Single-photon production in neutrino-hadron interactions at high energies

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Distinct approaches for the treatment of single-photon production in neutrino neutral-current interactions with hadrons at low and intermediate energies have been proposed over the last decades, mainly motivated by the fact that this process is one of the main backgrounds in $\nu_{\mu} \to \nu_{e}$ oscillation experiments. Such approaches disregard the contribution of the Pomeron (P) exchange, which becomes dominant at high energies. In this paper, the dipole formalism is extended for the exclusive photon production in the Z^0 -proton interactions at high energies and the contribution associated with the Pomeron exchange is estimated. Results for the squared transverse momentum distribution and total cross section are presented considering different models for the dipole-proton scattering amplitude, which imply a steep increase of the cross section with the energy.

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I. INTRODUCTION

One of the main challenges of particle physics is to obtain a deeper understanding of the properties of neutrinos and their interactions (for a recent review see, e.g., Ref. [[1\]](#page-7-0)). In recent years, a large number of studies have focused on studying the events observed in the IceCube observatory, mainly motivated by the desire to improve our understanding about the origin, propagation, and interaction of neutrinos at ultrahigh energies $[E_{\nu} \gtrsim \mathcal{O}(\text{TeV})]$ [[2](#page-7-1)]. Similarly, current and next-generation accelerator-based neutrino experiments, which consider neutrino interactions at low $[O(MeV)]$ and intermediate $[O(GeV)]$ energies, have motivated the search for answers to some of the fundamental questions about neutrinos, such as the precise determination of the neutrino mixing parameters as well as the neutrino mass ordering and leptonic CP violation [[3](#page-7-2)].

The description of the events observed in the neutrino observatories and accelerator-based experiments strongly depends on understanding the neutrino cross sections across many decades of energy, which is still an open question [[4](#page-7-3)]. In particular, the treatment of neutrino scattering off hadrons at low and intermediate energies is a hard task due to the large contribution of

nonperturbative effects [\[5](#page-7-4)], which implies that the calculations should be performed using phenomenological models based on, e.g., an effective Lagrangian motivated by the symmetries of QCD (see, e.g., Ref. [[6\]](#page-7-5)). Similarly, the description of the neutrino-proton cross section σ_{ν} at high energies is sensitive to the behavior of the parton distribution functions of the proton in the kinematical range beyond that probed by current colliders, where new QCD dynamical effects are expected to be present [[2](#page-7-1)]. In addition, several scenarios of beyond the Standard Model physics predict the modification of $\sigma_{\nu p}$ for large neutrino energies [[2\]](#page-7-1). In general, different approaches have been proposed for the distinct energy regimes and their limits of validity have not been clearly established, with results indicating that the predictions of a model proposed to treat the neutrino-hadron interactions in the low and intermediate energy ranges are incomplete or invalid in the high-energy limit. Such a conclusion motivates us to revisit single-photon production in neutral-current neutrinoproton interactions, which have only been discussed in the literature at low and intermediate energies [[7](#page-7-6)–[18\]](#page-7-7), and derive predictions for the high-energy regime. In addition, our study is motivated by the fact that observations in current and future neutrino observatories and acceleratorbased experiments are obtained using Cherenkov detectors, which implies that precise knowledge about photon production is fundamental to determine the irreducible background e.g.,. to the charged-current (CC) quasielastic

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signatures of $\nu e(\bar{\nu}e)$ appearance. This motivated the calculation of the QED radiative corrections to the (anti) neutrino-nucleon charged-current elastic scattering process performed in Refs. [\[19](#page-7-8)[,20\]](#page-7-9). In our case, we focus on exclusive single-photon production in the neutrino-nucleon neutral-current (NC) scattering process, where the nucleon remains intact and an energetic photon is produced in the final state. As a consequence, such a process will be characterized by an isolated diffusive ring created by the photon, unaccompanied by a hadronic shower and/or a leptonic track. Such distinct topology can be used, in principle, to separate the single-photon events produced in an exclusive process mediated by a Pomeron exchange (see below).

The production of a single photon in neutrino interactions with protons and nuclei was discussed by several groups in the last decades [[7](#page-7-6)–[18\]](#page-7-7), strongly motivated by the excess of events observed by the MiniBooNE experiment in the analysis of ν_e and $\bar{\nu}_e$ appearance searches [[21](#page-7-10)–[24](#page-7-11)], since the process is one of the largest backgrounds in $\nu_\mu \rightarrow$ ν_e ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) oscillation experiments (for related experimental analysis see, e.g., Refs. [[25](#page-7-12)–[28](#page-7-13)]). Another motivation for the analysis performed in these studies was that photon production is also important in measurements of the CP-violating phase (see, e.g., Refs. [[19](#page-7-8)[,20,](#page-7-9)[29\]](#page-7-14)). In Refs. [\[7](#page-7-6)–[18](#page-7-7)], results were derived considering an effective Lagrangian that describes the interaction between hadronic degrees of freedom and electroweak gauge fields, with the approaches differing in the assumptions for the subprocesses taken into account. In general, the contributions of distinct sets of diagrams, characterized by baryon or meson exchange, were included in the calculations and predictions for the cross section in the energy range $E_v \lesssim 3$ GeV. As is well known from hadronic physics (see, e.g., Ref. [\[30\]](#page-7-15)), the description of the interaction cross sections in terms of meson exchanges is a rather good approximation for low center-of-mass energies. However, as energy increases, the contribution associated with the exchange of a color-singlet object with vacuum quantum numbers, known as a Pomeron (P), becomes important and determines the behavior of the cross sections in the high-energy limit. Our goal in this paper is to estimate the contribution of Pomeron exchange to single-photon production in neutrino-proton interactions at high energies. In particular, we calculate the exclusive cross section associated with the process where a single photon is produced and the proton remains intact, with a rapidity gap between these two systems. In order to do that, we use the color dipole formalism [\[31\]](#page-7-16), which has been successfully used to describe the inclusive and exclusive processes observed in ep collisions at HERA (see, e.g., Refs. [\[32](#page-8-0)–[34\]](#page-8-1)). Such a formalism has already been used to derive estimates [\[35](#page-8-2)–[47\]](#page-8-3) for the total CC and NC neutrino-hadron cross sections that take into account nonlinear effects in the QCD dynamics, which are expected to be present at high energies [\[48\]](#page-8-4). In this paper, we extend this approach for the exclusive photon production in νp interactions and present predictions for the energy dependence of the associated cross section. As we demonstrate below, our results indicate that the cross section for this process increases with energy and is larger for smaller values of the virtuality of the gauge boson Z^0 .

This paper is organized as follows. In the next section we present a brief review of the dipole formalism. For completeness of our study, we first review the description of the inclusive neutrino-proton cross section. The extension of the formalism for exclusive photon production is presented, as well as the main ingredients used as input in our calculations. In Sec. [III](#page-4-0) we present our predictions for the energy dependence of the inclusive neutral-current neutrino-proton cross section, derived using distinct models for the forward dipole-proton amplitude considered in our analysis. Moreover, we present our predictions for the transverse momentum dependence of the differential cross section for single-photon production considering a fixed energy and different values for the Z^0 virtuality. In addition, we present our results for the energy dependence of the $Z^0 p \rightarrow \gamma p$ cross section. Finally, in Sec. [IV](#page-6-0) we summarize our main results and conclusions.

II. FORMALISM

In this section we present a brief review of the dipole formalism for the description of the NC neutrino-proton interactions at high energies (for previous studies see, e.g., Refs. [[35](#page-8-2)–[47](#page-8-3)]). Although our focus is on single-photon production in exclusive processes, where the proton target remains intact in the final state, we initially discuss the description of the inclusive neutrino-hadron interactions, which are typical deep inelastic scattering (DIS) processes and are characterized by the breakup of the proton. NC DIS is usually described in the Breit frame [\[49\]](#page-8-5), with a neutrino of four-momentum k interacting with a quark in the proton of four-momentum p via a virtual Z boson, producing in the final state a neutrino of the same flavor as the incoming lepton and a hadronic system X. The kinematics is determined by the squared four-momentum transfer $Q^2 \equiv$ $-q^2$ (where q is the four-momentum of the gauge boson), the Bjorken-x variable ($x \equiv Q^2/2p \cdot q$), and the inelasticity of the collision $y (y \equiv p \cdot q / p \cdot k)$. At high neutrino-proton center-of-mass energies, the value of the momentum fraction x carried by the quarks is typically small, which implies that we are probing sea quarks in the proton, which arise from the $g \rightarrow q_f \bar{q}_f$ process. This representation of the NC DIS process is presented in Fig. [1\(a\).](#page-2-0) A fully equivalent description of the process can be performed in the dipole frame, where the gauge boson Z^0 has enough energy to

FIG. 1. Representations of neutral-current deep inelastic neutrino-proton scattering at high energies in the Breit frame (a) and dipole frame (b).

fluctuate into a Fock state $|q_f\bar{q}_f\rangle$, which has color-charged constituents, and then scatter off of the color field of the target. At leading order, the dipole-target interaction can be described in terms of the gluon distribution of the proton, with the associated diagram shown in Fig. [1\(b\).](#page-2-0) One has that in the dipole frame the F_2^{NC} structure function is expressed in terms of the transverse and longitudinal structure functions, $F_2^{\text{NC}} = F_T^{\text{NC}} + F_L^{\text{NC}}$, which are given by

$$
F_{T,L}^{\text{NC}}(x,Q^2) = \frac{Q^2}{4\pi^2} \int_0^1 dz \int d^2 \mathbf{r} |\Psi_{T,L}^Z(\mathbf{r},z,Q^2)|^2 \sigma^{dp}(\mathbf{r},x), \tag{1}
$$

where r denotes the transverse size of the dipole, z is the longitudinal momentum fraction carried by a quark, and $\Psi_{T,L}^Z$ are the wave functions of the gauge boson corresponding to their transverse or longitudinal polarizations. Explicit expressions for $\Psi_{T,L}^Z$ were given in, e.g., Ref. [[35](#page-8-2)]. Furthermore, the dipole-proton cross section, σ_{dp} , is expressed as follows:

$$
\sigma_{dp}(x, \mathbf{r}^2) = 2 \int d^2 \mathbf{b}_p \mathcal{N}_p(x, \mathbf{r}, \mathbf{b}_p), \tag{2}
$$

where b_p is the transverse distance from the center of the proton to the center of mass of the $q\bar{q}$ dipole and $\mathcal{N}_p(x, r, b_p)$ denotes the nonforward scattering amplitude of a dipole of size r on the proton, which is directly related to the QCD dynamics. One of the main advantages of the description of DIS in the dipole frame is that nonlinear effects, associated with the high partonic density in the proton at small x , can be taken into account more easily in this frame. During the last decades, the description of the proton structure at high energies has been a topic of intense debate, mainly motivated by the expectation of the transition between the linear and nonlinear regimes of QCD dynamics (for reviews see, e.g., Ref. [[48](#page-8-4)]). While in the linear regime the dynamics is described by the emission processes, one expects that as the parton density at small x increases the physical process of recombination of partons becomes important in the parton cascade. This new regime is characterized by the limitation on the maximum phase-space parton density that can be reached in the hadron wave function (parton saturation) and its evolution is described by a nonlinear evolution equation for $\mathcal{N}_p(x, r, b_p)$. In recent years, several groups have proposed different phenomenological approaches to describe this quantity, which are based on the color glass condensate (CGC) formalism [\[50\]](#page-8-6) and successfully describe a large set of observables in ep , pp , pA , and AA collisions. In particular, the IP-Sat [\[32](#page-8-0)[,51\]](#page-8-7) and bCGC [\[33,](#page-8-8)[52\]](#page-8-9) models, which will be described below, are able to describe the ep HERA data for inclusive and exclusive processes.

Another advantage of the dipole formalism is that it can be easily extended to describe exclusive processes, where a given state is produced in the final state in the interaction of the gauge boson with the proton and it remains intact. The exclusive productions of vector mesons, photons, and Z^0 's in photon-hadron interactions are some examples of processes that have been largely studied in the last years (see, e.g., Refs. [[32](#page-8-0),[33](#page-8-8),[53](#page-8-10)–[56\]](#page-8-11)). In contrast, studies of these processes in neutrino-proton collisions are scarce and have mainly focused on heavy- and light-meson production in charged-current interactions (see, e.g., Refs. [[57](#page-8-12)–[69\]](#page-8-13)). In what follows, following Refs. [[53](#page-8-10)[,54\]](#page-8-14), we extend the dipole formalism for single-photon production in NC νp interactions. This process is characterized by a real photon and an intact proton in the final state, with a rapidity gap separating these systems. The basic idea in the dipole formalism is that the scattering amplitude for exclusive real photon production $Z^0 p \to \gamma p$ can be factorized in terms of the fluctuation of the virtual gauge boson Z^0 into a $q\bar{q}$ color dipole, the dipole-proton scattering by color-singlet exchange (P) , and the recombination into the exclusive final state γ , as represented in Fig. [2.](#page-3-0) Thus, it can be written as

$$
\mathcal{A}_T^{Z^0 p \to \gamma p}(x, \Delta, Q^2) = i \int d^2 \mathbf{r} \int \frac{dz}{4\pi} \int d^2 \mathbf{b}_p \sum_f (\Psi_{Z^0}^* \Psi_{\gamma})_T^f e^{-i[\mathbf{b}_p - (1-2z)r/2] \cdot \vec{\Delta}} 2\mathcal{N}_p(x, \mathbf{r}, \mathbf{b}_p)
$$

= $i \int dr (2\pi r) \int \frac{dz}{4\pi} \int db_p (2\pi b_p) J_0(b_p \Delta) J_0([1-2z]r\Delta/2) \sum_f (\Psi_{Z^0}^* \Psi_{\gamma})_T^f 2\mathcal{N}_p(x, r, b_p),$ (3)

 $\overline{}$

where $\Delta^2 = -t$, with t being the squared four-momentum transfer between the incoming and scattered proton, and J_0 is the Bessel function of the first kind. The transversely polarised overlap function, $(\Psi_{Z}^* \Psi_{\gamma})_T^f$, between the photon wave function and the Z^0 wave function for quark flavor $f = u$, d, s, c, b can be fully calculated using perturbation theory [\[54\]](#page-8-14) and is given by

$$
(\Psi_{Z^0}^* \Psi_{\gamma})_T^f = \frac{2N_c \alpha_{\text{em}}}{\pi} \frac{e_f g_{\nu}^f}{\sin 2\theta_W} \{ [z^2 + (1-z)^2] m_f K_1(m_f r) \varepsilon K_1(\varepsilon r) + m_f^2 K_0(m_f r) K_0(\varepsilon r) \}.
$$
 (4)

Here, the vector couplings are $g_v^{\mu,c} = 1/2 - 4/3\sin^2\theta_W$ and $g_v^{\mu,s,b} = -1/2 + 2/3\sin^2\theta_W$, where θ_W is the Weinberg angle, and $\varepsilon^2 = m_f^2 + Q^2 z (1 - z)$. Moreover, K_0 and K_1 are the modified Bessel functions. Finally, the total cross section for single-photon production can be estimated by

$$
\sigma^{Z^0 p \to \gamma p} (W, \mathcal{Q}^2) = \int_{-\infty}^0 dt \frac{d\sigma^{Z^0 p \to \gamma p}}{dt} = \int_{-\infty}^0 dt \frac{1}{16\pi} \Big| \mathcal{A}_T^{Z^0 p \to \gamma p} (x, \Delta, \mathcal{Q}^2) \Big|^2,\tag{5}
$$

where W is the Z^0 -proton center-of-mass energy, Q^2 is the Z^0 virtuality, and the amplitude is given by Eq. [\(3\)](#page-3-1). As in Ref. [\[51\]](#page-8-7), the differential cross section for a proton target will be multiplied by the factor $R_g^2(1 + \beta^2)$ in order to take
into account the skewness effect (R) and the real part of the into account the skewness effect (R_q) and the real part of the

FIG. 2. Scattering amplitude for single-photon production in a Z^0 -proton interaction.

scattering amplitude (β) . The skewness correction is related to the fact that in the two-gluon exchange limit, the gluons emitted from the quark and antiquark into the dipole can carry different momentum fractions. Such a correction was derived in the framework of collinear factorization [[70](#page-8-15)], and its application in the dipole approach is still under debate. However, the comparison of the dipole predictions with the HERA data indicate that the skewness and realpart corrections are needed to describe the data [\[32](#page-8-0)[,33\]](#page-8-8).

In the dipole formalism the main input for the calculations of NC structure functions and single-photon production is $\mathcal{N}_p(x, r, b_p)$, which is determined by the QCD dynamics at small x [[48](#page-8-4)]. In our analysis we consider the phenomenological models proposed in Refs. [[51](#page-8-7),[52](#page-8-9)] that successfully describe the ep HERA data for inclusive and exclusive processes [\[32,](#page-8-0)[51\]](#page-8-7). This will allow us to estimate the current theoretical uncertainties in the predictions for neutrino-hadron interactions. The bCGC and IP-Sat models are based on distinct approximations of the CGC formalism and satisfy its main properties: (a) for the interaction of a small dipole ($|r| \ll 1/Q_s$), $\mathcal{N}_p(x, r, b_p) \approx r^2$, implying that this system is weakly interacting; (b) for a large dipole $(|r| \gg 1/Q_s)$, the system is strongly absorbed and therefore

 $\mathcal{N}_p(x, r, b_p) \approx 1$. In particular, the bCGC model interpolates two analytical solutions of well-known evolution equations: the solution of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation near the saturation regime and the solution of the Balitsky-Kovchegov equation deep inside the saturation regime. The underlying assumption is that the interaction between gluonic ladders is taken into account by the bCGC model, with the saturation boundary being approached via the BFKL equation. Moreover, this model assumes that the saturation scale depends on the impact parameter, with the dipole-proton scattering amplitude being given by [\[51\]](#page-8-7)

$$
\mathcal{N}_p(x, r, b_p) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2} \right)^{2\left[r_s + (1/(\kappa \lambda Y)) \ln(2/rQ_s) \right]}, & rQ_s \le 2, \\ 1 - e^{-A \ln^2(BrQ_s)}, & rQ_s > 2 \end{cases}
$$
(6)

where $Y = \ln(1/x)$ and

$$
Q_s \equiv Q_s(x, b_p) = \left(\frac{x_0}{x}\right)^{\lambda/2} \left[\exp\left(-\frac{b_p^2}{2B_{CGC}}\right)\right]^{1/(2\gamma_s)} \quad (7)
$$

is the saturation scale of this model. Moreover, the coefficients A and B are determined by the continuity condition of N and its derivative in $rQ_s = 2$. The free parameters were fixed by fitting the HERA data and here we use the updated parameters obtained in Ref. [[33](#page-8-8)]. On the other hand, the IP-Sat model [[32](#page-8-0)] incorporates the saturation effects via the Glauber-Mueller approxima-tion [[71](#page-8-16)–[73\]](#page-8-17), assuming an eikonalized form for \mathcal{N}_p that depends on a gluon distribution evolved via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation. This model resums higher-twist contributions and, distinctly from the bCGC model, the saturation boundary is approached via the DGLAP evolution [[74](#page-8-18)]. In the IP-Sat model the dipole-proton scattering amplitude is given by

$$
\mathcal{N}_p(x, r, b_p) = 1 - \exp\left[\frac{\pi^2 r^2}{N_c} \alpha_s(\mu^2) x g\left(x, \mu^2 = \frac{C}{r^2} + \mu_0^2\right) \times T_G(b_p)\right],\tag{8}
$$

with a Gaussian profile

$$
T_G(b_p) = \frac{1}{2\pi B_G} \exp\left(-\frac{b_p^2}{2B_G}\right).
$$
 (9)

In this model, the gluon distribution evolves via the DGLAP equation, with the initial condition at μ_0^2 taken to be $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^6$. In this work we employ the parameters B_G , A_g , λ_g , C, and μ_0^2 obtained in Ref. [[34](#page-8-1)]. In order to quantify the impact of the nonlinear effects, we also present the predictions derived neglecting the nonlinear corrections, with the dipole-proton scattering amplitude being given by the linear part of the IP-Sat model, denoted hereafter as IPnonSat, which is

$$
\mathcal{N}_p(x, r, b_p) = \frac{\pi^2 r^2}{N_c} \alpha_s(\mu^2) x g(x, \mu^2) T_G(b_p), \quad (10)
$$

with the parameters obtained in Ref. [[34](#page-8-1)]. These three different phenomenological models for \mathcal{N}_p provide a rather good description of the ep HERA data, but its predictions for larger center-of-mass energies become distinct, in particular for exclusive processes, where the cross section is proportional to the square of the scattering amplitude and, consequently, is strongly sensitive to the underlying QCD dynamics. Such strong dependence has motivated a large number of studies of distinct final states produced in exclusive processes in electron-ion and ultraperipheral heavy-ion collisions [\[75](#page-8-19)–[77](#page-8-20)]. In the next section we analyze the impact of these distinct descriptions of QCD dynamics at high energies on the predictions for singlephoton production in neutral-current neutrino-proton interactions.

III. RESULTS

In order to illustrate the predictions of the dipole formalism and compare its results with those derived using the standard collinear formalism, we initially estimate the total neutral-current cross section, which is given by [\[49\]](#page-8-5)

$$
\sigma_{\nu\rho}^{\rm NC}(E_{\nu}) = \int_{Q_{\rm min}^2}^{s} dQ^2 \int_{Q^2/s}^{1} dx \frac{1}{xs} \frac{\partial^2 \sigma_{\nu\rho}^{\rm NC}}{\partial x \partial y},\qquad(11)
$$

where E_{ν} is the neutrino energy, $s = 2ME_{\nu}$ where M is the proton mass, $y = Q^2/(xs)$, and Q_{min}^2 is the minimum value
of Q^2 which is introduced in order to stay in the deep inelastic of Q^2 which is introduced in order to stay in the deep inelastic region. In what follows, we assume $Q_{\text{min}}^2 = 1 \text{ GeV}^2$.
Moreover the differential cross section is given by [49] Moreover, the differential cross section is given by [\[49\]](#page-8-5)

$$
\frac{\partial^2 \sigma_{\nu p}^{NC}}{\partial x \partial y} = \frac{G_F^2 M E_\nu}{\pi} \left(\frac{M_Z^2}{M_Z^2 + Q^2} \right)^2
$$

$$
\times \left[\frac{1 + (1 - y)^2}{2} F_2^{NC}(x, Q^2) - \frac{y^2}{2} F_L^{NC}(x, Q^2) + y \left(1 - \frac{y}{2} \right) x F_3^{NC}(x, Q^2) \right],
$$
 (12)

where G_F is the Fermi constant and M_Z denotes the mass of the neutral gauge boson. The calculation of $\sigma_{\nu p}$ involves integrations over x and Q^2 . At large energies, the x integral becomes dominated by the interaction with partons of small values of x and the Q^2 integral remains dominated by Q^2 values to the order of the electroweak boson mass squared. For Q^2 above M_Z^2 the integrand behaves as $1/Q^4$ and quickly

becomes irrelevant. In the collinear formalism, the structure functions $F_i(x, Q^2)$ are expressed in terms of the parton distribution functions of the proton (see, e.g., Ref. [\[49\]](#page-8-5)), which satisfy the linear DGLAP evolution equations [[74](#page-8-18)]. The associated prediction for the energy dependence of the total NC cross section, obtained using the parametrization derived by the CTEQ group in Ref.[\[78](#page-8-21)], is represented by the dotted line in Fig. [3](#page-5-0) and will be denoted by DGLAP(CT10) hereafter. Moreover, we also present in Fig. [3](#page-5-0) the predictions of the dipole formalism, derived considering the distinct phenomenological models for N_p discussed in the previous section. One has that the dipole predictions (bCGC, IP-Sat, and IPnonSat) underestimate the cross section at small energies in comparison to the standard collinear formalism, which is expected since the dipole formalism does not take into account the contribution of valence quarks. However, for large energies, where the proton structure is dominated by gluons and sea quarks, the IPnonSat prediction is similar to the DGLAP(CT10) one, which is also expected due to the fact that both results were derived disregarding nonlinear effects on the QCD dynamics. One has that the nonlinear effects imply the suppression of the cross section, with the bCGC model predicting the smaller neutrino-proton cross section at high energies.

We now discuss single-photon production in an exclusive neutrino-proton interaction. We focus on the description of the interaction between the virtual gauge boson Z^0 and the proton, and analyze the dependencies of our predictions on the squared transverse momentum t , energy W, and virtuality Q^2 . It is important to emphasize that the cross section for this process can, in principle, also be estimated in the framework of the collinear QCD approach where generalized gluon distributions, which provide a unique tool for addressing the longitudinal momentum and

FIG. 3. Total neutral-current neutrino-proton cross section as a function of the neutrino energy E_{ν} , derived considering the bCGC, IP-Sat, and IPnonSat approaches for the description of dipole-hadron interactions. The prediction obtained using the standard collinear approach, denoted DGLAP(CT10), is also presented for comparison.

spatial distributions of partons in hadrons, factorize from perturbatively calculable coefficient functions (for reviews see, e.g., Refs. [[79](#page-8-22),[80](#page-8-23)]). Such an approach was already applied for meson production in neutrino-hadron interactions in Refs. [[57](#page-8-12)–[69](#page-8-13)], but the treatment of single-photon production at high energies using this formalism is still a subject that deserves to be considered in detail, which we intend to do in a future publication. In what follows, we present the results derived using the dipole formalism presented in the previous section, which provides an intuitive description of the scattering amplitude as well as an efficient way of introducing nonlinear effects in the dipole-hadron interaction. Finally, its predictions are based on phenomenological models that are able to describe the current high-precision ep HERA data.

Initially, we present our predictions for the differential cross section $d\sigma/dt$, where t is the squared four-momentum transfer between the incoming and scattered hadron, which defines the typical transverse momentum of the photon in the final state. In recent years, several authors have pointed out that the analysis of this distribution in exclusive processes can be used to obtain the transverse spatial distributions of gluons in the target (see, e.g., Refs. [[81](#page-8-24)–[85](#page-9-0)]). One has that the behavior of $d\sigma/dt$ at small x is determined by the impact-parameter dependence of the scattering amplitude $\mathcal{N}_p(x, r, b_p)$ of a dipole off the proton. In Fig. [4](#page-6-1) we present our predictions considering a fixed value for the Z^0p center-of-mass energy $(W = 200 \text{ GeV})$ and two distinct values of the gauge boson virtuality ($Q^2 = 1$ and 10 GeV²). The predictions from the IP-Sat and bCGC models (see, e.g., Refs. [[32](#page-8-0)[,33,](#page-8-8)[51\]](#page-8-7)), which are phenomenological models based on CGC physics that assume distinct impact-parameter dependencies for the scattering amplitude, are compared with those obtained using the IPnonSat model, which can be derived from the IP-Sat model by disregarding the impact of the multiple scattering corrections that take into account the nonlinear QCD effects in this model. Therefore, the comparison between the IPnonSat predictions and those from the other models allows us to estimate the impact of the saturation effects for a proton target. The results presented in Fig. [4](#page-6-1) indicate that the distribution strongly depends on the model considered, as expected from similar analysis for exclusive processes in photon-hadron interactions. In particular, the IPnonSat model does not predict the presence of a dip in the $|t|$ distribution for $|t| \leq 3.0$ GeV². In contrast, the models based on CGC physics predict dips at large values of $|t|$, with their positions being dependent on the model considered. The first dip occurs for smaller values of $|t|$ when a smaller value of the gauge boson virtuality is assumed. One also has verified that the first dip occurs for smaller values of $|t|$ when the centerof-mass energy is increased. The large difference in the position of the dips predicted by the distinct models strongly motivates a future measurement of this observable which

FIG. 4. Predictions for the t distributions of single-photon production in Z^0p interactions, derived assuming $W = 200$ GeV and considering distinct models for the dipole-proton scattering amplitude and different values of the Z^0 virtuality ($Q^2 = 1$ and 10 GeV²).

FIG. 5. Predictions for the energy dependence of the single-photon production cross section in $Z^0 p$ interactions considering distinct models for the dipole-proton scattering amplitude and different values of the Z^0 virtuality ($Q^2 = 1$ and 10 GeV²).

could be able to discriminate between these different approaches for QCD dynamics at high energies.

In Fig. [5](#page-6-2) we present the predictions for the energy dependence of the total cross section for the $Z^0p \to \gamma p$ process, expressed in Eq. [\(5\)](#page-3-2), assuming different values for the gauge boson virtuality and considering the bCGC, IP-Sat, and IPnonSat models for the dipole-proton scattering amplitude. We predict the increase of the cross section with energy and a larger magnitude for smaller values of the gauge boson virtuality. The IP-Sat and IPnonSat predictions are steeper in energy in comparison to the bCGC one, with the IPnonSat predictions being a factor of ≈ 1.5 larger than the IP-Sat one for large energies and $Q^2 = 1$ GeV². For larger values of virtuality, one has that this difference decreases. The comparison between the predictions of the distinct approaches indicates that the difference between the predictions increases with energy and with decreasing gauge boson virtuality. Such results are expected, since the impact of the saturation effects is larger for small values of x and/or Q^2 . Although the results of the distinct approaches can differ by a factor of \approx 2 for large energies, the resulting predictions show that the cross section for single-photon production associated with Pomeron exchange is dominant in comparison to previous estimates for this process [[7](#page-7-6)–[18\]](#page-7-7) and its analysis must be considered in future experiments that explore the high-energy regime of neutral-current neutrino-proton interactions.

IV. SUMMARY

One of the main goals of current and future neutrino observatories and accelerator-based experiments is the improvement of our understanding of neutrino properties. In particular, the description of neutrino interactions in the low-, intermediate-, and high-energy regimes is one of the challenges of neutrino physics. In this paper, we focused on the derivation of predictions for single-photon production in neutral-current neutrino-proton interactions at high energies. This process has been discussed in the literature over the last decades for low and intermediate center-of-mass energies, motivated by the fact that this process is one of the largest backgrounds in $\nu_{\mu} \rightarrow \nu_{e}$ oscillation experiments. The proposed approaches include contributions from diagrams associated with baryon and meson exchanges, but disregard the contribution of Pomeron (P) exchange. In our analysis, we estimated this contribution using the dipole formalism, which has been largely used to describe inclusive and exclusive processes in ep collisions, and derived predictions for exclusive photon production in Z^0 -proton interactions at high energies considering three distinct phenomenological models for the dipole-proton scattering amplitude. We provided predictions for the squared transverse momentum distribution, $d\sigma/dt$, and demonstrated that this quantity is sensitive to the description of the QCD dynamics. Moreover, we presented our results for the energy dependence of the total cross section for different values of the Z^0 virtuality and demonstrated that it presents a steep increase with energy. Our results indicate that the Pomeron contribution for singlephoton production dominates at high energies and must be considered in future experiments.

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