# Toward inflation with $n_s = 1$ in light of the Hubble tension and implications for primordial gravitational waves

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Recently, it has been found that complete resolution of the Hubble tension might point to a scale-invariant Harrison-Zeldovich spectrum of primordial scalar perturbation, i.e., $n_s = 1$ for $H_0 \sim 73$ km/s/Mpc. We show that, for well-known slow-roll models, if inflation ends by a waterfall instability with respect to another field in the field space while inflaton is still at a deep slow-roll region, then $n_s$ can be lifted to $n_s = 1$. A surprise of our result is that with prerecombination early dark energy, chaotic $\phi^2$ inflation, strongly disfavored by Planck + BICEP/Keck in standard ΛCDM, can be revived, which is now well within testable region of upcoming cosmic microwave background B-mode experiments.

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## I. INTRODUCTION

Inflation [1–5] is the current paradigm of the early Universe, which predicts (nearly) scale-invariant scalar perturbation, as well as primordial gravitational waves. In well-known single field slow-roll inflation models, the spectral index $n_s$ of primordial scalar perturbation follows [6–9]

$$n_s - 1 = -\frac{\mathcal{O}(1)}{N_*} \tag{1}$$

in the large $N_*$ limit, where $N_* = \int H dt$ is the $e$-folds number before the end of inflation. The cosmic microwave background (CMB) perturbation modes exit the horizon at about $N_* \sim 60$ $e$-folds, if inflation ends around $\sim 10^{15}$ GeV. Recently, based on standard ΛCDM (cold dark matter) model the Planck collaboration obtains $n_s \approx 0.97$ [10], which is consistent with (1).

However, the current expansion rate of the Universe, the Hubble-Lemaitre constant $H_0$, inferred by the Planck collaboration [10] assuming ΛCDM is in $\gtrsim 5\sigma$ tension with that reported recently by the SH0ES collaboration [11] using Cepheid-calibrated supernovas. Currently, it is arriving at a consensus that this so-called Hubble tension likely signals new physics beyond ΛCDM [12,13], see also Refs. [14–17] for reviews and some recent developments.

As a promising resolution of Hubble tension, early dark energy (EDE) [18,19], which is non-negligible only for a few decades before recombination,[1] has been extensively studied, e.g., [20–30]. It is well known that Planck CMB observation precisely measured the angular scale

$$\theta = \frac{r_s}{D_A} \sim r_s H_0, \tag{2}$$

with $D_A$ being the angular diameter distance to the last scattering surface. The idea of EDE is that a dark energy component activating decades before recombination lowers the sound horizon $r_s = \int_{z_*}^{\infty} c_s dz/H(z)$ at that time so we have a high $H_0 \sim r_s^{-1}$, alleviating the Hubble tension. In original (axionlike) EDE [19], the scalar field with $V(\varphi) \sim (1 - \cos(\varphi/f_a))^3$ is responsible for EDE, which starts to oscillate at critical redshift, and dilutes away rapidly like a fluid with $w > 1/3$ before recombination. In AdS-EDE [25], since the potential has an anti–de Sitter (AdS) well (temporarily realizing $w > 1$ so EDE dilutes away faster), a larger EDE fraction and so higher $H_0 (\approx 73$ km/s/Mpc$)$ can be achieved without spoiling a fit to full Planck + BAO + Pantheon dataset. Recently, the combined analysis of Planck ($\ell_{TT} \lesssim 1000$) with ACT and SPT data for EDE has also been performed, such as Planck + SPTpol [31–33], Planck + ACT DR4 [34,35], and Planck + ACT

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[1]Actually, "EDE" corresponds to EDE + ΛCDM, which is a prerecombination modification to ΛCDM model, and the evolution after recombination must still be ΛCDM-like.

 

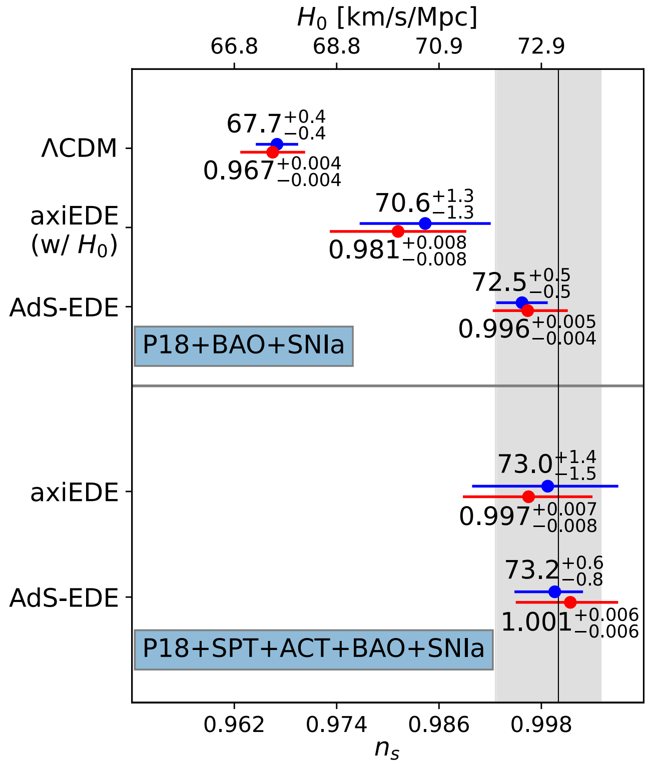


FIG. 1. $n_s$ vs $H_0$. In the upper panel with the fullPlanck + BAO + Pantheon dataset, we adopt the result in Ref. [19] for original axionlike EDE (the SH0ES result as a Gaussian prior on $H_0$), and Ref. [33] for AdS-EDE. In the lower panel with the Planck + ACT + SPT + BAO + Pantheon dataset, Ref. [37] (Planck $\ell_{\mathrm{TT}} < 650$) for axionlike EDE and Ref. [38] (Planck $\ell_{\mathrm{TT}} < 1000$) for AdS-EDE. Gray band represents the recent SH0ES result $H_0 = 73.04 \pm 1.04$ km/s/Mpc [11], and black solid line marks $n_s = 1$.

DR4 + SPT-3G [36–38], see also [39–42] for Planck + large scale structure data.

In Ref. [43], it has been found that, in corresponding Hubble-tension-free cosmologies, the best fit values of cosmological parameters acquired assuming ΛCDM must shift with $\delta H_0$, and with the full Planck + BAO + Pantheon dataset the shift of $n_s$ scales as

$$\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0}, \tag{3}$$

which suggests that prerecombination resolution of Hubble tension is pointing to a scale-invariant Harrison-Zeldovich primordial spectrum, i.e., $n_s = 1$ for $H_0 \sim 73$ km/s/Mpc, see also [44–46] for earlier discussion regarding $N_{\mathrm{eff}}$ and $n_T$. In Refs. [37,38], with the Planck + ACT + SPT + BAO + Pantheon dataset, similar results have also been found. We outlined the relevant results in Fig. 1. Thus it is significant to explore the implication of $n_s = 1$ on a primordial Universe.

How would $n_s = 1$ affect our understanding about inflation? At first thought, it seems that (1) is not compatible with the result in Hubble-tension-free cosmologies, since for $N_* \approx 60$ it is hardly possible to achieve $n_s = 1$ in the slow-roll models satisfying (1). Actually, $n_s \geq 0.99$ puts a lower bound $N_* > \mathcal{O}(10^2)$. Such perturbation modes are still far larger than our observable Universe today. This seems to pose a serious challenge to slow-roll inflation, implying that corresponding models might need to be reconsidered, e.g., [47–49], see also the recent Refs. [50–52].

However, inspired by recent Ref. [53], we might have a different story. In (1), $N_*$ is the "distance" between $\phi_*(\epsilon \ll 1)$ and $\phi_e(\epsilon = 1)$ at which inflation ends, $\phi$ being the inflaton. Typically $N_* \approx 60$. However, inflation can also be terminated at $\phi_c$ when $\epsilon \ll 1$ by waterfall instability with respect to another field $\sigma$, like in the hybrid inflation models [54,55], which suggests that $\Delta N \approx 60$ does not necessarily require $N_* \approx 60$, see Fig. 2. Thus we can actually have $n_s$ arbitrarily close to 1 by pushing $N_*$ to a sufficiently large value $N_* \gg \Delta N \approx 60$ while ending inflation by certain mechanism at $N_* - 60$.

We will present this possibility. In our (hybrid) uplift of $n_s \approx 0.97$ to $n_s = 1$, the potential of inflaton $\phi$ still preserves the shape of well-known single field slow-roll inflation models, see Sec. II, but inflation ends by the waterfall instability while $\phi$ is still in the deep slow-roll region. A surprise of our result is that certain models originally thought to be strongly disfavored by the tensor-to-scalar ratio $r$ upper bound from Planck + BICEP/Keck based on ΛCDM [56] and AdS-EDE [57], specially chaotic $\phi^2$ inflation [5], can be revived by this hybrid uplift to $n_s = 1$, which is now well within the testable region of upcoming CMB B-mode experiments, such as BICEP Array [58] and CMB-S4 [59].

## II. HYBRID UPLIFT TO $n_s = 1$

The scenario we consider is sketched in Fig. 2, in which

$$V(\phi, \sigma) = V_{\mathrm{inf}}(\phi) + \frac{1}{4\lambda}[(\lambda\sigma^2 - M^2)^2 - M^4] + \frac{g^2}{2}\sigma^2\phi^2, \tag{4}$$

and $V_{\mathrm{inf}}$ is some well-known inflation potentials satisfying (1). Initially, $\sigma = 0$ and $\partial_\sigma^2 V > 0$, the inflaton $\phi$ slow rolls along $V_{\mathrm{inf}}$ and $\epsilon \ll 0.01$. At $\phi = \phi_c \simeq M/g$, we still have $\epsilon(\phi_c) \ll 0.01$, but $\partial_\sigma^2 V \lesssim 0$ so that the inflation will rapidly end by a waterfall instability along $\sigma$,[2] see also [60,61] for the effective energy momentum tensor approach of multi-fields perturbations. In original hybrid inflation [54,55], when $\sigma = 0$, $V = V_{\mathrm{inf}} + \frac{M^4}{4\lambda}$ is lifted by $V_{\mathrm{up}} = M^4/4\lambda$. Generally, for $V_{\mathrm{inf}} \sim \phi^2$, when $V_{\mathrm{inf}} \ll V_{\mathrm{up}}$, one has $n_s - 1 > 0$ [62]. However, here we subtract out the

[2]At the minima of $\sigma$ when $\phi < \phi_c$, the corresponding potential $V = V_{\mathrm{inf}} - (M^2 - g^2\phi^2)^2/4\lambda$ might be negative for $|\phi| \ll M/g$ and certain $V_{\mathrm{inf}}$. In this case (4) should be thought of as an effective potential only captures the shape of field space for $\phi > \phi_c$ and $\phi \sim \phi_c$.

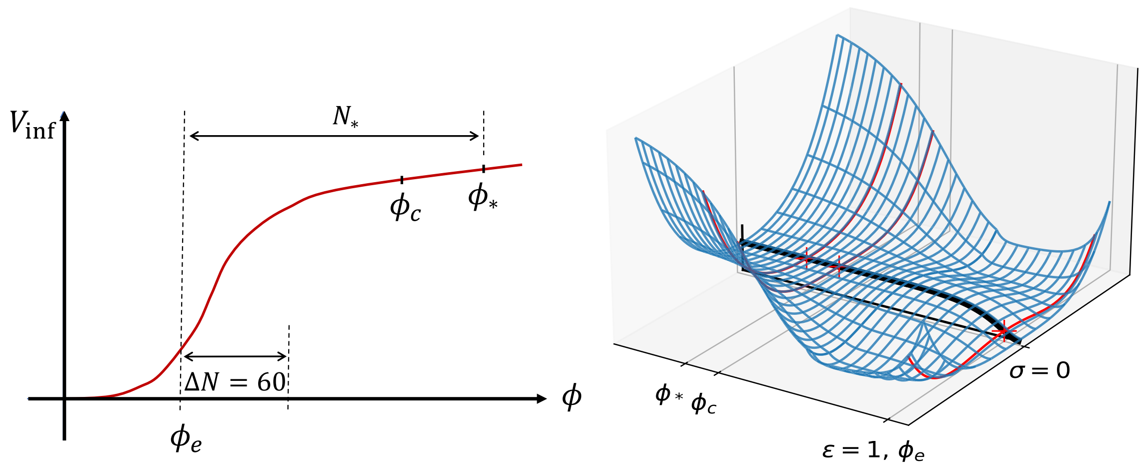


FIG. 2. Left panel: the slow-roll potential $V_{\rm inf}$. Right panel: $V(\phi, \sigma)$ obtained by hybrid lifting $V_{\rm inf}$ according to (4). $\Delta N$ is the $e$-folds number in the original slow-roll model. In well-known slow-roll models, inflation ends at $\phi_e$ where $\epsilon \simeq 1$. The perturbation modes exiting horizon near $N_* \gg \Delta N \approx 60$ is still far outside of our current Hubble horizon. After the hybrid uplift of $V_{\rm inf}$ to the $\phi - \sigma$ space, see the right panel, initially $\partial_\sigma^2 V|_{\sigma=0} > 0$, inflation follows the slow-roll trajectory ($\sigma = 0$) shown as the thick black line. However, contrary to standard slow-roll, inflation can now end at $\phi = \phi_c$ by a waterfall instability along $\sigma$, at which we still have $\epsilon(\phi_c) \ll 0.01$ but the slow-roll trajectory becomes unstable due to $\partial_\sigma^2 V|_{\sigma=0} < 0$. Thus the modes exiting horizon near $N_* \gg 60$ can be just at the CMB window, and $n_s = 1$ in light of (1).

uplift $V_{\rm up}$. We will see that, for $V_{\rm inf} \sim \phi^2$, we have $n_s - 1 \approx 0$ but $< 0$.

In slow-roll approximation, one has ($M_p = 1$)

$$N_* \approx \Delta N + \int_{\phi_e}^{\phi_c} \frac{d\phi}{\sqrt{2\epsilon}} = \left(\int_{\phi_c}^{\phi_*} + \int_{\phi_e}^{\phi_c}\right) \frac{d\phi}{\sqrt{2\epsilon}} \approx N(\phi_*). \tag{5}$$

The results of both $n_s$ and $r$ are determined by $\phi_*$, value of the inflaton field $\phi$ when the corresponding perturbation mode exits horizon during inflation, thus they are related to $N_*$ rather than $\Delta N$. This indicates that we can have $N_* > \mathcal{O}(10^2)$ in (1) and $n_s \simeq 1$, while still having $\Delta N \approx 60$. In certain sense, with (4), what we do corresponds to push the inflaton $\phi$ deeply into slow-roll region at which $N_* \gg \Delta N \approx 60$. Inflation will end at $N_* - 60$ so that the modes exiting horizon near $N_*$ can be just at CMB window.

Here, we show that the addition of a potential uplift $V_{\rm up}$ in Ref. [53] is actually equivalent to pushing the inflaton $\phi$ deeply into the slow-roll region without $V_{\rm up}$, i.e., Eq. (4). By lifting $V_{\rm inf} = V_0(1 - e^{-\gamma\phi})$ to $V_{\rm up} + V_{\rm inf}$, Ref. [53] found

$$n_s - 1 \approx -\frac{2}{(\frac{V_{\rm up}+V_0}{\gamma^2 V_0}) e^{\gamma\phi_*}} \approx -\frac{2}{\gamma^{-2} e^{\gamma\tilde{\phi}_*}} = -\frac{2}{\Delta N + \gamma^{-2} e^{\gamma\tilde{\phi}_c}},$$
$$\tilde{\phi} \equiv \phi + \frac{1}{\gamma}\ln(V_{\rm up}/V_0), \tag{6}$$

where the second approximate equality is obtained in the large $V_{\rm up} \gg V_0$ limit in the uplifted potential $V_{\rm up} + V_{\rm inf}$. This is simply equivalent to the large $N_*$ (or equivalently large $\phi_c$ and $\phi_*$) limit in $V_{\rm inf}$ (without $V_{\rm up}$) because in light of (5) we have

$$N_* \approx \int_{\phi_e}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon}} = \gamma^{-2} e^{\gamma\phi}|_e^*, \tag{7}$$

where $\epsilon = \frac{V_{\rm inf}'^2}{2V_{\rm inf}^2} = \frac{\gamma^2}{2} e^{-2\gamma\phi}$ is used. Thus combining (7) with (6), we obtain (1), which indicates that large $V_{\rm up}$ or $\phi_c$ limit in Ref. [53] actually is equivalent to the large $N_*$ limit. However, here we straightly push $N_* \approx 60$ to a sufficiently large value $N_* > \mathcal{O}(10^2)$, and do not make the uplift of $V_{\rm inf}$ to $V_{\rm up} + V_{\rm inf}$, so fully preserve the shape of single field slow-roll potentials satisfying (1), thus (1) can be directly applied. The advantage of our inflation model will be seen in the $\phi^p$ inflation.

### A. (Hybrid) Starobinski inflation

In Starobinsky ($R^2$) model [4], the effective potential is $V_{\rm inf}(\phi) \sim (1 - e^{-\sqrt{2/3}\phi})^2$, and

$$n_s - 1 \approx -\frac{2}{N_*}, \qquad r \approx \frac{12}{N_*^2}, \tag{8}$$

which corresponds to $\alpha = 1$ in $\alpha$-attractor inflation [7,63–65]. Equation (8) is compatible with the $\Lambda$CDM constraint $n_s \approx 0.967$ for $N_* = \Delta N \approx 60$ in standard slow-roll inflation. However, in Hubble-tension-free cosmologies, e.g., AdS-EDE, $n_s = 0.998 \pm 0.005$, see Fig. 1 for the full Planck + BAO + Pantheon result, this would require that, in the hybrid lifted Starobinski model, we need $N_* \gtrsim 300 \gg \Delta N$ and inflation ending at $N \approx N_* - 60$, thus $n_s \simeq 1–2/N_* \gtrsim 0.993$ and $r \simeq \frac{12}{N_*^2} \lesssim 1.3 \times 10^{-4}$.

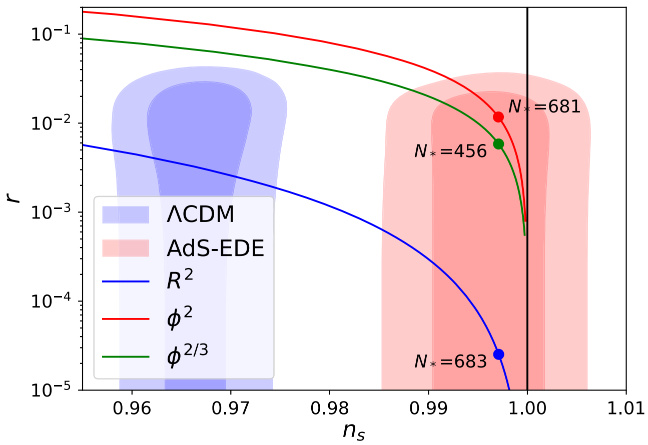


FIG. 3. Predictions of Starobinsky and $\phi^p$ ($p = 2, 2/3$) models with respect to 68% and 95% C.L. contour of $r - n_s$. Here, we adopt the result in recent BICEP/Keck Ref. [56] for ΛCDM and that in Ref. [57] (full Planck + BICEP/Keck + BAO + Pantheon) for AdS-EDE.

This tensor-to-scalar ratio is far smaller than that in the standard Starobinsky model.

### B. (Hybrid) $\phi^p$ inflation

In corresponding models, $V_{\rm inf}(\phi) \sim \phi^p$, and

$$n_s - 1 \approx -\frac{p/2+1}{N_*}, \qquad r \approx \frac{4p}{N_*}. \tag{9}$$

Here, $p = 2$ is the chaotic inflation [5], $p = 2/3$, 1 correspond to the monodromy inflation [66,67]. We have $n_s \approx 1\text{–}2/60 = 0.97$ for $p = 2$ and $N_* = 60$, which seems compatible with the ΛCDM constraint. However, since $r \approx 8/N_* = 0.13$, the $\phi^2$ model is strongly disfavored by Planck + BICEP/Keck data in ΛCDM [56].

In Fig. 3, we plot the $r - n_s$ posterior in flat ΛCDM and AdS-EDE (as an example of EDE) models, respectively. Following Ref. [57], the AdS-EDE results are obtained with the full Planck + BK18 + BAO + Pantheon dataset using the modified versions [68] of CLASS cosmology code [69,70] and MontePython-3.4 Monte Carlo Markov Chian (MCMC) sampler [71,72]. The ΛCDM results are directly produced from the public available BK18 chains [73] (full Planck + BK18 + BAO) [56]. See Appendix and Ref. [57] for detailed numeric results.

In Hubble-tension-free AdS-EDE model, $n_s = 0.998 \pm 0.005$, which requires that $N_* \gtrsim 300 \gg \Delta N$. It is interesting to note that the hybrid uplift to $n_s = 1$ generally lowers $r$, which has the potential to revive inflation models strongly disfavored by Planck + BICEP/Keck based on ΛCDM due to too large $r$. In Fig. 3, when $N_* > 300$, we have $0.993 \lesssim n_s \leq 1$ and $r \simeq 8/N_* \lesssim 0.03$ for chaotic $\phi^2$ inflation, perfectly consistent with current constraints. Since here $r \sim |n_s - 1|$ ($r \sim (n_s - 1)^2$ in Starobinski model), so in certain sense, $n_s = 1$ might also explain the nondetection of $r$ in current observations.

It is required that the effective field theory responsible for the evolution of our Universe must be UV complete, otherwise it belongs to the swampland. According to (5), we have $\Delta N \sim \frac{\Delta\phi}{\sqrt{2\epsilon}}$, where $\Delta\phi = \phi_* - \phi_c$. In $\phi^2$ inflation without hybrid uplift, we have $N_* = \Delta N$ and $\epsilon \sim 1/N_*$, so the field excursion of inflaton is $\Delta\phi \approx \sqrt{2N_*} > 1$, contradicting the swampland conjecture $\Delta\phi < 1$. However, for hybrid $\phi^2$ inflation, we have

$$\Delta\phi \approx \sqrt{\frac{2(\Delta N)^2}{N_*}}, \tag{10}$$

where $\Delta N \approx 60 \ll N_*$. Thus the swampland conjecture $\Delta\phi < 1$ requires $N_* \simeq 10^4$. In this case, $r \approx 8/N_* \sim 10^{-3}$, also consistent with the Lyth bound [74].

### C. (Hybrid) polynomial attractors

In corresponding models, $V_{\rm inf}(\phi) \sim 1 - (\frac{\mu}{\phi})^p$, see recent Ref. [75], which was invented in D-brane inflation [76–78], and

$$n_s - 1 \approx -\frac{2}{N_*}\left(\frac{p+1}{p+2}\right), \qquad r \approx \frac{8p^2}{[p(p+2)N_*]^{\frac{2p+2}{p+2}}}\mu^{\frac{2p}{p+2}}, \tag{11}$$

for $\mu \ll 1$. Thus for, e.g., $p = 2$, we have $n_s - 1 = -3/2N_*$ and $r \simeq N_*^{-3/2}\mu$, which is also compatible with the ΛCDM constraint $n_s \approx 0.97$ for $N_* = \Delta N \approx 60$. However, in Hubble-tension-free cosmologies, $n_s = 0.998 \pm 0.005$, this would require that $N_* \gtrsim 200 \gg \Delta N$ and inflation ends at $N \gtrsim 140$ (60 $e$-folds after the CMB modes exit horizon at $N_* \simeq 200$). Thus we have $n_s \simeq 1\text{–}3/2N_* = 0.993$, and $r \simeq 5 \times 10^{-4}\mu$, which is still consistent with current constraint but even smaller than that in hybrid Starobinsky model.

## III. CONCLUSION

The complete resolution of Hubble tension might be pointing to a scale-invariant Harrison-Zeldovich spectrum of primordial scalar perturbation, i.e., $n_s = 1$ for $H_0 \sim 73$ km/s/Mpc. We propose a scheme to lift $n_s$ predicted by well-known slow-roll inflation models to $n_s = 1$. In corresponding models satisfying (1), if inflation ends by a waterfall instability when inflaton is still at a deep slow-roll region, and $n_s$ can be lifted to $n_s = 1$. Particularly, it is found that chaotic $\phi^2$ inflation strongly disfavored by Planck+BICEP/Keck [56] can be revived by this hybrid uplift, which is testable with upcoming CMB B-mode experiments.

As a direct consequence, the spectral running $\frac{dn_s}{d(\Delta N)} = \frac{dn_s}{dN_*} \propto N_*^{-2}$, which will be suppressed due to $N_* \gg 60$. It would thus be interesting to explore the constraints of next-generation CMB experiment, i.e., CMB-S4 [59], on $n_s$ and its running.

It should be pointed out that the models relevant with the waterfall instability will be inevitably more complex than the standard slow-roll inflation. According to (5), the waterfall instability must happen at suitable value $\phi_c \approx \phi_* - \Delta N\sqrt{2\epsilon}$, which so sets the requirements for the waterfall parameters in (4). It should also be mentioned that though the waterfall instability would bring large non-Gassianities to perturbations, it affects only the small-scale perturbation modes leaving the horizon around the waterfall era.

Inflation might continue after the waterfall instability. It is possible that the waterfall instability is caused by the nucleations and collisions of vacuum bubbles [55]. This will yield a subhorizon stochastic gravitational wave background, which after being reddened by subsequent inflation can explain the recently observed NANOGrav signal [79,80]. It is also possible that EDE is the remnant after the waterfall instability along $\sigma$, so that inflaton, EDE, and the current dark energy could live harmoniously together in a landscape. In such a landscape, after inflation the field might be "trapped" on a barrier with eV scale by dark matter particles produced [81], hereafter at matter radiation equality the field starts to roll and play a role of EDE, as in Ref. [26]. Relevant models will be studied in upcoming work, which might imprint lots of observable signals to be explored.

Though our discussion is slightly simplified, it highlights a significant point that until the Hubble tension is solved completely, it seems premature to claim which model of inflation is favored or ruled out by current data, since new physics beyond ΛCDM might bring unforeseen impact on primordial Universe.

## ACKNOWLEDGMENTS

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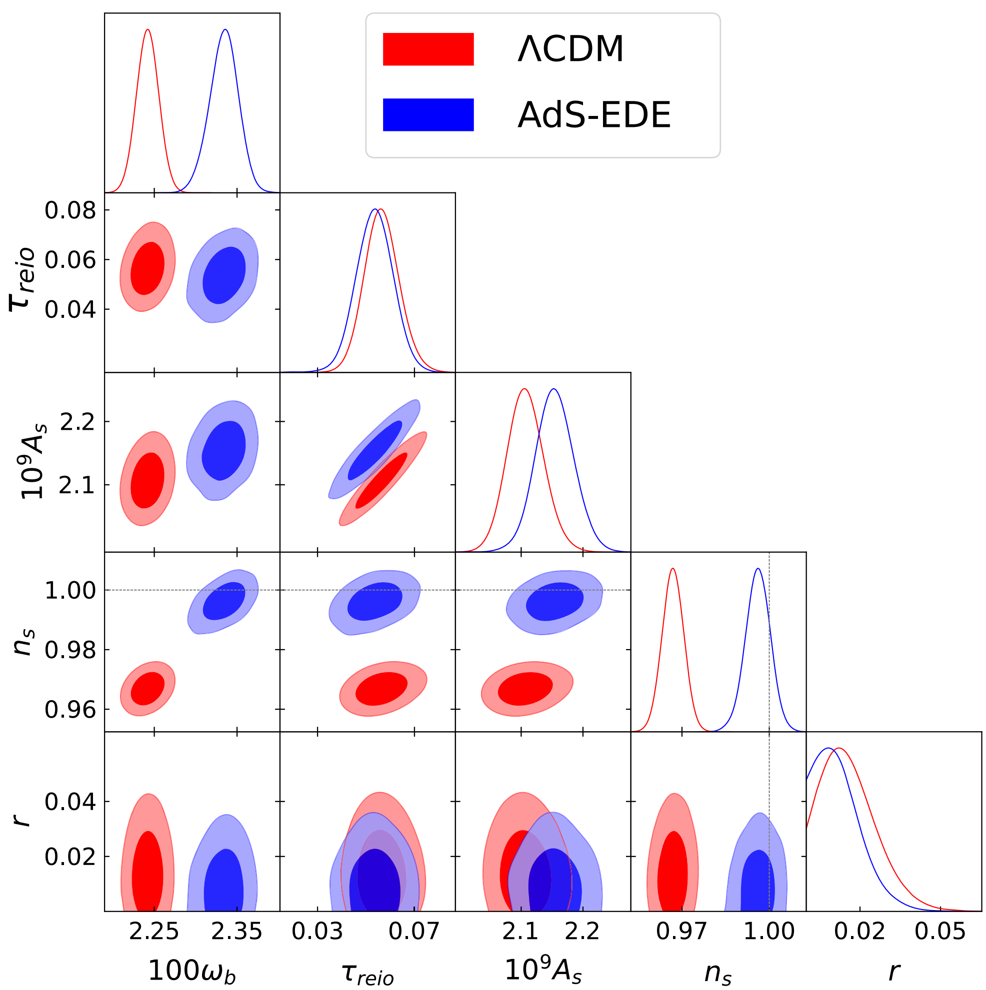


FIG. 4. The 68% and 95% posterior distribution of cosmological parameters in ΛCDM and AdS-EDE. Dashed lines mark the position of $n_s = 1$.

TABLE I. Bestfit $\chi^2$ per dataset for ΛCDM and AdS-EDE.

| Dataset | ΛCDM | AdS-EDE |
|---|---|---|
| Planck18 | 2778.10 | 2772.61 |
| BK18 | 537.25 | 535.93 |
| BAO | 5.78 | 5.46 |
| SNIa | ... | 1026.86 |

## APPENDIX: MCMC RESULTS

The EDE model we consider is AdS-EDE [25], which has a rolling potential $V(\phi) = V_0(\phi/M_p)^4 - V_{\rm AdS}$ glued to a cosmological constant $V(\phi) = {\rm const.} > 0$ at $\phi = (V_{\rm AdS}V_0)^{1/4}M_p$, where $V_{\rm AdS}$ is the depth of the AdS well. The evolution of the Universe after recombination is ΛCDM-like. In the MCMC analysis we sampled over the standard parameters $\{\omega_b, \omega_{\rm CDM}, H_0, \tau_{\rm reio}, \ln 10^{10}A_s, n_s, r\}$ for ΛCDM, plus two EDE parameters $\{f_{\rm EDE}, \ln(1+z_c)\}$ that parametrize the energy fraction $f_{\rm EDE}$ of EDE when it activates at redshift $z_c$. The primordial scalar and tensor power spectra are parametrized as $P_s = A_s(k/k_{\rm pivot})^{n_s-1}$ and $P_T = A_T(k/k_{\rm pivot})^{n_T}$ with scalar amplitude $A_s$ and tilt $n_s$, and tensor-to-scalar ratio $r = A_T/A_s$ defined at $k_{\rm pivot} = 0.05\ {\rm Mpc}^{-1}$ and $n_T = 0$. We assume spatially flat ($\Omega_K = 0$) throughout. For datasets we consider the full Planck18 high-l TTTEEE and low-l dataset as well as Planck lensing [10], postreconstructed BAO from BOSS DR12 [83], Type Ia supernova by Pantheon [84] as well as the CMB B-mode observation from BICEP/Keck18 [56].

Figure 4 plots the posterior distributions of relevant cosmological parameters with the ΛCDM results drawn from the publicly available BK18 chain (Planck18 + BK18 + BAO) [56]. Table I reports the bestfit $\chi^2$ per datasets. There is evident correlation between $n_s$ and $\omega_b$ in Fig. 4. This is because the increase of $n_s$ in the early resolutions of the Hubble tension (including AdS-EDE) actually stems from a compensation between the spectra tilt $n_s$ and the diffusion damping and baryon drag effects closely related to $\omega_b$. Since baryon drag effect is included, the compensation is most efficient around the first few CMB TT peaks in the spectrum, see Ref. [43] for details. This can also be readily seen in Fig. 5, where except for the first three acoustic peaks, the effect of having a more scale invariant tilt $n_s$ is not well compensated and becomes evident both on $l < 100$ and $l > 1000$. The derived reionization effective optical depth $\tau_{\rm reio}$ is very similar between ΛCDM and AdS-EDE, while also being consistent with Ref. [85] despite the slight shift in $A_s$. AdS-EDE produces a slightly tighter upper bound on $r$ due to the increase in $A_s$, see Ref. [57] for details.

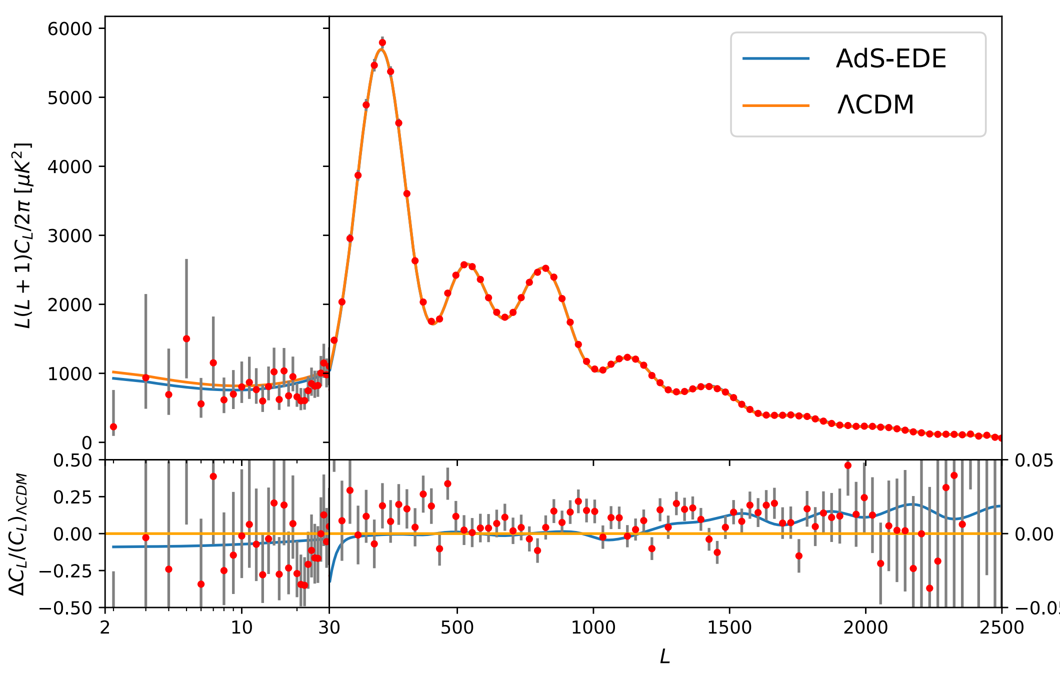


FIG. 5. CMB TT power spectra in the best fit ΛCDM and AdS-EDE models. Residuals are binned Planck2018 data points [10].

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