Spin correlations in τ -lepton pair production due to anomalous magnetic and electric dipole moments

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We present a simple algorithm for the calculation of event weights embedding the effects of anomalous electric and magnetic dipole moments in simulation of $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$ events, and the subsequent decay of the τ -leptons produced. The impact of these weights on the spin-correlation matrix and the total cross section is taken into account. The algorithm is prepared to work *in situ* the Monte Carlo KKMC program without the need for introducing any external change to the generator libraries. As an example, $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$, $\tau^- \rightarrow \rho^-\nu_{\tau} \rightarrow \pi^-\pi^0\nu_{\tau}$, $\tau^+ \rightarrow \rho^+\bar{\nu}_{\tau} \rightarrow \pi^+\pi^0\bar{\nu}_{\tau}$ events were simulated at a center-of-mass energy of 10.58 GeV. The distributions of the acoplanarity angle between the planes spanned by the $\pi^-\pi^0$ and the $\pi^+\pi^0$ momenta of, respectively, ρ^- and ρ^+ decays and in the rest frame of the entirely visible $\rho^-\rho^+$ system are presented for different values of the coupling constants incorporating anomalous electric and magnetic dipole moments in the $\tau^-\tau^+\gamma$ vertex.

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I. INTRODUCTION

The recently improved searches for electric dipole moments in τ -lepton pair production [1] have brought renewed attention to this topic in recent papers [2-4]. Furthermore, the deviation in the measurement of the anomalous magnetic moment of the muon [5] from its theoretical predictions has brought renewed attention to their measurement in τ -leptons [6], where the new physics contributions could be enhanced, as mentioned in Ref. [7] and references therein. Because of the short lifetime of the τ -lepton [8], the design of observables is complicated, and measurable signatures need to be combined from many τ -decay channels. Not only are all the final-state particles from decays of the τ -lepton not observed due to the presence of neutrinos, the kinematic constraints from energy-momentum conservation also need to be modified due to the presence of initial-state bremsstrahlung photons, some of which are often lost in the beam pipe. The use of the τ -decay vertex position [9] may be of help, but it is not a straightforward technique, since the details of experimental arrangements are critically important information, and experimental resolutions due to different detector responses need to be taken into account using simulations.

A short list of earlier searches for anomalous electromagnetic moments in the τ -pair production in e^-e^+ collisions include Refs. [10–14], and the role of the electron longitudinal polarization were noted in Refs. [15–18]. More recently, much attention has been paid to the LHC observations [19,20] for obtaining information on anomalous electromagnetic moments of the τ -lepton in peripheral collisions and $\gamma\gamma$ production of τ pairs [21,22], by using the phenomenon of spin precession in bent crystals [23,24], and in the γp collisions [25].

At present, for Belle II phenomenology, the KKMC Monte Carlo program [26] is widely used in the generation of event kinematics as input to detailed simulation of detector response. The program features all major τ -decay channels, complete with spin effects including longitudinal and transverse spin correlations between the two τ -leptons, higher-order QED corrections, and capable of ensuring that the $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$ cross sections reach precision at the per mille level [27]. The effects of anomalous dipole moments are not expected to be large, but the development of necessary calculations and tools which ensure the high precision of the Standard Model (SM) prediction is at present of renewed interest. Numerical effects of anomalous couplings need to be presented in the form in which all SM effects as well as detector resolution and acceptance effects are taken into

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account at the same time with sufficient precision. That is why it is useful to prepare theoretical predictions in a form suitable for use with Monte Carlo event generation programs. Here, we present such a solution for the KKMC program.

Our paper is organized as follows. In Sec. II, we present the amplitudes and spin-correlation matrix. The conventions and orientation of the quantization frames are emphasized. Compatibility with respect to the choices used in Refs. [28-31] are discussed in Sec. III, where some numerical results are discussed as well. The first group of results is oriented toward tests of the algorithm. For these results, all final-state momenta including the unobservable τ -neutrino momenta are used. We next discuss the results with a semirealistic observable, which is the acoplanarity angle φ between the two planes spanned by the decays $\rho^- \to \pi^- \pi^0$ and $\rho^+ \to \pi^+ \pi^0$, and defined in the $\rho^- \rho^+$ rest frame. The events of the process $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$, $\tau^- \to \rho^- \nu_\tau \to \pi^- \pi^0 \nu_\tau, \ \tau^+ \to \rho^+ \bar{\nu}_\tau \to \pi^+ \pi^0 \bar{\nu}_\tau$ are used to monitor the impact of anomalous magnetic and electric dipole moments. In Sec. IV, we demonstrate how the algorithm can be installed and used for weights of events simulated with the KKMC program. The summary Sec. V closes the paper.

II. AMPLITUDES AND SPIN CORRELATIONS

We consider electron-positron annihilation to a pair of τ -leptons

$$e^{-}(k_{-}) + e^{+}(k_{+}) \rightarrow \tau^{-}(p_{-}) + \tau^{+}(p_{+})$$
 (1)

with the four-momenta satisfying energy-momentum conservation $k_{-} + k_{+} = p_{-} + p_{+}$.

In the center-of-mass frame, the components of the momenta are

$$p_{-} = (E, \vec{p}), \qquad p_{+} = (E, -\vec{p}), \qquad \vec{p} = (0, 0, p),$$

$$k_{-} = (E, \vec{k}), \qquad k_{+} = (E, -\vec{k}),$$

$$\vec{k} = (E\sin(\theta), 0, E\cos(\theta)), \qquad (2)$$

so that the \hat{z} axis is along the momentum \vec{p} , the reaction plane $\hat{x}\hat{z}$ is defined by the momenta \vec{p} and \vec{k} , and the \hat{y} axis is along $\vec{p} \times \vec{k}$. Here, *E* is the beam energy related to the squared invariant energy *s* by relation $E = \sqrt{s}/2$, $p = \sqrt{E^2 - m^2}$ is the magnitude of the 3-momentum of the τ -lepton, *m* is the mass of the τ , and θ is the scattering angle. In the following, we use the Lorentz factor $\gamma = E/m$ and the τ -lepton velocity $\beta = \sqrt{1 - \gamma^{-2}}$. The mass of the electron is neglected.

The quantization frames of τ^- and τ^+ are connected to this reaction frame by the appropriate boosts along the \hat{z} direction. Note that the \hat{z} axis is parallel to momentum of τ^- but antiparallel to momentum of τ^+ . The momenta of the beams reside in the $\hat{x} \hat{z}$ plane. Only the reaction frame, the τ^- , and the τ^+ rest frames are used for calculations throughout the paper. The vector indices used in the program 1, 2, 3, 4 correspond to the \hat{x} , \hat{y} , \hat{z} , \hat{t} directions, respectively.

We assume that the electromagnetic vertex for the τ -lepton has the following structure:

$$\Gamma^{\mu} = \gamma^{\mu} + \frac{\sigma^{\mu\nu}q_{\nu}}{2m}[ia(s) + \gamma_5 b(s)], \qquad (3)$$

where $q = p_{-} + p_{+}$. The functions a(s) and b(s) are related to the Pauli and electric dipole form factors, respectively,

$$a(s) = F_2(s), \qquad b(s) = F_3(s).$$
 (4)

The form factors at $s \ge 4m^2$ acquire imaginary parts due to loop corrections and final-state interaction.¹ Thus, we choose the a(s) and the b(s) coefficients to be complex numbers. Up to two-loop corrections, one can use for the Dirac form factor $F_1(s)$, an approximation $F_1(s) = 1$, and therefore, we do not include $F_1(s)$ in the first term of Eq. (3).

The quantity a(0) is the anomalous magnetic dipole moment (AMDM), while b(0) is related to the electric dipole moment (EDM) d, namely,

$$a(0) = \frac{1}{2}(g-2), \qquad b(0) = \frac{2m}{e}d.$$
 (5)

In order to separate the contribution from new physics (NP), one can explicitly include the contribution from QED in a(s). To the first order in the fine-structure constant α , one has at $s \ge 4m^2$ [32]

$$a(s)_{\text{QED}} = \frac{\alpha m^2}{\pi s \beta} \left(\log \frac{1-\beta}{1+\beta} + i\pi \right). \tag{6}$$

This contribution at very large values of $s \ (s \gg 4m^2)$ behaves as

$$a(s)_{\text{QED}} = \frac{\alpha m^2}{\pi s} \left(-\log \frac{s}{m^2} + i\pi \right). \tag{7}$$

¹In general, an imaginary part arises at s > 0 due to the three-photon intermediate state in the higher-order loop corrections.

Therefore, in general, we can write

$$a(s) = a(s)_{\text{QED}} + a(s)_{NP}, \qquad b(s) = b(s)_{NP}, \quad (8)$$

neglecting the very small contribution to b(s) in the SM.

The matrix element for our reaction process can be written in terms of the spinors of initial- and final-state fermions as

$$\mathcal{M} = -\frac{e^2}{s} \bar{v}_e(k_+) \gamma_\mu u_e(k_-) \bar{u}_\tau(p_-) \Gamma^\mu v_\tau(p_+), \qquad (9)$$

where *e* is the positron charge satisfying $e^2 = 4\pi\alpha$. The differential cross section is related to the matrix element squared through

$$\frac{d\sigma}{d\Omega} = \frac{\beta}{64\pi^2 s} |\mathcal{M}|^2. \tag{10}$$

We consider production of the polarized τ^{-} and τ^{+} -leptons, which are characterized by the polarization 3-vectors in their rest-frames, respectively,

$$\vec{s}^- = (s_1^-, s_2^-, s_3^-), \qquad \vec{s}^+ = (s_1^+, s_2^+, s_3^+), \qquad (11)$$

where the Cartesian components are defined with respect to the chosen frame constructed from $\vec{p_{\pm}}$, $\vec{k_{\pm}}$ momenta and their vector products $\vec{p_{\pm}} \times \vec{k_{\pm}}$. It is convenient to introduce the fourth components of the spin vectors as follows:

$$s_i^- = (s_1^-, s_2^-, s_3^-, 1), \qquad s_j^+ = (s_1^+, s_2^+, s_3^+, 1)$$
(12)

with i, j = 1, 2, 3, 4.

After squaring the matrix element and averaging over the polarizations of the electron and positron, we obtain an expression in the form

$$|\mathcal{M}|^2 = \sum_{i,j=1}^4 R_{ij} s_i^- s_j^+.$$
 (13)

We keep only terms linear in a(s) and b(s).

The spin-correlation coefficients R_{ij} listed below depend on the energy *s* and the scattering angle θ , since the *s* and $\cos \theta$ differ for each event and its effective Born kinematic interpolation. However, for brevity of notation, the energy and scattering angle dependence is not explicitly indicated, and we denote $a(s) \equiv a$ and $b(s) \equiv b$ in the following expressions:

$$\begin{aligned} R_{11} &= \frac{e^4}{4\gamma^2} (4\gamma^2 \operatorname{Re}(a) + \gamma^2 + 1) \sin^2(\theta), \\ R_{12} &= -R_{21} = \frac{e^4}{2} \beta \sin^2(\theta) \operatorname{Re}(b), \\ R_{13} &= R_{31} = \frac{e^4}{4\gamma} [(\gamma^2 + 1) \operatorname{Re}(a) + 1] \sin(2\theta), \\ R_{22} &= -\frac{e^4}{4} \beta^2 \sin^2(\theta), \\ R_{23} &= -R_{32} = -\frac{e^4}{4} \beta \gamma \sin(2\theta) \operatorname{Re}(b), \\ R_{33} &= \frac{e^4}{4\gamma^2} [(4\gamma^2 \operatorname{Re}(a) + \gamma^2 + 1) \cos^2(\theta) + \beta^2 \gamma^2], \\ R_{14} &= -R_{41} = \frac{e^4}{4} \beta \gamma \sin(2\theta) \operatorname{Im}(b), \\ R_{24} &= R_{42} = \frac{e^4}{4} \beta^2 \gamma \sin(2\theta) \operatorname{Im}(b), \\ R_{34} &= -R_{43} = -\frac{e^4}{2} \beta \sin^2(\theta) \operatorname{Im}(b), \\ R_{44} &= \frac{e^4}{4\gamma^2} [4\gamma^2 \operatorname{Re}(a) + \beta^2 \gamma^2 \cos^2(\theta) + \gamma^2 + 1]. \end{aligned}$$

The imaginary parts of *a* and *b* lead to terms linear in polarizations which correspond to nonzero polarizations of the τ -leptons. The polarizations along the \hat{x} axis (called the transverse polarization) and \hat{z} axis (called the longitudinal polarization) induced by Im(*b*) are opposite in sign for τ^- and τ^+ , while those along the \hat{y} axis (called the normal polarization) induced by Im(*a*) have the same sign.

It is of interest to separate the contribution from the coupling b(s),

$$|\mathcal{M}|_{\text{EDM}}^{2} = \frac{e^{4}}{2} \beta \{ [(s_{1}^{-}s_{2}^{+} - s_{2}^{-}s_{1}^{+})\sin(\theta) \\ - \gamma(s_{2}^{-}s_{3}^{+} - s_{3}^{-}s_{2}^{+})\cos(\theta)] \text{Re}(b) \\ - [(s_{3}^{-} - s_{3}^{+})\sin(\theta) \\ - \gamma(s_{1}^{-} - s_{1}^{+})\cos(\theta)] \text{Im}(b) \} \sin(\theta), \quad (15)$$

which can be useful in the determination of b(s). Recently, new constraints on the value of EDM have been obtained by the Belle experiment at the KEKB e^+e^- collider [1]. These results improve upon the previous limits on EDM also obtained by the Belle Collaboration [33].

Finally, we address the issue of radiative corrections. In the KKMC program, beyond exclusive exponentiation, QED corrections up to the second order (real and virtual) are included or considered. The two-loop virtual corrections, not enhanced logarithmically, are not necessary in full and are taken into account only in part. In particular, the imaginary parts of the second-order contributions to the form factors belong to that ignored category. A more complete description of radiative corrections in KKMC and program limitations for use at relatively low energies are given in Ref. [27], where approximations for the box diagrams' contributions, and some other terms, also proportional to $\alpha m/\sqrt{s}$, are discussed. To reduce the ambiguities further, studies that are broader than those of Ref. [27] would be needed. This remains outside the scope of the present paper.

III. NUMERICAL RESULTS AND TESTS

We first need to check if Eq. (14) are properly installed in our program for the case a(s) = b(s) = 0. In this case, we obtain the expression

$$\begin{aligned} |\mathcal{M}|_{a=b=0}^{2} &= \frac{e^{4}}{4\gamma^{2}} \{ \gamma^{2} + 1 + \beta^{2}\gamma^{2}\cos^{2}(\theta) \\ &+ s_{3}^{-}s_{3}^{+}[\beta^{2}\gamma^{2} + (\gamma^{2} + 1)\cos^{2}(\theta)] \\ &+ [s_{1}^{-}s_{1}^{+}(\gamma^{2} + 1) - s_{2}^{-}s_{2}^{+}\beta^{2}\gamma^{2}]\sin^{2}(\theta) \\ &+ (s_{1}^{-}s_{3}^{+} + s_{3}^{-}s_{1}^{+})\gamma\sin(2\theta) \}, \end{aligned}$$
(16)

which is consistent with Ref. [34], and under the condition $E \gg m$ reduces to the known decomposition in Ref. [35]

$$|\mathcal{M}|^{2}_{a=b=0,E\gg m} = \frac{e^{4}}{4} \left[(1 + \cos^{2}(\theta))(1 + s_{3}^{-}s_{3}^{+}) + \sin^{2}(\theta)(s_{1}^{-}s_{1}^{+} - s_{2}^{-}s_{2}^{+}) \right].$$
(17)

These well-established formulas provide good intuition, but we have to check if the conventions of our code for the calculation of the weights implementing the effects of anomalous dipole moments, and in particular, the orientation of the reference frame, match what is used in KKMC and TAUOLA, as described in Refs. [28–31].

For our application, we calculate the following weights for events generated with KKMC+TAUOLA:

$$wt_{\rm spin}^{\rm SM} = R_{ij}^{\rm SM} h_i^- h_j^+ / R_{tt}^{\rm SM}, \qquad (18)$$

$$wt_{\rm spin} = R_{ij}h_i^-h_j^+/R_{tt}/wt_{\rm spin}^{\rm SM},$$
 (19)

$$wt = R_{tt}/R_{tt}^{\rm SM},\tag{20}$$

where *i*, *j* = 1, 2, 3, 4, and subscript *tt* stands for *i* = *j* = 4. The so-called polarimetric vectors h_i^- and h_j^+ in Eqs. (18)–(20) depend on the kinematics of, respectively, the τ^- - and τ^+ -decay products [36].

The R_{ij} elements are now calculated as functions of s and $\cos(\theta)$ which need to be interpolated from the kinematics of the generated events. Interpolation may be nontrivial for the case where hard bremsstrahlung photons are present, as clarified in the next section with technical details. R_{ij}^{SM} is obtained from R_{ij} by setting anomalous

couplings a_{NP} , b_{NP} , as well as the QED dipole moment, to zero.

We present two sets of tests in the subsections below.

A. Technical tests with no AMDM and EDM form factors

For the first set of tests, we check if spin correlations of Eq. (18) match those present in KKMC. For that purpose, we generate two samples of events: one with the spin effects included and the other with spin effects not included. For the latter case, spin effects are instead implemented with the event wights of Eq. (18). The following events were used: $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$, $\tau^- \rightarrow \pi^-\nu_{\tau}$, $\tau^+ \rightarrow \pi^+\overline{\nu}_{\tau}$. The distributions obtained for the two approaches need to coincide. In our technical test, we use unobservable momenta of τ^{\pm} for separation of the sample into subsamples where hard-, soft, or no-bremsstrahlung events are present, and for the definition of distributions as well.

For the preparation of the test distributions we perform the following steps:

- (1) define \hat{n}_3^{\pm} versors in the directions of τ^{\pm} in the lab frame,
- (2) define \hat{n}_2^{\pm} versors as following direction of vector products of τ^{\pm} with e^{\pm} beam momenta, also in the lab frame,
- (3) define $\hat{n}_{1}^{\pm} = \hat{n}_{2}^{\pm} \times \hat{n}_{3}^{\pm}$,
- (4) decompose the π[±] momenta denoted by q[±], respectively, in the n⁺ and n⁻ frames,
- (5) the components of \vec{q}^{\pm} obtained this way are used to monitor each of the R_{ij} elements. For histograming, the vectors \vec{q}^{\pm} are used, and events with $\cos(\theta)$ positive and negative are taken separately.

We note that the frames \hat{n}^+ and \hat{n}^- are obtained from each other by rotation around the \hat{y} axis on angle 180°. These frames are convenient for preparing histograms and are used below.

1. Born and soft photon case

The KKMC event samples with spin correlation included and with spin correlations absent, but using our $wt_{\rm spin}^{\rm SM}$, are compared. To be sensitive to the linear in $\cos(\theta)$ terms, we request that the τ^- momentum lies in the forward hemisphere ($\cos(\theta) \ge 0$). We first performed such a test for events in which no bremsstrahlung photons were present, and later for the ones with soft/collinear photons present as well. There were no distinguishable differences for spin correlations taken from KKMC and from our weights, both in the Born approximation and if the soft photons determined by the condition on the $\tau^-\tau^+$ invariant mass $m^2(\tau^-\tau^+)/s > 0.98$ were allowed.

2. Hard photon case

Now we turn our attention to events in which photons are present and choose the $\tau^-\tau^+(n\gamma)$ events in which $m^2(\tau^-\tau^+)$



FIG. 1. Correlation of $\pi^-\pi^+$ momenta (top left, top right, and bottom left), and distribution over $\pi^-\pi^+$ invariant mass (bottom right) for the e^-e^+ energy $\sqrt{s} = 10.58$ GeV. Top panels: correlation of components x - x (left) and components y - y(right). Bottom: correlation of components x - z (left). Anomalous couplings and QED contribution are not included. Two lines in each histogram show (i) KKMC event sample with spin correlations, but instead wt_{spin}^{SM} of Eq. (18) is used. The two lines of the histograms overlap almost completely.

was in the 0.2–0.98 range of the collision Mandelstam variable *s*. Most of these events include hard photons collinear to the beams, but we still check the impact of the dominant QED bremsstrahlung effects on our weighting method, and confirm that events of high p_T photons do not contribute sizable ambiguities.

The distribution of the invariant mass of the pion pair $m(\pi^{-}\pi^{+})$ is shown in Fig. 1 (bottom right). It is sensitive to the z - z correlation R_{33} . The z - z correlation is strongly positive; this is because the τ -leptons are relativistic. The z - z spin correlations are much better monitored by the distribution over $m(\pi^{-}\pi^{+})$. Spin correlations enhance the number of events for $m(\pi^{-}\pi^{+})$ around ~2 GeV and less visibly around ~8 GeV, as noted also in Fig. 2 of Ref. [37].

The purpose of the plots presented is purely technical. They are used to test elements of our code, first when photons are absent, then for events where soft photons are allowed, and finally, when only events of hard photons are selected.

For the case when $m^2(\tau^-\tau^+)/s > 0.98$, where only soft photons may be present, the comparison indicates statistically indistinguishable distribution. Only the correlations x - x, y - y, z - z, x - z, and z - x are nonzero, as noted in Eq. (16). We also checked that other correlations are zero. Thus, for the spin correlations of KKMC and our spin weights, only the plots with hard photons are included in the paper. It should be noted that for histograming purposes, the four momenta of only final states are used.

B. Results with AMDM and EDM form factors

Now we can turn our attention to the distributions sensitive to the dipole moments. Our aim is to check an extension to the KKMC event generator enabling inclusion of the AMDM and EDM form factors through the event weights listed in Eqs. (19) and (20). The selection of the actual distribution is not straightforward. For example, the ν_{τ} and the $\bar{\nu}_{\tau}$ momenta are not observed directly, but partially can be inferred through the reconstruction of the τ -decay vertex position. This is, of course, τ -decay-channel dependent. That is why the appropriate study including detection ambiguities requires simulation details of specific detector configurations, and it is out of the scope of this phenomenological paper. Our emphasis here is devoted to the extension of the KKMC event generation tool, a necessary prerequisite step to experimental studies involving detector simulations.

More than 25 years ago, it was demonstrated in Ref. [38] that the analyzing power of multimeson final states in semileptonic τ decays with respect to the τ spin is equal and maximal for all decay modes. Obviously, the $\tau^{\pm} \rightarrow \pi^{\pm}\nu_{\tau}$ decay mode provides signatures which are the easiest to interpret. In this connection, we should mention the studies in Refs. [39–42] of the τ -spin correlations in the $e^-e^+ \rightarrow$ $\tau^-\tau^+ \rightarrow h^-\nu_{\tau}h^+\bar{\nu}_{\tau}$ process, where h^{\pm} are hadrons and no secondary decays of h^{\pm} were taken into account. The authors constructed asymmetries sensitive to AMDM and EDM for the conditions of super-B/flavor factories. Inclusion of the electron beam polarization was essential for isolating the $\gamma\tau^-\tau^+$ anomalous form factors.

The main ambiguity of the measurement of the $\tau^{\pm} \rightarrow \pi^{\pm}\nu_{\tau}$ channel, and perhaps of any two-particle decay channel $\tau^{\pm} \rightarrow h^{\pm}\nu_{\tau}$, may come from the precision of the τ -decay vertex position reconstruction. From this point of view, the channel $\tau^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}\pi^{\mp}\nu_{\tau}$ may be better. On the other hand, systematic ambiguity from modeling of this τ -decay channel may need to be revisited. Thus, the work of Ref. [43] may need to be revised for the new application.

Our observable is independent of the details of the hadronic spectrum, as long as the scalar contribution to the hadronic current is negligible, which was shown by the Belle experiment for τ decays to two scalars in Ref. [44].

For these reasons, we have chosen to demonstrate the functionality of our code with the $\tau^{\pm} \rightarrow \rho^{\pm}\nu_{\tau} \rightarrow \pi^{\pm}\pi^{0}\nu_{\tau}$ channel. We may rely, in preparation of our test observable, only on the kinematic of secondary ρ decay. Then, there is no need to use τ -decay vertex reconstruction, and that is why this decay channel is of interest from the point of view of systematic ambiguity evaluation.

To demonstrate the potential of our approach, let us consider the cascade process: $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$, $\tau^- \rightarrow \rho^-\nu_{\tau} \rightarrow \pi^-\pi^0\nu_{\tau}$, $\tau^+ \rightarrow \rho^+\bar{\nu}_{\tau} \rightarrow \pi^+\pi^0\bar{\nu}_{\tau}$. We use the method of Refs. [45,46]. For constructing observables sensitive to the *CP* parity of the Higgs boson, it was suggested to measure the acoplanarity angle φ between the planes spanned on the decays $\rho^- \rightarrow \pi^-\pi^0$ and $\rho^+ \rightarrow \pi^+\pi^0$ and defined in the $\rho^-\rho^+$ rest frame. The momenta of all the pions are to be measured, which would yield the ρ^- and ρ^+ momenta, which are then boosted to the $\rho^-\rho^+$ rest frame.

In order to be sensitive to the transverse spin correlations, additional cuts need to be applied. Accordingly, we apply the following constraint on the sign of the product $y_1y_2 > 0$, where

$$y_1 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}, \qquad y_2 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}.$$
 (21)

Here, y_1 and y_2 are measured from the decay products of ρ^- and ρ^+ , respectively, and the energies of the pions are taken in the rest frame of the $\rho^-\rho^+$ pair. Further details are provided in the original papers [45,46].

In Fig. 2, we present results of the Monte Carlo simulation of the acoplanarity angle distribution using the weight method for a few choices of the couplings $a(s)_{NP}$ and $b(s)_{NP}$. We assume for simplicity that $a(s)_{NP}$ and $b(s)_{NP}$ take real values. Of course, absolute values are chosen arbitrarily, and these figures demonstrate the sensitivity of our method to the anomalous couplings.



FIG. 2. Distribution of the wt_{spin} as a function of the acoplanarity angle φ at $\sqrt{s} = 10.58$ GeV with the constraint $y_1y_2 > 0$. For the top left plot, $\text{Re}(a_{NP}) = 0.04$ and other couplings are zero. For the top right plot, $\text{Re}(b_{NP}) = 0.04$ and other couplings are zero. For the bottom plot, $\text{Re}(a_{NP}) = 0.04 \cos(\pi/4)$, $\text{Re}(b_{NP}) = 0.04 \sin(\pi/4)$, and other couplings are zero.

In these histograms, events without photons or with soft photons are selected using the constraint $m^2(\tau^-\tau^+)/s \ge 0.98$, although our reweighting technique still works even if this criterion is relaxed.

It is seen from these figures that the effect of the anomalous couplings on the distribution can reach about 0.005. Of course, this is related to the absolute value of couplings taken 0.04, which is already too large to be a realistic value. This value is also much larger than the leading-order QED correction $a(s)_{\text{QED}} = -0.000244 + i0.000219$.

The distributions for $\text{Re}(a_{NP}) = 0.04$ and $\text{Re}(b_{NP}) = 0.04$ have the form of a sinusoid and are shifted with respect to each other on the angle $\sim \pi/2$. The distribution for the case in which both couplings are present in Fig. 2 (bottom) is shifted on the angle $\sim \pi/4$ to the distributions in Fig. 2 (top). It may be useful to combine Re(a) and Re(b) in one complex coupling

$$c \equiv \operatorname{Re}(a) + i\operatorname{Re}(b) = \sqrt{(\operatorname{Re}(a))^2 + (\operatorname{Re}(b))^2 \exp(i\psi)},$$
$$\tan(\psi) = \operatorname{Re}(b)/\operatorname{Re}(a). \tag{22}$$

Then it appears that the angle ψ describes the shift of the distributions in Fig. 2. It gives a measure of *CP* violation in the photon interaction with the τ -leptons. Equation (22) can be extended to the case in which *a* and *b* are complex.

IV. CODE IMPLEMENTATION

In the future, one will be able to read in simulated events and use TauSpinner with pointer provided weights. One could also install R_{ij} in KORALB or play with the TRALOR routine of KKMC which define the relation between τ -lepton rest frames and laboratory frames.

At present, we assume that our solution will be applied simultaneously with KKMC event generation and its internal variables will be available. That is why we can proceed following the technique presented in Ref. [31]. The KKMC polarimetric vectors h_i^{\pm} are first boosted to the laboratory frame using the KKMC routine specifically prepared for the bremsstrahlung photons present in the event under consideration. Also, h_i^{\pm} are available for all τ -decay channels [36]. We simply need to boost them back to the τ^{\pm} rest frames but now with coordinate system axes as used in our code.

Here we present technical details related to the version of KKMC, which are used in the Belle II software [47], the so-called BASF2 library. In the file Taupair.fi, common block /c_Taupair/ resides. In this common spin polarimetric vectors for the first/second τ can be found: m_HvecTau1(4) m_HvecTau2(4). They are needed for our weight calculation. We copy these four vectors to the user program variables and boost them to the lab frame using KKMC SUBROUTINE GPS_TralorDolt(id,pp,q) residing in the GPS.F file. These spin polarimetric vectors then need to

be boosted back to the rest frames of τ^- and τ^+ , but now using the routines from our application. In this way, the τ leptons rest-frame axes orientation is adjusted.

Now all the information that our application needs, and in its own reference frames, is available.

It is now straightforward to calculate wt_{spin} and wt of Eqs. (19) and (20) with R_{ij} as a function of *s* and $\cos(\theta)$, which are calculated from the beam and τ -lepton momenta in the τ -pair rest frame, with simple call to

anomwt(iqed, Ar, Ai, Br, Bi, wtME, wtSPIN).

The iqed=1/0 activates/omits the QED SM part of anomalous magnetic moment. The Ar, Ai, Br, Bi denote, respectively, the real and imaginary parts of anomalous magnetic and electric dipole moments which are input for the calculation of the wtME and wtSPIN weights representing the ratio of the unpolarized cross section and spin effect factor for the anomalous coupling included and absent. For each event, such a calculation can be repeated several times for distinct numerical values of Ar, Ai, Br, Bi.

This can be useful for the fits and definition of optimal variables [48,49] for the anomalous moment observables or for machine learning applications. We hope that the solution similar to the ones we attempted in Ref. [50], and references therein, can be used.

Note that routines from KKMC are used to obtain h_i^- , h_j^+ for each event. Interpolation of the R_{ij} calculation to the case in which the photons are present, follows a similar recipe to that in [51] where the MUSTRAAL frame [52] was used.

V. SUMMARY

In this paper, we present a simple algorithm for the calculation of event weights embedding effects of the dipole anomalous magnetic and electric moments in simulations of $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$ events with the τ decays included. Impacts on the spin effects and on the cross section are taken into account with the help of two separate weights.

The algorithm is prepared to work with the KKMC Monte Carlo program without the need to introduce changes into generator libraries, but using internal information from KKMC common blocks. No internal information from KKMC is used, except through the calculation of *wt* and wt_{spin} . The solution is ready for use with the Belle II software KKMC installation [47].

The reweighting tool does not affect the distributions measured from final-state observable measured in real data from the experiment. However, for example, by the template fitting method, the experimental data can be used to measure the fraction of the dipole moment predicted by varying the strength of the coupling related to the dipole moments in KKMC simulated samples. This can confirm or rule out the strength of the dipole moments predicted by new physics models [53,54].

The option of reweighting previously generated events stored in data files is possible, but as the dependence on additional couplings is linear, this may not be necessary. The solution like the one used in Ref. [55] may be more convenient. Our code can be used to simultaneously calculate weights for four values of the dipole moments. Thereafter, the weight for any other configuration can be obtained from their linear combination.

To demonstrate the functioning of the algorithm, we calculate the effect of the anomalous dipole moments on the acoplanarity of two planes built with $\pi^+\pi^0$ and $\pi^-\pi^0$ momenta in the $\pi^+\pi^0\pi^-\pi^0$ system rest frame of decay products in $e^-e^+ \rightarrow \tau^-\tau^+n\gamma$; $\tau^{\pm} \rightarrow \pi^{\pm}\pi^0\nu_{\tau}$ events. The numerical results are included, but we expect that, as in case of the Higgs boson *CP* signatures, in this case also, the machine learning techniques improving sensitivity and enabling combination of contributions from all τ -decay channels can be efficient.

Finally, we like to mention that the present method can be extended to the case of a polarized electron. This is important in view of the planned upgrade of the SuperKEKB e^-e^+ collider with the polarized electron beam [56].

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