Erratum: QCD description of backward vector meson hard electroproduction [Phys. Rev. D 91, 094006 (2015)]

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We correct a sign mistake in our calculation of backward vector meson electroproduction amplitude within the transition distribution amplitude (TDA) framework presented in Table I. The same error has also propagated into Table 3 of Ref. [1]. We present the corrected version of this table here. The altered signs are marked in boldface.

A useful cross check for this result is provided by the calculation of the integral convolutions $\mathcal{I}^{(k)}$, k = 1, ..., 6, defined in Eq. (24) for the $\gamma^* p \to V^0 p$ reaction within the cross channel nucleon exchange model for *VN* TDAs considered in Appendix B. In this model, the convolution integrals $\mathcal{I}^{(k)}$ turn out to be proportional to the pQCD expression for the proton e.m. form factor F_1^p (see discussion in Ref. [2]):

TABLE I. 14 of the 21 diagrams contributing to the hard-scattering amplitude with their associated coefficient $T_{\alpha}^{(k)} \equiv D_{\alpha} \times N_{\alpha}^{(k)}$ (no summation over α assumed). The seven first ones with *u*-quark lines inverted are not drawn. The crosses represent the virtual-photon vertex.

α	Diagram D_{α}		Numerators
		$N^{(1)}_{lpha}$	$-(V^p - A^p)(V^{VN}_{1\mathcal{E}} - A^{VN}_{1\mathcal{E}}) + 2T^p(T^{VN}_{1\mathcal{E}} + T^{VN}_{2\mathcal{E}})$
1	$u(x_1) \qquad \qquad$	$N^{(2)}_{lpha}$ $N^{(3)}$	$-(V^{p} - A^{p})(V_{1n}^{VN} - A_{1n}^{VN}) + 4T^{p}(T_{1n}^{VN} + \frac{\Delta_{T}^{2}}{2M^{2}}T_{4n}^{VN}) -(V^{p} - A^{p})(V_{1n}^{VN} - A_{1n}^{VN} + V_{2n}^{VN} - A_{2n}^{VN})$
	$\begin{array}{c} u(x_2) \\ d(x_3) \end{array} \begin{array}{c} \hline \\ \hline \\ \hline \\ \hline \\ \end{array} \begin{array}{c} \\ \hline \\ \\ \end{array} \end{array} \begin{array}{c} u(y_2) \\ d(y_3) \end{array}$	Ίνα	$+4T^{p}(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4T}^{VN})$
	$\frac{Q_u(2\xi)^2}{(2\xi - x_1 - i\varepsilon)^2 (x_3 - i\varepsilon)(1 - y_1)^2 y_3}$	$N^{(4)}_{\alpha}$	$-(V^{p} - A^{p})(V_{2T}^{VN} - A_{2T}^{VN}) + 2T^{p}(T_{2T}^{VN} + T_{3T}^{VN})$ $(V^{p} - A^{p})(V^{VN} - A^{VN}) - 2T^{p}(T^{VN} - T^{VN})$
		$N^{(6)}_{lpha}$	$-(V^{p} - A^{p})(V_{2n}^{VN} - A_{2n}^{VN}) + 2T^{p}(T_{2n}^{VN} + T_{3n}^{VN})$
		$N^{(1)}_{lpha}$	0
		$N_{lpha}^{(2)}$	0
	$u(x_1)$ \longrightarrow $u(y_1)$	$N_{\alpha}^{(3)}$	0
2	$u(x_2)$ $\xrightarrow{\mathbf{q}}$ $u(y_2)$	$N_{\alpha}^{(4)}$	0
	$d(x_3)$ \overrightarrow{q} $d(y_3)$	$N_{\alpha}^{(5)}$	0
		$N^{(6)}_{lpha}$	0
	$u(x_1)$ $u(y_1)$	$N^{(1)}_{lpha}$	$-2T^p(T^{VN}_{1\mathcal{E}}+T^{VN}_{2\mathcal{E}})$
	$u(x_2)$ $\xrightarrow{\mathbf{O}}$ $u(y_2)$	$N^{(2)}_{lpha}$	$-4T^{p}(T_{1n}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4n}^{VN})$
2	$d(r_2)$ $d(u_2)$	$N_{lpha}^{(3)}$	$-4T^{p}(T_{1T}^{VN}+T_{2N}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4T}^{VN})$
3	~(~3) ~ ~(93)	$N_{\alpha}^{(4)}$	$-2T^{p}(T_{2T}^{VN}+T_{3T}^{oN})$
	$\frac{Q_u(2\xi)^{\epsilon}}{(x_1 - i\varepsilon)(2\xi - x_2 - i\varepsilon)(x_3 - i\varepsilon)y_1(1 - y_1)y_3}$	$N_{\alpha}^{(5)}$	$2T^p(T_{3\mathcal{E}}^{VN}-T_{4\mathcal{E}}^{VN})$
		$N^{(6)}_{lpha}$	$-2T^{p}(T_{2n}^{VN}+T_{3n}^{VN})$

(Table continued)

TABLE I.	(Continued)

α	Diagram D_{α}		Numerators
4	$u(x_1) \qquad u(y_1)$ $u(x_2) \qquad \qquad u(y_2)$ $d(x_3) \qquad \qquad \qquad d(y_3)$ $\frac{Q_u(2\xi)^2}{(x_1 - i\varepsilon)(2\xi - x_3 - i\varepsilon)(x_3 - i\varepsilon)y_1(1 - y_1)y_3}$	$N^{(1)}_{lpha} \ N^{(2)}_{lpha} \ N^{(3)}_{lpha} \ N^{(4)}_{lpha} \ N^{(5)}_{lpha} \ N^{(6)}_{lpha}$	$\begin{split} &-(V^{p}-A^{p})(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN})\\ &-(V^{p}-A^{p})(V_{1n}^{VN}-A_{1n}^{VN})\\ &-(V^{p}-A^{p})(V_{1n}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})\\ &-(V^{p}-A^{p})(V_{2\mathcal{T}}^{VN}-A_{2\mathcal{T}}^{VN})\\ &(V^{p}-A^{p})(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})\\ &-(V^{p}-A^{p})(V_{2n}^{VN}-A_{2n}^{VN}) \end{split}$
5	$u(x_1) \xrightarrow{u(y_1)} u(y_2)$ $u(x_2) \xrightarrow{(y_2)} u(y_2)$ $d(x_3) \xrightarrow{Q_{\mu}(2\xi)^2} d(y_3)$ $\frac{Q_{\mu}(2\xi)^2}{(x_1 - i\epsilon)(2\xi - x_3 - i\epsilon)(x_3 - i\epsilon)y_1(1 - y_2)y_3}$	$N^{(1)}_{lpha} N^{(2)}_{lpha} N^{(3)}_{lpha} N^{(3)}_{lpha} N^{(4)}_{lpha} N^{(5)}_{lpha} N^{(5)}_{lpha} N^{(6)}_{lpha}$	$ \begin{split} & (V^p + A^p)(V_{1\mathcal{E}}^{VN} + A_{1\mathcal{E}}^{VN}) \\ & (V^p + A^p)(V_{1n}^{VN} + A_{1n}^{VN}) \\ & (V^p + A^p)(V_{1T}^{VN} + A_{1T} + V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}) \\ & (V^p + A^p)(V_{2T}^{VN} + A_{2T}) \\ & -(V^p + A^p)(V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}) \\ & +(V^p + A^p)(V_{2n}^{VN} + A_{2n}^{VN}) \end{split} $
6	$u(x_1) \qquad \qquad u(y_1) \\ u(x_2) \qquad \qquad$	$N^{(1)}_{lpha} \ N^{(2)}_{lpha} \ N^{(3)}_{lpha} \ N^{(4)}_{lpha} \ N^{(5)}_{lpha} \ N^{(6)}_{lpha}$	0 0 0 0 0 0
7	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$N^{(1)}_{lpha} \ N^{(2)}_{lpha} \ N^{(3)}_{lpha} \ N^{(4)}_{lpha} \ N^{(5)}_{lpha} \ N^{(6)}_{lpha}$	$\begin{array}{c} -2(V^{p}V_{1\mathcal{E}}^{VN}+A^{p}A_{1\mathcal{E}}^{VN})\\ -2(V^{p}V_{1n}^{VN}+A^{p}A_{1n}^{VN})\\ -2(V^{p}(V_{1T}^{VN}+V_{2\mathcal{E}}^{VN})+A^{p}(A_{1T}^{VN}+A_{2\mathcal{E}}^{VN}))\\ -2(V^{p}V_{2T}^{VN}+A^{p}A_{2T}^{VN})\\ 2(V^{p}V_{2\mathcal{E}}^{VN}+A^{p}A_{2\mathcal{E}}^{VN})\\ -2(V^{p}V_{2\mathcal{E}}^{VN}+A^{p}A_{2\mathcal{E}}^{VN})\end{array}$
8	$u(x_1) u(y_1)$ $u(x_2) u(y_2)$ $d(x_3) d(y_3)$	$egin{array}{c} N^{(1)}_{lpha} \ N^{(2)}_{lpha} \ N^{(3)}_{lpha} \ N^{(4)}_{lpha} \ N^{(5)}_{lpha} \ N^{(6)}_{lpha} \end{array}$	0 0 0 0 0 0
9	$u(x_{1}) \xrightarrow{u(y_{1})} u(y_{2})$ $u(x_{2}) \xrightarrow{2} u(y_{2})$ $d(x_{3}) \xrightarrow{Q_{u}(2\xi)^{2}} d(y_{3})$ $\overline{Q_{u}(2\xi)^{2}}$	$N^{(1)}_{lpha} N^{(2)}_{lpha} N^{(3)}_{lpha} N^{(3)}_{lpha}$ $N^{(4)}_{lpha} N^{(5)}_{lpha} N^{(5)}_{lpha} N^{(6)}_{lpha}$	$\begin{split} &-(V^p - A^p)(V_{1\mathcal{E}}^{VN} - A_{1\mathcal{E}}^{VN}) + 2T^p(T_{1\mathcal{E}}^{VN} + T_{2\mathcal{E}}^{VN}) \\ &-(V^p - A^p)(V_{1n}^{VN} - A_{1n}^{VN}) + 4T^p(T_{1n}^{VN} + \frac{\Delta_r^2}{2M^2}T_{4n}^{VN}) \\ &-(V^p - A^p)(V_{1T}^{VN} - A_{1T}^{VN} + V_{2\mathcal{E}}^{V2} - A_{2\mathcal{E}}^{VN}) \\ &+ 4T^p(T_{1T}^{VN} + T_{3\mathcal{E}}^{VN} + \frac{\Delta_r^2}{2M^2}T_{4T}^{VN}) \\ &-(V^p - A^p)(V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN}) + 2T^p(T_{2T}^{VN} + T_{3T}^{VN}) \\ &(V^p - A^p)(V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN}) - 2T^p(T_{3\mathcal{E}}^{VN} - T_{4\mathcal{E}}^{VN}) \\ &-(V^p - A^p)(V_{2n}^{VN} - A_{2n}^{VN}) + 2T^p(T_{2n}^{VN} + T_{3n}^{VN}) \end{split}$

(Table continued)

TABLE I.	(Continued)

α	Diagram D_{α}		Numerators
		$N^{(1)}_{lpha}$	$-(V^p - A^p)(V^{VN}_{1\mathcal{E}} - A^{VN}_{1\mathcal{E}}) + 2T^p(T^{VN}_{1\mathcal{E}} + T^{VN}_{2\mathcal{E}})$
	$u(x_1)$ $u(y_1)$	$N^{(2)}_{lpha}$	$-(V^{p}-A^{p})(V_{1n}^{VN}-A_{1n}^{VN})+4T^{p}(T_{1n}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4n}^{VN})$
	$u(x_2) \longrightarrow u(y_2)$	$N^{(3)}_{lpha}$	$-(V^{p}-A^{p})(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$
10	$d(x_3)$ $\qquad \qquad $		$+4T^{p}(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4T}^{VN})$
10	$O(2^{\epsilon})^2$	$N_{lpha}^{(4)}$	$-(V^{p} - A^{p})(V_{2T}^{VN} - A_{2T}^{VN}) + 2T^{p}(T_{2T}^{VN} + T_{3T}^{VN})$
	$\frac{\mathcal{L}_{ll}(2\xi)}{(x_1 - i\varepsilon)(2\xi - x_2 - i\varepsilon)^2 y_1(1 - y_2)^2}$	$N^{(5)}_{lpha}$	$(V^p - A^p)(V^{VN}_{2\mathcal{E}} - A^{VN}_{2\mathcal{E}}) - 2T^p(T^{VN}_{3\mathcal{E}} - T^{VN}_{4\mathcal{E}})$
		$N^{(6)}_{lpha}$	$-(V^{p} - A^{p})(V_{2n}^{VN} - A_{2n}^{VN}) + 2T^{p}(T_{2n}^{VN} + T_{3n}^{VN})$
		$N_{\alpha}^{(1)}$	0
		$N_{\alpha}^{(2)}$	0
11	$u(x_1)$ — $u(y_1)$	$N_{\alpha}^{(3)}$	0
	$u(x_2)$ \longrightarrow $\overset{\frown}{\sim}$ $\overset{\frown}{\sim}$ $u(y_2)$	$N_{\alpha}^{(4)}$	0
	$d(x_3)$ $\overrightarrow{\partial}$ $\overrightarrow{\partial}$ $d(y_3)$	$N_{lpha}^{(5)}$	0
		$N^{(6)}_{lpha}$	0
		$N^{(1)}_{lpha}$	$(V^p+A^p)(V^{VN}_{1\mathcal{E}}{+}A^{VN}_{1\mathcal{E}})$
		$N_{\alpha}^{(2)}$	$(V^p + A^p)(V^{VN}_{1n} + A^{VN}_{1n})$
	$u(x_1)$ (y_1)	$N_{lpha}^{(3)}$	$(V^{p} + A^{p})(V_{1T}^{VN} + A_{1T}^{VN} + V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN})$
12	$u(x_2)$ \longrightarrow a a $u(y_2)$	$N^{(4)}_{lpha}$	$(V^p + A^p)(V_{2T}^{VN} + A_{2T}^{VN})$
	$d(x_3) \xrightarrow{\mathbf{Q}} \overrightarrow{\mathbf{Q}} d(y_3)$	$N^{(5)}_{lpha}$	$-(V^p+A^p)(V^{VN}_{2\mathcal{E}}+A^{VN}_{2\mathcal{E}})$
		$N^{(6)}_{lpha}$	$+(V^p+A^p)(V^{VN}_{2n}+A^{VN}_{2n})$
		$N^{(1)}_{lpha}$	$2T^p(T^{VN}_{1\mathcal{E}}+T^{VN}_{2\mathcal{E}})$
	$u(x_1)$ $u(y_1)$	$N^{(2)}_{lpha}$	$4T^{p}(T_{1n}^{VN}+rac{\Delta_{T}^{2}}{2M^{2}}T_{4n}^{VN})$
	$u(x_2)$ $aggin and u(y_2)$	$N^{(3)}_{lpha}$	$4T^{p}(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_{T}^{2}}{2M^{2}}T_{4T}^{VN})$
13	$d(x_3) \xrightarrow{\qquad \mathbf{y} \mathbf{y}} d(y_3)$	$N_{lpha}^{(4)}$	$2T^p(T_{2T}^{VN}+T_{3T}^{MN})$
	$\frac{Q_d(2\xi)^2}{(r_{-i}\epsilon)(2\xi-r_{-i}\epsilon)(r_{-i}\epsilon)(r_{-i}\epsilon)(r_{-i}\epsilon)}$	$N_{lpha}^{(5)}$	$-2T^p(T^{VN}_{3\mathcal{E}}-T^{VN}_{4\mathcal{E}})$
	$(x_1 - x_2)(x_2 - x_1 - x_2)(x_2 - x_2)(x_1 - y_2)(y_2 - y_2)(y_$	$N^{(6)}_{lpha}$	$+2T^p(T^{VN}_{2n}+T^{VN}_{3n})$
	$u(x_1)$ $u(y_1)$	$N_{\alpha}^{(1)}$	$(V^p - A^p)(V^{VN}_{1\mathcal{E}} - A^{VN}_{1\mathcal{E}})$
	$u(x_2)$ $u(y_2)$	$N_{lpha}^{(2)}$	$(V^p - A^p)(V^{VN}_{1n} - A^{VN}_{1n})$
14	$d(x_3)$ $d d d d d d d d d d d d d d d d d d d$	$N^{(3)}_{lpha}$	$(V^p - A^p)(V_{1T}^{VN} - A_{1T}^{VN} + V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN})$
	-(-5) ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	$N^{(4)}_{lpha}$	$(V^p - A^p)(V^{VN}_{2T} - A^{VN}_{2T})$
	$\frac{\mathcal{Q}_d(2\xi)^{-}}{(x_1 - i\varepsilon)(2\xi - x_1 - i\varepsilon)(x_2 - i\varepsilon)y_1y_2(1 - y_3)}$	$N^{(5)}_{lpha}$	$-(V^p-A^p)(V^{VN}_{2\mathcal{E}}-A^{VN}_{2\mathcal{E}})$
		$N^{(6)}_{lpha}$	$+(V^p-A^p)(V^{VN}_{2n}-A^{VN}_{2n})$

$$\begin{aligned} \mathcal{I}^{(1)}(\xi, \Delta^{2})|_{N(940)} &= -\mathcal{I}_{0} \frac{K_{1\ell}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}; \\ \mathcal{I}^{(2)}(\xi, \Delta^{2})|_{N(940)} &= -\mathcal{I}_{0} \frac{K_{1n}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}; \\ \mathcal{I}^{(3)}(\xi, \Delta^{2})|_{N(940)} &= -\mathcal{I}_{0} \frac{K_{1T}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T}) + K_{2\ell}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}; \\ \mathcal{I}^{(4)}(\xi, \Delta^{2})|_{N(940)} &= -\mathcal{I}_{0} \frac{K_{2T}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}; \\ \mathcal{I}^{(5)}(\xi, \Delta^{2})|_{N(940)} &= +\mathcal{I}_{0} \frac{K_{2\ell}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}; \\ \mathcal{I}^{(6)}(\xi, \Delta^{2})|_{N(940)} &= -\mathcal{I}_{0} \frac{K_{2n}^{VN}(\xi, \Delta^{2}, G_{VNN}^{V}, G_{VNN}^{T})}{2\xi}. \end{aligned}$$

$$(1)$$

Here G_{VNN}^V and G_{VNN}^T stand for the vector and tensor couplings of vector mesons to nucleons, and \mathcal{I}_0 is a well-known convolution of nucleon DAs with hard scattering kernel occurring in the leading order perturbative QCD description of the proton electromagnetic form factor $F_1^p(Q^2)$ [3]:

$$Q^4 F_1^p(Q^2) = \frac{(4\pi\alpha_s)^2 f_N^2}{54} \mathcal{I}_0.$$
 (2)

The explicit expressions for the functions $K_{1\mathcal{E}}^{VN}$, $K_{2\mathcal{E}}^{VN}$, K_{1T}^{VN} , K_{2T}^{VN} , are summarized in Appendix B. The modifications of the convolution integrals do not considerably change our cross section estimates within the cross channel nucleon exchange model for ωN , ρN , and ϕN TDAs. Therefore, our predictions stay consistent with those employed in Ref. [4]. As an example we show the replacement of Fig. 3 as Fig. 1 here presenting the unpolarized cross section $\frac{d^2 \sigma_T}{d\Omega_V}$ (in nb/sr) for backward $\gamma^* + p \rightarrow p + \omega$ for fixed W = 3.20 GeV as a function of Q^2 in the *u*-channel nucleon exchange model for ωN TDAs employing the Bonn 2000 set [6] for the $G_{\omega NN}^{V,T}$ couplings.

Let us also report a misprint in the last line of Eq. (23). The tensor structure $S_{s_1s_2}^{(6)\lambda_{\gamma}\lambda_V}$ must read

$$\mathcal{S}_{s_1s_2}^{(6)\lambda_{\gamma}\lambda_{V}} = (\mathcal{E}^*(p_V,\lambda_V)\cdot n)\bar{U}(p_2,s_2)\hat{\varepsilon}(q,\lambda_{\gamma})\hat{\Delta}_T U(p_1,s_1).$$
(3)



FIG. 1. Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_V}$ (in nb/sr) for backward $\gamma^* + p \rightarrow p + \omega$ for fixed W = 3.20 GeV as a function of Q^2 in the *u*-channel nucleon exchange model for ωN TDAs. COZ [3] (long-dashed blue line) and KS [5] (solid green line) solutions for the leading twist nucleon DA are used as the phenomenological input.

- [1] B. Pire, K. Semenov-Tian-Shansky, and L. Szymanowski, Phys. Rep. 940, 1 (2021).
- [2] B. Pire, K. M. Semenov-Tian-Shansky, A. A. Shaikhutdinova, and L. Szymanowski, Eur. Phys. J. C 82, 656 (2022).
- [3] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Z. Phys. C 42, 583 (1989); Yad. Fiz. 48, 1398 (1988) [Sov. J. Nucl. Phys. 48, 889 (1988)].
- [4] W. B. Li et al. (Jefferson Lab F π Collaboration), Phys. Rev. Lett. 123, 182501 (2019).
- [5] I. D. King and C. T. Sachrajda, Nucl. Phys. **B279**, 785 (1987).
- [6] R. Machleidt, Phys. Rev. C 63, 024001 (2001).