# Erratum: QCD description of backward vector meson hard electroproduction [Phys. Rev. D 91, 094006 (2015)]

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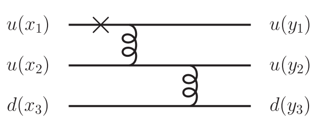
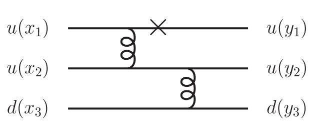
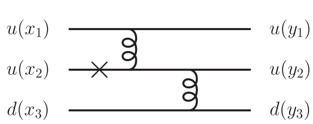
We correct a sign mistake in our calculation of backward vector meson electroproduction amplitude within the transition distribution amplitude (TDA) framework presented in Table I. The same error has also propagated into Table 3 of Ref. [1]. We present the corrected version of this table here. The altered signs are marked in boldface.

A useful cross check for this result is provided by the calculation of the integral convolutions $\mathcal{I}^{(k)}$, $k = 1, \ldots, 6$, defined in Eq. (24) for the $\gamma^* p \to V^0 p$ reaction within the cross channel nucleon exchange model for $VN$ TDAs considered in Appendix B. In this model, the convolution integrals $\mathcal{I}^{(k)}$ turn out to be proportional to the pQCD expression for the proton e.m. form factor $F_1^p$ (see discussion in Ref. [2]):

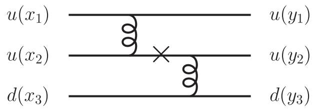
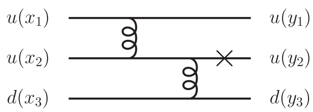
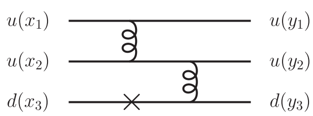
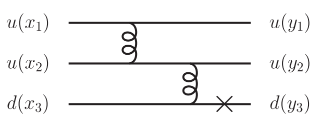
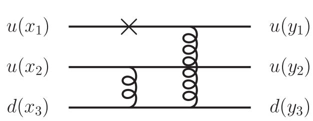
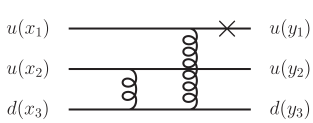
TABLE I. 14 of the 21 diagrams contributing to the hard-scattering amplitude with their associated coefficient $T_\alpha^{(k)} \equiv D_\alpha \times N_\alpha^{(k)}$ (no summation over $\alpha$ assumed). The seven first ones with $u$-quark lines inverted are not drawn. The crosses represent the virtual-photon vertex.

| $\alpha$ | Diagram $D_\alpha$ | | Numerators |
|---|---|---|---|
| 1 |  $\frac{Q_u(2\xi)^2}{(2\xi-x_1-i\epsilon)^2(x_3-i\epsilon)(1-y_1)^2y_3}$ | $N_\alpha^{(1)}$ | $-(V^p - A^p)(V_{1\mathcal{E}}^{VN} - A_{1\mathcal{E}}^{VN}) + 2T^p(T_{1\mathcal{E}}^{VN} + T_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $-(V^p - A^p)(V_{1n}^{VN} - A_{1n}^{VN}) + 4T^p(T_{1n}^{VN} + \frac{\Delta_T^2}{2M^2} T_{4n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-(V^p - A^p)(V_{1T}^{VN} - A_{1T}^{VN} + V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN})$ $+4T^p(T_{1T}^{VN} + T_{3\mathcal{E}}^{VN} + \frac{\Delta_T^2}{2M^2} T_{4T}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $-(V^p - A^p)(V_{2T}^{VN} - A_{2T}^{VN}) + 2T^p(T_{2T}^{VN} + T_{3T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $(V^p - A^p)(V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN}) - 2T^p(T_{3\mathcal{E}}^{VN} - T_{4\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-(V^p - A^p)(V_{2n}^{VN} - A_{2n}^{VN}) + 2T^p(T_{2n}^{VN} + T_{3n}^{VN})$ |
| 2 |  | $N_\alpha^{(1)}$ | 0 |
| | | $N_\alpha^{(2)}$ | 0 |
| | | $N_\alpha^{(3)}$ | 0 |
| | | $N_\alpha^{(4)}$ | 0 |
| | | $N_\alpha^{(5)}$ | 0 |
| | | $N_\alpha^{(6)}$ | 0 |
| 3 |  $\frac{Q_u(2\xi)^2)}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$ | $N_\alpha^{(1)}$ | $-2T^p(T_{1\mathcal{E}}^{VN} + T_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $-4T^p(T_{1n}^{VN} + \frac{\Delta_T^2}{2M^2} T_{4n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-4T^p(T_{1T}^{VN} + T_{3\mathcal{E}}^{VN} + \frac{\Delta_T^2}{2M^2} T_{4T}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $-2T^p(T_{2T}^{VN} + T_{3T}^{\omega N})$ |
| | | $N_\alpha^{(5)}$ | $2T^p(T_{3\mathcal{E}}^{VN} - T_{4\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-2T^p(T_{2n}^{VN} + T_{3n}^{VN})$ |

*(Table continued)*

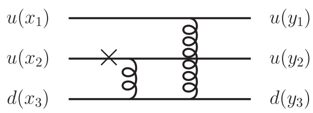
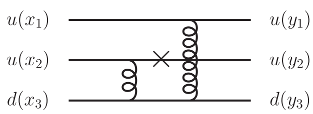
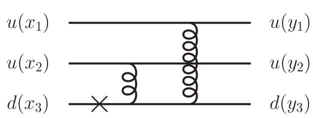
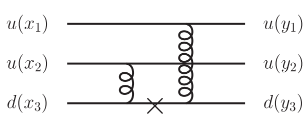
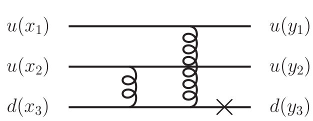
 

TABLE I. *(Continued)*

| $\alpha$ | Diagram $D_\alpha$ | | Numerators |
|---|---|---|---|
| 4 |   | $N_\alpha^{(1)}$ | $-(V^p-A^p)(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN})$ |
| | $\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$ | $N_\alpha^{(2)}$ | $-(V^p-A^p)(V_{1n}^{VN}-A_{1n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-(V^p-A^p)(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $-(V^p-A^p)(V_{2T}^{VN}-A_{2T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $(V^p-A^p)(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-(V^p-A^p)(V_{2n}^{VN}-A_{2n}^{VN})$ |
| 5 |   | $N_\alpha^{(1)}$ | $(V^p+A^p)(V_{1\mathcal{E}}^{VN}+A_{1\mathcal{E}}^{VN})$ |
| | $\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_2)y_3}$ | $N_\alpha^{(2)}$ | $(V^p+A^p)(V_{1n}^{VN}+A_{1n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $(V^p+A^p)(V_{1T}^{VN}+A_{1T}+V_{2\mathcal{E}}^{VN}+A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $(V^p+A^p)(V_{2T}^{VN}+A_{2T})$ |
| | | $N_\alpha^{(5)}$ | $-(V^p+A^p)(V_{2\mathcal{E}}^{VN}+A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $+(V^p+A^p)(V_{2n}^{VN}+A_{2n}^{VN})$ |
| 6 |   | $N_\alpha^{(1)}$ | 0 |
| | | $N_\alpha^{(2)}$ | 0 |
| | | $N_\alpha^{(3)}$ | 0 |
| | | $N_\alpha^{(4)}$ | 0 |
| | | $N_\alpha^{(5)}$ | 0 |
| | | $N_\alpha^{(6)}$ | 0 |
| 7 |   | $N_\alpha^{(1)}$ | $-2(V^pV_{1\mathcal{E}}^{VN}+A^pA_{1\mathcal{E}}^{VN})$ |
| | $\frac{Q_d(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)^2y_1(1-y_3)^2}$ | $N_\alpha^{(2)}$ | $-2(V^pV_{1n}^{VN}+A^pA_{1n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-2(V^p(V_{1T}^{VN}+V_{2\mathcal{E}}^{VN})+A^p(A_{1T}^{VN}+A_{2\mathcal{E}}^{VN}))$ |
| | | $N_\alpha^{(4)}$ | $-2(V^pV_{2T}^{VN}+A^pA_{2T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $2(V^pV_{2\mathcal{E}}^{VN}+A^pA_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-2(V^pV_{2n}^{VN}+A^pA_{2n}^{VN})$ |
| 8 |   | $N_\alpha^{(1)}$ | 0 |
| | | $N_\alpha^{(2)}$ | 0 |
| | | $N_\alpha^{(3)}$ | 0 |
| | | $N_\alpha^{(4)}$ | 0 |
| | | $N_\alpha^{(5)}$ | 0 |
| | | $N_\alpha^{(6)}$ | 0 |
| 9 |   | $N_\alpha^{(1)}$ | $-(V^p-A^p)(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN})+2T^p(T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN})$ |
| | $\frac{Q_u(2\xi)^2}{(2\xi-x_1-i\epsilon)^2(x_2-i\epsilon)(1-y_1)^2y_2}$ | $N_\alpha^{(2)}$ | $-(V^p-A^p)(V_{1n}^{VN}-A_{1n}^{VN})+4T^p(T_{1n}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-(V^p-A^p)(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ |
| | | | $+4T^p(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4T}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $-(V^p-A^p)(V_{2T}^{VN}-A_{2T}^{VN})+2T^p(T_{2T}^{VN}+T_{3T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $(V^p-A^p)(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})-2T^p(T_{3\mathcal{E}}^{VN}-T_{4\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-(V^p-A^p)(V_{2n}^{VN}-A_{2n}^{VN})+2T^p(T_{2n}^{VN}+T_{3n}^{VN})$ |

*(Table continued)*

TABLE I. *(Continued)*

| $\alpha$ | Diagram $D_\alpha$ | | Numerators |
|---|---|---|---|
| 10 |  $\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)^2 y_1(1-y_2)^2}$ | $N_\alpha^{(1)}$ | $-(V^p-A^p)(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN})+2T^p(T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $-(V^p-A^p)(V_{1n}^{VN}-A_{1n}^{VN})+4T^p(T_{1n}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $-(V^p-A^p)(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ $+4T^p(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4T}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $-(V^p-A^p)(V_{2T}^{VN}-A_{2T}^{VN})+2T^p(T_{2T}^{VN}+T_{3T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $(V^p-A^p)(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})-2T^p(T_{3\mathcal{E}}^{VN}-T_{4\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $-(V^p-A^p)(V_{2n}^{VN}-A_{2n}^{VN})+2T^p(T_{2n}^{VN}+T_{3n}^{VN})$ |
| 11 |  | $N_\alpha^{(1)}$ | 0 |
| | | $N_\alpha^{(2)}$ | 0 |
| | | $N_\alpha^{(3)}$ | 0 |
| | | $N_\alpha^{(4)}$ | 0 |
| | | $N_\alpha^{(5)}$ | 0 |
| | | $N_\alpha^{(6)}$ | 0 |
| 12 |  | $N_\alpha^{(1)}$ | $(V^p+A^p)(V_{1\mathcal{E}}^{VN}+A_{1\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $(V^p+A^p)(V_{1n}^{VN}+A_{1n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $(V^p+A^p)(V_{1T}^{VN}+A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}+A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $(V^p+A^p)(V_{2T}^{VN}+A_{2T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $-(V^p+A^p)(V_{2\mathcal{E}}^{VN}+A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $+(V^p+A^p)(V_{2n}^{VN}+A_{2n}^{VN})$ |
| 13 |  $\frac{Q_d(2\xi)^2)}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1(1-y_2)y_2}$ | $N_\alpha^{(1)}$ | $2T^p(T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $4T^p(T_{1n}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $4T^p(T_{1T}^{VN}+T_{3\mathcal{E}}^{VN}+\frac{\Delta_T^2}{2M^2}T_{4T}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $2T^p(T_{2T}^{VN}+T_{3T}^{\omega N})$ |
| | | $N_\alpha^{(5)}$ | $-2T^p(T_{3\mathcal{E}}^{VN}-T_{4\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $+2T^p(T_{2n}^{VN}+T_{3n}^{VN})$ |
| 14 |  $\frac{Q_d(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1y_2(1-y_3)}$ | $N_\alpha^{(1)}$ | $(V^p-A^p)(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(2)}$ | $(V^p-A^p)(V_{1n}^{VN}-A_{1n}^{VN})$ |
| | | $N_\alpha^{(3)}$ | $(V^p-A^p)(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(4)}$ | $(V^p-A^p)(V_{2T}^{VN}-A_{2T}^{VN})$ |
| | | $N_\alpha^{(5)}$ | $-(V^p-A^p)(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN})$ |
| | | $N_\alpha^{(6)}$ | $+(V^p-A^p)(V_{2n}^{VN}-A_{2n}^{VN})$ |

$$
\begin{aligned}
\mathcal{I}^{(1)}(\xi,\Delta^2)|_{N(940)} &= -\mathcal{I}_0 \frac{K_{1\mathcal{E}}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi};\\
\mathcal{I}^{(2)}(\xi,\Delta^2)|_{N(940)} &= -\mathcal{I}_0 \frac{K_{1n}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi};\\
\mathcal{I}^{(3)}(\xi,\Delta^2)|_{N(940)} &= -\mathcal{I}_0 \frac{K_{1T}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)+K_{2\mathcal{E}}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi};\\
\mathcal{I}^{(4)}(\xi,\Delta^2)|_{N(940)} &= -\mathcal{I}_0 \frac{K_{2T}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi};\\
\mathcal{I}^{(5)}(\xi,\Delta^2)|_{N(940)} &= +\mathcal{I}_0 \frac{K_{2\mathcal{E}}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi};\\
\mathcal{I}^{(6)}(\xi,\Delta^2)|_{N(940)} &= -\mathcal{I}_0 \frac{K_{2n}^{VN}(\xi,\Delta^2,G_{VNN}^V,G_{VNN}^T)}{2\xi}.
\end{aligned}
\tag{1}
$$

Here $G_{VNN}^V$ and $G_{VNN}^T$ stand for the vector and tensor couplings of vector mesons to nucleons, and $\mathcal{I}_0$ is a well-known convolution of nucleon DAs with hard scattering kernel occurring in the leading order perturbative QCD description of the proton electromagnetic form factor $F_1^p(Q^2)$ [3]:

$$
Q^4 F_1^p(Q^2) = \frac{(4\pi\alpha_s)^2 f_N^2}{54}\mathcal{I}_0. \tag{2}
$$

The explicit expressions for the functions $K_{1\mathcal{E}}^{VN}$, $K_{2\mathcal{E}}^{VN}$, $K_{1T}^{VN}$, $K_{2T}^{VN}$, are summarized in Appendix B.

The modifications of the convolution integrals do not considerably change our cross section estimates within the cross channel nucleon exchange model for $\omega N$, $\rho N$, and $\phi N$ TDAs. Therefore, our predictions stay consistent with those employed in Ref. [4]. As an example we show the replacement of Fig. 3 as Fig. 1 here presenting the unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_V}$ (in nb/sr) for backward $\gamma^* + p \to p + \omega$ for fixed $W = 3.20$ GeV as a function of $Q^2$ in the $u$-channel nucleon exchange model for $\omega N$ TDAs employing the Bonn 2000 set [6] for the $G_{\omega NN}^{V,T}$ couplings.

Let us also report a misprint in the last line of Eq. (23). The tensor structure $\mathcal{S}_{s_1 s_2}^{(6)\lambda_\gamma\lambda_V}$ must read

$$
\mathcal{S}_{s_1 s_2}^{(6)\lambda_\gamma\lambda_V} = (\mathcal{E}^*(p_V,\lambda_V)\cdot n)\bar{U}(p_2,s_2)\hat{\varepsilon}(q,\lambda_\gamma)\hat{\Delta}_T U(p_1,s_1). \tag{3}
$$

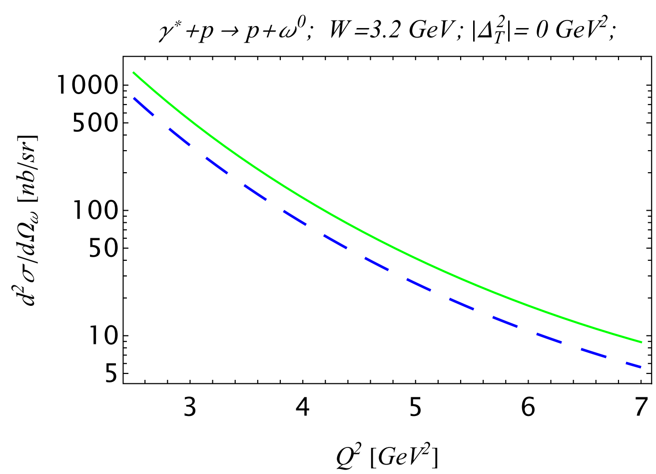



FIG. 1. Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_V}$ (in nb/sr) for backward $\gamma^* + p \to p + \omega$ for fixed $W = 3.20$ GeV as a function of $Q^2$ in the $u$-channel nucleon exchange model for $\omega N$ TDAs. COZ [3] (long-dashed blue line) and KS [5] (solid green line) solutions for the leading twist nucleon DA are used as the phenomenological input.

[1] B. Pire, K. Semenov-Tian-Shansky, and L. Szymanowski, Phys. Rep. **940**, 1 (2021).

[2] B. Pire, K. M. Semenov-Tian-Shansky, A. A. Shaikhutdinova, and L. Szymanowski, Eur. Phys. J. C **82**, 656 (2022).

[3] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Z. Phys. C **42**, 583 (1989); Yad. Fiz. **48**, 1398 (1988) [Sov. J. Nucl. Phys. **48**, 889 (1988)].

[4] W. B. Li *et al.* (Jefferson Lab F$\pi$ Collaboration), Phys. Rev. Lett. **123**, 182501 (2019).

[5] I. D. King and C. T. Sachrajda, Nucl. Phys. **B279**, 785 (1987).

[6] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).