

**Aether-quasi-dilaton massive gravity**

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Although quasi-dilaton massive gravity is a well-defined gravitational theory, it exhibits instabilities and suffers from the strong coupling problem. In this work we construct an extension of the theory through the inclusion of the aether field. Focusing on flat Friedmann-Lemaître-Robertson-Walker geometry, we show the existence of exact, self-accelerating solutions at the background level, characterized by an effective cosmological constant arising from the graviton mass. Additionally, we perform a detailed perturbation analysis, investigating separately the tensor, vector, and scalar perturbations, extracting the dispersion relation of gravitational waves, and determining the stability conditions for vector and scalar sectors. As we show, there are always regions in the parameter space in which the obtained solutions are free from ghost instabilities, as well as from the strong coupling problem. Hence, although the aether field does not play an important role in the background self-accelerating solutions, it does play a crucial role in the alleviation of the perturbation-related problems of the simple quasi-dilaton massive gravity.

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**I. INTRODUCTION**

The origin of the late-time accelerated expansion of the Universe, supported by accumulating observational data from supernova Ia [1,2], cosmic microwave background (CMB) radiation [3,4], baryon acoustic oscillations [5,6], etc, is one of the essential issues of the standard cosmological paradigm. It is noticeable that the accelerated expansion can be explained in the context of general relativity, which is a unique theory of a massless Lorentz-invariant spin-2 particle in four dimensions [7], by considering the cosmological constant [8,9], or the dark energy sector [10–13].

On the other hand, one may explain the accelerated expansion through the paradigm of modified gravity [14–18]. One direction within this framework is curvature-based gravity, such as  $f(R)$  gravity [19],  $f(G)$  gravity [20],  $f(P)$  gravity [21], Lovelock gravity [22], Horndeski/Galileon scalar-tensor theories [23,24] etc. Alternatively one may proceed with torsion-based modified gravity, such as  $f(T)$  gravity [25,26],  $f(T, T_G)$  gravity [27],  $f(T, B)$  gravity [28], scalar-torsion theories [29] etc.

One interesting subclass of gravitational modification is massive gravity, in which the propagation of gravity corresponds to a spin-2 massive graviton [30–35]. The first analysis to describe the massive spin-2 field theory was performed by Fierz and Pauli in 1939. They presented the unique Lorentz-invariant linear theory without ghosts in a flat spacetime, by considering a massive spin-2 particle that consists of a specific combination of the mass terms, resulting in five physical degrees of freedom [36]. In the following decades, van Dam, Veltman and Zakharov found that the Fierz-Pauli theory in the massless limit does not reduce to the massless theory, since there is a discontinuity (van Dam-Veltman-Zakharov (vDVZ) discontinuity) [37,38]. Hence, Vainshtein argued that in order to avoid the vDVZ discontinuity the theory should be extended to the nonlinear level [39]. However, Boulware and Deser reported that the nonlinear theory of Fierz and Pauli exhibits a ghost, namely an instability that was later called the Boulware-Deser ghost [40]. Finally, de Rham, Gabadadze, and Tolley (dRGT) presented a fully nonlinear massive gravity without the Boulware-Deser ghost in a certain decoupling limit, namely the dRGT massive gravity [30,31].

While dRGT massive gravity can explain the accelerated expansion of the Universe for an open Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, it cannot present any solutions for homogeneous and isotropic Universe [41].

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Furthermore, due to the strong coupling problem and the nonlinear ghost instability, the scalar and vector perturbations would vanish [42]. Thus, the quasi-dilaton massive gravity theory has been introduced in order to solve these problems [43,44] (see also [45–59]). Nevertheless, in the quasi-dilaton massive gravity there is an instability in the scalar perturbation analysis [60–62].

In order to solve the above issue, in this work we introduce the aether-quasi-dilaton massive gravity. In fact, we introduce the new extension of the quasi-dilaton massive gravity by considering the aether field in the action, and this novel extension exhibits instability-free perturbations. We mention here that although Lorentz violation has not been experimentally observed [63], it cannot be theoretically excluded, and thus gravitational models which violate Lorentz symmetry have been studied in detail [64–69]. In these lines, Einstein-aether theory is one of the Lorentz violating theories that has attracted attention [70–74]. In several studies, the Einstein-aether theory has been used to describe different aspects of the gravitational system [75–79]. We mention that this theory is a second-order one, and can explain the classical limit of Horava-Lifshitz gravity [80].

In the following we will show the existence of self-accelerating solutions, and we will perform the perturbation analysis for the aether-quasi-dilaton massive gravity. In particular, in the perturbations analysis we will extract the modified dispersion relation of gravitational waves, and we will present the stability conditions of vector and scalar perturbations. The paper is organized as follows. In Sec. II we present the aether-quasi-dilaton massive gravity, and we derive the background equations of motion, extracting self-accelerating solutions. In Sec. III we present the cosmological perturbations analysis, which consist of tensor, vector, and scalar perturbations. Finally, in Sec. IV we summarize the obtained results. Throughout the manuscript we consider natural units, where  $c = \hbar = 1$  and  $M_{\text{Pl}}^2 \equiv 8\pi G = 1$ , with  $G$  the Newton's constant.

## II. AETHER-QUASI-DILATON MASSIVE GRAVITY

In this section we introduce the new extension of quasi-dilaton massive gravity, which is constructed by adding the action of aether field. The total action is

$$S_{\text{Total}} = S_{\text{QDMG}} + S_{\text{Aether}}. \quad (1)$$

The quasi-dilaton massive gravity theory includes the massive graviton term and the quasi-dilaton term [43], namely it has the action

$$S_{\text{QDMG}} = \frac{1}{2} \int d^4x \{ \sqrt{-g} [R - \omega g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + 2m_g^2 U(\mathcal{K})] \}, \quad (2)$$

where  $R$  is the Ricci scalar,  $\omega$  is a dimensionless constant,  $\sigma$  is a scalar field,  $g_{\mu\nu}$  is the physical dynamical metric and  $\sqrt{-g}$  is its determinant. Note that the origin of the graviton mass  $m_g$  is the potential  $U$  which consists of three parts, i.e.,

$$U(\mathcal{K}) = U_2 + \alpha_3 U_3 + \alpha_4 U_4, \quad (3)$$

with  $\alpha_3$  and  $\alpha_4$  dimensionless free parameters. In the above expression we have [31]

$$\begin{aligned} U_2 &= \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2]), \\ U_3 &= \frac{1}{6} ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]), \\ U_4 &= \frac{1}{24} ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]), \end{aligned} \quad (4)$$

where “[.]” is construed as the trace of the tensor inside the brackets. Note that the building block tensor  $\mathcal{K}$  can be defined as

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - e^\sigma \sqrt{g^{\mu\alpha} f_{\alpha\nu}}, \quad (5)$$

where  $f_{\alpha\nu}$  is the fiducial metric defined through

$$f_{\alpha\nu} = \partial_\alpha \phi^c \partial_\nu \phi^d \eta_{cd}, \quad (6)$$

with  $\eta_{cd}$  the Minkowski metric ( $c, d = 0, 1, 2, 3$ ) and  $\phi^c$  the Stueckelberg fields which are introduced to restore general covariance. Notice that the theory is invariant under the global dilation transformation  $\sigma \rightarrow \sigma + \sigma_0$  [43].

In addition, the aether action in (1) corresponds to the aether field  $u^\mu$ , namely [70]

$$\begin{aligned} S_{\text{Aether}} &= -\frac{1}{2} \int d^4x \{ \sqrt{-g} [\beta_1(\sigma) u^{\nu;\mu} u_{\nu;\mu} \\ &\quad + \beta_2(\sigma) (g^{\mu\nu} u_{\mu;\nu})^2 + \beta_3(\sigma) u^{\nu;\mu} u_{\mu;\nu} \\ &\quad + \beta_4(\sigma) u^\mu u^\nu u_{;\mu} u_{;\nu} - \lambda (u^\mu u_\mu + 1)] \}, \end{aligned} \quad (7)$$

where  $\beta_1(\sigma)$ ,  $\beta_2(\sigma)$ ,  $\beta_3(\sigma)$ , and  $\beta_4(\sigma)$  are the coefficient functions that define the coupling between the aether field and the scalar field. Lastly,  $\lambda$  should be considered as a Lagrange multiplier, which ensures the aether-field unitarity, namely  $u^\mu u_\mu + 1 = 0$  [80,81].

### A. Background cosmological evolution

Let us apply the above theory in an FLRW metric at the background level. The dynamical and fiducial metrics are expressed as

$$g_{\mu\nu} = \text{diag}[-N^2, a^2, a^2, a^2], \quad (8)$$

$$f_{\mu\nu} = \text{diag}[-\dot{f}(t)^2, 1, 1, 1], \quad (9)$$

with  $a$  the scale factor and  $N$  the lapse function of the dynamical metric, which relates the coordinate-time  $dt$  to the proper-time  $d\tau$  via  $d\tau = Ndt$  [82,83]. Moreover, the function  $f(t)$  is the Stueckelberg scalar function, with  $\phi^0 = f(t)$  and  $\frac{\partial\phi^0}{\partial t} = \dot{f}(t)$  [84].

According to the above discussion, we result to the total Lagrangian

$$\begin{aligned} \mathcal{L} = & \frac{-3a\dot{a}^2}{N} \left[ 1 + \frac{A(\sigma)}{2} \right] + \frac{\omega a^3}{2N} \dot{\sigma}^2 \\ & + m_g^2 \{ Na^3(Y-1)[3(Y-2) \\ & - (Y-4)(Y-1)\alpha_3 - (Y-1)^2\alpha_4] \\ & + \dot{f}(t)a^4Y(Y-1)[3 - 3(Y-1)\alpha_3 + (Y-1)^2\alpha_4] \}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} A(\sigma) &= \beta_1(\sigma) + 3\beta_2(\sigma) + \beta_3(\sigma), \\ Y &\equiv \frac{e^\sigma}{a}. \end{aligned} \quad (11)$$

We proceed by considering the unitary gauge, namely  $f(t) = t$ , and thus a constraint equation is obtained by varying with respect to  $f$ , i.e.,

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta f} = & m_g^2 \frac{d}{dt} \{ a^4Y(Y-1)[3 - 3(Y-1)\alpha_3 \\ & + (Y-1)^2\alpha_4] \} = 0. \end{aligned} \quad (12)$$

By varying with respect to the lapse function  $N$ , we obtain the Friedmann equation

$$\begin{aligned} \frac{1}{a^3} \frac{\delta\mathcal{L}}{\delta N} = & 3H^2 \left[ 1 + \frac{A(\sigma)}{2} \right] - \frac{\omega}{2} \left( H + \frac{\dot{Y}}{NY} \right)^2 \\ & - m_g^2(Y-1)[(Y-4)(Y-1)\alpha_3 \\ & + (Y-1)^2\alpha_4 - 3(Y-2)] = 0. \end{aligned} \quad (13)$$

Similarly, the equation of motion related to the scalar field  $\sigma$  is given by

$$\begin{aligned} \frac{1}{a^3N} \frac{\delta\mathcal{L}}{\delta\sigma} = & -3H^2 \left\{ \frac{\omega}{N} + \frac{A'(\sigma)}{2} \right\} \\ & + m_g^2Y \{ 6(r+1)(\alpha_4 + 2\alpha_3 + 1)Y \\ & - (3+r)(3 + 3\alpha_3 + \alpha_4) \\ & - 3(3r+1)(\alpha_4 + \alpha_3)Y^2 + 4r\alpha_4Y^3 \} = 0, \end{aligned} \quad (14)$$

where  $r \equiv \frac{\dot{a}}{a}$  and  $H \equiv \frac{\dot{a}}{Na}$ . Furthermore, using the notation of (11), we can write the expression

$$\frac{\dot{\sigma}}{N} = H + \frac{\dot{Y}}{NY}, \quad \ddot{\sigma} = \frac{d}{dt} \left( NH + \frac{\dot{Y}}{Y} \right). \quad (15)$$

Since the Stueckelberg field  $f$  introduces a time reparametrization invariance, there is a Bianchi identity that relates the four equations of motion, namely

$$\frac{\delta S}{\delta\sigma} \dot{\sigma} + \frac{\delta S}{\delta f} \dot{f} - N \frac{d}{dt} \frac{\delta S}{\delta N} + \dot{a} \frac{\delta S}{\delta a} = 0. \quad (16)$$

Hence, the equation of motion corresponding to the scale factor  $a$  can be eliminated.

## B. Self-accelerating background solutions

We can now examine whether the above theory accepts self-accelerating solutions. By integrating the Stueckelberg constraint (12) we obtain

$$Y(Y-1)[3 - 3(Y-1)\alpha_3 + (Y-1)^2\alpha_4] \propto a^{-4}. \quad (17)$$

Hence, in an expanding universe the right-hand side of (17) will decrease as  $a^{-4}$ . Therefore,  $Y$  leads to a constant value,  $Y_{\text{SA}}$ , which is the saturate of  $Y$ , and it is clear that  $Y_{\text{SA}}$  is a root of the left-hand side of (17).

As we can see, one of the solutions of (17) is  $Y = 0$ . However, if we consider  $Y = 0$  the system leads to  $\sigma \rightarrow -\infty$ , which implies that this solution leads to strong coupling in the vector and scalar sectors, and thus we do not study it [43]. Hence, we have

$$(Y-1)[3 - 3(Y-1)\alpha_3 + (Y-1)^2\alpha_4]|_{Y=Y_{\text{SA}}} = 0. \quad (18)$$

Moreover, another obvious solution is  $Y = 1$ . However, by considering this solution the cosmological constant would vanish and the system would encounter inconsistency, and thus we do not study it either [43]. Therefore, the two remaining solutions of (17) are

$$Y_{\text{SA}}^\pm = \frac{3\alpha_3 + 2\alpha_4 \pm \sqrt{9\alpha_3^2 - 12\alpha_4}}{2\alpha_4}. \quad (19)$$

The modified Friedmann equation (13) leads to

$$3H^2 \left[ 1 + \frac{\tilde{A}}{2} - \frac{\omega}{6} \right] = \Lambda_{\text{SA}}^\pm, \quad (20)$$

where  $\tilde{A}$  is the saturate of  $A(\sigma)$ , and  $\Lambda_{\text{SA}}^\pm$  is given by

$$\begin{aligned} \Lambda_{\text{SA}}^\pm = & m_g^2(Y_{\text{SA}}^\pm - 1)[6 - 3Y_{\text{SA}}^\pm + (Y_{\text{SA}}^\pm - 4)(Y_{\text{SA}}^\pm - 1)\alpha_3 \\ & + (Y_{\text{SA}}^\pm - 1)^2\alpha_4]. \end{aligned} \quad (21)$$

Note that using (19), the above equation can be rewritten as

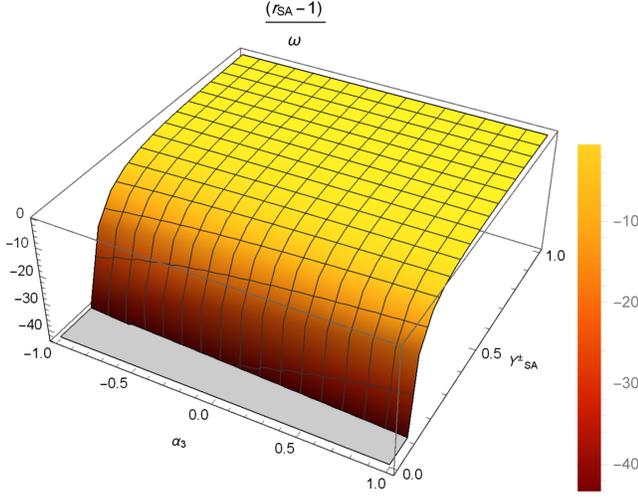


FIG. 1. The quantity  $\frac{r_{SA}-1}{\omega}$ , using (24), for  $m_g/H \simeq 1$ , in the case  $0 < Y_{SA}^\pm < 1$ . The excluded regions are illustrated in gray.

$$\Lambda_{SA}^\pm = \frac{3m_g^2}{2\alpha_3^3} \left[ 9\alpha_3^4 \pm 3\alpha_3^3 \sqrt{9\alpha_3^2 - 12\alpha_4} - 18\alpha_3^2 \alpha_4 \mp 4\alpha_3 \alpha_4 \sqrt{9\alpha_3^2 - 12\alpha_4 + 6\alpha_4^2} \right]. \quad (22)$$

We solve Eq. (20) to calculate the  $H^2$ , so we have

$$H^2 = \frac{2\Lambda_{SA}^\pm}{3\tilde{A} - \omega + 6}. \quad (23)$$

Additionally, from (14) we obtain  $r_{SA}$  as

$$r_{SA} = 1 + \frac{\omega H^2}{m_g^2 Y_{SA}^{\pm 2} (\alpha_3 Y_{SA}^\pm - \alpha_3 - 2)}. \quad (24)$$

We mention that we have acquired a result for  $r_{SA}$  similar with that of [60], which implies that the aether part of the theory does not affect  $r_{SA}$ . However, we stress that there is not any strong coupling in this condition, and thus this theory possesses well-behaved self-accelerating solutions with an effective cosmological constant. This is one of the main results of the present work.

In order to present the above results in a more transparent way, in Figs. 1 and 2 we illustrate the allowed parameter regions for (24). Note that these figures are generated by considering  $m_g/H \simeq 1$  [50]. We mention that by adjusting the value of the parameter  $\alpha_4$  it is possible to have a sizeable value of  $r_{SA}$ .

### III. PERTURBATIONS ANALYSIS

In this section we perform the perturbation analysis of the scenario at hand. The significance of such analysis is that the stability conditions of the solutions can be determined, too. Since we are interested in quadratic

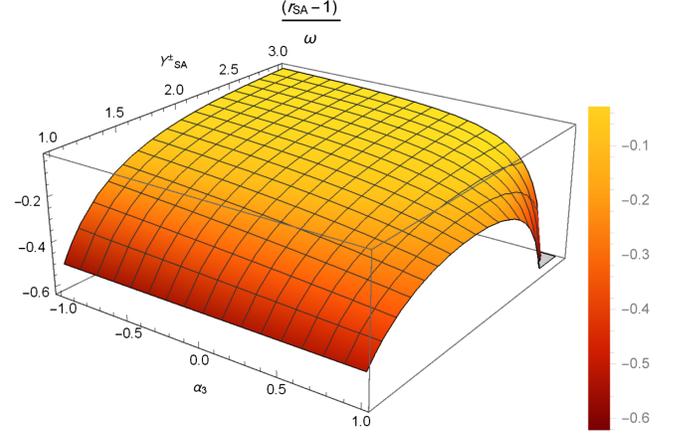


FIG. 2. The quantity  $\frac{r_{SA}-1}{\omega}$ , using (24), for  $m_g/H \simeq 1$ , in the case  $Y_{SA}^\pm > 1$ . The excluded regions are illustrated in gray.

perturbations, we expand the physical metric  $g_{\mu\nu}$  in terms of small fluctuations  $\delta g_{\mu\nu}^{(0)}$  around the background solution  $g_{\mu\nu}^{(0)}$ :

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}. \quad (25)$$

We keep all terms up to quadratic order, and as usual the metric perturbations can be split into three parts, namely scalar, vector, and tensor perturbations. Thus, we have

$$\begin{aligned} \delta g_{00} &= -2N^2 \Phi, \\ \delta g_{0i} &= Na(B_i + \partial_i B), \\ \delta g_{ij} &= a^2 \left[ h_{ij} + \frac{1}{2} (\partial_i E_j + \partial_j E_i) + 2\delta_{ij} \Psi \right. \\ &\quad \left. + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \partial_l \partial^l \right) E \right]. \end{aligned} \quad (26)$$

As usual, the tensor perturbations are transverse  $\partial^i h_{ij} = 0$ , and traceless  $h_i^i = 0$ , while the vector ones are transverse  $\partial^i E_i = \partial^i B_i = 0$ . Notice that all perturbations are functions of time and space, and they are consistent with the transformations under spatial rotations [50,85].

Additionally, we consider the perturbation of the scalar field  $\sigma$  as

$$\sigma = \sigma^{(0)} + \delta\sigma, \quad (27)$$

moreover, we perturb the aether field as [86,87],

$$u^\mu = u^{\mu(0)} + \delta u^\mu = \frac{1}{a} (1 - \Psi, \partial^i V + iS_i), \quad (28)$$

where

$$\delta u^\mu = \frac{1}{a} (-\Psi, \partial^i V + iS_i), \quad (29)$$

here  $V$  is the longitudinal scalar mode and  $S_i$  is the transverse vector mode i.e.,  $\partial^i S_i = 0$ . In the vector

perturbations, it can be possible to restrict ourselves to the condition where the Aether field would be defined by the Khronon [88–92]. The Khronometric model is a version of Einstein-Aether where the Aether field is constrained via a scalar field  $\sigma$ . This way, the field can be considered as

$$u_\mu = -\frac{\partial_\mu \sigma}{\sqrt{-g^{\alpha\beta} \partial_\alpha \sigma \partial_\beta \sigma}}, \quad (30)$$

and thus the timelike unit norm constraint should be satisfied automatically. This way, the Aether is restricted to be orthogonal to a set of spacelike surfaces defined by  $\sigma$ . At background order it can be proposed  $\sigma = \sigma(t)$  and therefore from the above equation we have  $u^\mu = (1, 0, 0, 0)$ . Consequently, the choice of the Khronon definition has no effect on background dynamics. In the khronometric model, the  $\sigma$  sets a preferred global time coordinate. In [88], it was investigated how this model explains the low energy limit of the consistent extension of Horava gravity which is a quantum theory of gravity. At low energies, this reduces to a Lorentz-violating scalar-tensor gravity theory. For the vector perturbations, so we have

$$\delta u_\mu = \frac{a}{\sigma'} \left[ -\partial_\mu \delta\sigma + \partial_\mu \sigma \left( \Psi + \frac{\delta\sigma'}{\sigma'} \right) \right], \quad (31)$$

where  $\delta\sigma$  is the perturbed field. The time component is then  $\delta u_0 = a\Psi$ , which is a result of the timelike unit norm constraint, as in Eq. (28). But, if we calculate the spatial component we have

$$\delta u_i = -\frac{a}{\sigma'} \partial_i \delta\sigma \rightarrow S_i = 0, \quad (32)$$

thus, there is no propagating transverse vector mode. For the scalar perturbation of aether field, we redefine  $\frac{1}{\sigma'} \partial_i \delta\sigma = \partial_i V$ . Thus, the scalar sector for generalized Einstein-Aether and the Khronon should be completely equivalent [89].

Furthermore, as usual the actions are expanded in Fourier plane waves, namely  $\vec{\nabla}^2 \rightarrow -k^2$ ,  $d^3x \rightarrow d^3k$ , while the spatial indices are raised and lowered by  $\delta^{ij}$  and  $\delta_{ij}$ . Lastly, since all calculations are performed in the unitary gauge, we do not need to specify gauge-invariant combinations [60].

### A. Tensor perturbations

We start our investigation by analyzing the tensor perturbations. Among others this analysis provides the speed of gravitational waves, and moreover it can determine the stability of the solutions.

For convenience, we calculate the perturbed action at second order separately for the different parts. The general relativity (GR) part is written as

$$S_{\text{GR}}^{(2)} = \frac{1}{8} \int d^3k dt a^3 N \left[ \frac{\dot{h}_{ij} \dot{h}^{ij}}{N^2} - \left( \frac{k^2}{a^2} + \frac{4\dot{H}}{N} + 6H^2 \right) h^{ij} h_{ij} \right]. \quad (33)$$

Additionally, the quasi-dilaton part of the perturbed action reads as

$$S_{\text{Quasi-dilaton}}^{(2)} = -\frac{1}{8} \int d^3k dt a^3 N \left[ \left( \frac{\omega}{N^2} \dot{\sigma}^2 \right) h^{ij} h_{ij} \right], \quad (34)$$

while the aether part is found to be

$$S_{\text{Aether}}^{(2)} = \frac{1}{16} \int d^3k dt a^3 N \left[ \frac{\dot{h}_{ij} \dot{h}^{ij}}{N^2} - \left( \frac{k^2}{a^2} + \frac{4\dot{H}}{N} + 6H^2 \right) h^{ij} h_{ij} \right] A(\sigma). \quad (35)$$

Finally, the massive gravity part becomes

$$S_{\text{massive}}^{(2)} = \frac{1}{8} \int d^3k dt a^3 N m_g^2 [(\alpha_3 + \alpha_4) r Y^3 - (1 + 2\alpha_3 + \alpha_4)(1 + 3r) Y^2 + (3 + 3\alpha_3 + \alpha_4)(3 + 2r) Y - 2(6 + 4\alpha_3 + \alpha_4) h^{ij} h_{ij}]. \quad (36)$$

In summary, assembling the above terms, the second-order perturbed action for tensor perturbations  $S_{\text{total}}^{(2)} = S_{\text{GR}}^{(2)} + S_{\text{quasi-dilaton}}^{(2)} + S_{\text{aether}}^{(2)} + S_{\text{massive}}^{(2)}$ , becomes

$$S_{\text{total}}^{(2)} = \frac{1}{8} \int d^3k dt a^3 N \left\{ \frac{\dot{h}_{ij} \dot{h}^{ij}}{N^2} [1 + A(\sigma)] - \left\{ \frac{k^2}{a^2} [1 + A(\sigma)] + M_{\text{GW}}^2 \right\} h^{ij} h_{ij} \right\}, \quad (37)$$

where

$$M_{\text{GW}}^2 = \left( \frac{4\dot{H}}{N} + 6H^2 \right) [1 + A(\sigma)] + \frac{\omega}{N^2} \dot{\sigma}^2 + \chi, \quad (38)$$

and with

$$\begin{aligned} \chi = & \frac{1}{(2Y_{\text{SA}}^\pm - 2)} \{ 2m_g^2 \{ Y_{\text{SA}}^\pm \{ Y_{\text{SA}}^\pm [Y_{\text{SA}}^\pm (r_{\text{SA}} + 1) - 6] + 6 \} - 2 \} \\ & - \frac{1}{(r_{\text{SA}} - 1) Y_{\text{SA}}^{\pm 2} N} \{ H^2 [Y_{\text{SA}}^\pm (Y_{\text{SA}}^\pm - 3) (Y_{\text{SA}}^\pm r_{\text{SA}} - 2) - 2] \\ & \cdot [2\omega + NA'(\sigma)] \} \}. \end{aligned} \quad (39)$$

The last relation is obtained using (19) and (24) to substitute  $\alpha_3$  and  $\alpha_4$ .

In summary, expression (38) determines the dispersion relation of gravitational waves in aether-quasi-dilaton

massive gravity. In particular, in order to guarantee the stability of long-wavelength gravitational waves, the mass square of gravitational waves should be positive, namely  $M_{\text{GW}}^2 > 0$ .

### B. Vector perturbations

We proceed by performing the vector perturbation analysis. We consider

$$B_i = \left[ A(\sigma)a + \frac{2k^2 a(r^2 - 1)}{2a^2 H^2 \omega + k^2(r^2 - 1)} \right] \frac{\dot{E}_i}{4N}. \quad (40)$$

Note that the field  $B_i$  is nondynamical, and thus we handle it as an auxiliary field in the main action. Thus, a single propagating vector is obtained, namely

$$S_{\text{vector}}^{(2)} = \frac{1}{8} \int d^3 k d t a^3 N \left( \frac{\beta}{N^2} |\dot{E}_i|^2 - \frac{k^2}{2} M_{\text{GW}}^2 |E_i|^2 \right), \quad (41)$$

where

$$\beta = \frac{k^2}{2} \left[ 1 + \frac{k^2(r^2 - 1)}{2a^2 H^2 \omega} \right]^{-1}. \quad (42)$$

It should be pointed out that in the case  $\frac{r^2-1}{\omega} \geq 0$ , we have no critical momentum scale. On the other hand, for  $\frac{r^2-1}{\omega} < 0$  we have a critical momentum scale which is  $k_c^2 = \frac{2a^2 H^2 \omega}{1-r^2}$ , to avoid a ghost. We mention that the physical critical momentum scale is vital in order to acquire stability, and this scale should be above the ultraviolet cutoff scale of effective field theory, namely

$$\Lambda_{\text{UV}}^2 \lesssim \frac{2H^2 \omega}{1-r^2}. \quad (43)$$

Moreover, the canonically normalized fields are defined to determine other instabilities in the vector modes:

$$\zeta_i = \frac{\beta E_i}{2}. \quad (44)$$

By substituting into (41), we have

$$S = \frac{1}{2} \int d^3 k d t a^3 N \left( \frac{|\dot{\zeta}_i|^2}{N^2} - c_V^2 |\zeta_i|^2 \right). \quad (45)$$

Thus, the sound speed for vector modes becomes

$$c_V^2 = M_{\text{GW}}^2(1+u^2) - \frac{H^2 u^2(1+4u^2)}{(1+u^2)^2}, \quad (46)$$

where the dimensionless quantity  $u^2$  is

$$u^2 \equiv \frac{k^2(r^2 - 1)}{2a^2 H^2 \omega}. \quad (47)$$

Lets us proceed by elaborating the stability conditions. Observing the first part of (46) we deduce that if  $M_{\text{GW}}^2 < 0$  and  $u^2 > 0$  then we encounter tachyonic instability. In order to avoid this condition, one requires

$$\Lambda_{\text{UV}}^2 \lesssim \frac{2H^2 \omega}{r^2 - 1}, \quad (48)$$

in the case  $\frac{r^2-1}{\omega} > 0$ . It is interesting to note that if all physical momenta are considered below the UV cutoff  $\Lambda_{\text{UV}}$ , then the rate of instability growth would be lower than the cosmological scale. Furthermore, by looking at the second part of (46), two cases can be ascertained. First, if we consider  $u^2 > 0$ , there are not any instabilities faster than the Hubble expansion. On the other hand, for  $u^2 < 0$ , due to the no-ghost condition (43), in order to avoid instabilities we require  $|u^2| \lesssim \frac{k^2}{a^2} \frac{1}{\Lambda_{\text{UV}}^2}$ . Hence, we have no instabilities in the second part of (46). In summary, in order to maintain the stability of the vector modes, we demand  $c_V^2 > 0$  and  $M_{\text{GW}}^2 > 0$ .

### C. Scalar perturbations

We proceed to the investigation of scalar perturbations, which are crucial for the growth of the Universe structure. Observing the perturbation form (26), we can handle  $\Phi$  and  $B$  as auxiliary fields, since they are free of time derivatives. In particular, we have

$$B = \frac{r^2 - 1}{\omega a H^2} \left[ H(\omega \delta \sigma - 2\Phi) + \frac{1}{3N} (k^2 \dot{E} + 6\dot{\Psi}) \right], \quad (49)$$

and

$$\begin{aligned} \Phi = & \frac{A(\sigma)(k^2 \dot{E} + 6\dot{\Psi})}{12HN} [1 - \Psi + V] + \frac{1}{48k^2(r^2 - 1) - 12H^2 a^2 \omega(\omega - 6)} \left\{ 4k^4 \omega^2 (E + 3) \right. \\ & \times \left( 2k^2(r^2 - 1) - \frac{3a^2 H^2 \omega}{r - 1} \right) \delta \sigma + 12\omega \left( 2k^2 + \frac{3a^2 H^2 \omega}{r - 1} \right) \Psi \\ & \left. - \frac{12Ha^2 \omega(\omega \delta \dot{\sigma} - 6\dot{\Psi})}{N} + \frac{8k^2(r^2 - 1)(k^2 \dot{E} + 6\dot{\Psi})}{HN} \right\}. \end{aligned} \quad (50)$$

Hence, substituting the above expressions into the action, we remain with four fields, namely  $E$ ,  $\Psi$ ,  $V$ , and  $\delta\sigma$ . In addition, we can use another nondynamical combination, i.e.,

$$\tilde{\Psi} = \frac{1}{\sqrt{2}}(\Psi + \delta\sigma). \quad (51)$$

Since  $\tilde{\Psi}$  is free of time derivatives, namely

$$\begin{aligned} \tilde{\Psi} = & \left( 1 + \frac{A(\sigma)}{12Nr} [144a^2H^2N(r+1)\tilde{\delta\sigma} - k^2Nr(6\tilde{\delta\sigma} + \sqrt{2}E) + 3a^2H(12(r+1)\dot{\tilde{\delta\sigma}} + \sqrt{2}r\dot{E})] \right)^{-1} \\ & \left\{ -\frac{2\sqrt{2}k^4E}{3(4k^2 - a^2H^2(6-\omega)\omega)} + \left( -k^2 - \frac{24a^2H^2}{r(r-1)} + \frac{2a^2H^2k^2(-\omega^2 + r(48 - (6-\omega)\omega))}{(r-1)(4k^2 - a^2H^2(6-\omega)\omega)} \right) \tilde{\delta\sigma} \right. \\ & + \frac{2a^2H}{N} \left( \frac{3}{r} + \frac{(6-\omega)(2k^2(r-1) + 3a^2H^2\omega)}{(r-1)(4k^2 - a^2H^2(6-\omega)\omega)} \right) \dot{\tilde{\delta\sigma}} + \frac{\sqrt{2}a^2Hk^2(6-\omega)\dot{E}}{3N(4k^2 - a^2H^2(6-\omega)\omega)} \\ & \left. + \frac{A(\sigma) + V}{12Nr} (144a^2H^2N(r+1)\tilde{\delta\sigma} - k^2Nr(6\tilde{\delta\sigma} + \sqrt{2}E) + 3a^2H(12(1+r)\dot{\tilde{\delta\sigma}} + \sqrt{2}r\dot{E})) \right\}, \quad (52) \end{aligned}$$

it can be used as an auxiliary field in order to eliminate the sixth degree of freedom. Finally, we consider the orthogonal combination

$$\tilde{\delta\sigma} = \frac{1}{\sqrt{2}k^2}(\Psi - \delta\sigma). \quad (53)$$

We can now write the action in terms of  $\tilde{\Psi}$ ,  $\tilde{\delta\sigma}$ ,  $E$ , and  $V$ . Introducing the notation  $P \equiv (\tilde{\delta\sigma}, E, V)$ , we have

$$S = \frac{1}{2} \int d^3k dt a^3 N \left[ \frac{\dot{P}^\dagger}{N} \mathcal{F} \frac{\dot{P}}{N} + \frac{\dot{P}^\dagger}{N} \mathcal{D} P + P^\dagger \mathcal{D}^T \frac{\dot{P}}{N} - P^T \varpi^2 P \right]. \quad (54)$$

In the above expression  $\mathcal{D}$  is a real antisymmetric  $2 \times 2$  matrix, and  $\mathcal{F}$  and  $\varpi^2$  are real symmetric  $2 \times 2$  matrices. The components of the  $\mathcal{F}$  matrix are

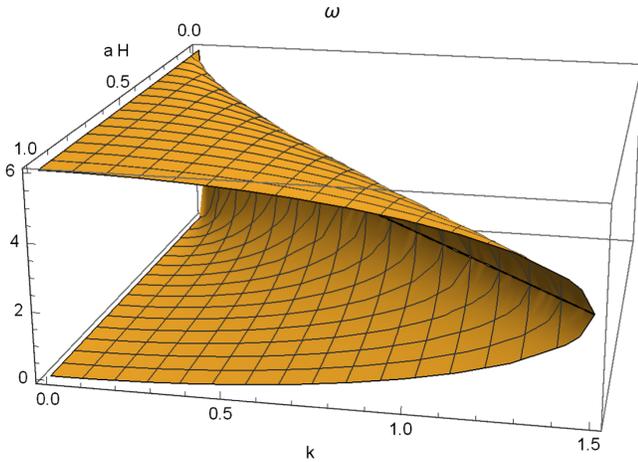


FIG. 3. The region corresponding to absence of ghost instabilities in the scalar perturbations, corresponding to positive determinant (58).

$$\mathcal{F}_{11} = 2k^4\omega + \frac{18k^2\omega a^2H^2}{(r-1)^2} - \frac{2k^4a^2H^2[\omega^3 + (6-\omega)\omega^2r]^2}{[4k^2 - (6-\omega)\omega a^2H^2](r-1)^2}, \quad (55)$$

$$\mathcal{F}_{12} = \frac{\sqrt{2}k^4r}{(r-1)} - \frac{2\sqrt{2}k^6[\omega^2 + (6-\omega)\omega r]}{[12k^2\omega - 3(6-\omega)\omega^2a^2H^2](r-1)}, \quad (56)$$

$$\mathcal{F}_{22} = \frac{k^4\omega}{36} - \frac{k^4a^2H^2(6-\omega)^2\omega}{144k^2 - 36a^2H^2(6-\omega)\omega}. \quad (57)$$

In order to examine the sign of the eigenvalues, we calculate the determinant of the kinetic matrix  $\mathcal{F}$  as

$$\det \mathcal{F} \equiv \mathcal{F}_{11}\mathcal{F}_{22} - \mathcal{F}_{12}^2 = \frac{3k^6\omega^2a^4H^4}{[\omega a^2H^2 - \frac{4k^2}{(6-\omega)}](r-1)^2}. \quad (58)$$

Hence, in order to avoid ghost instabilities in the scalar sector, we require

$$\frac{k}{aH} < \frac{\sqrt{6\omega - \omega^2}}{2}. \quad (59)$$

In Fig. 3 we depict the region corresponding to absence of ghost instabilities. As we observe, in order not to have ghosts we require  $0 < \omega < 6$ . Moreover, by demanding the left-hand side of (20) to be positive, we deduce that the aether field in the saturate condition  $\tilde{A}$  should be positive, i.e.,  $\tilde{A} > 0$ .

#### IV. CONCLUSIONS

In this manuscript we presented the aether-quasi-dilaton massive gravity. This theory arises from the inclusion of the aether field in the framework of quasi-dilaton massive gravity. After constructing the action of the theory, we

extracted the general field equations and then we applied them in a flat FLRW geometry.

We started our analysis at the cosmological background level, showing the existence of exact, self-accelerating solutions, characterized by an effective cosmological constant arising from the graviton mass.

However, the interesting feature of the scenario was revealed performing a detailed perturbation analysis. In particular, investigating separately the tensor, vector, and scalar perturbations we showed that the aether-quasi-dilaton massive gravity is free of ghost instabilities as well as of the strong coupling problem.

In particular, concerning the tensor perturbations we extracted the dispersion relation of gravitational waves. Additionally, performing the vector and scalar perturbation analysis we determined the stability conditions. As we saw, there are always regions in the parameter space in which the obtained solutions are well-behaved at both background and perturbation levels.

Hence, although the aether field does not play an important role in the background self-accelerating solutions, it does play a role in the alleviation of the perturbation-related problems of the simple quasi-dilaton massive gravity. This result is a good motivation for further investigation of the theory, and in particular of its early- and late-time cosmological application. Since such a study lies beyond the scope of this first work, it is left for a future project.

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