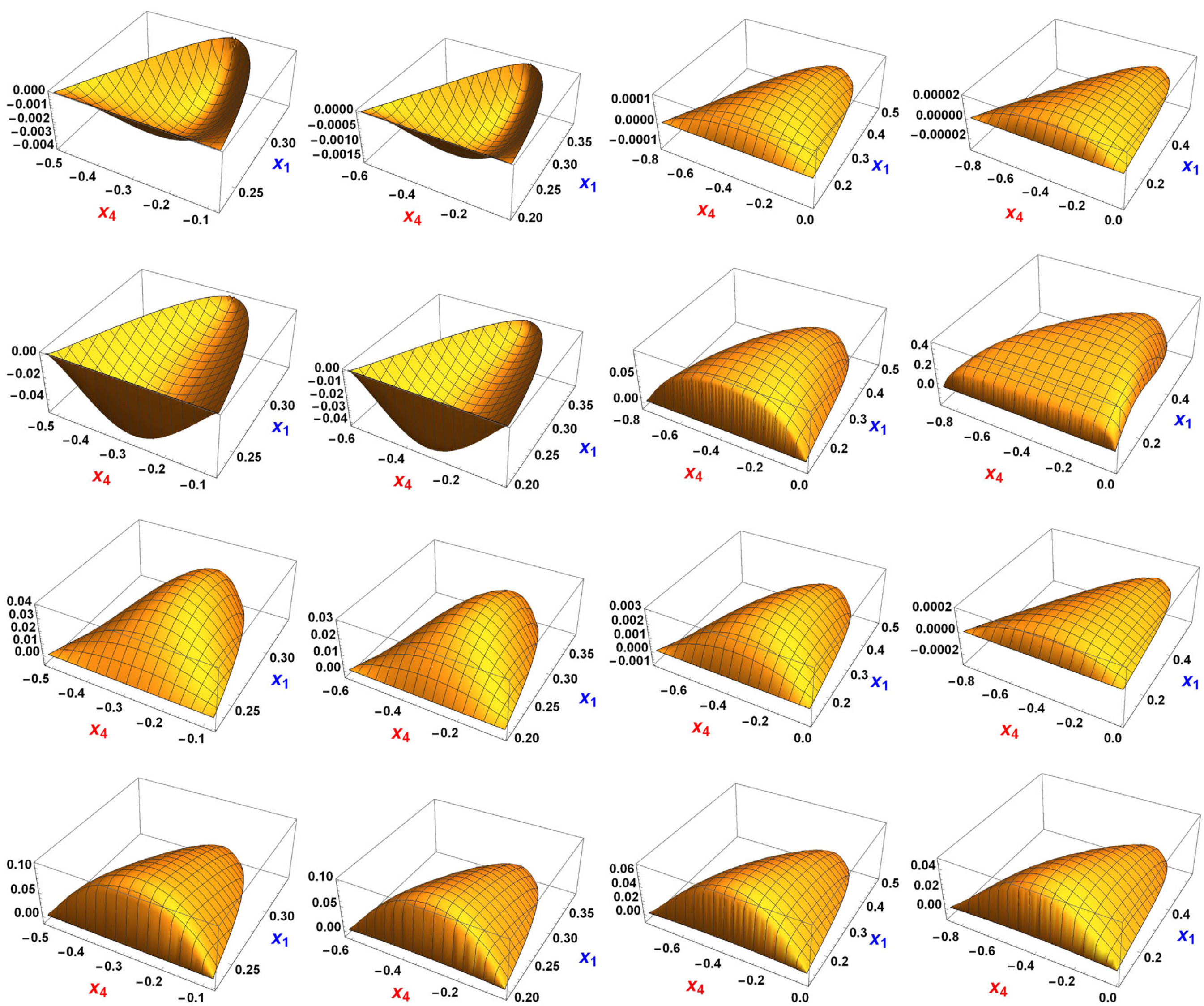


FIG. 4. First row: double differential distribution for the spin-dependent part of the cross section $d\sigma^N$ in nb units over the dimensionless invariant variables $x_1 = s_1/s$ and $x_4 = t_2/s$. Second row: corresponding proton normal polarization $P^N$ in the process $e^+ + e^- \to \pi^0 + p + \bar{p}$ for $s = 5$, 6, 10, 16 GeV$^2$ from left to right, respectively. Third and fourth rows: same as the first and second rows but for the process $e^+ + e^- \to \pi^0 + n + \bar{n}$.

## IV. DOUBLE DIFFERENTIAL DISTRIBUTIONS

We consider the double differential distributions $(t_2, s_1)$, $(s_1, s_2)$, and $(s_1, s_{12})$. We analyze the spin-dependent part of the cross section $d\sigma^N$ and the nucleon normal polarization defined as

$$P^N = \frac{d\sigma^N}{d\sigma}, \tag{18}$$

where $d\sigma$ is the unpolarized differential cross section. We obtain analytical expressions for these distributions, and we list below only the $(s_1, s_2)$ one [see Eq. (19)]. The $(s_1, s_{12})$ distribution can be derived from this one by simple algebraic exercise, and the expression for the $(t_2, s_1)$ distribution is too lengthy to be given in this paper.

The corresponding numerical results are plotted in Figs. 4–6 for both channels, $\pi^0 p\bar{p}-$ and $\pi^0 n\bar{n}-$, using the dimensionless invariant variables $x_4 = t_2/s$, $x_{1,2} = s_{1,2}/s$, and $x_{12} = s_{12}/s$.

Note that nucleon polarization depends on the single combination of electromagnetic form factors $\mathrm{Im}[G_E G_M^*]$ that is proportional to $\sin(\mathrm{Arg}[G_E G_M^*])$. It means that the corresponding measurements probe the phase difference between electric and magnetic form factors. This phase difference depends strongly on parametrization of the form factors. In Fig. 3, we show the dependence of the $\mathrm{Arg}[G_E G_M^*]$ on $q^2$ for two different choices of form factors, the one used in this paper and the one labeled in Ref. [1] as the "new" version [24]. These plots show that the predictions for the nucleon polarizations in the processes (1) depend radically on the form factor choice, which increases the interest of their measurement.

To decrease the number of curves in the figures, we make the choice of using the nucleon electromagnetic form factors labeled in Ref. [1] as the "old" version (only in the next section, we show plots of the polarization $P^N$ for single differential distributions over $x_4$, $x_1$, and $x_{12}$ calculated with the new version).

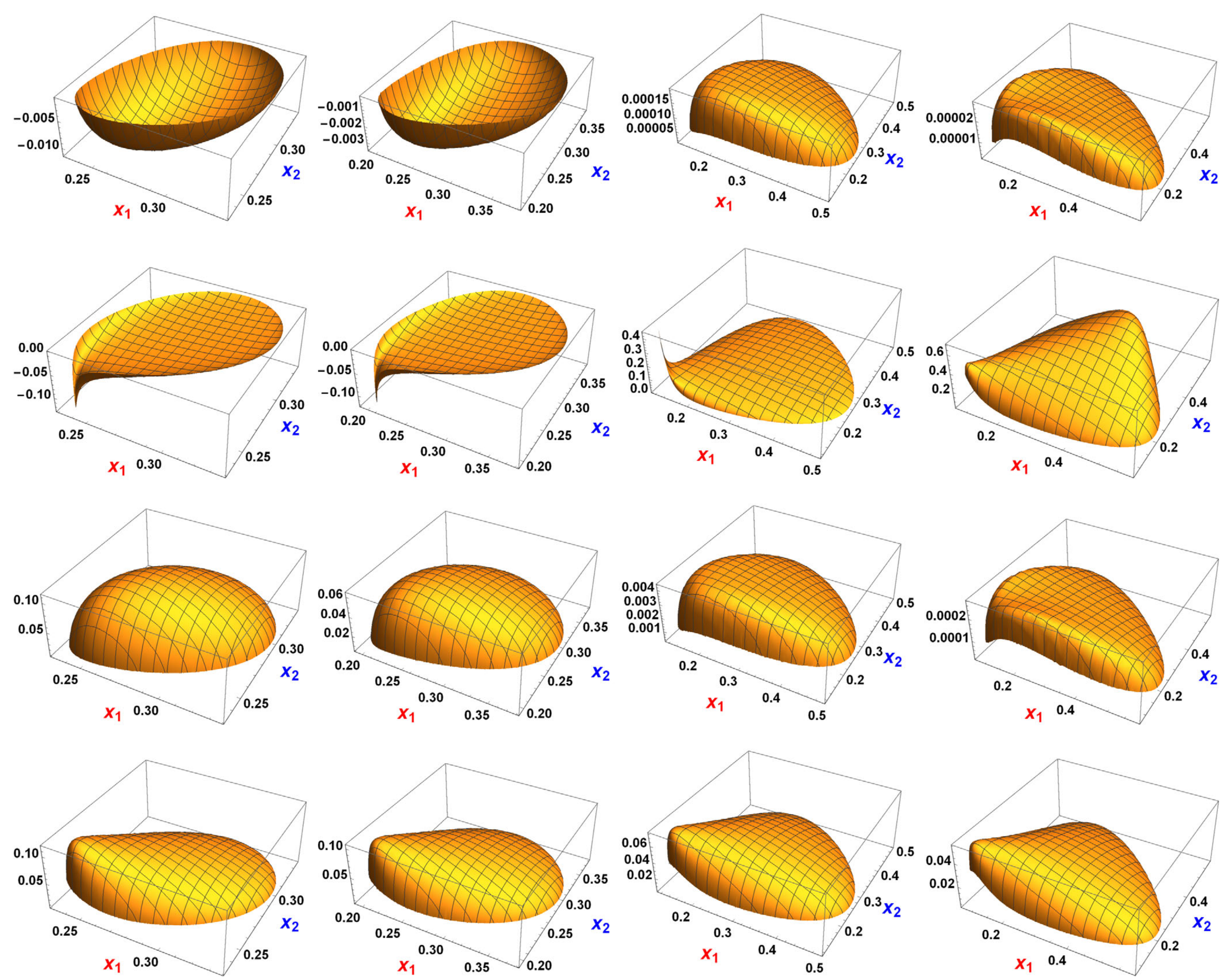



FIG. 5. The same as in Fig. 4 but for the double $(x_1, x_2)$ distribution.

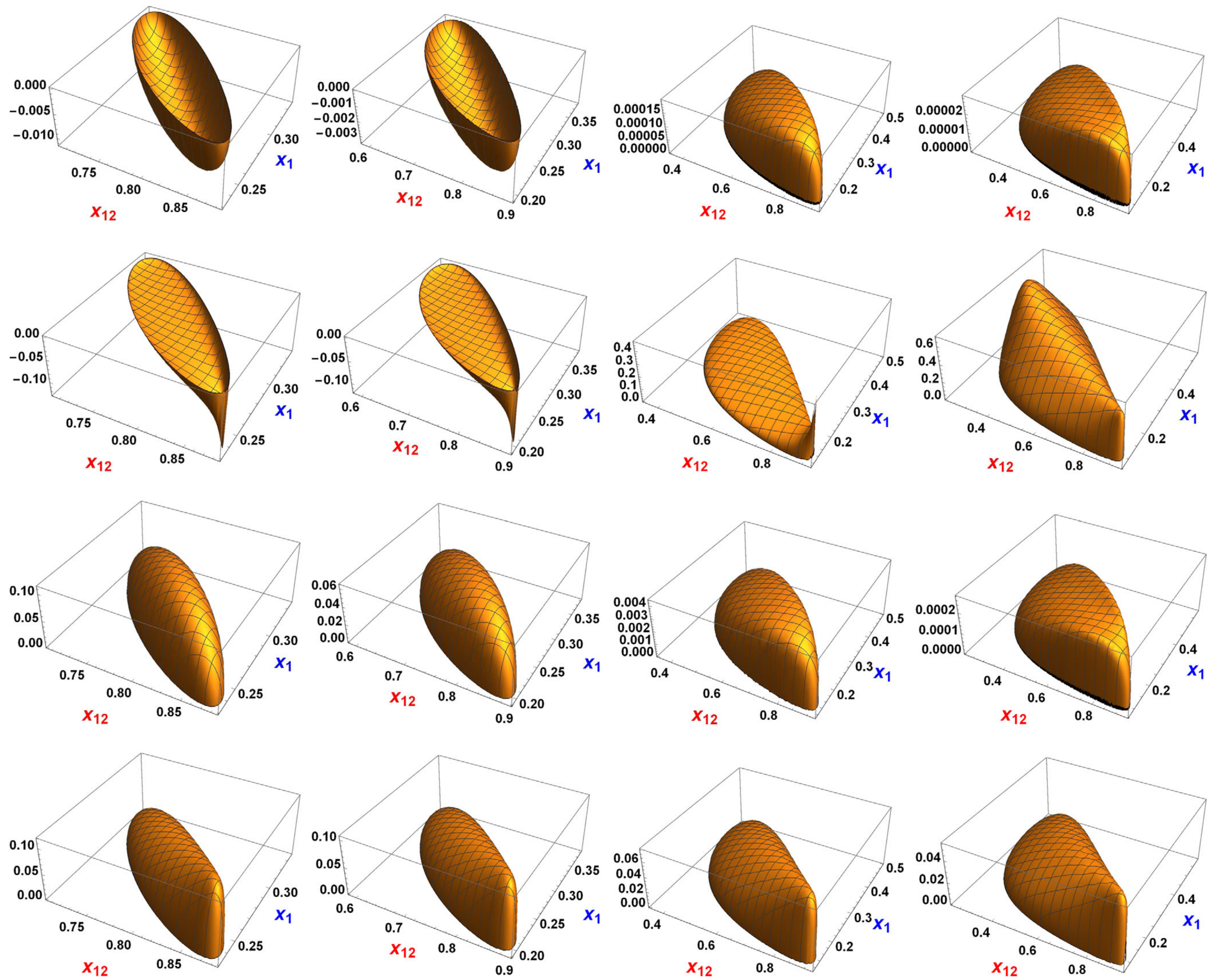


FIG. 6. The same as in Fig. 4 but for the $(x_1, x_{12})$ double distribution.

The double differential $(t_2, s_1)$ distribution of the spin-independent part of the cross section is symmetrical under the substitution $t_2 \to t_1$, $s_1 \to s_2$, but the spin-dependent part loses this symmetry.

Consider now the $(s_1, s_2)$ and $(s_1, s_{12})$ distributions. After integrating over $t_1$ and $t_2$, the spin-dependent part of the cross section simplifies,

$$\frac{d\sigma^N}{ds_1 ds_2} = \frac{g^2_{\pi^0 N\bar{N}}\alpha^2 MN(s_1 + s_2 - 2M^2 - 2m^2)\mathrm{Im}[G_E G_M^*]}{3\pi s^3(4M^2 - s)(M^2 - s_1)(M^2 - s_2)}, \tag{19}$$

in the limit $m_e \to 0$ and where the quantity $N$ is defined in Eq. (9). This distribution is symmetrical under the change $s_1 \rightleftarrows s_2$. The corresponding $(s_1, s_{12})$ distribution can be obtained from (19) by simple substitution $s_{12} = s + 2M^2 + m^2 - s_1 - s_2$.

In Figs. 5 and 6, the $(x_1, x_2)$ and the $(x_1, x_{12})$ distributions are plotted for both nucleon channels.

## V. SINGLE DIFFERENTIAL DISTRIBUTIONS

Let us consider the single differential distributions. In the case of the $s_1$ and the $s_{12}$ distributions, the expressions of the spin-dependent parts of the cross section are

$$\frac{d\sigma^N}{ds_{12}} = \frac{g^2_{\pi^0 N\bar{N}}\alpha^2 M(s - s_{12} - m^2)\mathrm{Im}[G_E G_M^*]}{6s^3(s - 4M^2)(s - s_{12} + m^2)} Z_1,$$
$$Z_1 = -(s + m^2 - s_{12})\sqrt{s_{12}} + 2\sqrt{M^2[(s - s_{12})^2 + m^4 - 2m^2(s + s_{12})] + m^2 s s_{12}}, \tag{20}$$

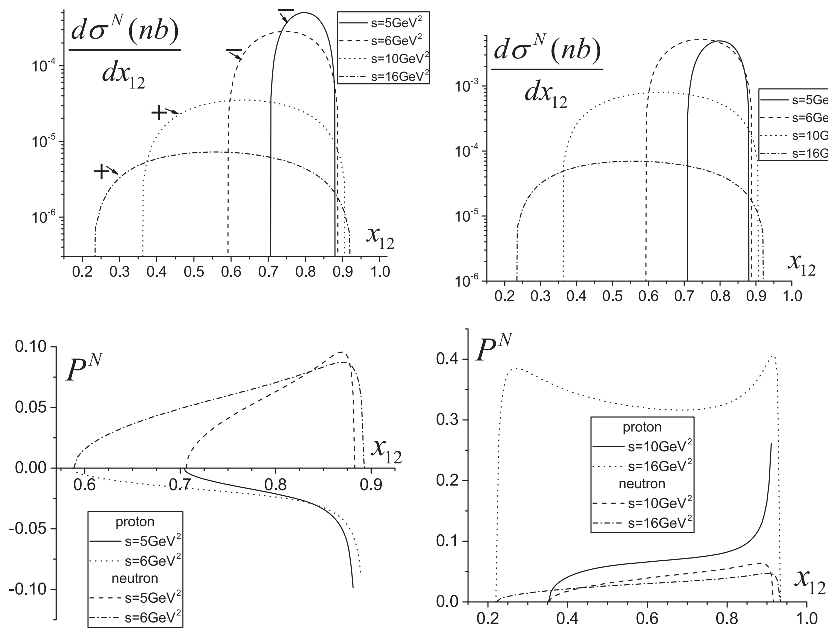



FIG. 7. First row: $x_{12}$ distribution of the spin-dependent part of the cross section for the $\pi^0 p\bar{p}$ channel (left panel) and $\pi^0 n\bar{n}$ channel (right panel) as defined by Eq. (20); the sign $+(-)$ indicates that the corresponding quantity is positive (negative). Second row: the corresponding nucleon normal polarization is plotted.

$$\frac{d\sigma^N}{ds_1} = \frac{g^2_{\pi^0 N\bar{N}}\alpha^2 M \mathrm{Im}[G_E G_M^*]}{48 s^3 (s-4M^2)(M^2-s_1)s_1^{3/2}} Z_2,$$

$$\begin{aligned} Z_2 = {} & M^8 - 2M^6(s+4s_1+m^2) + M^4[s^2+6ss_1+10s_1^2+m^2(4s-2s_1)+m^4] \\ & + s_1^2(s^2+2ss_1-3s_1^2) - 2M^2[ss_1(s+3s_1)+m^2(s^2-6ss_1+3s_1^2+m^4(s-3s_1)] \\ & + m^4(s^2-10ss_1-7s_1^2) - 2m^2 s_1(s^2-5s_1^2) \\ & + 8(M^2+2m^2-s_1)s_1^{3/2}\sqrt{M^6-2M^4 s_1+M^2(s_1^2-3m^2 s)+m^2 s(s-s_1+m^2)}. \end{aligned} \tag{21}$$

In Figs. 7 and 8, we plot the spin-dependent part of the cross section and the nucleon polarization for both channels, $\pi^0 p\bar{p}$ and $\pi^0 n\bar{n}$, as functions of the dimensionless variables $x_{12}$ and $x_1$, respectively. The differential distributions over $x_4$ are plotted in Fig. 9.

In Fig. 10, we plot the single distributions of the polarization $P^N$ with respect to the dimensionless variables $x_1$, $x_{12}$, and $x_4$ calculated with another choice of form factors (new version in Ref. [1]). Comparing with the corresponding curves in Figs. 7–9, one can see that the predictions for $P^N$ depend essentially on form factors; therefore, the measurement can give additional information about proton and neutron electromagnetic form factors.

## VI. DISCUSSION

A systematic investigation of the baryon resonances has begun at the Beijing Electron-Positron Collider (BEPCII) from the BESIII Collaboration [25,26]. Some results of these experiments are compiled in Ref. [27]. A number of experiments were devoted to the measurement of the reaction $e^+ + e^- \to p + \bar{p} + \pi^0$. In Ref. [28], this reaction was studied in the vicinity of the $\psi(3770)$ resonance. Later on, the measurement of this reaction was performed at higher energies, namely, for $(4.008 \leq \sqrt{s} \leq 4.600)$ GeV, in the vicinity of the $Y(4260)$ resonance [29]. The Born cross section of the reaction $e^+ + e^- \to R \to p + \bar{p} + \pi^0$, where R is the $\psi(3770)$ or $Y(4260)$ resonance, is the sum of two contributions: continuum (nonresonant) and resonant. The parameters of the continuum and the resonance (including the phase between the resonant and continuum production amplitudes) are free parameters of a fit of the data. Therefore, the precision of the determination of the resonance parameters depends on the knowledge of the continuum cross section. A number of single and double differential distributions (in the case of unpolarized particles) were calculated [1] analytically, and numerical estimates were given for the $p\bar{p}\pi^0$ and $n\bar{n}\pi^0$ channels, for the nonresonant contribution, in the energy range from

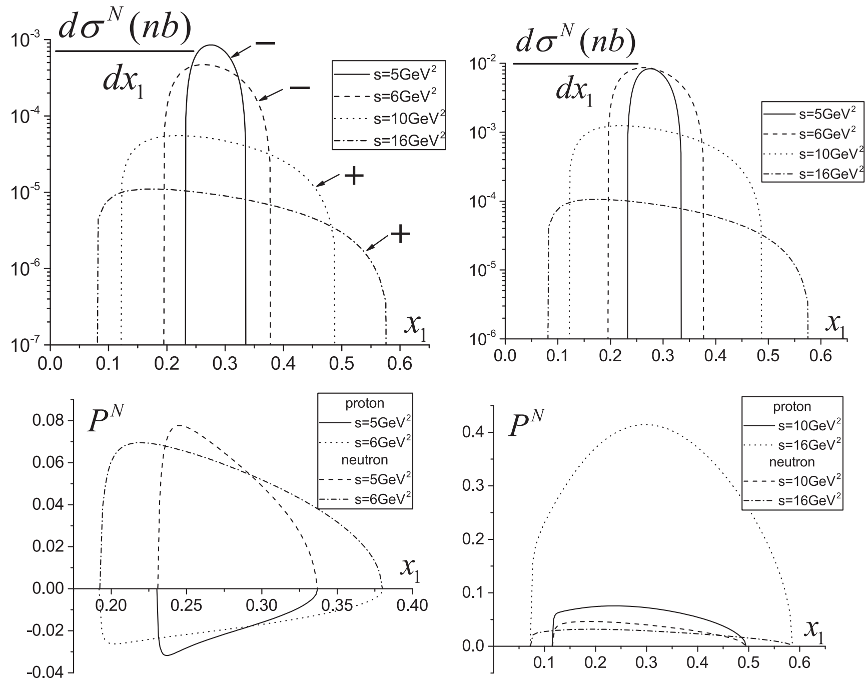



FIG. 8. The same as in Fig. 8 but for the $x_1$ distribution as defined by Eq. (21).

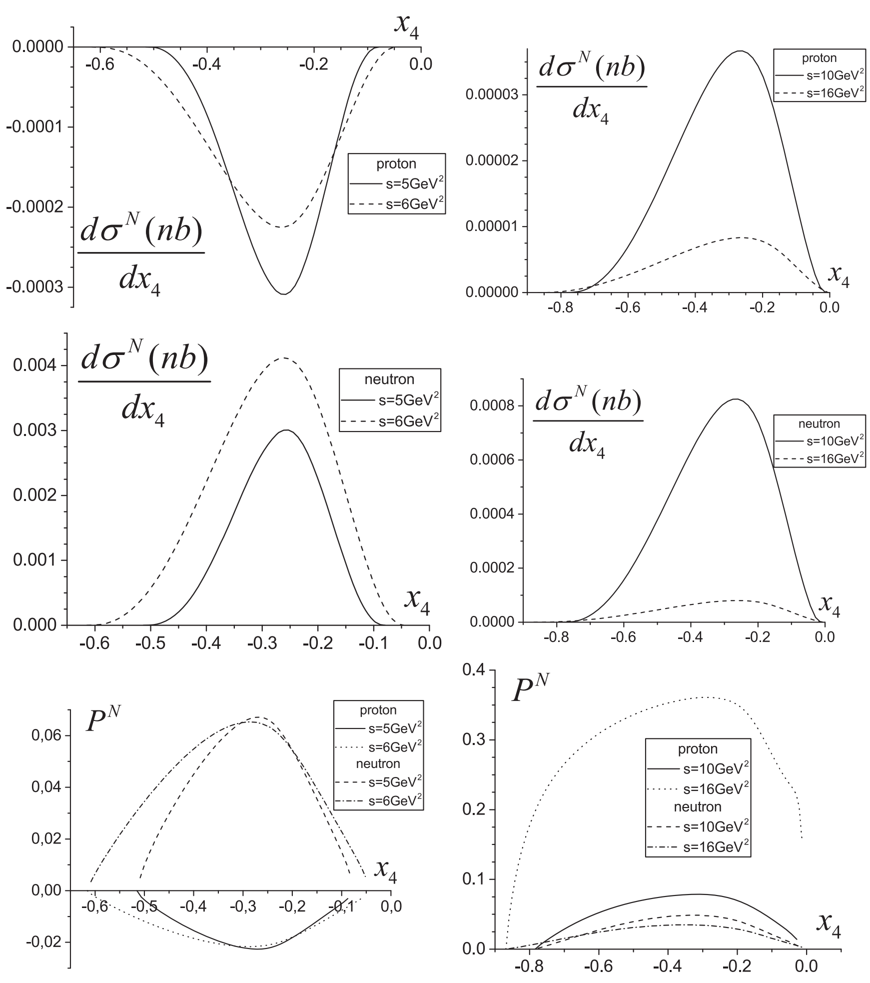



FIG. 9. Differential distributions over $x_4$ of the $d\sigma^N$ and corresponding nucleon polarization $d\sigma^N/d\sigma$.

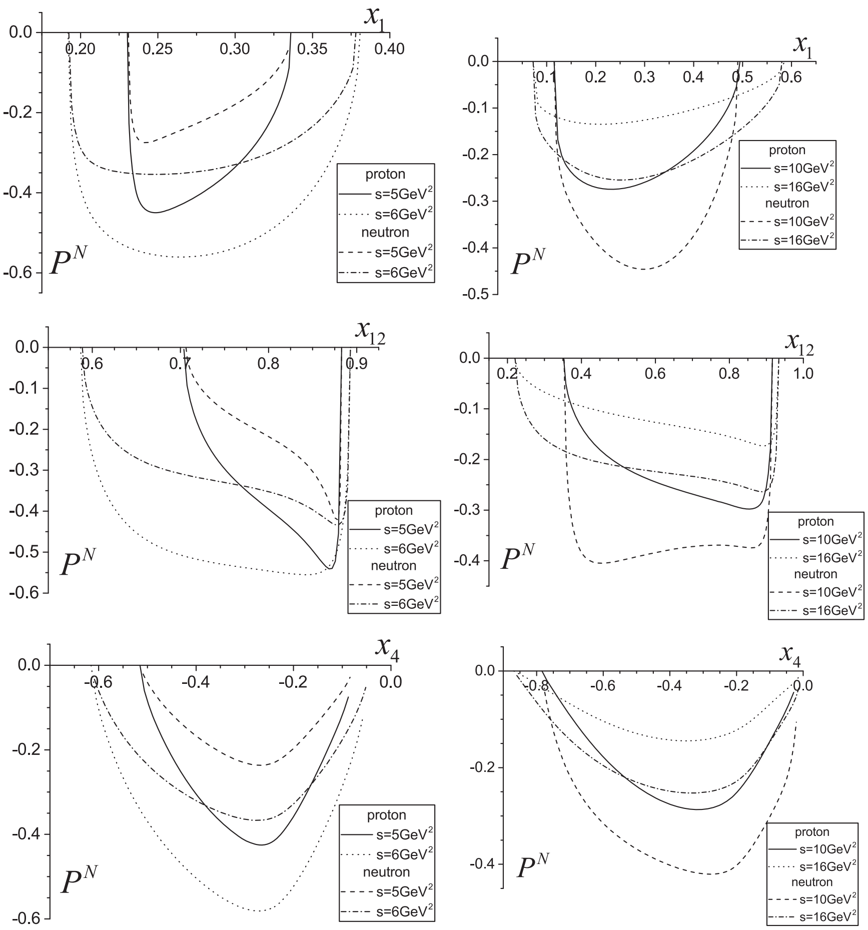


FIG. 10. Differential distributions over $x_1$, $x_{12}$, and $x_4$ of the nucleon polarization $P^N$ for both $e^+e^- \to p\bar{p}\pi^0$ and $e^+e^- \to n\bar{n}\pi^0$ channels calculated with form factors labeled in Ref. [1] as the new version.

the threshold up to $s = 16\ \mathrm{GeV}^2$. Here, we consider the contribution to the cross section from the polarization of the nucleon. In the frame of the developed approach, the longitudinal and transverse nucleon polarizations can be obtained up to their signs only.

The nucleon spin-dependent states: longitudinal $L$, transverse $T$, and normal $N$ are defined in such a way that, in the rest system, the nucleon spin 3-vector belongs to the hadronic plane $(\mathbf{p_1}, \mathbf{k})$ for longitudinal and transverse states, whereas in the case of the normal state, it is perpendicular to this plane. In an arbitrary system, it is convenient to express correspondingly the nucleon spin 4-vectors through the hadron momenta, giving the possibility to obtain all the nucleon polarizations in terms of invariant variables. The corresponding nucleon spin 4-vectors $S^L$, $S^T$, and $S^N$ are given by Eqs. (7), (9), and (13).

We choose the coordinate system in such a way that one of the final 3-momenta belongs to a definite plane (for example, the $zx$ one). Such a choice corresponds to integration over one azimuthal angle. Then, the full differential cross section is determined by Eqs. (2) and (3). The convolution of the symmetrical spin-independent part of the leptonic tensor and symmetrical spin-dependent part of the hadronic one is obtained analytically.

The different double and single differential distributions of the normal polarization in the reaction $e^+ + e^- \to p + \bar{p} + \pi^0 (e^+ + e^- \to n + \bar{n} + \pi^0)$ on various invariant variables, in the frame of the nonresonant mechanism, is derived. The numerical estimations are performed for energies ranging from the threshold up to $s = 16\ \mathrm{GeV}^2$, taking into account the contribution of the nonresonant mechanism in the unpolarized case, which was investigated earlier. The calculation is performed at different values of the variable $s$ (from 5 to $16\ \mathrm{GeV}^2$) using the nucleon electromagnetic form factors labeled in Ref. [1] as the old version. We plot the spin-dependent part of the cross sections and the corresponding nucleon polarizations to better understand the size of the cross section, which is useful for evaluating the number of events to be collected.

The polarizations $P^N(x_1)$, $P^N(x_{12})$, and $P^N(x_4)$ change (do not change) sign, increasing the energy near $s = 8\ \mathrm{GeV}^2$, in the case of the $\pi^0 p\bar{p}$ channel ($\pi^0 n\bar{n}$ channel), but they do not change sign varying $x_1$, $x_{12}$, and $x_4$, respectively. The magnitude of all polarizations $P^N$

is approximately the same for both channels except for the abnormally large proton polarizations at $s = 16\ \mathrm{GeV}^2$, which might be attributed to the smallness of the unpolarized cross section in this kinematics.

The spin-dependent part of the cross section $d\sigma^N/dx_{12}$ has a bell-like form, and the width of the peak becomes larger when $s$ increases. The magnitude of this quantity for the $n\bar{n}\pi^0$ channel is larger than for the $p\bar{p}\pi^0$ channel. The polarization $P^N$ for $n\bar{n}\pi^0$ is larger than $P^N$ for $p\bar{p}\pi^0$. The quantity $d\sigma^N/dx_{12}$ is positive for $n\bar{n}\pi^0$ and changes sign at $s \approx 8\ \mathrm{GeV}^2$ for the $p\bar{p}\pi^0$ channel. The polarization $P^N$ is positive for the $n\bar{n}\pi^0$ channel and changes sign for the $p\bar{p}\pi^0$ channel.

The dependences of $d\sigma^N/dx_1$ and of the polarization $P^N$ on the variable $x_1$ are similar to the cases above (for the variable $x_{12}$). The differential cross section $d\sigma^N/dx_4$ and the polarization $P^N(x_4)$ do not change sign as a function of the variable $x_4$, but they change sign as a function of the variable $s$ for the $p\bar{p}\pi^0$ channel.

## VII. CONCLUSIONS

The normal nucleon polarization in the reactions $e^+ + e^- \to p + \bar{p} + \pi^0$ and $e^+ + e^- \to n + \bar{n} + \pi^0$ is calculated in frame of the nonresonant mechanism. The corresponding contribution is illustrated in Fig. 1, in which the pion is emitted by the nucleon or the antinucleon. The present work extends the calculation of Ref. [1], in which the general analysis of the differential cross section and of different polarization observables was performed in the one-photon-annihilation approximation, taking into account the conservation of the hadron electromagnetic current and the P invariance of the hadron electromagnetic interaction.

We define the nucleon polarizations as the ratio of the spin-dependent parts of the cross section to the unpolarized cross section and study in detail their double and single distributions over selected invariant variables. The longitudinal and transverse polarizations are proportional to the factor $(k_1k_2p_1p_2)$, which can be expressed in terms of invariant variables up to the sign only; therefore, we do not give any numerical results for these observables.

The spin-dependent part of the cross section is driven by the factor $\mathrm{Im}[G_E G_M^*]$. The numerical results on the normal polarization depend on the choice of the nucleon electromagnetic form factors. In particular, this observable gives additional information about the phase difference between the electric and magnetic form factors and strongly constrains nucleon models.

Several parametrizations of form factors exist (for a review, see Ref. [30]). We choose here two parametrizations based on vector meson dominance [24,31]. They contain a small number of parameters that are fitted to reproduce the data (known at that time) both for proton and neutron, in spacelike as well as in timelike regions, but it turns out that they also reproduce qualitatively the most recent data on the proton timelike electric and magnetic form factors (in moduli) [32]. It is also noticeable that, by construction, an imaginary part arises naturally by analytical prolongation in the timelike region and that the large-$q^2$ behavior predicted by perturbative QCD is fulfilled. Choosing two versions of this vector meson dominance model allows us to point out the effect of the relative phase of electric and magnetic form factors.

The present work is useful for modeling the background contribution in the study of nucleon resonances driving Monte Carlo simulations in the experimental analysis. The complexity of these analysis is due to the fact that the final particles can be produced in different intermediate states. An interplay among experimental distributions and Monte Carlo input, following chosen physics-driven assumptions, is necessary [28]. The significance of the normal polarization pointed out in the present paper suggests future experiments including final hadron polarimetry.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