Hidden-flavor pentaquarks

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We have recently studied hidden-charm pentaquarks, $c\bar{c}qqq$, using the dynamical correlations between the heavy quarks arising from the Coulomb-like nature of the short-range interaction. A pattern was obtained that compares well with the experimental data. We extend our description to other flavor sectors which can be framed within the same type of structures discussed in the original paper. A detailed comparison is made with other results in the literature and with experimental data. The predictions will be a useful tool to discriminate between different models of multiquark system dynamics.

I. INTRODUCTION

The recent findings in the heavy-hadron spectra have become both a theoretical challenge and a suitable test bench for trying to understand QCD realizations in the nonperturbative regime [1–12]. It appears to be undeniable that more complex quark structures allowed by QCD than the simplest quark-antiquark (meson) or three-quark (baryon) clusters proposed by Gell-Mann [13], the so-called multiquarks, are being found in the heavy-hadron experimental data [14,15].

For multiquark states with manifestly exotic quantum numbers, as is the case of doubly-heavy tetraquarks, both lattice QCD approaches [16,17] and constituent models [18,19] predict a very small number of states restricted to very specific configurations. To give the big picture of multiquark states with ordinary quantum numbers it has been suggested the possibility of the existence of correlations between the constituents [20–24].

In a recent paper [25], we explored a theoretical scenario where the dynamics of a multiquark system remains marked by correlations between heavy flavors dictated by QCD [26], that turn the five-body problem into a more tractable three-body problem. The most suitable system for developing and testing our model was the hidden-charm pentaquarks, $c\bar{c}qqq$. We obtained a pattern that compares well with the experimental data available in this sector. This

work is a natural extension of the analysis performed in Ref. [25] to study hidden-flavor pentaquarks which present the same type of structure in other flavor sectors and, therefore, can benefit from the correlations induced by the color-Coulomb potential. In particular, we address $b\bar{b}qqq$ states and also hidden-charm and hidden-bottom pentaquarks with a strange quark, $c\bar{c}qqs$ and $b\bar{b}qqs$.

The structure of the paper is the following. In the next section we briefly review the model; the interacting potential between the quarks, the Hilbert space arising from the correlations between the heavy flavors, and the solution of the Faddeev equations for the bound state three-body problem. In Sec. III we present and discuss our results. Finally, our conclusions are summarized in Sec. IV.

II. THE MODEL

A. Pentaquarks wave function

We study hidden-flavor pentaquarks $Q\bar{Q}qq'$, with $Q = b$ or c, $q = u$ or d, and $q' = u\,d$ or s. Models based on the attractive character of a qq pair in a color-$\bar{3}$ state have been widely explored in the literature [20–23]. If a Qq color-$\bar{3}$ diquark has a binding proportional to m_q, in the same units a $Q\bar{Q}$ color-1 has a binding proportional to $2M_Q$. Thus, the color Coulomb-like interaction between the components of a hidden-flavor pentaquark favors a $Q\bar{Q}$ color singlet instead of a color octet [26], uniquely determining its color wave function. Antisymmetrization constraints allow to identify the different vectors that contribute to any (I, J) pentaquark for the lowest-lying states, i.e., in the case of a fully symmetric radial wave function,

$$
\begin{aligned}
\Psi^{(I,J)}_{\text{Pentaquark}} = &\{\mathbf{3}_c, i_1, s_1 = 1/2\}_q \otimes \{\mathbf{1}_c, i_2 = 0, s_2\}_{(Q\bar{Q})} \\
&\otimes \{\bar{\mathbf{3}}_c, i_3 = s_3, s_3\}_{(qq)},
\end{aligned} \tag{1}
$$

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TABLE I. Quantum numbers of the different components resulting in a (I,J) $Q\bar{Q}qqq$ hidden-flavor pentaquark containing one of the most attractive QCD channels, according to Eq. (1). The numbers in parenthesis stand for the isospin of $Q\bar{Q}qqs$ pentaquarks. See text for details.

I	J	s_1	s_2	s_3	Vector
	1/2		0	0	v_1
	1/2		1	0	v_2
1/2(0)	1/2	1/2	0	1	v_3
	3/2		1	0	w_1
3/2(1)	3/2	1/2	0	1	w_3

where $i_1 = 1/2$ for $Q\bar{Q}qqq$ pentaquarks and $i_1 = 0$ for $Q\bar{Q}qqs$ states.

Table I of Ref. [25] summarizes the possible value of the quantum numbers leading to an allowed (I,J) hidden-flavor pentaquark. Quark correlations dominating the QCD phenomena [26] hint to the most favorable states that can be observed in nature. First, the very strong quark-antiquark correlation in the color-, flavor-, and spin-singlet channel $\{\mathbf{1}_c, \mathbf{1}_f, 0_s\}$ which can be viewed as the responsible for chiral symmetry breaking. The attractive forces in this channel are so strong that condenses in the vacuum, breaking $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry. The next most attractive channel in QCD seems to be the color antitriplet, flavor antisymmetric, spin singlet $\{\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f, 0_s\}$, that would select the qq configurations most important spectroscopically. Thus, we summarize in Table I those states that contain at least one the most attractive QCD channels, i.e., a diquark with spin zero. s_1 stands for the spin of the single light quark (with isospin $1/2$ for u, d, and 0 for s), s_2 denotes the spin of the heavy quark-antiquark pair (with isospin zero), and finally, s_3 represents the spin of the light quark pair (with the restrictions imposed by the Pauli principle such that $s_3 = i_3$). The notation in the last column will later be used to identify the wave function of the different pentaquarks.

B. Quark-quark interaction

To perform exploratory studies of systems with more than three-quarks it is of basic importance to work with models that correctly describe the two- and three-quark problems of which thresholds are made of. In Ref. [25] we have adopted a generic constituent model, containing chromoelectric and chromomagnetic contributions, tuned to reproduce the masses of the mesons and baryons entering the various vectors, the so-called AL1 model by Semay and Silvestre-Brac [27]. It has been widely used in a number of exploratory studies of multiquark systems [18,19,28–32]. It includes a standard Coulomb-plus-linear central potential, supplemented by a smeared version of the chromomagnetic interaction,

$$V(r) = -\frac{3}{16}\tilde{\lambda}_i.\tilde{\lambda}_j\left[\lambda r - \frac{\kappa}{r} - \Lambda + \frac{V_{SS}(r)}{m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j\right],$$

$$V_{SS} = \frac{2\pi\kappa'}{3\pi^{3/2}r_0^3}\exp\left(-\frac{r^2}{r_0^2}\right), \quad r_0 = A\left(\frac{2m_i m_j}{m_i + m_j}\right)^{-B}, \quad (2)$$

where $\lambda = 0.1653$ GeV^2, $\Lambda = 0.8321$ GeV, $\kappa = 0.5069$, $\kappa' = 1.8609$, $A = 1.6553$ GeV^{B-1}, $B = 0.2204$, $m_u = m_d = 0.315$ GeV, $m_s = 0.577$ GeV, $m_c = 1.836$ GeV, and $m_b = 5.227$ GeV. Here, $\tilde{\lambda}_i.\tilde{\lambda}_j$ is a color factor, suitably modified for the quark-antiquark pairs. Note that the smearing parameter of the spin-spin term is adapted to the masses involved in the quark-quark or quark-antiquark pairs. The parameters of the AL1 potential are constrained in a simultaneous fit of 36 well-established mesons and 53 baryons, with a remarkable agreement with the data, as can be seen in Table 2 of Ref. [27]. It is worth to note that although the χ^2 obtained in Ref. [27] with the AL1 potential is slightly larger than the one obtained with other models, this is essentially because a number of resonances with high angular momenta were considered. The AL1 model is very well-suited to study the low-energy hadron spectra [33]. The spin-color algebra of the five-quark system has been worked elsewhere [29,34].

C. Faddeev equations

The flavor independence of the interacting potential makes the five-body problem to factorize into a three-body problem that can be exactly solved by means of the Faddeev equations. We follow the method developed in Ref. [35], that it is described in detail in Ref. [25] for S- and P-wave states. Three-body states in which a particle has a given spin can only couple to other three-body states in which that particle has the same spin, since the spinors corresponding to different eigenvalues are orthogonal. The same applies for isospin. This leads to a decoupling of the integral equations in various sets in which the spin and isospin of each particle remains the same. The different sets contributing to each (I,J) state are listed in Ref. [25].

For S-wave states one finally gets

$$T_{i;IJ}^{I_i S_i}(x_i q_i) = \sum_n P_n(x_i) T_{i;IJ}^{nI_i S_i}(q_i), \quad (3)$$

where $T_{i;IJ}^{nI_i S_i}(q_i)$ satisfies the one-dimensional integral equation,

$$T_{i;IJ}^{nI_i S_i}(q_i) = \sum_{j\neq i}\sum_{mI_j S_j}\int_0^\infty dq_j K_{ij;IJ}^{nI_i S_i;mI_j S_j}(q_i,q_j;E)T_{j;IJ}^{mI_j S_j}(q_j),$$

$$(4)$$

with

$$K_{ij;IJ}^{nI_iS_i;mI_jS_j}(q_i, q_j; E)$$

$$= \sum_r \tau_{i;I_iS_i}^{nr}(E - q_i^2/2\nu_i)\frac{q_j^2}{2}$$

$$\times \int_{-1}^{1} d\cos\theta\, h_{ij;IJ}^{I_iS_i;I_jS_j}\frac{P_r(x_i')P_m(x_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j}. \quad (5)$$

The three amplitudes $T_{1;IJ}^{rI_1S_1}(q_1)$, $T_{2;IJ}^{mI_2S_2}(q_2)$, and $T_{3;IJ}^{nI_3S_3}(q_3)$ in Eq. (4) are coupled together.

In these equations $\tau_{i;I_iS_i}^{nr}(e)$ are expansion coefficients given in terms of Legendre polynomials and the two-body amplitudes $t_{i;I_iS_i}$,

$$\tau_{i;I_iS_i}^{nr}(e) = \frac{2n+1}{2}\frac{2r+1}{2}\int_{-1}^{1} dx_i$$

$$\times \int_{-1}^{1} dx_i' P_n(x_i) t_{i;I_iS_i}(x_i, x_i'; e) P_r(x_i'), \quad (6)$$

$h_{ij;IJ}^{I_iS_i;I_jS_j}$ are the spin-isospin coefficients

$$h_{ij;IJ}^{I_iS_i;I_jS_j} = (-)^{I_j+i_j-I}\sqrt{(2I_i+1)(2I_j+1)}W(i_ji_kIi_i;I_iI_j)$$

$$\times(-)^{S_j+s_j-J}\sqrt{(2S_i+1)(2S_j+1)}W(s_js_kJs_i;S_iS_j), \quad (7)$$

where W is a Racah coefficient and i_i, I_i, and I (s_i, S_i, and J) are the isospins (spins) of particle i, of the pair jk, and of the three-body system. η_i and ν_i are the corresponding reduced masses,

$$\eta_i = \frac{m_j m_k}{m_j + m_k},$$

$$\nu_i = \frac{m_i(m_j + m_k)}{m_i + m_j + m_k}. \quad (8)$$

$\vec{p}_i'$ is the momentum of the pair jk (with ijk an even permutation of 123) and $\vec{p}_j$ is the momentum of the pair ki which are given by

$$\vec{p}_i' = -\vec{q}_j - \alpha_{ij}\vec{q}_i,$$

$$\vec{p}_j = \vec{q}_i + \alpha_{ji}\vec{q}_j, \quad (9)$$

where

$$\alpha_{ij} = \frac{\eta_i}{m_k},$$

$$\alpha_{ji} = \frac{\eta_j}{m_k}, \quad (10)$$

so that,

$$p_i' = \sqrt{q_j^2 + \alpha_{ij}^2 q_i^2 + 2\alpha_{ij}q_iq_j\cos\theta},$$

$$p_j = \sqrt{q_i^2 + \alpha_{ji}^2 q_j^2 + 2\alpha_{ji}q_iq_j\cos\theta}. \quad (11)$$

Finally,

$$x_i = \frac{p_i - b}{p_i + b}, \quad (12)$$

where and b is a scale parameter that has no effect on the solution.

To solve the Faddeev equations for P-wave states, we write them symbolically as

$$T_i = t_i h_{ij} G_0 T_j, \quad (13)$$

that has to be generalized to a matrix equation,

$$\begin{pmatrix} T_i^{01} \\ T_i^{10} \end{pmatrix} = \begin{pmatrix} t_i^0 \\ t_i^1 \end{pmatrix} h_{ij} G_0 \begin{pmatrix} \hat{q}_i \cdot \hat{q}_j & \hat{q}_i \cdot \hat{p}_j \\ \hat{p}_i' \cdot \hat{q}_j & \hat{p}_i' \cdot \hat{p}_j \end{pmatrix} \begin{pmatrix} T_j^{01} \\ T_j^{10} \end{pmatrix}, \quad (14)$$

where, from Eq. (9),

$$\hat{q}_i \cdot \hat{q}_j = \cos\theta,$$

$$\hat{q}_i \cdot \hat{p}_j = \frac{q_i^2 + \alpha_{ji}q_iq_j\cos\theta}{q_ip_j},$$

$$\hat{p}_i' \cdot \hat{q}_j = \frac{-q_j^2 - \alpha_{ij}q_iq_j\cos\theta}{p_i'q_j},$$

$$\hat{p}_i' \cdot \hat{p}_j = \frac{-(1 + \alpha_{ij}\alpha_{ji})q_iq_j\cos\theta - \alpha_{ji}q_j^2 - \alpha_{ij}q_i^2}{p_i'p_j}, \quad (15)$$

and p_i' and p_j are given by Eq. (11).

In general, the two-body amplitudes $t_{i;I_iS_i}$ are obtained by solving the Lippmann-Schwinger equation,

$$t = V + VG_0t, \quad (16)$$

where V is the interaction given by Eq. (2). Due to the reduction from five to three particles, some pairs of two-body amplitudes are coupled together. Therefore, in this case one has to solve the coupled equations,

$$t_{11} = V_{11} + V_{11}G_0t_{11} + V_{12}G_0t_{21},$$

$$t_{21} = V_{21} + V_{21}G_0t_{11} + V_{22}G_0t_{21}, \quad (17)$$

where the diagonal interactions V_{11} and V_{22} show contributions from the chromoelectric and chromomagnetic terms of the interaction, while the off-diagonal interactions V_{12} and V_{21} contain only the contribution of the chromomagnetic part of the interacting potential. As expected, the confinement and Coulomb terms are the dominant ones such that the spin-spin term is just a small perturbation. The

TABLE II. Binding energy, in MeV, of the different $Q\bar{Q}qqq'$ pentaquarks.

Q	q	q'	v_1	v_2	v_3	w_1	w_3
c	u, d	u, d	7	17	24	12	12
c	u, d	s	133	138	143	134	134
b	u, d	u, d	39	41	52	40	40
b	u, d	s	165	167	175	166	166

TABLE IV. Predictions of different models for the mass, in MeV, of the $b\bar{b}qqq$ pentaquarks in Table I.

Vector	$(I)J^P$	Our model	Ref. [24]	Ref. [41]	Ref. [42]
v_1	$(1/2)1/2^-$	11062	11137.1	11080 (11078)	10605
v_2	$(1/2)1/2^-$	11121	11148.9	11115 (11043)	10629
w_1	$(1/2)3/2^-$	11122	11237.5	11124 (11122)	10629
v_3	$(1/2)1/2^-$	11195	11205.0	$\cdots$	$\cdots$
w_3	$(3/2)3/2^-$	11207	11370.6	11112 (10999)	$\cdots$

effect of the nondiagonal terms is very small and it can be safely neglected.

III. RESULTS

We have solved the three-body problem for the different (I, J) $Q\bar{Q}qqq'$ states as discussed in Sec. II. We show in Table II the binding energy of the most favorable five-quark states that could be observed in nature.

Let us first note the degeneracy existing between $I = 1/2$ and $I = 3/2$ $Q\bar{Q}qqq$ $J = 3/2$ pentaquarks ($I = 0$ and $I = 1$ $Q\bar{Q}qqs$ $J = 3/2$ pentaquarks), as could have been expected *a priori* due to the isospin independence of the potential model in Eq. (2), although the result is not trivial due to the requirements of the Pauli principle. Secondly, it has been checked that the conclusions dealing with stability or instability of multiquarks survive variations of the parameters; we have specifically checked that the pattern remains for different strengths of the spin-spin interaction by modifying the regularization parameter, r_0 in Eq. (2).

One can determine the general properties of the multiquarks favored by the quark correlations dominating the QCD phenomena shown in Table I. In the charmonium sector, the mass difference between the $Q\bar{Q}$ $\{\mathbf{1}_c, \mathbf{1}_f, 1_s\}$ and $\{\mathbf{1}_c, \mathbf{1}_f, 0_s\}$ correlated states could be assimilated to the $J/\Psi - \eta_c$ mass difference. Likewise, in the bottomonium sector it corresponds to the $\Upsilon - \eta_b$ mass difference. The mass difference between the qq $\{\bar{\mathbf{3}}_c, \mathbf{6}_f, 1_s\}$ and $\{\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f, 0_s\}$ has been estimated from full lattice QCD simulations to be in the range of 100–200 MeV [36–38]. Thus, we have fixed the effective mass difference of the correlated structures considering the following realistic values,

$$\Delta M^{c\bar{c}} = M^{c\bar{c}}_{\{\mathbf{1}_c,\mathbf{1}_f,1_s\}} - M^{c\bar{c}}_{\{\mathbf{1}_c,\mathbf{1}_f,0_s\}} = 86 \text{ MeV},$$

$$\Delta M^{b\bar{b}} = M^{b\bar{b}}_{\{\mathbf{1}_c,\mathbf{1}_f,1_s\}} - M^{b\bar{b}}_{\{\mathbf{1}_c,\mathbf{1}_f,0_s\}} = 61 \text{ MeV},$$

$$\Delta M^{qq} = M^{qq}_{\{\bar{\mathbf{3}}_c,\mathbf{6}_f,1_s\}} - M^{qq}_{\{\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_f,0_s\}} = 146 \text{ MeV}. \quad (18)$$

Then, denoting by $M_0^{Q\bar{Q},q}$ the sum of the masses of a spin zero $Q\bar{Q}$ diquark, a spin zero qq diquark and a light quark, the mass of the $Q\bar{Q}qqq$ states in Table I would be given by

$$M_i = M_0^{Q\bar{Q},q} - B_i + \Delta M^{Q\bar{Q}}\delta_{s_2,1} + \Delta M^{qq}\delta_{s_3,1}, \quad (19)$$

where B_i is the binding energy calculated above, see Table II. For $Q\bar{Q}qqs$ states we have a similar expression,

$$M_i = M_0^{Q\bar{Q},s} - B_i + \Delta M^{Q\bar{Q}}\delta_{s_2,1} + \Delta M^{qq}\delta_{s_3,1}. \quad (20)$$

By taking $M_0^{c\bar{c},q} = 4319$ MeV, one gets the results shown in Table III. As can be seen, there is a good agreement between theoretical states showing the most important correlations dictated by the QCD phenomena and the experimental data [39,40]. Thus, Table III presents a theoretical spin-parity assignment for the existing hidden-charm pentaquarks. A careful analysis of the results and a detailed comparison with other approaches in the literature was performed in Ref. [25].

Now, we can make parameter-free predictions for the lowest-lying hidden-bottom pentaquarks, for which there is still no experimental data. The results are shown in Table IV, compared to other results available in the literature. Reference [24] considers a color-magnetic interaction to estimate the mass splitting of the different states with respect to a reference mass adjusted to experimental data. Reference [41] makes use of a chiral quark model and solves the five-body bound state problem by the Gaussian expansion method. We quote the results corresponding to the color-singlet calculation, which, from a theoretical point of view, would be the closest to our model, and between parenthesis the coupled channel calculation including hidden-color channels. Reference [42] presents results of a hadroquarkonium model with baryons of $I = 1/2$ and two different chromoelectric polarizability strengths. We show the results of the model with lower attraction, in which the hidden-bottom pentaquarks are

TABLE III. Properties of the $c\bar{c}qqq$ pentaquarks in Table I.

Vector	$(I)J^P$	M_{Th} (MeV)	State	M_{Exp} (MeV)
v_1	$(1/2)1/2^-$	4312	$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$
v_2	$(1/2)1/2^-$	4388	$P_c(4380)^+$	$4380 \pm 8 \pm 29$
w_1	$(1/2)3/2^-$	4393		
v_3	$(1/2)1/2^-$	4441	$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$
w_3	$(3/2)3/2^-$	4453	$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$

TABLE V. Properties of the $c\bar{c}qqs$ pentaquarks in Table I.

Vector	$(I)J^P$	M_{Th} (MeV)	State	M_{Exp} (MeV)	$M_{\mathrm{Exp}}^{\dagger}$ (MeV)
v_1	$(0)1/2^-$	4338	$P_{\Psi_s}^{\Lambda}(4338)$	4338.2 ± 0.7	4338 ± 0.7
v_2	$(0)1/2^-$	4419			
w_1	$(0)3/2^-$	4423			
v_3	$(0)1/2^-$	4474	$P_{cs}(4459)$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	4454.9 ± 2.7
w_3	$(1)3/2^-$	4483			4467.8 ± 3.7

located in the 10.6–10.9 GeV energy region.[1] Positive parity states have smaller binding energies and appear about 150 MeV above the negative parity states. In the first two references, which use quark degrees of freedom, there is a rich spectra of pentaquarks with different isospins, $I = 1/2$ and $3/2$, and spins, $J^P = 1/2^-$, $3/2^-$ and $5/2^-$. We only show the lowest-lying states to compare with those obtained with our model. It is interesting to note that most of the hidden-bottom pentaquarks predicted by quark substructure models are in the same energy region, 11.0–11.2 GeV. The hadroquarkonium model finds more deeply bound states. These differences support a future experimental effort to look for hidden-bottom pentaquarks in this energy region, what would be a clear signal to discriminate between the different dynamics that may drive to hidden-bottom pentaquarks.

Very recently the LHCb Collaboration announced the observation of a new strange pentaquark $P_{\Psi_s}^{\Lambda}(4338)$ in the decay $B^- \to J/\Psi\Lambda\bar{p}$ as a resonance in the $J/\Psi\Lambda$ invariant mass distribution [43]. It has a mass of 4338.2 ± 0.7 and a minimal quark content $c\bar{c}uds$. One can take advantage of the recent discovery of this hidden-charm pentaquark with strangeness to tune the free parameter in the strange sector, by taking $M_0^{c\bar{c},s} = 4471$ MeV. Thus, one gets the results shown in Table V.

Moreover, the LHCb Collaboration has reported evidence of a structure in the $J/\Psi\Lambda$ invariant mass distribution obtained from an amplitude analysis of $\Xi_b^- \to J/\Psi\Lambda K^-$ decays [44]. The observed structure, with mass $4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV, is consistent with being due to a charmonium pentaquark with strangeness, i.e., minimal quark content $c\bar{c}uds$. These two states are collected in the penultimate column, M_{Exp}, of Table V. However, the structure observed in Ref. [44] is also consistent with being due to two resonances, with masses 4454.9 ± 2.7 MeV and 4467.8 ± 3.7 MeV. This experimental situation is reflected in the last column, $M_{\mathrm{Exp}}^{\dagger}$, of Table V. The existence of two states resembles the situation found in the nonstrange sector with the $P_c(4440)^+$ and the $P_c(4457)^+$ [45], two resonances predicted as $J = 1/2$ and $3/2$ states, but different isospin in our model, see Table III.

As can be seen, there is a good agreement between theoretical states showing the most important correlations dictated by the QCD phenomena and the experimental data [43,44]. Therefore, Table V presents a theoretical spin-parity assignment for the existing strange hidden-charm pentaquarks deduced from our model together with the prediction of a few new states in the same energy region.

In Table VI we compare our results with others available in the literature. The perturbative color-magnetic calculation of Ref. [24] predicts large splittings among the lowest-lying states. The chiral effective field theory potentials of Ref. [46] are closer to the results of our model, with the lowest-lying states in the 4.3–4.4 GeV energy region. They do not find bound state solutions for $I = 1$ channels. Reference [47] presents results of a chiral quark model using a variational method with radial wave functions expanded in terms of Gaussians. The model, successful in describing the nonstrange pentaquarks, predicts very small binding energies in the different baryon-meson channels. They focus on $I = 0$ states and do not show results for $I = 1$ systems. The hadroquarkonium model of Ref. [48] obtains the larger masses for the hidden-charm pentaquarks with strangeness, unlike nonstrange and strange hidden-bottom pentaquarks where they found much smaller masses than quark-based models.

Similarly to the hidden-charm sector, we can now make parameter-free predictions for the lowest-lying strange hidden-bottom pentaquarks, for which experimental data are not yet available. The results are shown in Table VII, compared to other results in the literature.

The strange hidden-bottom pentaquarks are obtained close to the nonstrange case. They are found to be in the 11.1–11.3 GeV energy region. This is the same conclusion of the lowest-lying states of the perturbative chromomagnetic

TABLE VI. Predictions of different models for the mass, in MeV, of the $c\bar{c}qqs$ pentaquarks in Table I.

Vector	$(I)J^P$	Our model	Ref. [24]	Ref. [46]	Ref. [47]	Ref. [48]
v_1	$(0)1/2^-$	4338	4362.3	$4319.4^{+2.8}_{-3.0}$	4330	4474
v_2	$(0)1/2^-$	4419	4548.2	$4456.9^{+3.2}_{-3.3}$	4475	4522
w_1	$(0)3/2^-$	4423	4556.1	$4423.7^{+6.4}_{-6.8}$	4440	4522
v_3	$(0)1/2^-$	4474	4571.4	$4463.0^{+2.8}_{-3.0}$	4476	$\cdots$
w_3	$(1)3/2^-$	4483	4846.4	$\cdots$	$\cdots$	$\cdots$

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[1]The model with higher attraction, derived by considering charmonia as a pure Coulombic system, predicts the lowest-lying hidden-bottom pentaquarks in the 10.4–10.7 GeV energy region.

TABLE VII. Predictions of different models for the mass, in MeV, of the $b\bar{b}qqs$ pentaquarks in Table I.

Vector	$(I)J^P$	Our model	Ref. [24]	Ref. [48]
v_1	$(0)1/2^-$	11088	11117.7	10671
v_2	$(0)1/2^-$	11147	11183.8	10695
w_1	$(0)3/2^-$	11148	11180.2	10695
v_3	$(0)1/2^-$	11224	11301.2	$\cdots$
w_3	$(1)3/2^-$	11233	11509.0	$\cdots$

TABLE VIII. Mass of the first positive parity states, two degenerate states with quantum numbers $J^P = 1/2^+$ and $3/2^+$, for the different $Q\bar{Q}qq'$ pentaquarks.

Q	q	q'	Mass (MeV)
c	u, d	u, d	4527
c	u, d	s	4552
b	u, d	u, d	11273
b	u, d	s	11291

model of Ref. [24]. Analogously to the hidden-bottom nonstrange case, the hadroquarkonium model of Ref. [48] reports smaller masses.[2] It also predicts smaller splittings than quark-based model between the lowest-lying states. Thus, hidden-bottom pentaquarks, either with or without strangeness, seems to be an adequate tool to discriminate among different models for the dynamics of multiquark systems. Models based on the quark substructure indicate that future searches of nonstrange and strange hidden-bottom pentaquarks should be carried out in the 11 GeV energy region.

It is worth mentioning that other alternatives have been used for the study of strange and nonstrange hidden-flavor pentaquarks. Among them a diquark-triquark model [49] has been applied to the same systems studied in this work. The model exploits the color attractive configurations of a not pointlike triquark with a light cone distribution amplitude for the pentaquark. An effective diquark-triquark Hamiltonian based on a spin-orbital interaction is established. The results of this model suggest that the $P_c(4380)^+$ could correspond to a $J^P = 3/2^-$ state with a mass of 4349 MeV and the state originally reported as $P_c(4450)^+$ (see Ref. [25] for details) would correspond to a $J^P = 5/2^+$ with a mass of 4453 MeV. The lightest state is a $J^P = 3/2^-$ hidden-charm pentaquark with a mass of 4085 MeV. $J^P = 1/2^-$ states are theoretically discussed but their masses are not shown in the paper. Following the same strategy strange hidden-charm and hidden-bottom pentaquarks were studied in Ref. [49]. For the sake of completeness we quote the lowest-lying state for each quantum numbers in the different flavor sectors: hidden-charm $(3/2^-, 5/2^-, 5/2^+) = (4085, 4433, 4453)$ MeV; strange hidden-charm $(3/2^-, 5/2^-, 5/2^+) = (4314, 4624, 4682)$ MeV; hidden-bottom $(3/2^-, 5/2^-, 5/2^+) = (10723, 11045, 1146)$ MeV; strange hidden-bottom $(3/2^-, 5/2^-, 5/2^+) = (10981, 11264, 11413)$ MeV.

Preliminary analysis of the experimental data in the hidden-charm sector suggested the coexistence of negative and positive parity pentaquarks in the same energy region [39]. We have calculated the mass of the lowest positive parity state, the first orbital angular momentum excitation of the v_1 state. The technical details have been described in

Sec. II C. We chose this state because it is made up of the most strongly-correlated structures, $Q\bar{Q}\{\mathbf{1}_c, \mathbf{1}_f, 0_s\}$ and qq $\{\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f, 0_s\}$. Then, it might have a similar mass to negative parity states made up of spin-1 structures. As can be seen in Table VIII, the lowest-lying positive parity hidden-charm pentaquarks appear above 4.5 GeV, a mass slightly larger than that of the states measured so far. The orbital excitation is smaller in the hidden-bottom case such that positive parity states appear closer to the negative parity ground states. Similarly, most of the theoretical works prefer to assign the lowest-lying pentaquarks to negative parity states. Almost degenerate negative and positive parity states may occur for hidden-flavor pentaquarks that have been detected in the same channel but that were formed by different pairs of quarkonium-nucleon states [50], one of them radially excited. The assignment of negative and positive parity states to different parity Born-Oppenheimer multiplets has already been suggested as a plausible solution in the triquark-diquark picture of Ref. [21]. Nevertheless, this issue remains one of the most challenging problems in the pentaquark phenomenology that should be first confirmed experimentally.

Multiquark states would show very different decay patterns regarding its internal structure [24]. The decays of the pentaquarks in Table I into an heavy meson + heavy baryon are strongly suppressed since decays into open flavor channels can go only via t-channel exchange by a heavy meson. Due to the content of the pentaquarks states they would follow the decays of quarkonium excited states, $\Psi(nS)[\Upsilon(nS)]$ and $\eta_c(nS)[\eta_b(nS)]$. Thus, multiquarks containing a spin-0 heavy quark-antiquark pair (v_1, v_3, and w_3 in Table I) would be narrower than those with a spin one heavy quark-antiquark pair (v_2 and w_1 in Table I). This corresponds nicely with the experimental observations up to now. However, besides the contribution to the width of the substructures that form each pentaquark, one should also consider the width due to the bound nature of the system. At this point it is worth to mention that the final width of a resonance does not come only determined by its internal content, but there are significant corrections due to an interplay between the phase space for its decay to the detection channel and its mass with respect to the hadrons generating the state [51].

Finally, it is worth noting that the correlations used do not lead to stable multiquarks for any quark substructure, in

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[2]The model with higher attraction, derived by considering charmonia as a pure Coulombic system, predicts the lowest-lying hidden-bottom pentaquarks in the 10.4 GeV energy region.

the same way the *NN* short-range repulsion induced by the one-gluon exchange dynamics is not universal and disappears for other two-hadron channels [52]. Thus, for example, the QCD correlations used in this work would not constrain the color wave function of pentaquarks with anticharm or beauty, $\bar{Q}qqqq$. Therefore, such systems would not present bound states, as recently discussed in Ref. [53], due to a nonfavorable interplay between chromoelectric and chromomagnetic effects.

IV. SUMMARY

In brief, we have studied hidden-flavor pentaquarks which can be framed within the same type of structures discussed in our recent work of Ref. [25]. In particular, we addressed hidden-flavor pentaquarks $Q\bar{Q}qqq'$, with $Q = b$ or c, $q = u$ or d, and $q' = u$ d or s. The color Coulomb-like nature of the short-range one-gluon exchange interaction leads to a frozen color wave function of the five-body system, which allows to reduce the problem to a more tractable three-body problem. The three-body problem has been exactly solved by means of the Faddeev equations. The interactions between the constituents are deduced from a generic constituent model, the AL1 model, that gives a nice description of the low-energy baryon and meson spectra.

For the nonstrange and strange hidden-charm pentaquarks a pattern was obtained that compares well with the experimental data. The tentative spin-parity assignment of the different pentaquarks agrees well with other approaches dedicated to study a particular set of states. Under the assumption that nature favors multiquarks which are made up of correlated substructures dictated by QCD, we have estimated the mass of the lowest-lying pentaquarks. We have considered realistic values for the mass difference of the correlated quark pairs. A good description of the experimental data has been obtained.

For the nonstrange and strange hidden-bottom pentaquarks we present parameter-free predictions. The splitting between the lowest-lying states is smaller than in the charm sector, due to the larger mass of the bottom quark appearing in the chromomagnetic potential. The hidden-bottom pentaquarks are found to be in the 11.0–11.2 GeV energy region; this is a general conclusion for models based on the quark substructure. Hadroquarkonium models predict smaller masses for hidden-bottom pentaquarks, which makes them an excellent test bench for testing the dynamics of multiquark systems.

The coexistence of negative and positive parity states in the same energy region appears to be more likely in the bottom sector. It is worth noting that the correlations used do not lead to stable multiquarks for any quark substructure. Thus, for example, the QCD correlations used in this work would not constraint the color wave function of pentaquarks with anticharm or beauty.

Bound states and resonances are usually very sensitive to model details and therefore theoretical investigations with different phenomenological models are highly desirable. We have tried to minimize the influence of the interacting potential by using a standard constituent model and we have explored the consequences of dynamical correlations arising from the Coulomb-like nature of the short-range potential. The pattern obtained could be scrutinized against the future experimental results providing a great opportunity for extending our knowledge to some unreached part of the hadron spectra. More such exotic baryons are expected, and needed, to make reliable hypotheses on the way the interactions in the system are shaping the spectra.

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