# Implications for cosmic domain walls from the first three observing runs of LIGO-Virgo

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We put constraints on the normalized energy density in gravitational waves from cosmic domain walls by searching for the stochastic gravitational-wave background in the data of Advanced LIGO and Virgo's first three observing runs. By adopting a phenomenological broken power-law model, we obtain the upper limit on the amplitude of stochastic gravitational-wave background generated by domain walls in the peak frequency band 10–200 Hz and get the most stringent limitation $\Omega_{\mathrm{DW}}(f_* = 35\ \mathrm{Hz}) < 1.4 \times 10^{-8}$ at 95% confidence level at the peak frequency $f_* = 35$ Hz. Subsequently, we work out the constraints on the parameter space in the appealing realization of DW structure—the heavy axion model which can avoid the so-called quality problem.

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## I. INTRODUCTION

The successful observation of gravitational-wave (GW) signal from compact binary coalescence [1] by Advanced LIGO [2] and Advanced Virgo [3] has open up a new era of GW astronomy. The superposition of a large number of not loud enough GW incidences can produce a stochastic gravitational-wave background (SGWB) that encodes information about the physics of astrophysical sources. Until the O3 observing period, the LIGO and Virgo Scientific Collaboration has not detected such a SGWB, and therefore they set the upper limits of SGWB [4].

Besides the origin of astronomy like compact binary coalescence, SGWB can also have the cosmological origins, including phase transitions [5–8], primordial perturbations during inflation [9–11], cosmic string [12–16] etc. Detecting SGWB signals from these cosmological origins is of great significance for understanding the physics of the early Universe. In this paper, we focus on SGWB signal from cosmic domain wall (DW) [17,18] network—laminar topological defects that form when a discrete symmetry is spontaneously broken. Such discrete symmetries arise in various particle physics frameworks beyond the Standard Model (SM), such as Higgs models [19–21], grand unification [22,23], supersymmetry [24–26], non-Abelian discrete symmetries [27], axion models [18,28], and so on. Once the breaking of symmetry occurs after inflation, DWs begin to form and leave different Hubble patches in different degenerate vacua. Although the formation of DW networks cause potential conflict with cosmological observations if they dominate the energy and then overclose the Universe [29], one can expect that the discrete symmetry is only approximate [30] and the energy bias of the potential can induce the annihilation of the DWs. Along with the motion and annihilation of DWs, GWs are radiated and form the SGWB for present observers.

An appealing mechanism forming such a DW network is the spontaneous breaking of $U(1)$ Peccei-Quinn (PQ) symmetry [31,32]. The resulting QCD axion [33] explains the undetectable $CP$ violation in strong interaction. However, PQ mechanism depends particularly on the axion potential from QCD instantons and it is susceptible to additional global symmetry breaking. It is widely accepted that there is no continuous global symmetry in quantum gravity [34] and thus PQ solution is spoiled. This so-called quality problem has always been perplexing. To avoid the problem, heavy axion [35–40] is taken to be consideration. Its heavy sector provides a larger contribution to the axion potential and lift the quality of PQ symmetry. In general, the heavy axion decays more easily and leave few detectable objects. Therefore, detecting GW radiation becomes a crucial way to understand this model.

In this paper, we provide a first search for the SGWB from DWs using the data of Advanced LIGO and

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Advanced Virgo's first three observing runs. First of all, we take a model-independent analysis and figure out the constraints on the parameters characterizing the SGWB from DWs. Then, like [40], we focus on the heavy axion [35,36] for explaining strong *CP* problems and avoiding the vulnerable PQ mechanism. In the standard QCD axion model, the DW network makes up a very tiny fraction of energy density of the Universe and leave SGWB undetectable [41]. But GWs from the DW network in the heavy axion model can be much stronger because of the large tension carried by heavy axion DWs.

Recently, based on the evidence for a stochastic common-spectrum process in the pulsar timing array (PTA) datasets [42–47], the energy spectrum of SGWB from DWs has been searched and used to put constraint on axionlike particles in [48]. They have found that DWs annihilating at temperature 20–50 MeV with tension $(40\text{–}100\ \mathrm{TeV})^3$ are well fitted to the NANOGrav 12.5 years dataset and International PTA Data Release 2. If the DW mechanism is realized by QCD heavy axions, part of parameter space fitted by PTA will be checked by future collider experiments like DUNE ND [49], MATHUSLA [50], and HL-LHC [51]. In our analysis, instead, the data from Advanced LIGO and Advanced Virgo's first three observing runs is adopted to probe the mechanism of DWs at much higher energy scale because the frequency band of LIGO and Virgo is much higher than that of PTA.

## II. SGWB FROM DWS

The energy spectrum of SGWB normalized by the critical density of Universe is defined as

$$\Omega_{\mathrm{gw}}(f) = \frac{1}{\rho_{\mathrm{c}}}\frac{\mathrm{d}\rho_{\mathrm{gw}}}{\mathrm{d}\ln f}, \tag{1}$$

where $\rho_{\mathrm{c}}$ is present critical density, $\rho_{\mathrm{gw}}$ is the energy density of GWs, and $f$ is the frequency of GWs. In the absence of significant friction from surrounding plasma, the DW network quickly achieves a scaling regime [5,12] with the energy density $\rho_{\mathrm{DW}} = c\sigma H$, where $c \sim \mathcal{O}(1)$ is a model-dependent prefactor, $\sigma$ is the tension or surface energy density of DW, and $H$ is the Hubble rate. This corresponds to the fractional energy density:

$$\alpha = \frac{\rho_{\mathrm{DW}}}{3H^2M_p^2}, \tag{2}$$

where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck energy scale. Notice that DWs can possess large energy and become dangerous to cosmology [29,52]. Therefore, a mechanism inducing the annihilation of DW network at certain time is usually needed in practice.

Due to the time-varying quadrupole, a DW network emits GWs, and the simple estimations show that $\rho_{\mathrm{gw}} \sim \rho_{\mathrm{DW}}^2/(32\pi H^2 M_p^2)$ [53–59]. Most GWs are radiated at the frequency $\tilde{f}_* \simeq H$ corresponding to the time of annihilation and then redshifted to today with the peak frequency:

$$f_* \simeq 1.5\left(\frac{g_*}{106.75}\right)^{1/6}\left(\frac{T_*}{10^7\ \mathrm{GeV}}\right)\ \mathrm{Hz}, \tag{3}$$

where $g_*$ is the number of relativistic degrees of freedom at annihilation. We take the number of (entropy) relativistic degrees of freedom $g_{*,s} = g_* = 106.75$ in the whole paper. By assuming annihilation happens at the radiation domination period, numerical simulations [41,55] show that the normalized energy density of SGWB takes the form

$$\Omega_{\mathrm{DW}}(f)h^2 \simeq 4.7\times 10^{-7}\tilde{\epsilon}\alpha_*^2\left(\frac{106.75}{g_*}\right)^{1/3}\mathcal{S}\left(\frac{f}{f_*}\right). \tag{4}$$

Here $\alpha_*$ represents the fractional energy density at annihilation. $\tilde{\epsilon} \simeq 0.1\text{–}1$ is a numerical efficiency factor that is suggested in [55], and its value is fixed to be 0.7 in our analysis. The causality ensures $\Omega_{\mathrm{DW}} \sim f^3$ when $f < f_*$. Besides, $\Omega_{\mathrm{DW}} \sim f^{-1}$ when $f \to \infty$ until the cutoff depicted by the inverse of wall width. The order of $\mathcal{O}(1)$ width around the maximum is suggested by numerical studies [55]. Therefore, the shape of the SGWB spectrum can be parametrized as a broken power law:

$$\mathcal{S}(x) = \frac{4}{x^{-3} + 3x}. \tag{5}$$

We adopt the cross-correlation spectrum of isotropic SGWB during Advanced LIGO's and Advanced Virgo's O1 ~ O3 observing runs [4] and follow the method described in [60,61] to do a Bayesian analysis and estimate the probability of models. The likelihood is given by

$$p(\hat{C}|\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\sum_{IJ}\sum_{f}\frac{(\hat{C}_{IJ}(f) - \Omega_{\mathrm{gw}}(f;\boldsymbol{\theta}))^2}{\sigma_{IJ}^2(f)}\right]. \tag{6}$$

The model of energy spectrum $\Omega_{\mathrm{gw}}(f;\boldsymbol{\theta})$ is parametrized by a series parameters $\boldsymbol{\theta}$. $\hat{C}_{IJ}$ is the cross-correlation spectrum and $\sigma_{IJ}$ denotes its variance. Likelihoods obtained by each detector pair are multiplied together to obtain Eq. (6). The Bayes factor between SGWB signal and pure noise is used to show the fitness of the model. Here $\boldsymbol{\theta} = (\alpha_*, f_*)$ are free parameters and their priors are listed in Table I. The Bayesian analysis makes use of PYTHON

TABLE I. Priors of the parameters adopted in broken power law model.

| Parameter | Prior |
|---|---|
| $\alpha_*$ | Uniform [0, 0.3] |
| $f_*$(Hz) | Log Uniform [10, 200] |

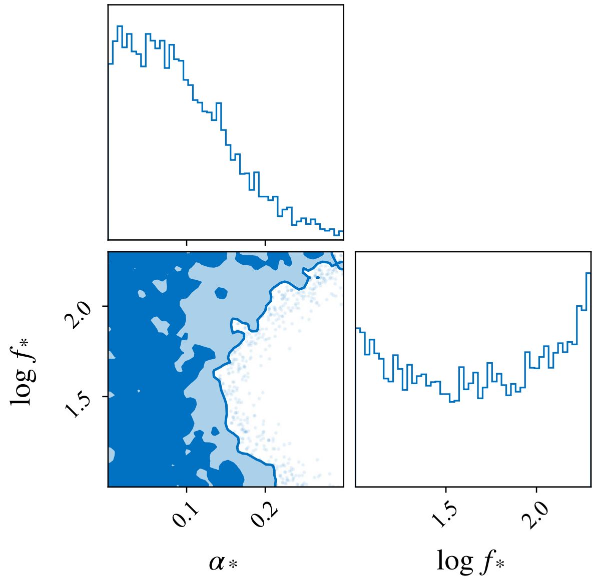

FIG. 1. Posterior distribution of $\alpha_*$ and $f_*$ for the broken power law model. The 68% and 95% C.L. exclusion contours are shown with blue shaded region.

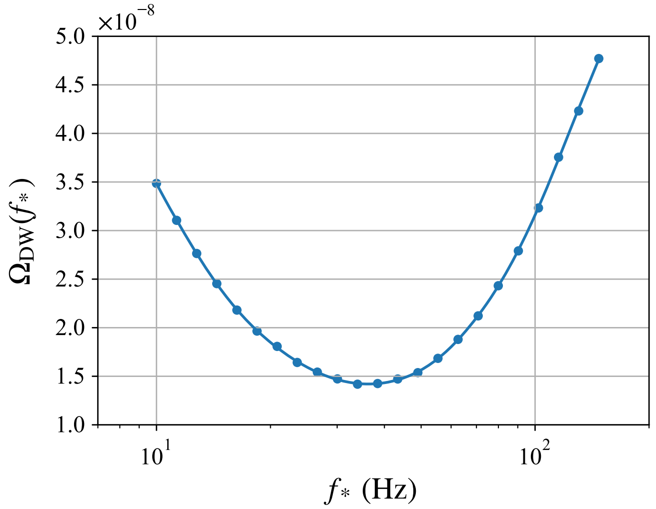

FIG. 2. Upper limit of $\Omega_{\rm DW}(f_*)$ at 95% C.L. for different peak frequency $f_*$.

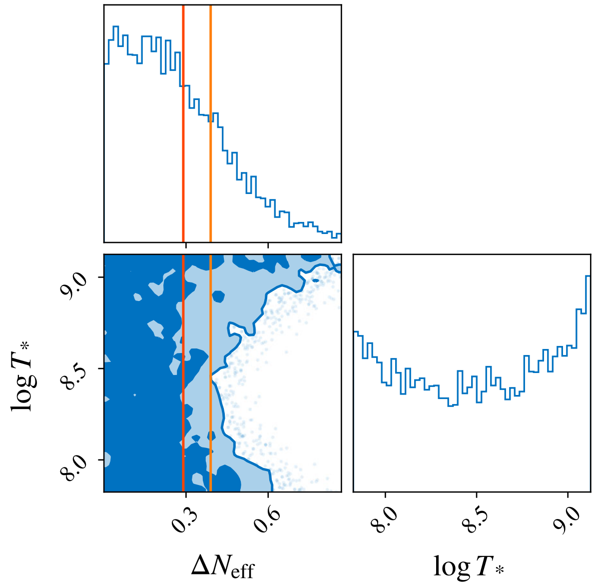

FIG. 3. Posterior distribution of $\Delta N_{\rm eff}$ and $T_*$ (GeV) for the broken power law model. The 68% and 95% C.L. exclusion contours are shown with blue shaded region. The vertical orange (red) line denotes the constraint from BBN (CMB + BAO).

BILBY package [62] and dynamic nested sampling package DYNESTY [63]. The posterior distributions of parameters in the broken power law model is presented in Fig. 1. The Bayes factor is $\log \mathcal{B}^{\rm DW}_{\rm noise} = -0.27$, indicating that no evidence of such signal in the strain data. In fact, the posterior of $\alpha_*$ allows us to put constraint on the amplitude of the energy spectrum of GWs from DW network. In Fig. 2, the upper limit of $\Omega_{\rm DW}(f_*)$ at 95% confidence level (C.L.) is illustrated for different peak frequency $f_*$. The most stringent limitation is $\Omega_{\rm DW}(f_* = 35~{\rm Hz}) < 1.4 \times 10^{-8}$ at 95% C.L.

After the annihilation, DWs may decay into SM particles or dark radiation (DR). In literature the abundance of DR is described by the effective number of neutrino species $\Delta N_{\rm eff} \coloneqq \rho_{\rm DR}/\rho_\nu$. If DWs decay into DR entirely, $\rho_{\rm DR} \simeq \rho_{\rm DW}$ and then

$$\Delta N_{\rm eff,decay} \simeq 13.6 g_*^{-1/3} \alpha_*. \tag{7}$$

In addition, GWs emitted by the DW network also make a contribution to DR, namely

$$\Delta N_{\rm eff,gw} \simeq 0.09 \alpha_*^2 \left(\frac{g_*}{106.75}\right)^{-1/3}. \tag{8}$$

In this scenario, $\Delta N_{\rm eff} = \Delta N_{\rm eff,decay} + \Delta N_{\rm eff,gw}$. It is worth mentioning that big bang nucleosynthesis (BBN) (CMB + BAO cosmic microwave background (CMB) and baryon acoustic oscillation (BAO)) also put constraints on the abundance of DR with $\Delta N_{\rm eff} \lesssim 0.39$ [64] (0.29 [65]) at 95% C.L. This may also provide a constraint on the energy density of DW network. In Fig. 3, we transform the posterior of $(\alpha_*, f_*)$ to $(\Delta N_{\rm eff}, T_*)$ using Eqs. (3), (7), and (8). It implies that the constraint from LIGO-Virgo is comparable to those from BBN and CMB + BAO in the scenario where DWs are all assumed to decay into DR.

## III. HEAVY QCD AXION MODEL

From now on, we will use the former results to constrain the parameter space in the heavy axion model. The composite potential of heavy axion periodic potential and the contribution from QCD is given by

$$V_a = (\kappa_{\rm Q}^2\Lambda_{\rm Q}^4 + \kappa_{\rm H}^2\Lambda_{\rm H}^4)\left[1-\cos\left(\frac{a}{F_a}\right)\right], \tag{9}$$

where $F_a$ is axion decay constant, $\Lambda_{\rm Q,H}$ are coupling scales of QCD and heavy sector. The factors $\kappa_{\rm Q,H} \leq 1$ are related to mass of fermion. After the spontaneous breaking of $U(1)$ PQ symmetry, axionic string-DW network forms and then GWs are emitted. In the form of Eq. (9), periodic potential from the heavy sector is aligned with that of QCD. It is typically ensured by the $\mathbb{Z}_2$ symmetry [50]. To that end, QCD cannot induce the annihilation of DWs. However, regardless of its specific physical origin (see some models in [66–69]), we could expect a term misaligned with $V_a$ as follows

$$V_b \simeq -\mu_b^4 \cos\left(\frac{N_b}{N_{\rm DW}}\frac{a}{F_a} - \delta\right), \tag{10}$$

where $\mu_b$ is related to the specific model under consideration, $N_b, N_{\rm DW}$ are integers describing discrete symmetry subgroup. Note that energy bias of potential only appears when $N_b = 1$ or coprime with $N_{\rm DW}$. $\delta$ is the generic misaligned phase. $CP$ violation caused by $V_b$ is depicted by

$$\Delta\theta = \theta - \theta_{\rm Q} \simeq r^4\left(\frac{N_b}{N_{\rm DW}}\right)\left(\frac{\sin\delta}{\kappa_H^2}\right), \tag{11}$$

where $r = \mu_b/\Lambda_{\rm H}$. Once $N_{\rm DW} > 1$, a long-lasting DW network could form, and the fractional energy density is

$$\alpha_* \simeq 0.1\left(\frac{\sqrt{d}c\kappa_{\rm H}}{\sin(N_b\pi/N_{\rm DW})}\right)^2\left(\frac{F_a}{10^{12}\ {\rm GeV}}\right)^2\left(\frac{0.002}{r}\right)^4, \tag{12}$$

and the temperature at annihilation reads

$$T_* \simeq 10^8\frac{\sin(N_b\pi/N_{\rm DW})}{\sqrt{dc\kappa_{\rm H}}}\left(\frac{106.75}{g_*}\right)^{1/4}\left(\frac{r}{0.005}\right)^2 \times\left(\frac{10^{12}\ {\rm GeV}}{F_a}\right)^{1/2}\left(\frac{\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)\ {\rm GeV}. \tag{13}$$

Different from the standard QCD axion, the heavy axion is unstable and tends to decay into SM particles [70]: gluons, photons, or fermions, depending on the energy scale, and then the heavy axion avoids the constraints on the abundance of DR from BBN and CMB + BAO. If the heavy axion does not decay fast enough, then there is a temporary period of matter domination (MD) and hence Eqs. (3) and (4) should be modified. To evaluate whether the situation takes place, we compare the characteristic temperature of efficient decay with that of MD, where, according to [40], the effective characteristic temperature of decay is

$$T_{\rm d} \simeq 10^6\left(\frac{\sqrt{\kappa_{\rm H}}\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)^3\left(\frac{10^{12}\ {\rm GeV}}{F_a}\right)^{5/2}\ {\rm GeV}, \tag{14}$$

and the temperature of MD is

$$T_{\rm MD} = 8\times10^6\left(\frac{\sqrt{d}(c\kappa_{\rm H})^{3/2}}{\sin(N_b\pi/N_{\rm DW})}\right)\left(\frac{F_a}{10^{12}\ {\rm GeV}}\right)^{3/2} \times\left(\frac{\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)\left(\frac{0.001}{r}\right)^2\left(\frac{106.75}{g_*}\right)^{1/4}\ {\rm GeV}. \tag{15}$$

If $T_{\rm d} < T_{\rm MD}$, then there is a MD phase, and the peak frequency and the energy spectrum of SGWB from DWs become [40]

$$f_* \to f_*\left(\frac{T_{\rm d}}{T_{\rm MD}}\right)^{1/3}, \tag{16}$$

$$\Omega_{\rm DW} \to \Omega_{\rm DW}\left(\frac{g_*(T_{\rm d})}{g_*(T_{\rm MD})}\right)^{1/3}\left(\frac{T_{\rm d}}{T_{\rm MD}}\right)^{4/3}. \tag{17}$$

In fact, there are also some inherent limitations which should be taken into account in our analysis. First, the characteristic temperature ($T_{\text{DW-dom}}$) of string wall dominating the Universe can be estimated by $\alpha_* \simeq 1$ and

$$T_{\text{DW-dom}} \simeq 5\times10^6\sqrt{\frac{c\kappa_{\rm H}F_a}{10^{12}\ {\rm GeV}}} \times\left(\frac{\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)\left(\frac{106.75}{g_*}\right)^{1/4}\ {\rm GeV}. \tag{18}$$

In the case of $T_{\text{DW-dom}} > T_*$, DWs will dominate our Universe, which may conflict with the standard cosmology. Even if the contradiction might be avoided [71], there is a lack of knowledge about the evolution of DWs in the situation. Second, the validity of the axion effective theory requires $\sqrt{\kappa_{\rm H}}\Lambda_{\rm H} < F_a$. At last, neutron electric dipole moment measurements [72] require $\Delta\theta \lesssim 10^{-10}$.

In this paper we mainly focus on constraining the three physical parameters (e.g., the coupling scale $\Lambda_{\rm H}$ of heavy

TABLE II. Priors of the parameters adopted in the heavy axion model.

| $\Lambda_{\rm H} = 10^{10}, 10^{10.5}, 10^{11}$ GeV | |
|---|---|
| Parameter | Prior |
| $F_a$(GeV) | LogUniform $[10^{10}, 10^{12.2}]$ |
| $\Delta\theta$ | LogUniform $[10^{-14.5}, 10^{-10}]$ |
| $\Delta\theta = 10^{-13}, 10^{-12}, 10^{-11}$ | |
| Parameter | Prior |
| $F_a$(GeV) | LogUniform $[10^{10}, 10^{12.2}]$ |
| $\Lambda_{\rm H}$(GeV) | LogUniform $[10^9, 10^{12.2}]$ |

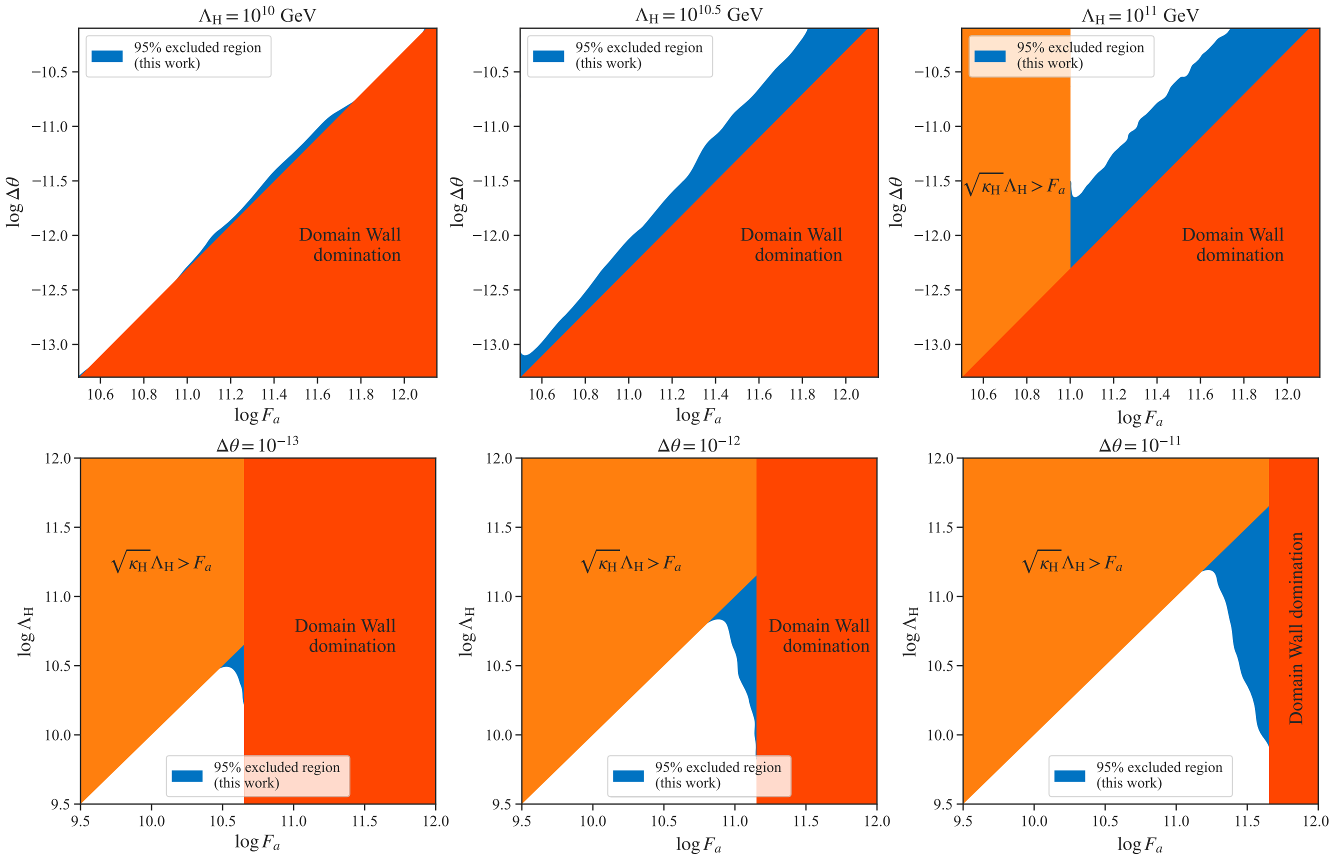

FIG. 4. Parameter space excluded by Advanced LIGO and Advanced Virgo O1 ∼ O3 observing runs. The blue shaded regions denote the 95% exclusion contours. The orange-red and orange shaded regions correspond to DW domination and violation of axion effective theory, respectively.

sector, axion decay constant $F_a$, and $CP$ violation coefficient $\Delta\theta$) in the heavy axion model. From Eqs. (12) and (13), the former constraints on $\alpha_*$ and $T_*$ can be used to constrain these three parameters. Here we set the factor $\kappa_H \simeq 1$ providing that the heavy sector contains a light fermion. Similar to [73], we take $N_b = 1$, $N_{DW} = 6$, and then $c = 4.48$, $d = 6.28$. Since the dependence of $\delta$ only appears in Eq. (11) as a factor between $\Delta\theta$ and $r$, we set $\delta = 0.3$ and the results for other values can be obtained by simply rescaling $\Delta\theta$. Since there are three free parameters, our strategy of analysis is to constrain the other two parameters by keeping $\Lambda_H$ or $\Delta\theta$ fixed, respectively. The priors adopted in our analysis are listed in Table II, and the excluded parameter spaces in the heavy axion model are illustrated by the blue shaded regions in Fig. 4.

## IV. CONCLUSIONS

By adopting the cross-correlation spectrum of Advanced LIGO and Advanced Virgo's first three observing runs, we search for the GW signal from DW network in the peak frequency range 10–200 Hz. Since no such signal in the strain data has been observed, we place constraints on the energy spectrum of SGWB from cosmic DW network. The most stringent limitation is $\Omega_{DW}(f_* = 35\ \text{Hz}) < 1.4 \times 10^{-8}$ at 95% C.L. The posterior of $f_*$ does not show obvious preference for any certain peak frequency, and the fractional energy density of DWs with $\alpha_* > 0.2$ is excluded at 95% C.L. for $f_* = 20$–$100$ Hz corresponding to the most sensitive band of LIGO and Virgo.

Furthermore, for fixed values of $\Lambda_H = 10^{10}$, $10^{10.5}$ or $10^{11}$ GeV, the $CP$ violation coefficient $\Delta\theta$ in the blue shaded regions in the upper panel of Fig. 4 is excluded; for fixed values of $\Delta\theta = 10^{-13}$, $10^{-12}$ or $10^{-11}$, the coupling scale $\Lambda_H$ of the heavy sector in the blue shaded regions in the bottom panel of Fig. 4 is excluded. In all, even though SGWB has not been detected by Advanced LIGO and Advanced Virgo, the nondetection of the SGWB from cosmic DWs already places constraints on the heavy axion model.

Our results have shown the ability of LIGO/Virgo/KAGRA interferometers to understand physics related to DWs. With the improvement of the detector sensitivity and the construction of next-generation gravitational detectors like Cosmic Explorer [74] and Einstein Telescope [75], we expect more elaborate results according to the predictions in [40].

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[1] R. Abbott *et al.* (LIGO Scientific, VIRGO, and KAGRA Collaborations), GWTC-3: Compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run, arXiv:2111.03606.
[2] J. Aasi *et al.* (LIGO Scientific Collaboration), Advanced LIGO, Classical Quantum Gravity **32**, 074001 (2015).
[3] F. Acernese *et al.* (VIRGO Collaboration), Advanced Virgo: A second-generation interferometric gravitational wave detector, Classical Quantum Gravity **32**, 024001 (2015).
[4] R. Abbott *et al.* (KAGRA, Virgo, and LIGO Scientific Collaborations), Upper limits on the isotropic gravitational-wave background from Advanced LIGO and Advanced Virgo's third observing run, Phys. Rev. D **104**, 022004 (2021).
[5] T. W. B. Kibble, Some implications of a cosmological phase transition, Phys. Rep. **67**, 183 (1980).
[6] Edward Witten, Cosmic separation of phases, Phys. Rev. D **30**, 272 (1984).
[7] C. J. Hogan, Gravitational radiation from cosmological phase transitions, Mon. Not. R. Astron. Soc. **218**, 629 (1986).
[8] Anupam Mazumdar and Graham White, Review of cosmic phase transitions: their significance and experimental signatures, Rep. Prog. Phys. **82**, 076901 (2019).
[9] A. A. Starobinskiĭ, Spectrum of relict gravitational radiation and the early state of the universe, Sov. J. Exp. Theor. Phys. Lett. **30**, 682 (1979).
[10] Michael S. Turner, Detectability of inflation-produced gravitational waves, Phys. Rev. D **55**, R435 (1997).
[11] Rennan Bar-Kana, Limits on direct detection of gravitational waves, Phys. Rev. D **50**, 1157 (1994).
[12] T. W. B. Kibble, Topology of cosmic domains and strings, J. Phys. A **9**, 1387 (1976).
[13] Saswat Sarangi and S.-H.Henry Tye, Cosmic string production towards the end of brane inflation, Phys. Lett. B **536**, 185 (2002).
[14] Thibault Damour and Alexander Vilenkin, Gravitational radiation from cosmic (super)strings: Bursts, stochastic background, and observational windows, Phys. Rev. D **71**, 063510 (2005).
[15] Xavier Siemens, Vuk Mandic, and Jolien Creighton, Gravitational-Wave Stochastic Background from Cosmic Strings, Phys. Rev. Lett. **98**, 111101 (2007).
[16] R. Abbott *et al.* (LIGO Scientific, Virgo, and KAGRA Collaborations), Constraints on Cosmic Strings Using Data from the Third Advanced LIGO–Virgo Observing Run, Phys. Rev. Lett. **126**, 241102 (2021).
[17] Alexander Vilenkin and Allen E. Everett, Cosmic Strings and Domain Walls in Models with Goldstone and Pseudo-Goldstone Bosons, Phys. Rev. Lett. **48**, 1867 (1982).
[18] P. Sikivie, Axions, Domain Walls, and the Early Universe, Phys. Rev. Lett. **48**, 1156 (1982).
[19] Richard A. Battye, Apostolos Pilaftsis, and Dominic G. Viatic, Domain wall constraints on two-Higgs-doublet models with $Z_2$ symmetry, Phys. Rev. D **102**, 123536 (2020).
[20] Z. Chacko, Hock-Seng Goh, and Roni Harnik, The Twin Higgs: Natural Electroweak Breaking from Mirror Symmetry, Phys. Rev. Lett. **96**, 231802 (2006).
[21] Luca Di Luzio, Michele Redi, Alessandro Strumia, and Daniele Teresi, Coset cosmology, J. High Energy Phys. 06 (2019) 110.
[22] G. Lazarides, Q. Shafi, and T. F. Walsh, Cosmic strings and domains in unified theories, Nucl. Phys. **B195**, 157 (1982).
[23] Allen E. Everett and Alexander Vilenkin, Left-right symmetric theories and vacuum domain walls and strings, Nucl. Phys. **B207**, 43 (1982).
[24] S. A. Abel, Subir Sarkar, and P. L. White, On the cosmological domain wall problem for the minimally extended supersymmetric standard model, Nucl. Phys. **B454**, 663 (1995).
[25] G. R. Dvali and Mikhail A. Shifman, Domain walls in strongly coupled theories, Phys. Lett. B **396**, 64 (1997); **407**, 452(E) (1997).
[26] A. Kovner, Mikhail A. Shifman, and Andrei V. Smilga, Domain walls in supersymmetric Yang-Mills theories, Phys. Rev. D **56**, 7978 (1997).
[27] Francesco Riva, Low-scale leptogenesis and the domain wall problem in models with discrete flavor symmetries, Phys. Lett. B **690**, 443 (2010).
[28] Giovanni Grilli di Cortona, Edward Hardy, Javier Pardo Vega, and Giovanni Villadoro, The QCD axion, precisely, J. High Energy Phys. 01 (2016) 034.
[29] Ia. B. Zel'dovich, I. Iu. Kobzarev, and Lev Borisovich Okun, Cosmological consequences of a spontaneous breakdown of a discrete symmetry, J. Exp. Theor. Phys. **67**, 401 (1974).
[30] Sebastian E. Larsson, Subir Sarkar, and Peter L. White, Evading the cosmological domain wall problem, Phys. Rev. D **55**, 5129 (1997).
[31] R. D. Peccei and Helen Quinn, *CP* Conservation in the Presence of Pseudoparticles, Phys. Rev. Lett. **38**, 1440 (1977).

[32] Roberto D. Peccei and Helen R. Quinn, Constraints imposed by *CP* conservation in the presence of pseudoparticles, Phys. Rev. D **16**, 1791 (1977).

[33] Steven Weinberg, A New Light Boson?, Phys. Rev. Lett. **40**, 223 (1978).

[34] Tom Banks and Nathan Seiberg, Symmetries and strings in field theory and gravity, Phys. Rev. D **83**, 084019 (2011).

[35] Bob Holdom and Michael E. Peskin, Raising the axion mass, Nucl. Phys. **B208**, 397 (1982).

[36] Bob Holdom, Strong qcd at high energies and a heavy axion, Phys. Lett. **154B**, 316 (1985).

[37] V. A. Rubakov, Grand unification and heavy axion, JETP Lett. **65**, 621 (1997).

[38] Savas Dimopoulos, Anson Hook, Junwu Huang, and Gustavo Marques-Tavares, A collider observable QCD axion, J. High Energy Phys. 11 (2016) 052.

[39] Tony Gherghetta and Minh D. Nguyen, A composite Higgs with a heavy composite axion, J. High Energy Phys. 12 (2020) 094.

[40] Ricardo Zambujal Ferreira, Alessio Notari, Oriol Pujolàs, and Fabrizio Rompineve, High Quality QCD Axion at Gravitational Wave Observatories, Phys. Rev. Lett. **128**, 141101 (2022).

[41] Takashi Hiramatsu, Masahiro Kawasaki, Ken'ichi Saikawa, and Toyokazu Sekiguchi, Axion cosmology with long-lived domain walls, J. Cosmol. Astropart. Phys. 01 (2013) 001.

[42] Zaven Arzoumanian *et al.* (NANOGrav Collaboration), The NANOGrav 12.5 yr data set: Search for an isotropic stochastic gravitational-wave background, Astrophys. J. Lett. **905**, L34 (2020).

[43] Zu-Cheng Chen, Yu-Mei Wu, and Qing-Guo Huang, Searching for isotropic stochastic gravitational-wave background in the international pulsar timing array second data release, arXiv:2109.00296.

[44] Zu-Cheng Chen, Chen Yuan, and Qing-Guo Huang, Non-tensorial gravitational wave background in NANOGrav 12.5-year data set, Sci. China Phys. Mech. Astron. **64**, 120412 (2021).

[45] Boris Goncharov *et al.*, On the evidence for a common-spectrum process in the search for the nanohertz gravitational-wave background with the parkes pulsar timing array, Astrophys. J. Lett. **917**, L19 (2021).

[46] S. Chen *et al.*, Common-red-signal analysis with 24-yr high-precision timing of the European Pulsar Timing Array: Inferences in the stochastic gravitational-wave background search, Mon. Not. R. Astron. Soc. **508**, 4970 (2021).

[47] J. Antoniadis *et al.*, The International Pulsar Timing Array second data release: Search for an isotropic gravitational wave background, Mon. Not. R. Astron. Soc. **510**, 4873 (2022).

[48] Ricardo Z. Ferreira, Alessio Notari, Oriol Pujolas, and Fabrizio Rompineve, Gravitational waves from domain walls in pulsar timing array datasets, arXiv:2204.04228.

[49] Babak Abi *et al.* (DUNE Collaboration), Deep underground neutrino experiment (DUNE), far detector technical design report, Volume II: DUNE physics, arXiv:2002.03005.

[50] Anson Hook, Soubhik Kumar, Zhen Liu, and Raman Sundrum, High Quality QCD Axion and the LHC, Phys. Rev. Lett. **124**, 221801 (2020).

[51] John Paul Chou, David Curtin, and H. J. Lubatti, New detectors to explore the lifetime frontier, Phys. Lett. B **767**, 29 (2017).

[52] Ya. B. Zel'Dovich, I. Yu. Kobzarev, and L. B. Okun', Cosmological consequences of a spontaneous breakdown of a discrete symmetry, Sov. J. Exp. Theor. Phys. **40**, 1 (1975).

[53] Alexander Vilenkin, Gravitational field of vacuum domain walls and strings, Phys. Rev. D **23**, 852 (1981).

[54] Masahiro Kawasaki and Ken'ichi Saikawa, Study of gravitational radiation from cosmic domain walls, J. Cosmol. Astropart. Phys. 09 (2011) 008.

[55] Takashi Hiramatsu, Masahiro Kawasaki, and Ken'ichi Saikawa, On the estimation of gravitational wave spectrum from cosmic domain walls, J. Cosmol. Astropart. Phys. 02 (2014) 031.

[56] John Preskill, Sandip P. Trivedi, Frank Wilczek, and Mark B. Wise, Cosmology and broken discrete symmetry, Nucl. Phys. **B363**, 207 (1991).

[57] Sanghyeon Chang, C. Hagmann, and P. Sikivie, Studies of the motion and decay of axion walls bounded by strings, Phys. Rev. D **59**, 023505 (1998).

[58] Marcelo Gleiser and Ronald Roberts, Gravitational Waves from Collapsing Vacuum Domains, Phys. Rev. Lett. **81**, 5497 (1998).

[59] E. Babichev, D. Gorbunov, S. Ramazanov, and A. Vikman, Gravitational shine of dark domain walls, J. Cosmol. Astropart. Phys. 04 (2022) 028.

[60] V. Mandic, E. Thrane, S. Giampanis, and T. Regimbau, Parameter Estimation in Searches for the Stochastic Gravitational-Wave Background, Phys. Rev. Lett. **109**, 171102 (2012).

[61] Joseph D. Romano and Neil J. Cornish, Detection methods for stochastic gravitational-wave backgrounds: A unified treatment, Living Rev. Relativity **20**, 2 (2017).

[62] Gregory Ashton *et al.*, Bilby: A user-friendly bayesian inference library for gravitational-wave astronomy, Astrophys. J. Suppl. Ser. **241**, 27 (2019).

[63] Sergey Koposov *et al.*, joshspeagle/dynesty: v1.2.3, 10.5281/zenodo.6609296 (2022).

[64] Brian D. Fields, Keith A. Olive, Tsung-Han Yeh, and Charles Young, Big-bang nucleosynthesis after Planck, J. Cosmol. Astropart. Phys. 03 (2020) 010; 11 (2020) E02.

[65] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).

[66] Gia Dvali, Three-form gauging of axion symmetries and gravity, arXiv:hep-th/0507215.

[67] Michael Dine and Nathan Seiberg, String theory and the strong *CP* problem, Nucl. Phys. **B273**, 109 (1986).

[68] Peter Svrcek and Edward Witten, Axions in string theory, J. High Energy Phys. 06 (2006) 051.

[69] Marc Kamionkowski and John March-Russell, Planck scale physics and the Peccei-Quinn mechanism, Phys. Lett. B **282**, 137 (1992).

[70] David J. E. Marsh, Axion cosmology, Phys. Rep. **643**, 1 (2016).

[71] Masahiro Kawasaki and Fuminobu Takahashi, Late-time entropy production due to the decay of domain walls, Phys. Lett. B **618**, 1 (2005).

[72] C. Abel *et al.*, Measurement of the Permanent Electric Dipole Moment of the Neutron, Phys. Rev. Lett. **124**, 081803 (2020).

[73] Masahiro Kawasaki, Ken'ichi Saikawa, and Toyokazu Sekiguchi, Axion dark matter from topological defects, Phys. Rev. D **91**, 065014 (2015).

[74] Benjamin P Abbott *et al.* (LIGO Scientific Collaboration), Exploring the sensitivity of next generation gravitational wave detectors, Classical Quantum Gravity **34**, 044001 (2017).

[75] Michele Maggiore *et al.*, Science case for the Einstein telescope, J. Cosmol. Astropart. Phys. 03 (2020) 050.