Production of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in B decays as $D^{(*)}K$ and $D_s^{(*)}\eta$ molecules

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The molecular nature of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ have been extensively studied from the perspective of their masses, decay properties, and production rates. In this work, we study the weak decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ by invoking triangle diagrams where the B meson first decays weakly into $\bar{D}^{(*)} D_s^{(*)}$ and $J/\psi K(\eta_c K)$, and then the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are dynamically generated by the final-state interactions of $D_s^{(*)}\eta$ and $D^{(*)}K$ via exchanges of η and $D^{(*)}$ mesons. The obtained absolute branching fractions of $\mathrm{Br}[B \to \bar{D}^{(*)} D_{s0}^*(2317)]$ are in reasonable agreement with the experimental data, while the branching fractions of $\mathrm{Br}[B \to \bar{D}^{(*)} D_{s1}(2460)]$ are smaller than the experimental central values by almost a factor of two to three. We tentatively attribute such a discrepancy to either reaction mechanisms missing in the present work or the likely existence of a relatively larger $c\bar{s}$ component in the $D_{s1}(2460)$ wave function.

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I. INTRODUCTION

In 2003, the *BABAR* Collaboration discovered a quite narrow state near 2.32 GeV in the inclusive $D_s^+\pi^0$ invariant mass distribution [1], named $D_{s0}^*(2317)$, which was subsequently confirmed by the CLEO [2] and Belle Collaborations [3]. Taken as a $c\bar{s}$ state with the quantum number of $I(J^P) = 0(0^+)$, its mass is lower by 160 MeV than the prediction of the Godfrey-Isgur quark model [4]. Such a large deviation has also appeared within the lattice QCD simulations [5,6]. To explain the discrepancy, many different interpretations of the $D_{s0}^*(2317)$ have been proposed, such as a P-wave $c\bar{s}$ excited state [7–9], a compact tetraquark state [10], or a hadronic molecule [11–15]. Among them, the hadronic molecular interpretation has attracted considerable attention.

In Refs. [16,17], the authors interpreted $D_{s0}^*(2317)$ as a hadronic molecule generated by the DK and $D_s\eta$ coupled-channel interactions in the chiral unitary approach, which is also supported by many other studies [18–21]. The DK coupled-channel interactions [22–24] have been simulated on the lattice, and a bound state below the DK mass threshold is found, which can be identified as $D_{s0}^*(2317)$. In addition, a D^*K molecule as the partner of $D_{s0}^*(2317)$ is predicted via the heavy quark spin symmetry (HQSS), and it can be identified as $D_{s1}(2460)$ [15,18,25,26], discovered by the CLEO Collaboration in the $D_s^*\pi$ mass distribution [2] and confirmed by the Belle Collaboration [3]. Up to now, only the upper limits for the widths of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are known, i.e., $\Gamma_{D_{s0}^*(2317)} < 3.7$ MeV and

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$\Gamma_{D_{s1}(2460)} < 3.5$ MeV [27]. In the molecular picture, Faessler *et al.* took the effective Lagrangian approach to estimate the dominant partial decay widths of $D_{s0}^*(2317) \to D_s\pi$ and $D_{s1}(2460) \to D_s^*\pi$ to be 80 keV and 50–79 keV [28,29]. Very recently, an effective field theory study estimated their partial decay widths to be 120 keV and 102 keV, respectively [30].

Recently, we proposed a novel approach to verify the molecular nature of exotic states from the existence of relevant three-hadron molecules (see Refs. [31,32] for reviews). The molecular nature of $D_{s0}^*(2317)$ can be verified by searching for the three-body molecule DDK, where the DK interaction is determined by reproducing the mass of $D_{s0}^*(2317)$ and plays a dominant role in forming the DDK molecule [33,34]. In Ref. [35], assuming $D_{s0}^*(2317)$ as a DK molecule, we employed the one-kaon exchange potential and predicted the existence of a $DD_{s0}^*(2317)$ molecule, whose mass and quantum numbers are consistent with those of the DDK molecule. Moreover, we have investigated the $\bar{D}DK$ system [36], and it was found that the $\bar{D}D_{s0}^*(2317)$ configuration accounts for about 87% of the $\bar{D}DK$ configuration, which indicates that the DK interaction plays the most important role in forming the $\bar{D}DK$ molecule as well [37]. If the $\bar{D}DK$ molecule is discovered by experiments, it will also verify the molecular nature of $D_{s0}^*(2317)$. It should be noted that although the DK molecular interpretation is the most favorable, the $c\bar{s}$ component is found to play a non-negligible role in describing the mass of $D_{s0}^*(2317)$ in the unquenched quark models [38–42]. In a recent work [43], by fitting to the lattice QCD finite volume spectra, Yang *et al.* found that the $c\bar{s}$ component accounts for about 32% of the wave function of $D_{s0}^*(2317)$, while the $c\bar{s}$ component accounts for more than half of the $D_{s1}(2460)$ wave function, which is consistent with a number of earlier studies [44–46].

The production of $D_{s0}^*(2317)$ in the molecular picture has also been extensively investigated. In Ref. [47], assuming $D_{s0}^*(2317)$ as either a conventional $c\bar{s}$ state, a compact multiquark state or a hadronic molecule, Cho *et al.* adopted the coalescence model and statistical model to estimate the corresponding yield of $D_{s0}^*(2317)$ in heavy ion collisions, which would help probe its nature in future experiments. On the other hand, the production of $D_{s0}^*(2317)$ in the weak decays of B and B_s mesons also provides a very good platform to study the meson-meson interactions and the nature of $D_{s0}^*(2317)$. In Ref. [48], Miguel *et al.* investigated the nature of $D_{s0}^*(2317)$ by extracting the DK interaction via the DK invariant mass distributions of the processes $B^+ \to \bar{D}^0 D^0 K^+$, $B^0 \to D^- D^0 K^+$, and $B_s^0 \to \pi^+ \bar{D}^0 K^-$. In Ref. [49], Navarra *et al.* investigated the molecular nature of $D_{s0}^*(2317)$ in the semileptonic B_s^0 and B decays taking into account the DK and $D_s\eta$ rescattering.

On the experimental side, the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ have been found in the weak decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$, and their branching fractions can be found in Ref. [27]. In Ref. [50], Cheng *et al.* employed the covariant light-front quark model to study the weak decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ using the factorization approach, where $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are treated as P-wave $c\bar{s}$ states. Later, Segovia *et al.* adopted a similar approach to study the decays $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ [51]. Recently, Zhang *et al.* calculated the decay $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ in the pQCD approach [52]. In addition, the production rates of $D_{s1}^*(2317)$ and $D_{s1}(2460)$ in the semileptonic decays $B_s \to D_{s0}^*(2317)(D_{s1}^*(2460)) l\bar{\nu}$ [53] and in the nonleptonic decays $\Lambda_b \to \Lambda_c D_{s0}^*(2317)(D_{s1}(2460))$ [54] have been predicted.

Assuming $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as DK and D^*K molecules, Faessler *et al.* calculated the branching ratios of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ in the naive factorization approach [55], where the couplings $f_{D_{s0}^*}$ and $f_{D_{s1}}$ are estimated in the molecular picture, different from Refs. [50,51]. In the present work, we will revisit the $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ decays in the triangle mechanism, where $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are dynamically generated by the coupled-channels $D^{(*)}K$ and $D_s^{(*)}\eta$. We note that a similar approach has earlier been employed to study $a_0(980)$ generated by the coupled-channels $\pi\eta$ and $K\bar{K}$ in the process $D_s \to \pi\pi\eta$ [56], where the theoretical results are found in good agreement with the experimental data.

This work is organized as follows. We briefly introduce the triangle mechanism for the decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$ and the effective Lagrangian approach in Sec. II. Results and discussions are given in Sec. III, followed by a short summary in the last section.

II. THEORETICAL FORMALISM

The mesonic weak transition form factors and decay constants are the two main ingredients in the study of hadronic weak decays of mesons, which are less certain for P-wave charmed mesons than for S-wave charmed mesons. Here, we adopt the triangle mechanism to study the weak decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$, where the form factors and decay constants of S-wave mesons are stringently constrained by experiments. This way, we can largely reduce the theoretical uncertainties. In the following, we explain in detail the triangle mechanism accounting for the weak decays of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$.

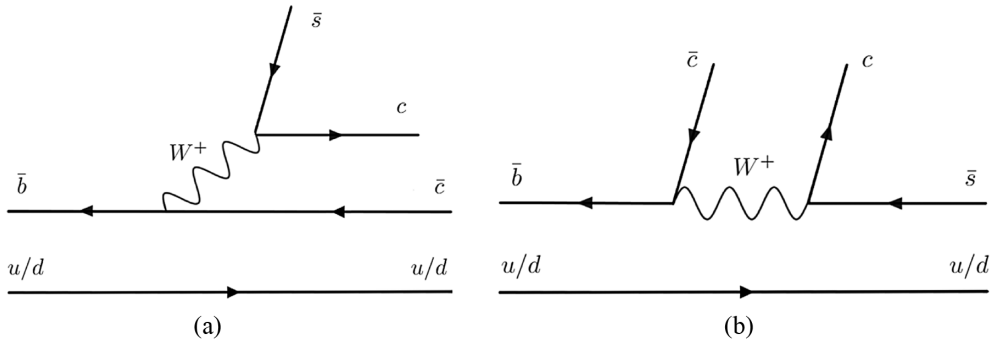

FIG. 1. (a) External W-emission for $B^{+(0)} \to D_s^{(*)+}\bar{D}^{*0}(D^{*-})$ and (b) internal W-conversion for $B^{+(0)} \to J/\psi/\eta_c K^{+(0)}$.

A. Triangle diagrams

At the quark level, the decays of $B^{+(0)} \to D_s^{(*)+}\bar{D}^{(*)0}(D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c)K^{+(0)}$ can proceed via the external W-emission and the internal W-conversion mechanisms as shown in Fig. 1(a) and 1(b), respectively. Referring to the review of particle physics (RPP) [27], the absolute branching fractions of the processes $B^{+(0)} \to D_s^{(*)+}\bar{D}^{(*)0}(D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c)K^{+(0)}$ are tabulated in Table I, which follows the topological classification of weak decays where the strength of the external W-emission mechanism is larger than that of the internal W-conversion mechanism [57–59].

Taking into account the scattering vertices of $\bar{D}^* \to \bar{D}\eta$, $J/\psi \to \bar{D}D$, $\bar{D} \to \bar{D}^*\eta$ and $\eta_c \to \bar{D}^*D$, the $D_{s0}^*(2317)$ state can be dynamically generated by the DK and $D_s\eta$ coupled-channel interactions. We illustrate the decays of $B^+ \to \bar{D}^{(*)0}D_{s0}^*(2317)^+$ and $B^0 \to D^{(*)-}D_{s0}^*(2317)^+$ at the hadronic level via the triangle diagrams shown in Fig. 2. Similarly, we depict the triangle diagrams of the decays of $B^{+(0)} \to \bar{D}^0(D^-)D_{s1}(2460)^+$ in Fig. 3, and $B^+ \to \bar{D}^{*0}(D^{*-})D_{s1}(2460)^+$ in Fig. 4.

B. Effective Lagrangians

To compute the contributions of the triangle diagrams shown in Figs. 2–4, we introduce the effective Lagrangians. The effective Hamiltonian describing the weak decays of $B^{+(0)} \to D_s^{(*)+}\bar{D}^{(*)0}(D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c)K^{+(0)}$ has the following form

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}[c_1^{\rm eff}\mathcal{O}_1 + c_2^{\rm eff}\mathcal{O}_2] + \text{H.c.}, \quad (1)$$

where G_F is the Fermi constant, V_{bc} and V_{cs} are the Cabibbo-Kobayashi-Maskawa matrix elements, $c_{1,2}^{\rm eff}$ are the effective Wilson coefficients, and $\mathcal{O}_1$ and $\mathcal{O}_2$ are the four-fermion operators of $(s\bar{c})_{V-A}(c\bar{b})_{V-A}$ and $(\bar{c}c)_{V-A}(s\bar{b})_{V-A}$ with $(q\bar{q})_{V-A}$ standing for $q\gamma_\mu(1-\gamma_5)\bar{q}$ [60–62].

The effective Lagrangians accounting for the interactions between the charmonium states (J/ψ, η_c) and a pair of charmed mesons read [63,64]

$$\begin{aligned}
\mathcal{L}_{\psi DD} &= ig_{\psi DD}\psi_\mu(\partial^\mu D\bar{D} - D\partial^\mu\bar{D}),\\
\mathcal{L}_{\psi DD^*} &= -g_{\psi DD^*}\epsilon^{\alpha\beta\mu\nu}\partial_\alpha\psi_\beta(\partial_\mu D_\nu^*\bar{D} + D\partial_\mu\bar{D}_\nu^*),\\
\mathcal{L}_{\psi D^*D^*} &= -ig_{\psi D^*D^*}[\psi^\mu(\partial_\mu D_\nu^*\bar{D}^{*\nu} - D^{*\nu}\partial_\mu\bar{D}_\nu^*)\\
&\quad + (\partial_\mu\psi_\nu D^{*\nu} - \psi_\nu\partial_\mu D^{*\nu})\bar{D}^{*\mu}\\
&\quad + D^{*\mu}(\psi^\nu\partial_\mu\bar{D}_\nu^* - \partial_\mu\psi_\nu\bar{D}^{*\nu})],\\
\mathcal{L}_{\eta_c D^*D} &= ig_{\eta_c D^*D}[D^{*\mu}(\partial_\mu\eta_c\bar{D} - \eta_c\partial_\mu\bar{D})\\
&\quad - (\partial_\mu\eta_c D - \eta_c\partial_\mu D)\bar{D}^{*\mu}],\\
\mathcal{L}_{\eta_c D^*D^*} &= -g_{\eta_c D^*D^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu D_\nu^*\partial_\alpha\bar{D}_\beta^*\eta_c,
\end{aligned} \quad (2)$$

where $g_{\psi DD}$, $g_{\psi D^*D}$, $g_{\psi D^*D^*}$, $g_{\eta_c D^*D}$, and $g_{\eta_c D^*D^*}$ are the couplings of the charmonium mesons to the charmed mesons. The coupling constants are determined as follows: $g_{\psi DD} = g_{\psi D^*D^*} = m_\psi/f_\psi$, $g_{\psi D^*D} = \frac{2}{m_D}g_{\psi DD}$ [64,65], $g_{\eta_c D^*D} = \frac{m_D}{2}g_{\eta_c D^*D^*} = g_2\sqrt{m_{\eta_c}}m_D$, and $g_2 = 2.36\ \text{GeV}^{-3/2}$ [66].

The effective Lagrangian describing the interaction between the charmed mesons (D and D^*) and η are written as [67]

$$\begin{aligned}
\mathcal{L}_{DD^*\eta} &= ig_{DD^*\eta}(D_\mu^*\partial^\mu\eta\bar{D} - D\partial^\mu\eta\bar{D}_\mu^*),\\
\mathcal{L}_{D^*D^*\eta} &= -g_{D^*D^*\eta}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu D^{*\nu}\partial^\alpha\bar{D}^{*\beta}\eta,
\end{aligned} \quad (3)$$

TABLE I. Branching ratios (10^{-3}) of $B^{+(0)} \to D_s^{(*)+}\bar{D}^{(*)0} \times (D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c)K^{+(0)}$.

Decay mode	RPP [27]	Decay mode	RPP [27]
$B^+ \to \bar{D}^0 D_s^+$	9.0 ± 0.9	$\bar{B}^0 \to D^- D_s^+$	7.2 ± 0.8
$B^+ \to \bar{D}^0 D_s^{*+}$	7.6 ± 1.6	$\bar{B}^0 \to D^- D_s^{*+}$	7.4 ± 1.6
$B^+ \to \bar{D}^{*0} D_s^+$	8.2 ± 1.7	$\bar{B}^0 \to D^{*-} D_s^+$	8.0 ± 1.1
$B^+ \to \bar{D}^{*0} D_s^{*+}$	17.1 ± 2.4	$\bar{B}^0 \to D^{*-} D_s^{*+}$	17.7 ± 1.4
$B^+ \to J/\psi K^+$	1.010 ± 0.029	$\bar{B}^0 \to J/\psi K^0$	0.873 ± 0.032
$B^+ \to \eta_c K^+$	1.09 ± 0.09	$\bar{B}^0 \to \eta_c K^0$	0.79 ± 0.12

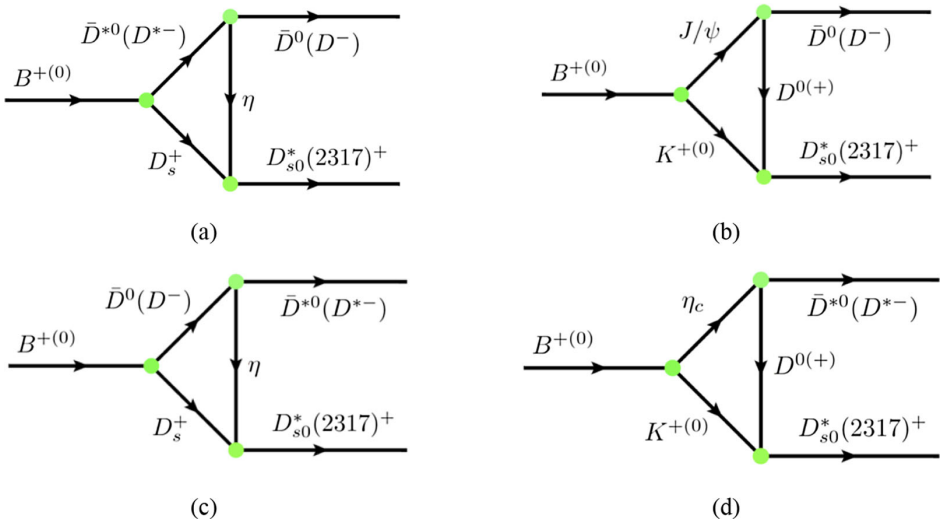

FIG. 2. Triangle diagrams accounting for the four B decays: (a) $B^{+(0)} \to D_s^+ \bar{D}^{*0}(D^{*-}) \to D_{s0}^*(2317)\bar{D}^0(D^-)$, (b) $B^{+(0)} \to J/\psi K^{+(0)} \to D_{s0}^*(2317)\bar{D}^0(D^-)$, (c) $B^{+(0)} \to D_s^+ \bar{D}^0(D^-) \to D_{s0}^*(2317)\bar{D}^{*0}(D^{*-})$ and (d) $B^{+(0)} \to \eta_c K^{+(0)} \to D_{s0}^*(2317)\bar{D}^{*0}(D^{*-})$.

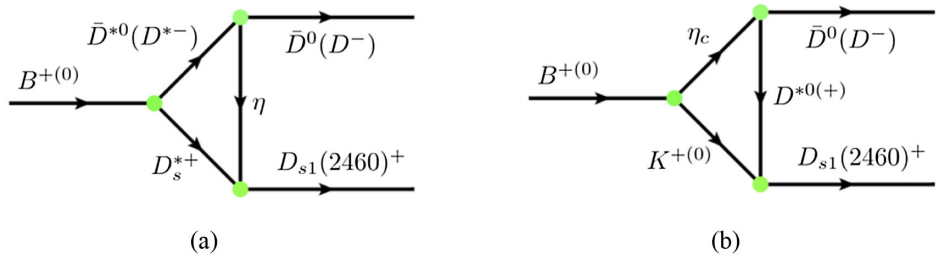

FIG. 3. Triangle diagrams accounting for the two B decays: (a) $B^{+(0)} \to D_s^{*+}\bar{D}^{*0}(D^{*-}) \to D_{s1}(2460)^+\bar{D}^0(D^-)$ and (b) $B^{+(0)} \to \eta_c K^{+(0)} \to D_{s1}(2460)^+\bar{D}^0(D^-)$.

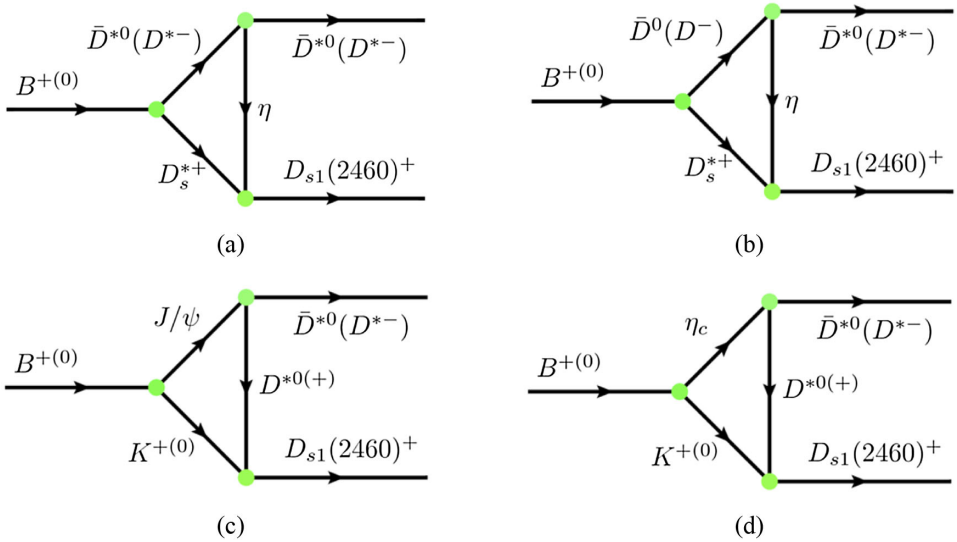

FIG. 4. Triangle diagrams accounting for the four B decays: (a) $B^{+(0)} \to D_s^{*+}\bar{D}^{*0}(D^{*-}) \to D_{s1}(2460)^+\bar{D}^{*0}(D^{*-})$, (b) $B^{+(0)} \to D_s^{*+}\bar{D}^0(D^-) \to D_{s1}(2460)^+\bar{D}^{*0}(D^{*-})$, (c) $B^{+(0)} \to J/\psi K^{+(0)} \to D_{s1}(2460)^+\bar{D}^{*0}(D^{*-})$, and (d) $B^{+(0)} \to \eta_c K^{+(0)} \to D_{s1}(2460)^+\bar{D}^{*0}(D^{*-})$.

where $g_{DD^*\eta}$ and $g_{D^*D^*\eta}$ are the couplings between charmed mesons and light mesons. For the couplings between the charmed mesons and η, $g_{D^{*0}D^0\eta} = g_{D^{*-}D^-\eta} = \frac{g_{D^{*0}D^0\pi^0}}{\sqrt{3}}$ are derived by the SU(3)-flavor symmetry, and the coupling $g_{D^{*0}D^0\pi^0} = 11.7$ is obtained from the decay width of $D^{*0} \to D^0\pi^0$ [27]. The coupling of $g_{\bar{D}^{*0}\bar{D}^{*0}\eta}$ is obtained by the relationship $g_{\bar{D}^{*0}\bar{D}^{*0}\eta} = g_{D^{*0}D^0\eta}/m_D$ [67].

Assuming that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are dynamically generated by the S-wave $DK - D_s\eta$ and $D^*K - D_s^*\eta$ coupled-channel interactions, respectively, the relevant Lagrangians can be written as [28,29]

$$\begin{aligned}
\mathcal{L}_{D_{s0}^*DK} &= g_{D_{s0}^*DK} D_{s0}^* DK, \\
\mathcal{L}_{D_{s0}^*D_s\eta} &= g_{D_{s0}^*D_s\eta} D_{s0}^* D_s\eta, \\
\mathcal{L}_{D_{s1}D^*K} &= g_{D_{s1}D^*K} D_{s1}^\mu D_\mu^* K, \\
\mathcal{L}_{D_{s1}D_s^*\eta} &= g_{D_{s1}D_s^*\eta} D_{s1}^\mu D_{s\mu}^* \eta,
\end{aligned} \tag{4}$$

where $g_{D_{s0}^*DK}$ and $g_{D_{s0}^*D_s\eta}$ represent the couplings of $D_{s0}^*(2317)$ to DK and $D_s\eta$, and $g_{D_{s1}D^*K}$ and $g_{D_{s1}D_s^*\eta}$ represent the couplings of $D_{s1}(2460)$ to D^*K and $D_s^*\eta$. The values of $g_{D_{s0}^*DK}$ and $g_{D_{s0}^*D_s\eta}$ are determined from the residues of $D_{s0}^*(2317)$ on the complex plane, where it is treated as a molecule dynamically generated by the DK and $D_s\eta$ coupled-channel interactions. In this work, we take $g_{D_{s0}^*DK} = 9.4$ GeV and $g_{D_{s0}^*D_s\eta} = 7.4$ GeV given in the effective field theory approach [30], in agreement with those obtained in Ref. [17]. The $D_{s1}(2460)$ is regarded as the HQSS partner of $D_{s0}^*(2317)$, which is dynamically generated by the D^*K and $D_s^*\eta$ coupled-channel interactions. The couplings of $g_{D_{s1}D^*K} = 10.1$ GeV and $g_{D_sD_s^*\eta} = 7.9$ GeV are also taken from Ref. [30]. Taking into account isospin symmetry, the relevant couplings are obtained as $g_{D_{s0}^{*+}D^+K^0} = g_{D_{s0}^{*+}D^0K^+} = \frac{1}{\sqrt{2}} g_{D_{s0}^*DK}$ and $g_{D_{s1}^+D^{*+}K^0} = g_{D_{s1}^+D^{*0}K^+} = \frac{1}{\sqrt{2}} g_{D_{s1}D^*K}$.

C. Decay amplitudes and partial decay widths

The decay amplitudes of $B^{+(0)} \to D_s^{(*)+}\bar{D}^{(*)0}(D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c)K^{+(0)}$ can be written as the products of two hadronic matrix elements [68,69]

$$\begin{aligned}
&\mathcal{A}(B^+ \to D_s^+\bar{D}^{*0}) \\
&= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_1 \langle D_s^+|(s\bar{c})|0\rangle \langle \bar{D}^{0*}|(c\bar{b})|B^+\rangle,
\end{aligned} \tag{5}$$

$$\mathcal{A}(B^+ \to D_s^+\bar{D}^0) = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_1' \langle D_s^+|(s\bar{c})|0\rangle \langle \bar{D}^0|(c\bar{b})|B^+\rangle, \tag{6}$$

$$\begin{aligned}
&\mathcal{A}(B^+ \to D_s^{*+}\bar{D}^0) \\
&= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_1^* \langle D_s^{*+}|(s\bar{c})|0\rangle \langle \bar{D}^0|(c\bar{b})|B^+\rangle,
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\mathcal{A}(B^+ \to D_s^{*+}\bar{D}^{*0}) \\
&= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_1'^* \langle D_s^{*+}|(s\bar{c})|0\rangle \langle \bar{D}^{*0}|(c\bar{b})|B^+\rangle,
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\mathcal{A}(B^+ \to J/\psi K^+) \\
&= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_2 \langle J/\psi|(\bar{c}c)|0\rangle \langle K^+|(s\bar{b})|B^+\rangle,
\end{aligned} \tag{9}$$

$$\mathcal{A}(B^+ \to \eta_c K^+) = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs} a_2' \langle \eta_c|(\bar{c}c)|0\rangle \langle K^+|(s\bar{b})|B^+\rangle, \tag{10}$$

where $a_1 = c_1^{\rm eff} + c_2^{\rm eff}/N_c$ and $a_2 = c_1^{\rm eff}/N_c + c_2^{\rm eff}$ with N_c the number of colors. It should be noted that a_1 and a_2 can be obtained in the factorization approach [70].

The current matrix elements between a pseudoscalar meson or vector meson and the vacuum have the following form,

$$\begin{aligned}
&\langle D_s^+|(s\bar{c})|0\rangle = f_{D_s^+} p_{D_s^+}^\mu, \quad \langle D_s^{*+}|(s\bar{c})|0\rangle = m_{D_s^{*+}} f_{D_s^{*+}} \epsilon_\mu^*, \\
&\langle \eta_c|(c\bar{c})|0\rangle = f_{\eta_c} p_{\eta_c}^\mu, \quad \langle J/\psi|(\bar{c}c)|0\rangle = m_{J/\psi} f_{J/\psi} \epsilon_\mu^*,
\end{aligned} \tag{11}$$

where $f_{D_s^+}$, $f_{D_s^{*+}}$, f_{η_c}, and $f_{J/\psi}$ are the decay constants for D_s^+, D_s^{*+}, η_c, and J/ψ, respectively, and ϵ_μ^* denotes the polarization vector of a vector particle. In this work, we take $G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$, $V_{cb} = 0.041$, $V_{cs} = 0.987$, $f_{D_s} = 250$ MeV, $f_{D_s^{*+}} = 272$ MeV, $f_{J/\psi} = 405$ MeV, and $f_{\eta_c} = 420$ MeV as in Refs. [27,71–74].

The hadronic matrix elements can be parametrized in terms of form factors [71]

$$\begin{aligned}
\langle \bar{D}^{*0}|(c\bar{b})|B^+\rangle = \epsilon_\alpha^* \Big\{ &-g^{\mu\alpha}(m_{\bar{D}^{*0}} + m_{B^+}) A_1(q^2) + P^\mu P^\alpha \frac{A_2(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} \\
&+ i\epsilon^{\mu\alpha\beta\gamma} P_\beta q_\gamma \frac{V(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} + q^\mu P^\alpha \left[\frac{m_{\bar{D}^{*0}} + m_{B^+}}{q^2} A_1(q^2) - \frac{m_{B^+} - m_{\bar{D}^{*0}}}{q^2} A_2(q^2) - \frac{2m_{\bar{D}^{*0}}}{q^2} A_0(q^2) \right] \Big\},
\end{aligned} \tag{12}$$

$$\langle \bar{D}^0|(c\bar{b})|B^+\rangle = \left[(p_{B^+}+p_{\bar{D}^0})^\mu - \frac{m^2_{B^+}-m^2_{\bar{D}^0}}{q'^2}q'_\mu\right]F_{1D}(q'^2) + \frac{m^2_{B^+}-m^2_{\bar{D}^0}}{q'^2}q'_\mu F_{0D}(q'^2), \tag{13}$$

$$\langle K^+|(s\bar{b})|B^+\rangle = \left[(p_{B^+}+p_{K^+})^\mu - \frac{m^2_{B^+}-m^2_{K^+}}{q''^2}q''_\mu\right]F_{1K}(q''^2) + \frac{m^2_{B^+}-m^2_{K^+}}{q''^2}q''_\mu F_{0K}(q''^2), \tag{14}$$

where q, q', and q'' represent the momentum transfer of $p_{B^+}-p_{\bar{D}^{*0}}$, $p_{B^+}-p_{\bar{D}^0}$, and $p_{B^+}-p_{K^+}$, respectively, and $P=p_{B^+}+p_{\bar{D}^{*0}}$.

The form factors of $F_{1,0D}(t)$, $F_{1,0K}(t)$, $A_0(t)$, $A_1(t)$, $A_2(t)$, and $V(t)$ with $t\equiv q'^{(\prime\prime)2}$ can be parametrized as [71]

$$X(t) = \frac{X(0)}{1-a(t/m_B^2)+b(t^2/m_B^4)}. \tag{15}$$

For these form factors, we adopt those of the covariant light-front quark model, i.e., $(F_1(0),a,b)^{B\to\bar{D}} = (0.67,1.22,0.36)$, $(F_0(0),a,b)^{B\to\bar{D}} = (0.67,0.63,0.01)$, $(F_1(0),a,b)^{B\to K} = (0.34,1.60,0.73)$, $(F_0(0),a,b)^{B\to K} = (0.34,0.78,0.05)$, $(A_0(0),a,b)^{B\to\bar{D}^*} = (0.68,1.21,0.36)$, $(A_1(0),a,b)^{B\to\bar{D}^*} = (0.65,0.60,0.00)$, $(A_2(0),a,b)^{B\to\bar{D}^*} = (0.61,1.12,0.31)$, and $(V_0(0),a,b)^{B\to\bar{D}^*} = (0.77,1.25,0.38)$ [71].

With the above relevant Lagrangians, one can easily compute the corresponding decay amplitudes of Fig. 2,

$$\mathcal{A}_a = g_{D^*_{s0}D_s\eta}\int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to D_s\bar{D}^*)\mathcal{A}(\bar{D}^*\to\bar{D}\eta)}{(q_1^2-m^2_{\bar{D}^*})(q_2^2-m^2_{D_s})(q_3^2-m^2_\eta)}, \tag{16}$$

$$\mathcal{A}_b = g_{D^*_{s0}DK}\int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to J/\psi K)\mathcal{A}(J/\psi\to\bar{D}D)}{(q_1^2-m^2_\psi)(q_2^2-m^2_K)(q_3^2-m^2_D)}, \tag{17}$$

$$\mathcal{A}_c = g_{D^*_{s0}D_s\eta}\int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to D_s\bar{D})\mathcal{A}(\bar{D}\to\bar{D}^*\eta)}{(q_1^2-m^2_{\bar{D}})(q_2^2-m^2_{D_s})(q_3^2-m^2_\eta)}, \tag{18}$$

$$\mathcal{A}_d = g_{D^*_{s0}DK}\int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to\eta_cK)\mathcal{A}(\eta_c\to\bar{D}^*D)}{(q_1^2-m^2_{\eta_c})(q_2^2-m^2_K)(q_3^2-m^2_D)}, \tag{19}$$

where q_1, q_2, and q_3 denote the momenta of $\bar{D}^*$, D_s, and η for Fig. 2(a), J/ψ, K, and D for Fig. 2(b), $\bar{D}$, D_s, and η for Fig. 2(c), and η_c, K, and D for Fig. 2(d), and p_1 and p_2 represent the momenta of $\bar{D}^{(*)}$ and $D^*_{s0}(2317)$.

Similarly, the corresponding decay amplitudes of Fig. 3 are written as

$$\mathcal{A}_a = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to D_s^*\bar{D})\mathcal{A}(\bar{D}^*\to\bar{D}\eta)\mathcal{A}(D_s^*\eta\to D_{s1})}{(q_1^2-m^2_{\bar{D}^*})(q_2^2-m^2_{D_s^*})(q_3^2-m^2_\eta)}, \tag{20}$$

$$\mathcal{A}_b = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to\eta_cK)\mathcal{A}(\eta_c\to\bar{D}D^*)\mathcal{A}(D^*K\to D_{s1})}{(q_1^2-m^2_\psi)(q_2^2-m^2_K)(q_3^2-m^2_{D^*})}, \tag{21}$$

and the corresponding amplitudes of Fig. 4 are written as

$$\mathcal{A}_a = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to D_s^*\bar{D}^*)\mathcal{A}(\bar{D}^*\to\bar{D}^*\eta)\mathcal{A}(D_s^*\eta\to D_{s1})}{(q_1^2-m^2_{\bar{D}^*})(q_2^2-m^2_{D_s^*})(q_3^2-m^2_\eta)}, \tag{22}$$

$$\mathcal{A}_b = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to D_s^*\bar{D})\mathcal{A}(\bar{D}\to\bar{D}^*\eta)\mathcal{A}(D_s^*\eta\to D_{s1})}{(q_1^2-m^2_{\bar{D}})(q_2^2-m^2_{D_s})(q_3^2-m^2_\eta)}, \tag{23}$$

$$\mathcal{A}_c = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to J/\psi K)\mathcal{A}(J/\psi\to\bar{D}^*D^*)\mathcal{A}(D^*K\to D_{s1})}{(q_1^2-m^2_\psi)(q_2^2-m^2_K)(q_3^2-m^2_{D^*})}, \tag{24}$$

$$\mathcal{A}_d = \int\frac{d^4q_3}{(2\pi)^4}\frac{i\mathcal{A}(B\to\eta_cK)\mathcal{A}(\eta_c\to\bar{D}^*D^*)\mathcal{A}(D^*K\to D_{s1})}{(q_1^2-m^2_{\eta_c})(q_2^2-m^2_K)(q_3^2-m^2_{D^*})}, \tag{25}$$

where the representation of momenta are the same as Eqs. (16)–(19).

The weak decay amplitudes of $B\to D_s^{(*)}\bar{D}^{(*)}$ and $B\to J/\psi(\eta_c)K$ are written as

$$\mathcal{A}(B\to D_s\bar{D}^*)=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_1 f_{D_s}\Big\{-q_2\cdot\varepsilon(q_1)(m_{\bar{D}^{*0}}+m_{B^+})A_1(q_2^2)+(k_0+q_1)\cdot\varepsilon(q_1)q_2\cdot(k_0+q_1)\frac{A_2(q_2^2)}{m_{\bar{D}^{*0}}+m_{B^+}}$$
$$+(k_0+q_1)\cdot\varepsilon(q_1)[(m_{\bar{D}^{*0}}+m_{B^+})A_1(q_2^2)-(m_{B^+}-m_{\bar{D}^{*0}})A_2(q_2^2)-2m_{\bar{D}^{*0}}A_0(q_2^2)]\Big\},$$
$$\mathcal{A}(B\to D_s\bar{D})=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_1' f_{D_s}(m_B^2-m_D^2)F_{0D}(q_2^2),$$
$$\mathcal{A}(B^+\to D_s^{*+}\bar{D}^0)=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_1^* m_{D_s^{*+}}f_{D_s^{*+}}(k_0+q_1)^\mu F_1(q_2^2),$$
$$\mathcal{A}(B^+\to D_s^{*+}\bar{D}^{*0})=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_1^{*\prime} m_{D_s^{*+}}f_{D_s^{*+}}\left[(-g^{\mu\alpha}(m_{\bar{D}^{*0}}+m_{B^+})A_1(q_2^2)+P^\mu P^\alpha\frac{A_2(q_2^2)}{m_{\bar{D}^{*0}}+m_{B^+}}+i\epsilon^{\mu\alpha\beta\gamma}P_\beta q_\gamma\frac{V(q_2^2)}{m_{\bar{D}^{*0}}+m_{B^+}}\right],$$
$$\mathcal{A}(B\to J/\psi K)=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_2 m_\psi f_\psi\varepsilon(q_1)\cdot(k_0+q_2)F_{1K}(q_1^2),$$
$$\mathcal{A}(B\to\eta_c K)=\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}a_2 f_{\eta_c}(m_B^2-m_K^2)F_{0K}(q_1^2). \tag{26}$$

With these branching ratios of $B^{+(0)}\to D_s^{(*)+}\bar{D}^{(*)0}(D^{(*)-})$ and $B^{+(0)}\to J/\psi(\eta_c)K^{+(0)}$ in Table I, we determine $a_1=0.93(0.95)$, $a_1'=0.80(0.74)$, $a_1^*=0.81(0.83)$, and $a_1'^*=0.83(0.88)$ as well as $a_2=0.27(0.26)$ and $a_2'=0.24(0.21)$, consistent with the estimates of Ref. [68].

The vertices representing the $\bar{D}^{(*)}$ mesons scattering into $\bar{D}^{(*)}$ and η mesons and $J/\psi(\eta_c)$ mesons scattering into $\bar{D}^{(*)}$ and $D^{(*)}$ mesons are written as

$$\mathcal{A}(\bar{D}^*\to\bar{D}\eta)=g_{\bar{D}^*\bar{D}\eta}q_3\cdot\varepsilon(q_1), \tag{27}$$
$$\mathcal{A}(\bar{D}\to\bar{D}^*\eta)=-g_{\bar{D}^*\bar{D}\eta}q_3\cdot\varepsilon(q_1), \tag{28}$$
$$\mathcal{A}(\bar{D}^*\to\bar{D}^*\eta)=g_{\bar{D}^*\bar{D}^*\eta}\varepsilon_{\mu\nu\alpha\beta}q_1^\mu\varepsilon^\nu(q_1)p_1^\alpha\varepsilon^\beta(p_1), \tag{29}$$
$$\mathcal{A}(J/\psi\to\bar{D}D)=-m_\psi/f_\psi(q_3-p_1)\cdot\varepsilon(q_1), \tag{30}$$
$$\mathcal{A}(\eta_c\to\bar{D}^*D)=g_{\eta_c\bar{D}^*D}(q_3+q_1)\cdot\varepsilon(p_1), \tag{31}$$
$$\mathcal{A}(J/\psi\to\bar{D}^*D^*)=g_{J/\psi\bar{D}^*D^*}[\varepsilon(q_1)^\mu(p_1-q_3)_\mu\varepsilon(q_3)^\nu\varepsilon(p_1)_\nu+\varepsilon(p_1)^\mu(q_1+q_3)_\mu\varepsilon(q_1)^\nu\varepsilon(q_3)_\nu-\varepsilon(q_3)^\mu(p_1+q_1)_\mu\varepsilon(q_1)^\nu\varepsilon(p_1)_\nu], \tag{32}$$
$$\mathcal{A}(\eta_c\to\bar{D}^*D^*)=g_{\bar{D}^*\bar{D}^*\eta_c}\varepsilon_{\mu\nu\alpha\beta}q_3^\mu\varepsilon^\nu(q_3)p_1^\alpha\varepsilon^\beta(p_1). \tag{33}$$

The vertices describing the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ molecules generated by $D^{(*)}K$ and $D_s^{(*)}\eta$ coupled channels are expressed as

$$\mathcal{A}(DK\to D_{s0}^*)=g_{D_{s0}^*DK}, \tag{34}$$
$$\mathcal{A}(D_s\eta\to D_{s0}^*)=g_{D_{s0}^*D_s\eta}, \tag{35}$$
$$\mathcal{A}(D_s^*\eta\to D_{s1})=g_{D_s^*\eta D_{s1}}\varepsilon(p_2)\cdot\varepsilon(q_3), \tag{36}$$
$$\mathcal{A}(D^*K\to D_{s1})=g_{D^*KD_{s1}}\varepsilon(p_2)\cdot\varepsilon(q_3). \tag{37}$$

With the above amplitudes determined as specified above, the corresponding partial decay widths can be finally written as

$$\Gamma=8\pi\frac{|\vec{p}|}{m_B^2}\overline{|\mathcal{M}|^2}, \tag{38}$$

where the overline indicates the sum over the polarization vectors of final states, and $|\vec{p}|$ is the momentum of either final state in the rest frame of the B meson.

III. NUMERICAL RESULTS AND DISCUSSION

With the above preparation and the masses of relevant particles given in Table II, we can obtain the decay widths of $B\to\bar{D}^{(*)}D_{s0}^*(2317)$ shown in Table III. We note that the branching ratios of $B^+\to\bar{D}^0D_{s0}^{*+}(2317)$, $B^0\to D^-D_{s0}^{*+}(2317)$, $B^+\to\bar{D}^{*0}D_{s0}^{*+}(2317)$, and $B^0\to D^{*-}D_{s0}^{*+}(2317)$ are consistent with the experimental data within uncertainties [27]. The theoretical uncertainties originate from the breaking of $SU(3)$-flavor symmetry and heavy quark spin symmetry, which are used in deriving the couplings of $g_{\bar{D}^*\bar{D}\eta}$ and $g_{\eta_c/J/\psi\bar{D}^{(*)}D}$. We assume that the breaking of $SU(3)$-flavor symmetry is at the order of 20% and that of heavy quark spin symmetry is at the level of 20% [75]. Adding them in quadrature, we obtain the theoretical uncertainty of 28% given in Table III.

In Ref. [55], the authors estimated the branching ratios of $B\to\bar{D}^{(*)}D_{s0}^*(2317)$ by the naive factorization approach, where the coupling $f_{D_{s0}^*}$ is determined treating $D_{s0}^*(2317)$

TABLE II. Masses and quantum numbers of mesons relevant to the present work [27].

Meson	$I(J^P)$	M (MeV)	Meson	$I(J^P)$	M (MeV)
B^0	$\frac{1}{2}(0^-)$	5279.65	B^+	$\frac{1}{2}(0^-)$	5279.34
D^0	$\frac{1}{2}(0^-)$	1864.84	D^+	$\frac{1}{2}(0^-)$	1869.66
D^{*0}	$\frac{1}{2}(1^-)$	2006.85	D^{*+}	$\frac{1}{2}(1^-)$	2010.26
D_s^+	$0(0^-)$	1968.34	D_s^{*+}	$0(1^-)$	2112.2
D_{s0}^*	$0(0^+)$	2317.8	D_{s1}	$0(1^+)$	2459.5
K^+	$\frac{1}{2}(0^-)$	493.677	K^0	$\frac{1}{2}(0^-)$	497.611
η	$0(0^-)$	547.862	J/ψ	$0(1^-)$	3096.9

TABLE III. Branching ratios (10^{-3}) of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$.

Decay modes	Our results	[55]	Exp [27]	BarBar
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	0.677 ± 0.190	1.03 ± 0.14	$0.80^{+0.16}_{-0.13}$	$1.0 \pm 0.3 \pm 0.1$
$B^0 \to D^- D_{s0}^{*+}(2317)$	0.637 ± 0.178	0.96 ± 0.13	$1.06^{+0.16}_{-0.16}$	$1.8 \pm 0.4 \pm 0.3$
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	1.210 ± 0.339	0.50 ± 0.07	$0.90^{+0.70}_{-0.70}$	$0.9 \pm 0.6 \pm 0.2$
$B^0 \to D^{*-} D_{s0}^{*+}(2317)$	0.889 ± 0.249	0.47 ± 0.06	$1.50^{+0.60}_{-0.60}$	$1.5 \pm 0.4 \pm 0.2$
$B^+ \to \bar{D}^0 D_{s1}^+(2460)$	1.255 ± 0.351	2.54 ± 0.39	$3.1^{+1.0}_{-0.9}$	$2.7 \pm 0.7 \pm 0.5$
$B^0 \to D^- D_{s1}^+(2460)$	1.158 ± 0.324	2.36 ± 0.36	3.5 ± 1.1	$2.8 \pm 0.8 \pm 0.5$
$B^+ \to \bar{D}^{*0} D_{s1}^+(2460)$	3.065 ± 0.858	7.33 ± 1.12	12.0 ± 3.0	$7.6 \pm 1.7 \pm 1.8$
$B^0 \to D^{*-} D_{s1}^+(2460)$	2.709 ± 0.759	6.85 ± 1.05	9.3 ± 2.2	$5.5 \pm 1.2 \pm 1.0$

as a pure DK molecule. Their branching ratios are shown in Table III. We note that the branching ratios of $B \to \bar{D} D_{s0}^*(2317)$ and $B \to \bar{D}^* D_{s0}^*(2317)$ are consistent with ours, but those of $B \to \bar{D}^{*0} D_{s0}^*(2317)$ and $B \to \bar{D}^{*-} D_{s0}^*(2317)$ are smaller than ours and in worse agreement with the experimental data. We note that many recent works claim that $D_{s0}^*(2317)$ contains a $c\bar{s}$ component of 30%, which is not explicitly taken into account in both our work and that of Ref. [55]. Considering such an uncertainty, both our results and those of Ref. [55] are consistent with the experimental data.

For the $D_{s1}(2460)$ state, our predictions for all the four processes studied are smaller than the PDG averages by about a factor of 3 and than the *BABAR* results by roughly a factor of 2. On the other hand, the results of Ref. [55] are in better agreement with the data. In Ref. [55], the authors estimated such branching ratios via a naive factorization approach, where the determination of the couplings $f_{D_{s0}^*}$ and $f_{D_{s1}}$ depends on the choice of cutoff parameter and relies on the SU(4) symmetry which relates the weak vertices $D^* \to KW$ and $D \to K^* W$. Furthermore, in Ref. [55], $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are treated as pure DK and D^*K molecules, while in our approach it is shown that the coupled channel $D^{(*)}\eta$ plays an important role as well. The discrepancy between our results and the experimental data can be attributed to either missing reaction mechanisms or the neglect of the likely existence of a relatively large $c\bar{s}$ component in the wave function of $D_{s1}(2460)$. In most of the unquenched quark models, both $D_{s1}(2460)$ and $D_{s0}^*(2317)$ contain sizable $c\bar{s}$ components, while the former contains a larger $c\bar{s}$ component. In addition, in the molecular picture, other reaction mechanisms than the triangle mechanism studied here can also contribute to the production of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in B decays, such as those studied in Refs. [48,76].

We decompose the contributions of the η and $D^{(*)}$ exchanges in Table IV. Note that the processes mediated

TABLE IV. Branching ratios (10^{-3}) of $B \to \bar{D}^{(*)} D_{s0}^*(2317)$ and $B \to \bar{D}^{(*)} D_{s1}(2460)$.

Decay modes	Total results	η meson exchange	$D^{(*)}$ meson exchange
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	0.677	0.414	0.033
$B^0 \to D^- D_{s0}^{*+}(2317)$	0.637	0.401	0.028
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	1.210	0.246	0.382
$B^0 \to D^{*-} D_{s0}^{*+}(2317)$	0.889	0.194	0.264
$B^+ \to \bar{D}^0 D_{s1}^+(2460)$	1.255	0.209	0.442
$B^0 \to D^- D_{s1}^+(2460)$	1.158	0.202	0.309
$B^+ \to \bar{D}^{*0} D_{s1}^+(2460)$	3.065	1.263	0.648
$B^0 \to D^{*-} D_{s1}^+(2460)$	2.709	1.298	0.446

by the η meson contain stronger weak-interaction vertices but weaker strong-interaction scattering vertices with respect to those mediated by the $D^{(*)}$ meson, while the couplings of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ molecules to their constituents $D^{(*)}K$ and $D_s^{(*)}\eta$ are approximately the same in the particle basis, i.e., $g_{D_{s0}^*D^+K^0} \approx g_{D_{s0}^*D_s^+\eta}$ ($g_{D_{s1}D^{*+}K^0} \approx g_{D_{s1}D_s^{*+}\eta}$). From Table IV, one can see that among the eight branching ratios studied, the contribution of the η exchange is comparable to that of the $D^{(*)}$ exchange except for the processes $B \to \bar{D}D_{s0}^*(2317)$, where the $D^{(*)}$ contribution is accidentally one order of magnitude smaller that of the η exchange.

IV. SUMMARY AND DISCUSSION

To distinguish the nature of $D_{s0}^*(2317)$ as either a DK molecule, a $c\bar{s}$ state, or a combination of both has motivated a lot of experimental and theoretical studies. In this work, we utilized the triangle mechanism to describe the decays of $B \to \bar{D}D_{s0}^*(2317)$ and $B \to \bar{D}^*D_{s0}^*(2317)$, assuming that the B meson first weakly decays into $\bar{D}^*D_s$ and $J/\psi K$, then $\bar{D}^*$ and J/ψ mesons scatter to $\bar{D}^{(*)}\eta$ and $\bar{D}^*D$, and finally $D_{s0}^*(2317)$ is dynamically generated by the DK and $D_s\eta$ coupled-channel interactions. Without any unknown parameters, we take the effective Lagrangian approach to calculate the branching ratios as $\mathrm{Br}[B^+ \to \bar{D}^0D_{s0}^{*+}(2317)] = 0.677 \times 10^{-3}$ ($\mathrm{Br}[B^0 \to D^-D_{s0}^{*+}(2317)] = 0.676 \times 10^{-3}$), and $\mathrm{Br}[B^+ \to \bar{D}^{*0}D_{s0}^{*+}(2317)] = 1.210 \times 10^{-3}$ ($\mathrm{Br}[B^0 \to D^{*-}D_{s0}^{*+}(2317)] = 0.889 \times 10^{-3}$), which are in reasonable agreement with the experimental data.

In the same approach, we also investigated the decays of $B \to \bar{D}D_{s1}(2460)$ and $B \to \bar{D}^*D_{s1}(2460)$, where $D_{s1}(2460)$ is dynamically generated by the D^*K and $D_s^*\eta$ coupled-channel interactions. Our results, $\mathrm{Br}[B^+ \to \bar{D}^0D_{s1}^+(2460)] = 1.255 \times 10^{-3}$ ($\mathrm{Br}[B^0 \to D^-D_{s1}^+(460)] = 1.158 \times 10^{-3}$), and $\mathrm{Br}[B^+ \to \bar{D}^{*0}D_{s1}^+(2460)] = 3.065 \times 10^{-3}$ ($\mathrm{Br}[B^0 \to D^{*-}D_{s1}^+(2460)] = 2.709 \times 10^{-3}$), are smaller than the experimental central values by almost a factor of 2–3. Such a deviation can be attributed to either a smaller molecular component in the $D_{s1}(2460)$ wave function or reaction mechanisms missing in the present work.

We note that the degree of agreement between our predictions and the experimental data indeed provides further support for the molecular nature of $D_{s0}^*(2317)$ and $D_{s1}(2460)$. However, more precise data and further theoretical studies are needed in order to pin down the precise percentage of the $c\bar{s}$ and $D^*K/D_s^{(*)}\eta$ components in their wave functions.

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