Nested radiative seesaw masses for dark matter and neutrinos

Talal Ahmed Chowdhury
Department of Physics, University of Dhaka, P.O. Box 1000, Dhaka, Bangladesh

Shaaban Khalil
Center for Fundamental Physics, Zewail City of Science and Technology, 6 October City, Giza 12588, Egypt

Ernest Ma
Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

(Received 1 June 2022; accepted 28 October 2022; published 18 November 2022)

The scotogenic model of neutrino mass is modified so that the dark Majorana fermion singlet S which makes the neutrino massive is itself generated in one loop. This is accomplished by having Z_6 lepton symmetry softly broken to Z_2 in the scalar sector by a unique quadratic term.

DOI: 10.1103/PhysRevD.106.095020

I. INTRODUCTION

The origin of neutrino mass [1] may be dark matter [2]. A one-loop radiative mechanism [3] is the possible connection, known now widely as the scotogenic model. They may also be indirectly related through lepton parity [4] or lepton number [5]. There are many variants of this basic idea. Here it is proposed that the dark-matter mass itself is also radiative [6]. This scenario is very suitable for freeze-in light dark matter through Higgs decay [7]. It arises as the result of softly broken lepton symmetry and serves as a comprehensive framework for understanding neutrinos and dark matter as belonging in the same category of fundamental particles.

To implement this idea, a heavy right-handed neutrino N is assumed, but it is prevented from coupling directly to the left-handed neutrino ν by a symmetry. Nevertheless, both ν and N couple to the dark fermion S. The imposed symmetry is then softly broken so that S gets a radiative mass from N, and ν pairs up with N in one loop through S. This scenario is very suitable for the freeze-in mechanism [8] where the dark matter interacts very weakly and slowly builds up its relic abundance, from the decay of a massive particle, in this case the Higgs boson of the Standard Model (SM), before the latter itself goes out of thermal equilibrium. The direct detection of dark matter in underground experiments then becomes very difficult, which is consistent with the mostly null results obtained so far.

It is well known that baryon number B and lepton number L are automatically conserved in the standard model (SM) of particle interactions in the case of massless ν. The simplest way for it to become massive is to add a singlet right-handed fermion N_R, then they pair up through the term $\bar{N}_R(\nu_L\phi^0 - e_L\phi^+)$, where $\Phi = (\phi^+, \phi^0)$ is the SM Higgs scalar doublet. This renders the neutrino a Dirac mass from the vacuum expectation value $\langle\phi^0\rangle = v$, and N_R is naturally assigned $L = 1$. On the other hand, gauge invariance also allows the $N_R N_R$ Majorana mass term, hence L naturally breaks to $(-1)^L$ and a seesaw mass for ν_L is obtained. In this paper, we assume L to be an input symmetry of the Lagrangian, so that other choices of L for N_R are also possible [9]. In particular, a Z_6 symmetry is softly broken to Z_2.

II. MODEL

Each family of the SM is extended to include a right-handed fermion singlet N_R and a left-handed fermion singlet S_L. The scalar sector consists of the SM Higgs doublet Φ and a second doublet $\eta = (\eta^+, \eta^0)$ together with a neutral singlet χ^0. The discrete symmetry Z_6 is imposed on these fields as shown in Table 1, and is respected by all dimension-four terms of the Lagrangian. It is softly broken by the quadratic scalar mass term $\chi^0\chi^0$, resulting in a radiative mass for S_L, which then induces a radiative Majorana mass for ν. A residual Z_2 discrete symmetry $D = (-1)^{L+2j}$ remains [4], where j is the intrinsic spin of the particle in question.

TABLE I. Particle content of model with $\omega^6 = 1$.

Fermion/scalar	$SU(2)_L \times U(1)_Y$	Z_6	L	D
$(\nu, e)_L$	$(2, -1/2)$	ω	1	+
N_R	$(1, 0)$	ω^3	1	+
S_L	$(1, 0)$	ω^{-2}	0	−
(ϕ^+, ϕ^0)	$(2, 1/2)$	1	0	+
(η^+, η^0)	$(2, 1/2)$	ω	−1	−
χ^0	$(1, 0)$	ω	−1	−

The resulting Higgs potential is given by

$$\begin{aligned} V = {} & m_1^2\Phi^\dagger\Phi + m_2^2\eta^\dagger\eta + m_3^2\bar{\chi}^0\chi^0 + \frac{1}{2}m_4^2\chi^{02} + \text{H.c.} \\ & + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\eta^\dagger\eta)^2 + \frac{1}{2}\lambda_3(\bar{\chi}^0\chi^0)^2 \\ & + \lambda_{12}(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_{13}(\Phi^\dagger\Phi)(\bar{\chi}^0\chi^0) \\ & + \lambda_{23}(\eta^\dagger\eta)(\bar{\chi}^0\chi^0) + \mu\eta^\dagger\Phi\chi^0 + \text{H.c.} \end{aligned} \quad (1)$$

Let $\langle\phi^0\rangle = v$, then the mass of the Higgs boson H is given by $m_H^2 = 2\lambda_1 v^2$. Note that the Z_6 symmetry is respected by all dimension-four terms, but is softly broken to Z_2 by the m_4^2 term. Together with the following allowed Yukawa terms,

$$\mathcal{L} \supset f_\chi \bar{S}_L N_R \chi^0 + f_\eta(\nu_L\eta^0 - e_L\eta^+)S_L + \text{H.c.}, \quad (2)$$

the lepton number L may be assigned as shown in Table 1. However, since the $N_R N_R$ Majorana mass term is allowed by Z_6, only lepton parity $(-1)^L$ is strictly conserved, as is dark parity $D = (-1)^{L+2j}$ [4].

III. RADIATIVE DARK MATTER MASS

The fermion singlet N_R has an allowed Majorana mass m_N under Z_6, but S_L is massless at tree level. However, the breaking of Z_6 through the soft quadratic scalar term $\chi^0\chi^0$ allows S_L to acquire a radiative Majorana mass in one loop, as shown in Fig. 1. The residual symmetry of this model is then Z_2, which may be understood as dark parity derived from lepton parity [4], as shown in Table 1.

Let $\chi^0 = (\chi_R + i\chi_I)/\sqrt{2}$ and $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then the 2×2 mass-squared matrices spanning (χ_R, η_R) and (χ_I, η_I) are given by

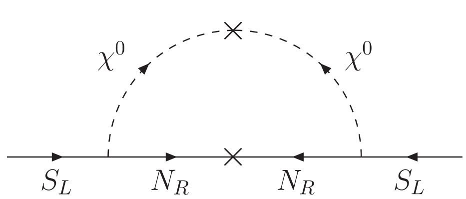

FIG. 1. One-loop radiative Majorana mass for the dark fermion S.

$$\mathcal{M}_{R,I}^2 = \begin{pmatrix} m_3^2 + \lambda_{13}v^2 \pm m_4^2 & \mu v \\ \mu v & m_2^2 + \lambda_{12}v^2 \end{pmatrix}. \quad (3)$$

This means that the radiative m_S comes from the difference in the contributions of $\mathcal{M}_R^2$ and $\mathcal{M}_I^2$. Let (ψ_{R1}, ψ_{R2}) be the mass eigenstates of $\mathcal{M}_R^2$ with eigenvalues (m_{R1}^2, m_{R2}^2):

$$\psi_{R1} = c_R\chi_R + s_R\eta_R, \qquad \psi_{R2} = -s_R\chi_R + c_R\eta_R, \quad (4)$$

and similarly for $\mathcal{M}_I^2$. Of the 6 parameters $m_{R1}^2, m_{R2}^2, m_{I1}^2, m_{I2}^2, s_R, s_I$, only 4 are independent. In the limit that m_4^2 is very small, s_I differs from s_R by only a small amount, say

$$s_I = s_R + \delta, \qquad c_I = c_R - \delta(s_R/c_R), \quad (5)$$

then

$$\begin{aligned} m_{I1}^2 &= m_{R1}^2 - (\delta/s_R)(m_{R1}^2 - m_{R2}^2), \\ m_{I2}^2 &= m_{R2}^2 - (\delta s_R/c_R^2)(m_{R1}^2 - m_{R2}^2). \end{aligned} \quad (6)$$

Now the radiative m_S mass is given by

$$\begin{aligned} m_S = \frac{f_\chi m_N}{32\pi^2}[& c_R^2 F(m_{R1}^2, m_N^2) - c_I^2 F(m_{I1}^2, m_N^2) \\ & + s_R^2 F(m_{R2}^2, m_N^2) - s_I^2 F(m_{I2}^2, m_N^2)] f_\chi^T, \end{aligned} \quad (7)$$

where $F(a,b) = a\ln(a/b)/(a-b)$ and f_χ is the $\bar{S}_L N_R \chi^0$ coupling.

IV. RADIATIVE NEUTRINO MASS

Since S_L gets a radiative mass, ν_L is now connected to N_R as shown in Fig. 2. Call this Dirac mass m_D, then the neutrino gets the usual seesaw Majorana mass m_D^2/m_N.

Since m_S itself is suppressed by m_N^{-1} from Fig. 1, m_ν gets suppressed by m_N^{-3} in this case. On the other hand, there is a diagram for Majorana m_ν directly as shown in Fig. 3, which is suppressed by only m_N^{-1}.

The radiative neutrino mass m_ν is then generated from m_S in exact analogy to m_S from m_N, i.e.

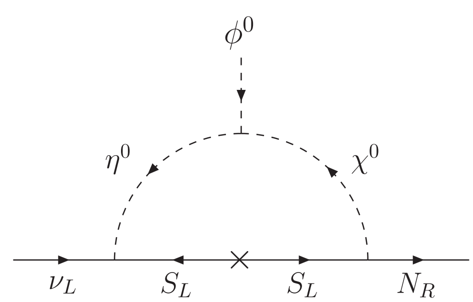

FIG. 2. One-loop radiative Dirac mass linking ν_L to N_R.

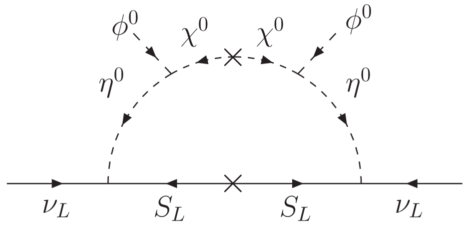

FIG. 3. Scotogenic Majorana mass for ν.

$$
\begin{aligned}
m_\nu &= \frac{f_\eta m_S}{32\pi^2}[s_R^2 F(m_{R1}^2, m_S^2) - s_I^2 F(m_{I1}^2, m_S^2) \\
&\quad + c_R^2 F(m_{R2}^2, m_S^2) - c_I^2 F(m_{I2}^2, m_S^2)] f_\eta^T, \\
&= f_\eta . \Lambda . f_\eta^T
\end{aligned}
\tag{8}
$$

where f_η is the $\nu_L S_L \eta^0$ coupling and Λ is the diagonal matrix containing the loop functions. Just as m_S is a function of f_χ, s_L, δ, m_{R1}, m_{R2}, and m_N, m_ν is a function of f_η, s_L, δ, m_{R1}, m_{R2}, and m_S. We explore below the possible parameter space for the dark matter mass m_S and the neutrino mass m_ν.

V. THE VIABLE PARAMETER SPACE

For a general complex Yukawa f_χ, the mass eigenstates for the S fermion with corresponding mass eigenvalues can be determined from Eq. (7) by following the diagonalization procedure of a complex symmetric matrix. For simplicity, we consider the Yukawa matrix to be, $f_\chi = \mathrm{diag}(1,1,1)$, and the lightest fermion denoted here as S_1, is considered as the DM candidate. Besides, the masses of $S_{1,2,3}$ depend on the RH neutrino masses, $m_{N_{1,2,3}}$ and the scalar masses m_{R_i}, m_{I_i}, s_R and δ as seen in Fig. 4. We can see from Fig. 4 that the mass of the DM candidate S_1 increases if the mass-splitting between

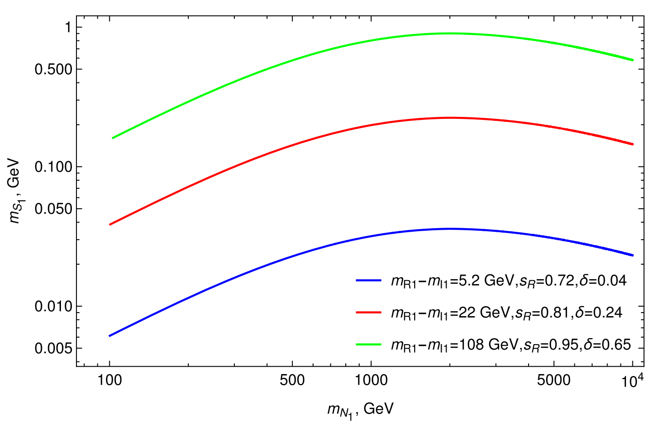

FIG. 4. Correlation between the lightest RH neutrino mass, m_{N_1} and the DM mass, m_{S_1} for three fixed mass separations, $m_{R1} - m_{I1}$, s_R and δ. Here, $m_{N_2} = m_{N_1} + 100$ GeV and $m_{N_3} = m_{N_2} + 200$ GeV. Moreover, the mass parameters m_2 and m_3 are fixed at $m_2 = m_3 = 1000$ GeV, and the cubic term, $\mu = 500$ GeV.

ψ_R and ψ_I increases which in turn depends on the larger value of mass parameter, m_4 associated with the quadratic scalar mass term that softly breaks the Z_6 to Z_2 resulting the radiative mass for the S_i.

Besides, the Yukawa matrix f_η, introduced in Eq. (2), can be written using the Casas-Ibarra parametrization [10] in the following way,

$$
f_\eta = U_{\mathrm{PMNS}} \sqrt{\hat{m}} R \sqrt{\Lambda^{-1}}
\tag{9}
$$

where, U_{PMNS} is the PMNS matrix of the neutrino mixing, $\hat{m} = \mathrm{diag}(m_1, m_2, m_3)$ is the neutrino masses and R is an complex orthogonal matrix whose angles are taken to be real in our case for simplicity. This expression is 3×3 matrix $(f_\eta)_{ij}$, which corresponds to three flavors ν_i and $(S_L)_j$. As a result, f_η is a function of all neutrino masses and mixing would lead to the charged lepton flavor violation (LFV) in our model. We consider the constraints on the following charged LFV processes: $\mathrm{Br}(\mu\to e\gamma) < 4.2\times 10^{-13}$ [11], $\mathrm{Br}(\tau\to e\gamma) < 5.6\times 10^{-8}$ and $\mathrm{Br}(\tau\to \mu\gamma) < 4.2\times 10^{-8}$ [12], $\mathrm{Br}(\mu\to 3e) < 10^{-12}$ [13], $\mathrm{Br}(\tau\to 3e) < 2.7\times 10^{-8}$ and $\mathrm{Br}(\tau\to 3\mu) < 2.1\times 10^{-8}$ [14], $\mu - e$ conversion in Ti $< 1.7\times 10^{-12}$ [15] and $\mu - e$ conversion in Au $< 7\times 10^{-13}$ [16] to determine the viable region of the parameter space with our numerical analysis.

VI. PRODUCTION OF THE DARK FERMION S_L

The dark fermions, S_L can have effective interaction, $f_h \bar{S} S h$ with the Higgs boson generated at one-loop shown in Fig. 5, where the relevant Yukawa coupling, f_h is given by,

$$
\begin{aligned}
f_h &= \frac{\lambda_{13} v}{32\pi^2} f_\chi m_N [c_R^2 G(m_{R1}^2, m_N^2) - c_I^2 G(m_{I1}^2, m_N^2) \\
&\quad + s_R^2 G(m_{R2}^2, m_N^2) - s_I^2 G(m_{R1}^2, m_N^2)] f_\chi^T,
\end{aligned}
\tag{10}
$$

with $G(a,b) = 1/(a-b) - b\ln(a/b)/(a-b)^2$.

Based on this effective interaction between the Higgs and the dark matter candidate S_1, one can consider the freeze-in mechanism [8,17] to achieve the correct relic abundance of the DM with the following considerations,

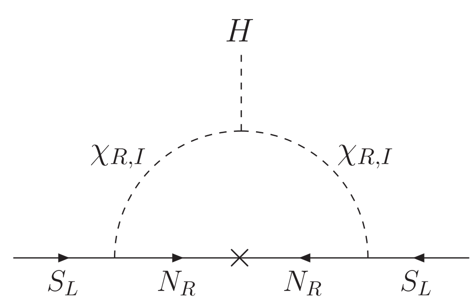

FIG. 5. One-loop diagram of the interaction among the Higgs boson and the dark fermions, S_L.

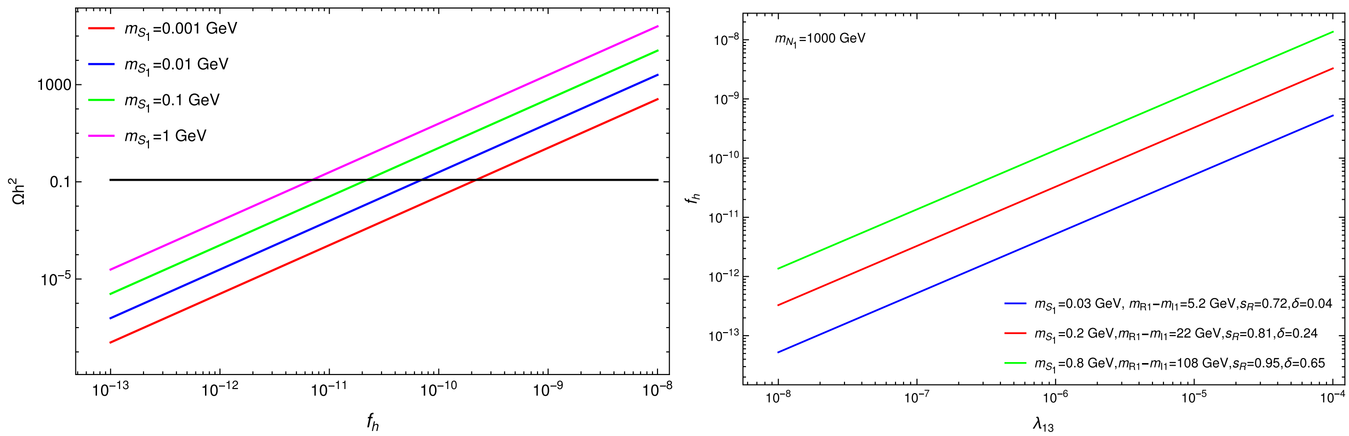

FIG. 6. Left: DM relic abundance with respect to the effective Higgs-DM coupling, f_h for different DM masses via the freeze-in mechanism. The horizontal black line represent the observed DM relic abundance, $\Omega h^2 = 0.12 \pm 0.001$ (68% Confidence Level) [20]. Right: correlation between the couplings, λ_{13} and f_h for fixed DM mass, m_{N_1}, $m_{R1} - m_{I1}$, s_R, and δ.

(i) The reheating temperature has been set as $T_R \ll m_{N_{1,2,3}}, m_{R1,R2}, m_{I1,I2}, m_{\eta^+}$ so that during the gradual increase of the abundance of feebly interacting massive particle, S_1 from an initially negligible value at the early universe (i.e. at T_R) through the decay and scattering from the thermal bath particles, the abundances of N_i, ψ_R and ψ_I are already Boltzmann suppressed as their presence in the thermal bath would led to excessive relic abundance of S_1 via the processes, controlled by the Yukawa couplings, f_η and f_χ, noted below,

$$\bar{S}_1 S_1 \leftrightarrow \overline{N_i} N_j, \psi_{R_i}\psi_{R_j}, \psi_{I_i}\psi_{I_j}, \psi_{R_i}\psi_{I_j}, \eta^+\eta^-, \overline{\nu}\nu, l^+l^-. \tag{11}$$

Therefore, T_R is set at the $T_R \sim T_c = 159.5$ GeV [18], which is the Standard Model cross-over temperature.

(ii) Now the relevant processes which contribute to the freeze-in of the DM are the decay of the Higgs into dark fermions, S_1, and the scattering of the SM fermions, f and gauge bosons, V via Higgs boson at s-channel as follows,

$$h \to \overline{S_1} S_1, \qquad \bar{f} f \to \overline{S_1} S_1, \qquad VV \to \overline{S_1} S_1 \tag{12}$$

Among these processes, the decay channel dominates the DM production, because the other channels are much more suppressed by the squared mass of Higgs.

We calculate the relic abundance following the formalism presented in [19].[1] We find out that for the DM mass varying from 0.001 GeV to 1 GeV, the required value of the f_h coupling is 2×10^{-10} to 8×10^{-12}, respectively, as seen from Fig. 6 (left). Besides, we can see from Fig. 6 (right) that for a fixed value of the DM mass or in other words for fixed values of m_{N_1}, $m_{R1} - m_{I1}$, s_R, and δ, one can adjust the value of the scalar coupling λ_{13} to achieve the f_h value for the correct relic abundance. It is worth noting that the radiative DM mass in our analysis is typically less than one GeV, as shown in Fig. 4, and if the DM mass is of the order of one GeV, then the S_1 annihilation to electron positron or the first family of quarks is possible. However, the cross section of these processes is very suppressed (being proportional to m_f^2) and would result in an excessive relic abundance. Therefore, we avoid the freeze-out scenario, in which a S_1 particle in thermal equilibrium would overclose the universe.

VII. CONCLUSION

In this work, we address a modification of the scotogenic model of the neutrino mass where the mass of the fermionic dark matter is radiatively generated at one-loop along with the mass of the neutrino. We determine the viable parameter space which satisfies the current limits on the charged lepton flavor violating processes. Afterwards, within that parameter space, we calculate the relic abundance of the DM via freeze-in mechanism involving the decay of the Higgs boson and the $2 \to 2$ scatterings of SM fermions and gauge bosons into DM pairs. We find out that for the light DM in our model with mass ranging from $m_{S_1} = 0.001$ GeV to $m_{S_1} = 1$ GeV, the required effective coupling between the Higgs and the DM generated at one-loop, f_h has to be from $f_h = 2 \times 10^{-10}$ to $f_h = 8 \times 10^{-12}$, respectively to obtain the correct relic abundance. So we can see that the increasing value of the DM mass requires smaller value of f_h.

[1]Micromega can do freeze-in analysis in principle; however, we used the mathematical formalism of the micromegas in *Mathematica* and for some specific channels involving Higgs because we were taking reheating temperatures much lower than N_R to calculate our freeze-in relic density.

Furthermore, we observe that such loop suppressed small value of the effective coupling f_h can be achieved for the DM mass range, $m_{S_1} = 0.001 - 1$ GeV by adjusting the scalar quartic coupling, λ_{13} which turns out be relatively larger than the f_h, and within the range, $O(10^{-8} - 10^{-6})$.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy Grant No. DE-SC0008541. Also, S. K. acknowledges support from Science, Technology & Innovation Funding Authority (STDF) Egypt, under Grant No. 37272.

[1] See, for example, A. de Gouvea, Annu. Rev. Nucl. Part. Sci. **66**, 197 (2016).
[2] See, for example, B.-L. Young, Front. Phys. **12**, 121201 (2017).
[3] E. Ma, Phys. Rev. D **73**, 077301 (2006).
[4] E. Ma, Phys. Rev. Lett. **115**, 011801 (2015).
[5] E. Ma, Phys. Lett. B **809**, 135736 (2020).
[6] E. Ma and V. De Romeri, Phys. Rev. D. **104**, 055004 (2021).
[7] E. Ma, Lett. High Energy Phys. **2**, 103 (2019).
[8] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, J. High Energy Phys. 03 (2010) 080.
[9] E. Ma, Mod. Phys. Lett. A **32**, 173007 (2017).
[10] J. A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001).
[11] A. M. Baldini *et al.* (MEG Collaboration), Eur. Phys. J. C **76**, 434 (2016).
[12] A. Abdesselam *et al.* (Belle Collaboration), J. High Energy Phys. 10 (2021) 19.
[13] U. Bellgardt *et al.* (SINDRUM Collaboration), Nucl. Phys. **B299**, 1 (1988).
[14] K. Hayasaka, K. Inami, Y. Miyazaki, K. Arinstein, V. Aulchenko, T. Aushev, A. M. Bakich, A. Bay, K. Belous, V. Bhardwaj *et al.*, Phys. Lett. B **687**, 139 (2010).
[15] J. Kaulard *et al.* (SINDRUM II Collaboration), Phys. Lett. B **422**, 334 (1998).
[16] W. H. Bertl *et al.* (SINDRUM II Collaboration), Eur. Phys. J. C **47**, 337 (2006).
[17] J. McDonald, Phys. Rev. Lett. **88**, 091304 (2002).
[18] M. D'Onofrio and K. Rummukainen, Phys. Rev. D **93**, 025003 (2016).
[19] G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov, and B. Zaldivar, Comput. Phys. Commun. **231**, 173 (2018).
[20] N. Aghanim *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).