

Heavy quark contribution to the electromagnetic properties of the nucleon

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Quantum chromodynamics (QCD) predicts the existence of both nonperturbative intrinsic and perturbative extrinsic heavy quark contributions to the fundamental structure of hadrons. The existence of intrinsic charm at the 3-standard-deviation level in the proton has recently been established from structure function measurements by the NNPDF Collaboration. Here, we revisit the physics of intrinsic heavy quarks using *light-front holographic QCD (LFHQCD)*—a novel comprehensive approach to hadron structure which provides detailed predictions for dynamical properties of the hadrons, such as form factors, distribution amplitudes, structure functions, etc. We will extend this nonperturbative light-front QCD approach to study the heavy quark-antiquark contribution to the electromagnetic properties of nucleon. Our framework is based on a study of the eigenfunctions of the QCD light-front Hamiltonian, the frame-independent light-front wave functions (LFWFs) underlying hadron dynamics. We analyze the heavy quark content in the proton, induced either directly by the nonperturbative $|uud + Q\bar{Q}\rangle$ Fock state or by the $|uud + g\rangle$ Fock state, where the gluon splits into a heavy quark-antiquark pair. The specific form of these LFWFs are derived from LFHQCD. Using these LFWFs, we construct light-front representations for the heavy quark-antiquark asymmetry, the electromagnetic form factors of nucleons induced by heavy quarks, including their magnetic moments and radii.

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One of the rigorous predictions of quantum chromodynamics (QCD) is the existence of both nonperturbative intrinsic and perturbative extrinsic heavy quark contents of the nucleon [1,2]. The existence of intrinsic charm at the 3-standard-deviation level has recently been established from a comprehensive analysis of structure function measurements by the NNPDF Collaboration [3].

The full heavy quark parton distribution (PDF) is given by the sum of intrinsic (in) and extrinsic (ex) components: $Q(x) = Q_{\text{in}}(x) + Q_{\text{ex}}(x)$, where $Q = c, b$ and $x = \frac{k^+}{p^+}$ is the light-front variable. It was assumed in Refs. [1,2] that the intrinsic contribution is induced by the twist-5 Fock state $|uud + Q\bar{Q}\rangle$. For a review see, e.g., Ref. [4]. The light-front wave functions (LFWFs) are the Fock state projections of the QCD light-front Hamiltonian.

We will use the light-front QCD approach for the study of the heavy quark-antiquark contribution to the electromagnetic properties of nucleon. Our framework is based on the boost-invariant LFWFs, effectively describing the heavy quark-antiquark content in the nucleon. We will focus on two nonperturbative Fock states $|uud + Q\bar{Q}\rangle$ and $|uud + g\rangle$ (see Fig. 1). The latter gives rise to the heavy quark-antiquark contribution via gluon splitting into a $Q\bar{Q}$ pair. The analytic form of these LFWFs is predicted by light-front holographic QCD (LFHQCD) [5–9] which we will apply to the heavy quark contribution to the heavy quark-antiquark asymmetry and the electromagnetic properties of nucleons: form factors, magnetic moment, and radii.

We will derive the LFWFs $\psi_{Q:\lambda_Q}^{\lambda_N}(x, \mathbf{k}_\perp)$ and $\psi_{\bar{Q}:\lambda_{\bar{Q}}}^{\lambda_N}(x, \mathbf{k}_\perp)$ describing a heavy quark-antiquark contribution in the proton with specific helicities for the proton $\lambda_N = \uparrow$ and \downarrow and for the struck heavy quark $\lambda_Q = \pm\frac{1}{2}$ or antiquark $\lambda_{\bar{Q}} = \pm\frac{1}{2}$.

The LFWFs, which determine the heavy struck quark distribution $\psi_{Q:\lambda_Q}^{\lambda_N}(x, \mathbf{k}_\perp)$, are listed as

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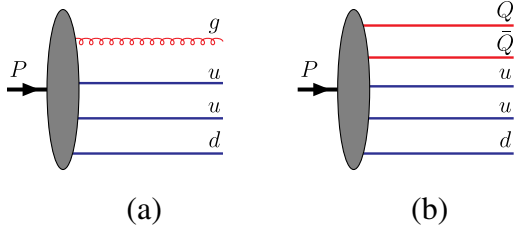


FIG. 1. Proton Fock states producing heavy quark-antiquark contributions: (a) $|uud + g\rangle$ state and (b) $|uud + Q\bar{Q}\rangle$ state.

$$\begin{aligned}
 \psi_{Q;+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) &= \varphi(x, \mathbf{k}_\perp), \\
 \psi_{Q;-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) &= -\frac{k^1 + ik^2}{\kappa} \varphi(x, \mathbf{k}_\perp), \\
 \psi_{Q;+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) &= -\left[\psi_{Q;-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp)\right]^\dagger = \frac{k^1 - ik^2}{\kappa} \varphi(x, \mathbf{k}_\perp), \\
 \psi_{Q;-\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) &= \left[\psi_{Q;+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp)\right]^\dagger = \varphi(x, \mathbf{k}_\perp), \quad (1)
 \end{aligned}$$

where

$$\varphi(x, \mathbf{k}_\perp) = \frac{2\pi\sqrt{2}}{\kappa} \sqrt{Q_{\text{in}}(x)} \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\right], \quad (2)$$

$\kappa = 500$ MeV is the scale dilaton parameter, and $Q_{\text{in}}(x)$ is the intrinsic heavy quark PDF, which is expressed in terms of the derived LFWFs as

$$\begin{aligned}
 Q_{\text{in}}(x) &= \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi_{Q;+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp)|^2 + |\psi_{Q;-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp)|^2 \right] \\
 &= \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi_{Q;+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp)|^2 + |\psi_{Q;-\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp)|^2 \right]. \quad (3)
 \end{aligned}$$

We can assume that the probability of intrinsic heavy quarks in the proton scales as $\frac{1}{m_Q^2}$ [4], a result that comes from a two-gluon intermediate state perturbative contribution, in contrast to the logarithmic dependence of the extrinsic contribution generated by pQCD evolution.

For the $Q_{\text{in}}(x)$ we can use the result obtained in LFQCD [1]:

$$Q_{\text{in}}(x) = N_{Q_{\text{in}}} x^2 [(1-x)(1+10x+x^2) + 6x(1+x)\log(x)], \quad (4)$$

where $N_{Q_{\text{in}}}$ is the normalization constant, fixed from data. We have assumed that the normalization constant of the intrinsic heavy quark distribution, $N_{c_{\text{in}}}$ in Eq. (4), is equal to 6, corresponding to a 1% intrinsic charm contribution to the proton PDF. This choice was motivated by an estimate of the magnitude of the diffractive production of the Λ_c baryon in the $pp \rightarrow p\Lambda_c X$ reaction [1], which is consistent

with the MIT bag-model estimate [10] of the probability of finding a five-quark $|uud + Q\bar{Q}\rangle$ configuration the nucleon at the order of 1%–2%. In the case of the bottom distribution, $N_{b_{\text{in}}} = 6(m_c/m_b)^2$ [4].

The LFWFs with a heavy struck antiquark $\psi_{\lambda_{\bar{Q}}}^{\lambda_N}(x, \mathbf{k}_\perp)$ are obtained from the LFWFs for a heavy struck quark, upon replacement of the heavy quark PDF by the heavy antiquark PDF, as

$$Q_{\text{in}}(x) \rightarrow \bar{Q}_{\text{in}}(x) = N(1-x)Q_{\text{in}}(x). \quad (5)$$

We have assumed that the $\bar{Q}_{\text{in}}(x)$ has an at least $(1-x)$ falloff at large x in comparison with $Q_{\text{in}}(x)$; i.e., $\bar{Q}_{\text{in}}(x) \sim (1-x)^6$ at $x \rightarrow 1$. The normalization constant N will be fixed using an explicit form of the $Q_{\text{in}}(x)$.

Following Ref. [11], we introduce the asymmetric heavy-antiheavy quark distribution function $Q_{\text{asym}}(x) = Q_{\text{in}}(x) - \bar{Q}_{\text{in}}(x)$. Using the explicit form of $Q_{\text{in}}(x)$ (4), the condition that the zero moments of the $Q_{\text{in}}(x)$ and $\bar{Q}_{\text{in}}(x)$ PDFs should be equal, or that the zero moment of the asymmetry PDF $Q_{\text{asym}}(x)$ should vanish, gives us that $N = 7/5$ and

$$\bar{Q}_{\text{in}}(x) = \frac{7}{5}(1-x)Q_{\text{in}}(x). \quad (6)$$

Finally, the LFWFs with a heavy struck antiquark $\psi_{\bar{Q};\lambda_{\bar{Q}}}^{\lambda_N}(x, \mathbf{k}_\perp)$ are related to the corresponding LFWFs with a heavy struck quark as

$$\psi_{\bar{Q};\lambda_{\bar{Q}}}^{\lambda_N}(x, \mathbf{k}_\perp) = \sqrt{\frac{7}{5}}(1-x)\psi_{Q;\lambda_Q}^{\lambda_N}(x, -\mathbf{k}_\perp). \quad (7)$$

We can also consider the sum of the heavy quark and antiquark PDFs:

$$Q^+(x) = Q_{\text{in}}(x) + \bar{Q}_{\text{in}}(x), \quad (8)$$

and the fraction of the proton momentum $[Q]$ carried by heavy quark and antiquark:

$$[Q] = \int_0^1 dx x Q^+(x) \quad (9)$$

which were recently extracted by the NNPDF Collaboration [3].

In Fig. 2, we present numerical results for the asymmetric heavy-antiheavy quark distribution functions $Q_{\text{asym}}(x)$ for charm and bottom quarks. We note that our results for the intrinsic charm quark asymmetry $c_{\text{asym}}(x)$ are in very good agreement with the predictions in Ref. [11] based on lattice gauge theory. It is also interesting to

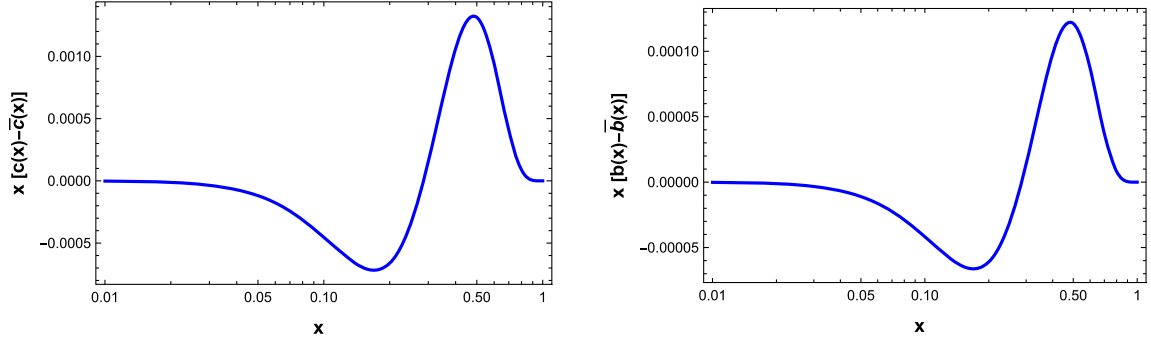


FIG. 2. Plots of the $xc_{\text{asym}}(x)$ (left panel) and $xb_{\text{asym}}(x)$ (right panel) asymmetric distributions.

compare the LFHQCD prediction for the first moment of the asymmetry:

$$\langle x \rangle_{Q-\bar{Q}} = \int_0^1 dx x Q_{\text{asym}}(x) = \frac{1}{3500} \quad (10)$$

with the constraint on the charm-anticharm asymmetry in the nucleon obtained from lattice QCD. Our prediction for the $\langle x \rangle_{c-\bar{c}} = 0.00029$ is in order of magnitude agreement with the prediction of Ref. [11] $\langle x \rangle_{c-\bar{c}} = 0.00047(15)$. Our result for the first moment of the bottom quark asymmetry is suppressed by a factor $(m_c/m_b)^2$ due to the corresponding ratio of the normalization constants $N_{b_m}/N_{c_{\text{in}}} = (m_c/m_b)^2$. This dependence on the mass of the heavy quark reflects the twist of the operators controlling the probability for intrinsic heavy quarks in hadrons. For the heavy quark masses, we have used the central values from the Particle Data Group [12]: $m_c = 1.27$ GeV and $m_b = 4.18$ GeV. This gives $\langle x \rangle_{b-\bar{b}} = 0.000026$.

In Fig. 3, we present numerical results for the symmetric heavy-antiheavy quark distribution functions $xQ^+(x)$, for both charm and bottom quarks. One should stress that our results for the charm quark are in very good agreement with the empirical results obtained by the NNPDF Collaboration [3].

In the case of the charm quark, we assume the normalization constant $N_{c_{\text{in}}} = 6 \pm 3$, which corresponds to a variation of the intrinsic charm probability in the interval $(1 \pm 0.5)\%$ to the proton PDF. Our central curve corresponds to the central value of $N_{c_{\text{in}}} = 6$; the shaded band corresponds to variation of $N_{c_{\text{in}}}$ from 3 to 9. Increasing $N_{c_{\text{in}}}$ leads to increasing the normalization for $xQ^+(x)$. The corresponding results of Ref. [3], shown as a central value curve and shaded band, take into account different flavor number schemes, different orders in the α_s , and uncertainties of the empirical extraction. The main difference with our analysis is that at $x=0$ and small x our results are consistent with the physical meaning of the PDFs: They are distributions and must be positively defined quantities (for both heavy quark and antiquark), whereas the NNPDF results allow for negative distributions, due to the uncertainties mentioned above.

It is interesting to compare the predictions of our approach and NNPDF Collaboration for the fraction of the proton momentum $[Q]$. Our prediction for the central value $N_{c_{\text{in}}} = 6$ is $[Q] = 0.54\%$ [with the variation of $N_{c_{\text{in}}}$ we obtain $[Q] = (0.54 \pm 0.27)\%$]. This is in good agreement with the NNPDF result: $(0.62 \pm 0.28)\%$ in the case when only PDF uncertainties have been included and $(0.62 \pm 0.61)\%$ in the case when the effect of missing higher-order uncertainties have also been included.

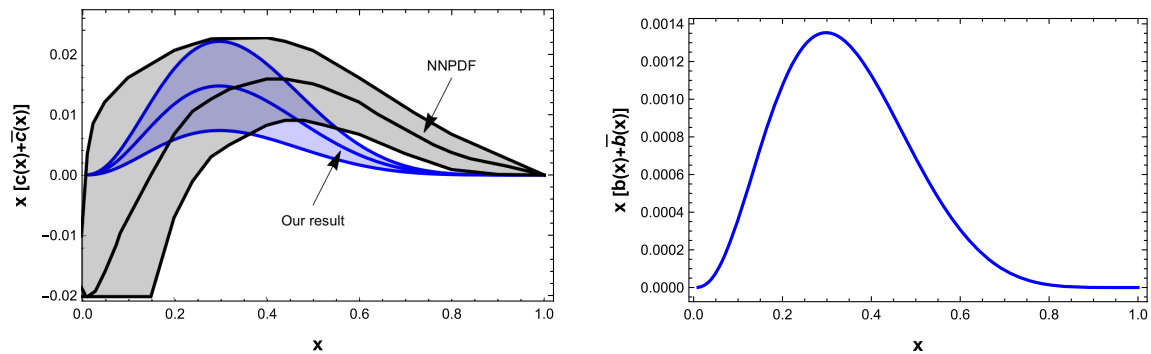


FIG. 3. Plots of the $xc^+(x)$ (left panel) and $xb^+(x)$ (right panel) symmetric distributions. In the case of charm quarks, we present a comparison with results of the NNPDF Collaboration [3] and vary our normalization constant in the region $N_{c_{\text{in}}} = 6 \pm 3$.

As the next step, we calculate the intrinsic contribution of heavy quarks to the nucleon electromagnetic form factors. In LF QCD, the Dirac and Pauli electromagnetic form factors of the nucleon are identified with the spin-conserving and spin-flip current matrix elements and are calculated using the Drell-Yan-West (DYW) [13] and Brodsky-Drell (BD) [14] formulas:

$$F_1^Q(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi_{Q;+\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{Q;+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) + \psi_{Q;-\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{Q;-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) \right], \quad (11)$$

$$F_2^Q(Q^2) = -\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi_{Q;+\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{Q;+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) + \psi_{Q;-\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{Q;-\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) \right] \quad (12)$$

for heavy quark contributions and

$$F_1^{\bar{Q}}(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi_{\bar{Q};+\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{\bar{Q};+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) + \psi_{\bar{Q};-\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{\bar{Q};-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) \right], \quad (13)$$

$$F_2^{\bar{Q}}(Q^2) = -\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi_{\bar{Q};+\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{\bar{Q};+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) + \psi_{\bar{Q};-\frac{1}{2}}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{\bar{Q};-\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp) \right] \quad (14)$$

for the heavy antiquark contributions. Here, $\mathbf{k}'_\perp = \mathbf{k}_\perp \pm \mathbf{q}_\perp(1-x)$.

A straightforward calculation gives the following expressions for the heavy quark-antiquark contributions to the proton Dirac and Pauli form factors:

$$\begin{aligned} F_1^{Q-\bar{Q}}(Q^2) &= F_1^Q(Q^2) - F_1^{\bar{Q}}(Q^2) \\ &= \int_0^1 dx Q_{\text{asym}}(x) \left[1 - \frac{Q^2(1-x)^2}{8\kappa^2} \right] \exp \left[-\frac{Q^2(1-x)^2}{4\kappa^2} \right], \end{aligned} \quad (15)$$

$$\begin{aligned} F_2^{Q-\bar{Q}}(Q^2) &= F_2^Q(Q^2) - F_2^{\bar{Q}}(Q^2) \\ &= \frac{M_N}{\kappa} \int_0^1 dx (1-x) Q_{\text{asym}}(x) \exp \left[-\frac{Q^2(1-x)^2}{4\kappa^2} \right]. \end{aligned} \quad (16)$$

One can see that the Dirac form factors are properly normalized at $Q^2 = 0$:

$$\begin{aligned} F_1^Q(0) &= \int_0^1 dx Q_{\text{in}}(x) = \frac{1}{100}, \quad F_1^{\bar{Q}}(0) = \int_0^1 dx \bar{Q}_{\text{in}}(x) = \frac{1}{100}, \\ F_1^{Q-\bar{Q}}(0) &= F_1^Q(0) - F_1^{\bar{Q}}(0) = \int_0^1 dx Q_{\text{asym}}(x) = 0. \end{aligned} \quad (17)$$

The heavy quark-antiquark contributions to the nucleon Sachs form factors $G_{E/M}^{Q-\bar{Q}}(Q^2)$ and the electromagnetic radii $\langle r_{E/M}^2 \rangle^{Q-\bar{Q}}$ are given in terms of the Dirac and Pauli form factors $F_{1,2}^{Q-\bar{Q}}(Q^2)$ as

$$\begin{aligned} G_E^{Q-\bar{Q}}(Q^2) &= F_1^{Q-\bar{Q}}(Q^2) - \frac{Q^2}{4M_N^2} F_2^{Q-\bar{Q}}(Q^2), \\ G_M^{Q-\bar{Q}}(Q^2) &= F_1^{Q-\bar{Q}}(Q^2) + F_2^{Q-\bar{Q}}(Q^2), \\ \langle r_E^2 \rangle^{Q-\bar{Q}} &= -6 \frac{G_E^{Q-\bar{Q}}(Q^2)}{dQ^2} \Big|_{Q^2=0}, \\ \langle r_M^2 \rangle^{Q-\bar{Q}} &= -\frac{6}{\mu_p} \frac{G_M^{Q-\bar{Q}}(Q^2)}{dQ^2} \Big|_{Q^2=0}, \end{aligned} \quad (18)$$

where $G_M^{Q-\bar{Q}}(0) \equiv F_2^{Q-\bar{Q}}(0) = \mu^{Q-\bar{Q}}$ is the heavy quark-antiquark contribution to the proton magnetic moment. We have normalized the slope $\langle r_M^2 \rangle^{Q-\bar{Q}}$ using the proton magnetic moment $\mu_p = 2.793$. One can see that the magnetic moment $\mu^{Q-\bar{Q}}$ is proportional to the first moment of the asymmetry PDF $Q_{\text{asym}}(x)$ and directly related to the first moment of the asymmetry distribution $\langle x \rangle_{Q-\bar{Q}}$:

$$\begin{aligned} \mu^{Q-\bar{Q}} &= F_2^{Q-\bar{Q}}(0) = \frac{M_N}{\kappa} \int_0^1 dx (1-x) Q_{\text{asym}}(x) \\ &= -\frac{M_N}{\kappa} \langle x \rangle_{Q-\bar{Q}}. \end{aligned} \quad (19)$$

Our prediction for the charm-anticharm and bottom-antibottom contributions to the proton magnetic moment are negative, consistent with other theoretical predictions (see, e.g., discussion in Ref. [11]) and are equal to

$$\mu^{c-\bar{c}} = -5.36 \times 10^{-4}, \quad \mu^{b-\bar{b}} = -4.94 \times 10^{-5}. \quad (20)$$

It is clear why $\mu^{Q-\bar{Q}}$ is negative. The integrand is positive and proportional to $(1-x)$. The 1 contributes to the zero moment, which is zero, while the $-x$ contributes to the first moment, leading to the negative results for $\mu^{Q-\bar{Q}}$.

We note that our prediction for $\mu^{c-\bar{c}}$ is suppressed by a factor of 2.4 in comparison with the result of Ref. [11]: $\mu^{c-\bar{c}} = -1.27 \times 10^{-3}$. Our prediction for the relative scaling of the charm-anticharm and bottom-antibottom contributions to the magnetic moment of proton is $\mu^{b-\bar{b}}/\mu^{c-\bar{c}} \sim (m_c/m_b)^2 \sim 0.1$. Note that a negative contribution to the magnetic moment of the heavy quarks is also a property of the strange quark contribution. A detailed discussion is given in Ref. [15] in the framework of the perturbative chiral quark model (PCQM). The predictions of the PCQM for the strange quark contribution to the nucleon properties have been successfully confirmed from data [16] and lattice calculations [17,18].

Analytic result for the intrinsic heavy quark contributions to the electromagnetic radii of the proton can be obtained using derivatives of the Dirac and Pauli form factors:

$$F_1^{r;Q-\bar{Q}}(0) = -\frac{3}{8\kappa^2} \int_0^1 dx (1-x)^2 Q_{\text{asym}}(x),$$

$$F_2^{r;Q-\bar{Q}}(0) = -\frac{M_N}{4\kappa^3} \int_0^1 dx (1-x)^3 Q_{\text{asym}}(x). \quad (21)$$

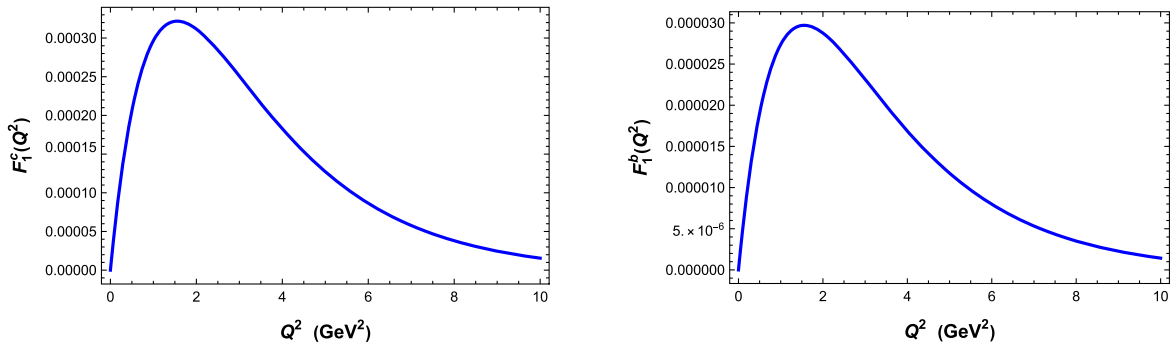


FIG. 4. Heavy quark-antiquark contributions to the Dirac form factor of proton. Left panel: charm-anticharm. Right panel: bottom-antibottom.

One gets

$$\langle r_E^2 \rangle^{Q-\bar{Q}} = -6F_1^{r;Q-\bar{Q}}(0) + \frac{3}{2M_N^2} \mu^{Q-\bar{Q}},$$

$$\langle r_M^2 \rangle^{Q-\bar{Q}} = -6[F_1^{r;Q-\bar{Q}}(0) + F_2^{r;Q-\bar{Q}}(0)]. \quad (22)$$

Numerical results for the charm-anticharm and bottom-antibottom contributions to the electromagnetic slopes are

$$\langle r_E^2 \rangle^{c-\bar{c}} = -1.70 \times 10^{-4} \text{ fm}^2, \quad \langle r_M^2 \rangle^{c-\bar{c}} = -1.12 \times 10^{-4} \text{ fm}^2,$$

$$\langle r_E^2 \rangle^{b-\bar{b}} = -1.57 \times 10^{-5} \text{ fm}^2, \quad \langle r_M^2 \rangle^{b-\bar{b}} = -1.03 \times 10^{-5} \text{ fm}^2. \quad (23)$$

Notice that in our approach the following scaling laws for the ratios of the charm-anticharm and bottom-antibottom contributions to the proton radii are $\langle r_{E,M}^2 \rangle^{b-\bar{b}}/\langle r_{E,M}^2 \rangle^{c-\bar{c}} \sim (m_c/m_b)^2 \sim 0.1$. We also note that our results for the charm sector are in agreement with the results of Ref. [11] by the same sign (negative) and in order of magnitude: $\langle r_E^2 \rangle^{c-\bar{c}} = -0.0005(1) \text{ fm}^2$ and $\langle r_M^2 \rangle^{c-\bar{c}} = -0.0003(1) \text{ fm}^2$. We plot our results for the charm-anticharm and bottom-antibottom contributions to the Dirac, Pauli, and Sachs form factors in Figs. 4–8. In particular, in Fig. 8 we present a comparison of our results for the Sachs form factors $G_E^c(Q^2)$ (left panel) and $G_M^c(Q^2)$ (right panel) form factors with lattice predictions [11] (shaded band and data points with errors).

An important check of our approach concerns the model-independent result obtained in Ref. [19] for the vanishing of the anomalous gravitomagnetic moment $B(0) = 0$ for each Fock state contributing to a composite system, consistent with the general theorem of Okun and Kobzarev [20]. In particular, we will verify that the contribution of the Fock state describing a presence of the heavy quark-antiquark in the proton to the $B(0)$ is zero.

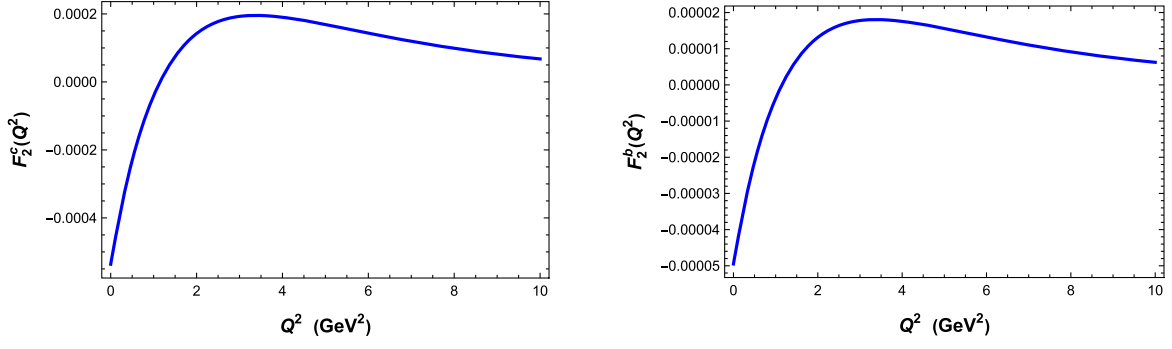


FIG. 5. Heavy quark-antiquark contributions to the Pauli form factor of proton. Left panel: charm-anticharm. Right panel: bottom-antibottom.

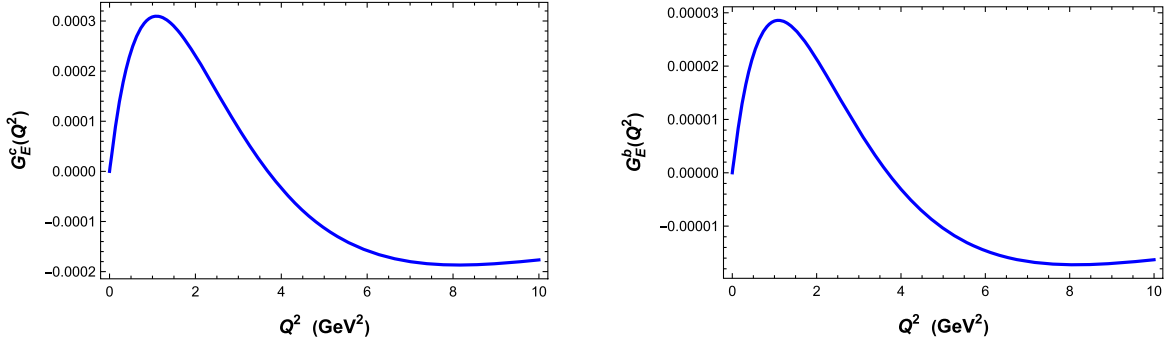


FIG. 6. Heavy quark-antiquark contributions to the Sachs charge form factor of proton. Left panel: charm-anticharm. Right panel: bottom-antibottom.

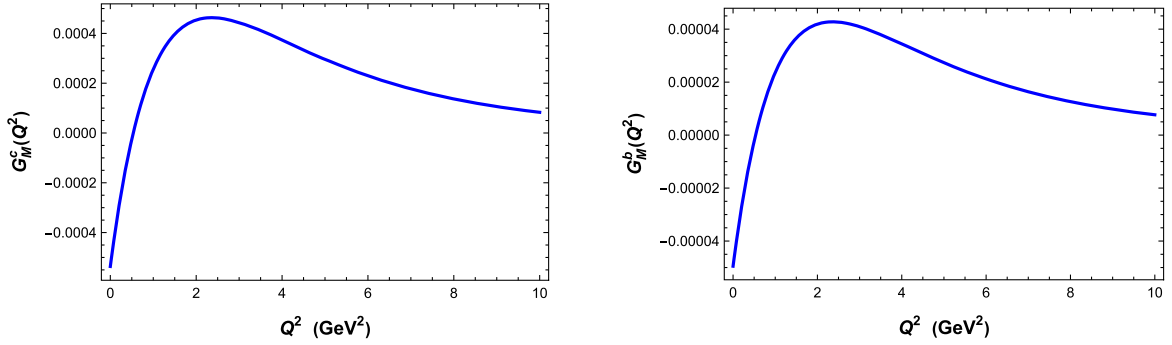


FIG. 7. Heavy quark-antiquark contributions to the Sachs magnetic form factor of proton. Left panel: charm-anticharm. Right panel: bottom-antibottom.

Using the master formula for the $B(0)$ derived in Ref. [19], one has

$$\begin{aligned}
 -\frac{B_Q(0)}{2M_N} &= \lim_{q_\perp^1 \rightarrow 0} \frac{\partial}{\partial q_\perp^1} \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_Q = \pm \frac{1}{2}} [x \psi_{Q:\lambda_Q}^{\dagger\uparrow}(x, \mathbf{k}'_\perp) \psi_{Q:\lambda_Q}^\downarrow(x, \mathbf{k}_\perp) + (1-x) \psi_{Q:\lambda_Q}^{\dagger\uparrow}(x, \mathbf{k}''_\perp) \psi_{Q:\lambda_Q}^\downarrow(x, \mathbf{k}_\perp)] \\
 &= \lim_{q_\perp^1 \rightarrow 0} \frac{\partial}{\partial q_\perp^1} \left[\frac{q_\perp^1}{\kappa} \int_0^1 dx Q_{\text{in}}(x) x(1-x) \underbrace{(1-1)}_{=0} \exp\left[-\frac{(q_\perp^1)^2}{\kappa^2}\right] \right] = 0
 \end{aligned} \tag{24}$$

for the struck heavy quark and

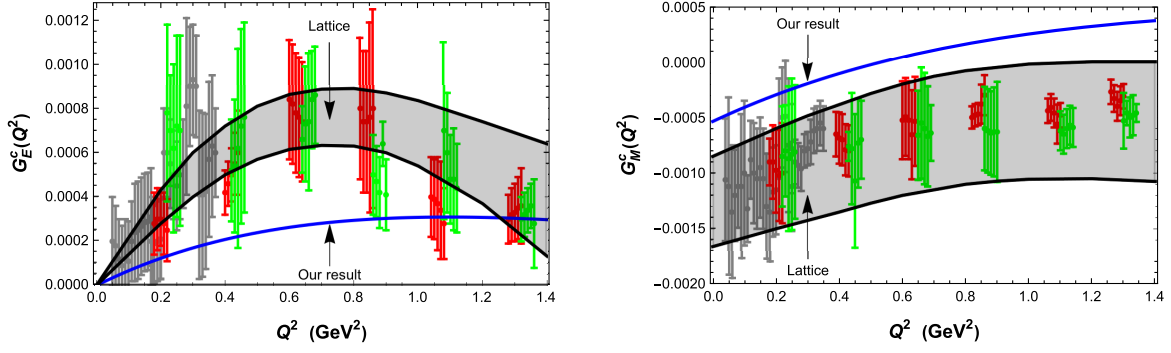


FIG. 8. Comparison of our results for the charm-anticharm contributions to the Sachs form factors $G_E^c(Q^2)$ (left panel) and $G_M^c(Q^2)$ (right panel) with lattice predictions [11] (shaded band and data points with errors).

$$\begin{aligned}
 -\frac{B_{\bar{Q}}(0)}{2M_N} &= \lim_{q_{\perp}^1 \rightarrow 0} \frac{\partial}{\partial q_{\perp}^1} \int_0^1 dx \int \frac{d^2 \mathbf{k}_{\perp}}{16\pi^3} \sum_{\lambda_{\bar{Q}} = \pm \frac{1}{2}} [x \psi_{Q;\lambda_{\bar{Q}}}^{\dagger\uparrow}(x, \mathbf{k}_{\perp}^{\prime-}) \psi_{Q;\lambda_{\bar{Q}}}^{\downarrow}(x, \mathbf{k}_{\perp}) + (1-x) \psi_{Q;\lambda_{\bar{Q}}}^{\dagger\uparrow}(x, \mathbf{k}_{\perp}^{\prime-}) \psi_{Q;\lambda_{\bar{Q}}}^{\downarrow}(x, \mathbf{k}_{\perp})] \\
 &= \lim_{q_{\perp}^1 \rightarrow 0} \frac{\partial}{\partial q_{\perp}^1} \left[\frac{q_{\perp}^1}{\kappa} \int_0^1 dx \frac{7}{5} Q_{\text{in}}(x) x (1-x)^2 \underbrace{(1-1)}_{=0} \exp\left[-\frac{(q_{\perp}^1)^2}{\kappa^2}\right] \right] = 0
 \end{aligned} \tag{25}$$

for the struck heavy antiquark. Here, $\mathbf{k}_{\perp}^{\pm} = (k_{\perp}^1 \pm q_{\perp}^1(1-x), k_{\perp}^2)$ and $\mathbf{k}_{\perp}^{\prime\pm} = (k_{\perp}^1 \mp q_{\perp}^1 x, k_{\perp}^2)$.

In conclusion, we derived LFWFs describing heavy quark-antiquark content in the proton. Using these LFWFs, we construct light-front representations for the proton properties induced by heavy quarks and made numerical applications.

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