Decay behaviors of the fully bottom and fully charm tetraquark states

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We study the decay behaviors of the fully bottom tetraquark states within the diquark-antidiquark picture and calculate their relative branching ratios through the Fierz rearrangement. Our results suggest that the $C = +$ states can be searched for in the $\mu^+\mu^-\Upsilon(1S)$ and $\mu^+\mu^-\Upsilon(2S)$ channels with the relative branching ratio $\mathcal{B}(X \to \mu\mu\Upsilon(2S))/\mathcal{B}(X \to \mu\mu\Upsilon(1S)) \approx 0.4$. Our results also suggest that the $C = -$ states can be searched for in the $\mu^+\mu^-\eta_b(1S)$ and $\mu^+\mu^-\eta_b(2S)$ channels with the similar relative branching ratio $\mathcal{B}(X \to \mu\mu\eta_b(2S))/\mathcal{B}(X \to \mu\mu\eta_b(1S)) \approx 0.4$. We also reanalyze the fully charm tetraquark states and study the $X(6900)$ decay into the $J/\psi\psi(2S)$ channel to obtain the relative branching ratio $\mathcal{B}(X \to J/\psi\psi(2S))/\mathcal{B}(X \to J/\psi J/\psi) \approx 0.1$.

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I. INTRODUCTION

In 2020, the LHCb Collaboration reported their observation of two exotic structures in the di-J/ψ invariant mass spectrum [1], i.e., a broad structure ranging from 6.2 to 6.8 GeV and a narrow structure at around 6.9 GeV. They described the latter as a resonance with the Breit-Wigner line shape, whose mass and width were measured to be

$$X(6900)\colon\ M = 6905 \pm 11 \pm 7\ \text{MeV},\quad \Gamma = 80 \pm 19 \pm 33\ \text{MeV}. \tag{1}$$

These values were obtained under the assumption that no interference with the nonresonant single-parton scattering continuum is present. Assuming that the continuum interferes with the broad structure, the above values were shifted to be

$$X(6900)\colon\ M = 6886 \pm 11 \pm 11\ \text{MeV},\quad \Gamma = 168 \pm 33 \pm 69\ \text{MeV}. \tag{2}$$

The above two structures are good candidates for the fully charm tetraquark states, and their observation immediately attracted much attention from the particle physics community [2–50]. We refer to our recent review [51] as well as the reviews [52–72] and the reports [73–75] for their detailed discussions. Especially, some theorists reanalyzed the LHCb data on the di-J/ψ spectrum [1] and proposed the existence of more structures; e.g., the authors of Ref. [76] reproduced three peak structures at near 6.5, 6.9, and 7.3 GeV, while the authors of Ref. [77] proposed the existence of a near-threshold state in the di-J/ψ system at near 6.2 GeV. We refer to Refs. [78–87] and the reviews [51,71] for more discussions.

Very recently, the CMS and ATLAS Collaborations also investigated the di-J/ψ invariant mass spectrum, and both of them confirmed the existence of the $X(6900)$ [88,89]. Besides, the CMS Collaboration observed two new structures, the $X(6600)$ and $X(7200)$, in the di-J/ψ invariant mass spectrum. Their masses and widths were measured to be [88]

$$X(6600)\colon\ M = 6552 \pm 10 \pm 12\ \text{MeV},\quad \Gamma = 124 \pm 29 \pm 34\ \text{MeV}; \tag{3}$$

$$X(6900)\colon\ M = 6927 \pm 9 \pm 5\ \text{MeV},\quad \Gamma = 122 \pm 22 \pm 19\ \text{MeV}; \tag{4}$$

$$X(7200)\colon\ M = 7287 \pm 19 \pm 5\ \text{MeV},\quad \Gamma = 95 \pm 46 \pm 20\ \text{MeV}. \tag{5}$$

The ATLAS Collaboration investigated the di-J/ψ invariant mass spectrum, and their best fit was performed with three

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interfering resonances, whose masses and widths were measured to be [89]

$$X(6200):\ M = 6.22 \pm 0.05^{+0.04}_{-0.05}\ \text{GeV},\quad \Gamma = 0.31 \pm 0.12^{+0.07}_{-0.08}\ \text{GeV}; \tag{6}$$

$$X(6600):\ M = 6.62 \pm 0.03^{+0.02}_{-0.01}\ \text{GeV},\quad \Gamma = 0.31 \pm 0.09^{+0.06}_{-0.11}\ \text{GeV}; \tag{7}$$

$$X(6900):\ M = 6.87 \pm 0.03^{+0.06}_{-0.01}\ \text{GeV},\quad \Gamma = 0.12 \pm 0.04^{+0.03}_{-0.01}\ \text{GeV}. \tag{8}$$

The ATLAS Collaboration also investigated the $J/\psi\psi(2S)$ invariant mass spectrum. They reported the evidence for an enhancement at 6.9 GeV and a resonance at 7.2 GeV, whose masses and widths were measured to be [89]

$$X(6900):\ M = 6.78 \pm 0.36^{+0.35}_{-0.54}\ \text{GeV},\quad \Gamma = 0.39 \pm 11^{+0.11}_{-0.07}\ \text{GeV}; \tag{9}$$

$$X(7200):\ M = 7.22 \pm 0.03^{+0.02}_{-0.03}\ \text{GeV},\quad \Gamma = 0.10^{+0.13}_{-0.07}{}^{+0.06}_{-0.05}\ \text{GeV}. \tag{10}$$

Actually, the fully heavy tetraquark states were already studied by some theorists in the 1980s [90–98], but there have not been many relevant experiments from that time till now. Besides the above experiments [1,88,89], in 2017 the CMS Collaboration found an excess in the $\Upsilon(1S)\mu^+\mu^-$ invariant mass spectrum near 18.5 GeV with a global significance of 3.6σ [99,100], and in 2019 the ANDY Collaboration at the Relativistic Heavy Ion Collider reported an evidence of a significance peak at around 18.12 GeV [101]. These structures are good candidates for the fully bottom tetraquark states, and they also attracted some attention from the particle physics community [102–117]. Since these structures were not confirmed by some other experiments [118,119], they require more investigations crucially.

In Ref. [120], we have studied the two-body decay behaviors of the fully charm tetraquark states within the diquark-antidiquark picture and calculated their relative branching ratios through the Fierz rearrangement of the Dirac and color indices. In this paper, we shall further study the decay behaviors of the fully bottom tetraquark states. As summarized in Table I, our previous QCD sum rule results suggest that the fully charm tetraquark states lie above the dicharmonium thresholds, while some of the fully bottom tetraquark states lie below the dibottomonium thresholds [121]. Accordingly, in this paper, we shall investigate not the fall-apart two-body decays, but the three-body decays of the fully bottom tetraquark states. We shall study their decays into one bottomonium meson and one muon-antimuon pair, with the muon-antimuon pair produced by another intermediate vector bottomonium meson. This method has been applied in Refs. [122–126] to investigate some other exotic hadrons, such as the $Z_c(3900)$ and the P_c states, etc. Especially, our results obtained in Ref. [122] for the $Z_c(3900)$ are consistent with those obtained in Refs. [127–129] using the QCD sum rule method and the nonrelativistic effective field theory. Besides, a similar arrangement of the spin and color indices in the nonrelativistic case was applied in Refs. [130–135] to study the decay properties of exotic hadrons, and our results obtained in Ref. [124] for the P_c states are consistent with those obtained in Ref. [130] through the heavy quark spin symmetry.

TABLE I. Mass spectra of the fully charm and fully bottom tetraquark states, calculated in Ref. [121] through the QCD sum rule method.

J^{PC}	Currents	$T_{cc\bar{c}\bar{c}}$ (GeV)	$T_{bb\bar{b}\bar{b}}$ (GeV)
0^{++}	$J_1^{0^{++}}$	6.44 ± 0.15	18.45 ± 0.15
	$J_2^{0^{++}}$	6.46 ± 0.16	18.46 ± 0.14
1^{+-}	$J_{3\alpha}^{1^{+-}}$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{4\alpha\beta}^{2^{++}}$	6.51 ± 0.15	18.53 ± 0.15
0^{-+}	$J_5^{0^{-+}}$	6.84 ± 0.18	18.77 ± 0.18
	$J_6^{0^{-+}}$	6.85 ± 0.18	18.79 ± 0.18
0^{--}	$J_7^{0^{--}}$	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{8\alpha}^{1^{-+}}$	6.84 ± 0.18	18.80 ± 0.18
	$J_{9\alpha}^{1^{-+}}$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{10\alpha}^{1^{--}}$	6.84 ± 0.18	18.77 ± 0.18
	$J_{11\alpha}^{1^{--}}$	6.83 ± 0.18	18.77 ± 0.16

This paper is organized as follows. In Sec. II, we construct the fully bottom tetraquark currents within the diquark-antidiquark picture and apply the Fierz rearrangement to transform them into the meson-meson currents. Based on the obtained Fierz identities, we study the decay behaviors of the fully bottom tetraquark states in Sec. III and calculate their relative branching ratios. The results are summarized and discussed in Sec. IV.

II. CURRENTS AND FIERZ IDENTITIES

In Refs. [120,121], we have systematically constructed all the fully heavy tetraquark currents without derivatives. We briefly summarize them in this section, which will be used to study the decay behaviors of the fully bottom tetraquark states in the next section.

Besides, in Ref. [136], we have systematically constructed all the P-wave fully strange tetraquark currents by explicitly adding the covariant derivative operator. Their corresponding fully heavy tetraquark currents with derivatives can be similarly constructed. See Refs. [137,138] for more discussions. However, we shall not investigate them in the present study, since many relevant decay constants are not known yet. We also refer to Refs. [2,139], where the authors systematically classified the S- and P-wave fully heavy and strange tetraquark states using the nonrelativistic quark model within the diquark-antidiquark picture.

A. Currents of the positive parity

There are altogether 12 fully bottom tetraquark currents of the positive parity, four of which correspond to the S-wave fully bottom tetraquark states within the diquark-antidiquark picture:

$$J_1^{0^{++}} = b_a^T C\gamma_5 b_b \bar{b}_a \gamma_5 C \bar{b}_b^T, \tag{11}$$

$$J_2^{0^{++}} = b_a^T C\gamma_\mu b_b \bar{b}_a \gamma^\mu C \bar{b}_b^T, \tag{12}$$

$$J_{3\alpha}^{1^{+-}} = b_a^T C\gamma^\mu b_b \bar{b}_a \sigma_{\alpha\mu}\gamma_5 C \bar{b}_b^T - b_a^T C\sigma_{\alpha\mu}\gamma_5 b_b \bar{b}_a \gamma^\mu C \bar{b}_b^T, \tag{13}$$

$$J_{4\alpha\beta}^{2^{++}} = \mathcal{P}_{\alpha\beta}^{\mu\nu} b_a^T C\gamma_\mu b_b \bar{b}_a \gamma_\nu C \bar{b}_b^T. \tag{14}$$

In the above expressions, b_a is the bottom quark field with the color index a, and $\mathcal{P}_{\alpha\beta}^{\mu\nu}$ is the projection operator:

$$\mathcal{P}^{\alpha\beta;\mu\nu} = g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu} - \frac{1}{2} g^{\alpha\beta} g^{\mu\nu}. \tag{15}$$

After applying the Fierz transformation, we obtain

$$J_1^{0^{++}} = -\frac{1}{4}\xi_1^{0^{++}} - \frac{1}{4}\xi_2^{0^{++}} - \frac{1}{4}\xi_3^{0^{++}} - \frac{1}{4}\xi_4^{0^{++}} + \frac{1}{8}\xi_5^{0^{++}}, \tag{16}$$

$$J_2^{0^{++}} = \xi_1^{0^{++}} - \xi_2^{0^{++}} + \frac{1}{2}\xi_3^{0^{++}} - \frac{1}{2}\xi_4^{0^{++}}, \tag{17}$$

$$J_{3\alpha}^{1^{+-}} = 3i\xi_{6\alpha}^{1^{+-}} - \xi_{7\alpha}^{1^{+-}}, \tag{18}$$

$$J_{4\alpha\beta}^{2^{++}} = \frac{1}{2}\xi_{8\alpha\beta}^{2^{++}} - \frac{1}{2}\xi_{9\alpha\beta}^{2^{++}} + \frac{1}{2}\xi_{10\alpha\beta}^{2^{++}}, \tag{19}$$

where

$$\begin{aligned}
\xi_1^{0^{++}} &= \bar{b}_a b_a \bar{b}_b b_b, \\
\xi_2^{0^{++}} &= \bar{b}_a \gamma_5 b_a \bar{b}_b \gamma_5 b_b, \\
\xi_3^{0^{++}} &= \bar{b}_a \gamma_\mu b_a \bar{b}_b \gamma^\mu b_b, \\
\xi_4^{0^{++}} &= \bar{b}_a \gamma_\mu\gamma_5 b_a \bar{b}_b \gamma^\mu\gamma_5 b_b, \\
\xi_5^{0^{++}} &= \bar{b}_a \sigma_{\mu\nu} b_a \bar{b}_b \sigma^{\mu\nu} b_b, \\
\xi_{6\alpha}^{1^{+-}} &= \bar{b}_a \gamma_5 b_a \bar{b}_b \gamma_\alpha b_b, \\
\xi_{7\alpha}^{1^{+-}} &= \bar{b}_a \gamma^\mu\gamma_5 b_a \bar{b}_b \sigma_{\alpha\mu} b_b, \\
\xi_{8\alpha\beta}^{2^{++}} &= \mathcal{P}_{\alpha\beta}^{\mu\nu} \bar{b}_a \gamma_\mu\gamma_5 b_a \bar{b}_b \gamma_\nu\gamma_5 b_b, \\
\xi_{9\alpha\beta}^{2^{++}} &= \mathcal{P}_{\alpha\beta}^{\mu\nu} \bar{b}_a \gamma_\mu b_a \bar{b}_b \gamma_\nu b_b, \\
\xi_{10\alpha\beta}^{2^{++}} &= \mathcal{P}_{\alpha\beta}^{\mu\nu} \bar{b}_a \sigma_{\mu\rho} b_a \bar{b}_b \sigma_{\nu\rho} b_b.
\end{aligned} \tag{20}$$

B. Currents of the negative parity

There are altogether seven fully bottom tetraquark currents of the negative parity:

$$J_5^{0^{-+}} = b_a^T C b_b \bar{b}_a \gamma_5 C \bar{b}_b^T + b_a^T C\gamma_5 b_b \bar{b}_a C \bar{b}_b^T, \tag{21}$$

$$J_6^{0^{-+}} = b_a^T C\sigma_{\mu\nu} b_b \bar{b}_a \sigma^{\mu\nu}\gamma_5 C \bar{b}_b^T, \tag{22}$$

$$J_7^{0^{--}} = b_a^T C b_b \bar{b}_a \gamma_5 C \bar{b}_b^T - b_a^T C\gamma_5 b_b \bar{b}_a C \bar{b}_b^T, \tag{23}$$

$$J_{8\alpha}^{1^{-+}} = b_a^T C\gamma_\alpha\gamma_5 b_b \bar{b}_a \gamma_5 C \bar{b}_b^T + b_a^T C\gamma_5 b_b \bar{b}_a \gamma_\alpha\gamma_5 C \bar{b}_b^T, \tag{24}$$

$$J_{9\alpha}^{1^{-+}} = b_a^T C\sigma_{\alpha\mu} b_b \bar{b}_a \gamma^\mu C \bar{b}_b^T + b_a^T C\gamma^\mu b_b \bar{b}_a \sigma_{\alpha\mu} C \bar{b}_b^T, \tag{25}$$

$$J_{10\alpha}^{1^{--}} = b_a^T C\gamma_\alpha\gamma_5 b_b \bar{b}_a \gamma_5 C \bar{b}_b^T - b_a^T C\gamma_5 b_b \bar{b}_a \gamma_\alpha\gamma_5 C \bar{b}_b^T, \tag{26}$$

$$J_{11\alpha}^{1^{--}} = b_a^T C\sigma_{\alpha\mu} b_b \bar{b}_a \gamma^\mu C \bar{b}_b^T - b_a^T C\gamma^\mu b_b \bar{b}_a \sigma_{\alpha\mu} C \bar{b}_b^T. \tag{27}$$

After applying the Fierz transformation, we obtain

$$J_5^{0^{-+}} = -\xi_{11}^{0^{-+}} + \frac{1}{4}\xi_{12}^{0^{-+}}, \tag{28}$$

$$J_6^{0^{-+}} = 6\xi_{11}^{0^{-+}} - \frac{1}{2}\xi_{12}^{0^{-+}}, \tag{29}$$

$$J_7^{0^{--}} = -\xi_{13}^{0^{--}}, \tag{30}$$

$$J_{8\alpha}^{1^{-+}} = -\xi_{14\alpha}^{1^{-+}} + i\xi_{15\alpha}^{1^{-+}}, \tag{31}$$

$$J_{9\alpha}^{1^{-+}} = -3i\xi_{14\alpha}^{1^{-+}} + \xi_{15\alpha}^{1^{-+}}, \tag{32}$$

$$J_{10\alpha}^{1^{--}} = \xi_{16\alpha}^{1^{--}} - i\xi_{17\alpha}^{1^{--}}, \tag{33}$$

$$J_{11\alpha}^{1^{--}} = -3i\xi_{16\alpha}^{1^{--}} + \xi_{17\alpha}^{1^{--}}, \tag{34}$$

where

$$\begin{aligned}
\xi_{11}^{0^{-+}} &= \bar{b}_a b_a \bar{b}_b \gamma_5 b_b, \\
\xi_{12}^{0^{-+}} &= \bar{b}_a \sigma_{\mu\nu} b_a \bar{b}_b \sigma^{\mu\nu}\gamma_5 b_b, \\
\xi_{13}^{0^{--}} &= \bar{b}_a \gamma_\mu b_a \bar{b}_b \gamma^\mu\gamma_5 b_b, \\
\xi_{14\alpha}^{1^{-+}} &= \bar{b}_a \gamma_5 b_a \bar{b}_b \gamma_\alpha\gamma_5 b_b, \\
\xi_{15\alpha}^{1^{-+}} &= \bar{b}_a \gamma^\mu b_a \bar{b}_b \sigma_{\alpha\mu} b_b, \\
\xi_{16\alpha}^{1^{--}} &= \bar{b}_a b_a \bar{b}_b \gamma_\alpha b_b, \\
\xi_{17\alpha}^{1^{--}} &= \bar{b}_a \gamma^\mu\gamma_5 b_a \bar{b}_b \sigma_{\alpha\mu}\gamma_5 b_b.
\end{aligned} \tag{35}$$

III. RELATIVE BRANCHING RATIOS

In this section, we study possible decay channels of the fully bottom tetraquark states and calculate their relative branching ratios. The same method has been applied in

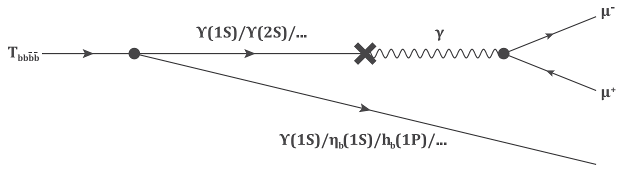

FIG. 1. Decay mechanism of a fully bottom tetraquark state into one bottomonium meson and one intermediate $\Upsilon(1S)/\Upsilon(2S)/\ldots$ meson, with the intermediate $\Upsilon(1S)/\Upsilon(2S)/\ldots$ meson annihilating to be a photon and then transferring into a muon-antimuon pair.

Ref. [120] to study the fully charm tetraquark states. We separately investigate the S- and P-wave fully bottom tetraquark states as follows.

(i) According to our previous QCD sum rule calculations summarized in Table I, we assume the masses of the S-wave fully bottom tetraquark states to be about 18.5 GeV. This value is below the $\eta_b(1S)\eta_b(1S)/\eta_b(1S)\Upsilon(1S)/\Upsilon(1S)\Upsilon(1S)$ thresholds, so the S-wave fully bottom tetraquark states cannot fall-apart decay into these two-body channels. Instead, they can decay into one bottomonium meson and one intermediate $\Upsilon(1S)/\Upsilon(2S)/\ldots$ meson, with the intermediate meson annihilating to be a photon and then transferring into a muon-antimuon pair. We depict this decay process in Fig. 1.

(ii) According to our previous QCD sum rule calculations summarized in Table I, we assume the masses of the P-wave fully bottom tetraquark states to be about 18.8 GeV. This value is below the $\eta_b(1S)\Upsilon(1S)/\Upsilon(1S)\Upsilon(1S)$ thresholds, so the P-wave fully bottom tetraquark states cannot fall-apart decay into these two-body channels. This value is above the $\eta_b(1S)\eta_b(1S)$ threshold, but the P-wave fully bottom tetraquark states of $J^{PC} = 0^{-\pm}/1^{-\pm}$ cannot decay into this two-body channel either, due to either the C-parity conservation or the Bose-Einstein statistics. Accordingly, we shall also investigate the three-body decay process depicted in Fig. 1.

As an example, we apply the Fierz rearrangement given in Eq. (16) to investigate the decay properties of the fully bottom tetraquark state of $J^{PC} = 0^{++}$ corresponding to the current $J_1^{0^{++}}$ defined in Eq. (11). We denote this state as $|X_1; 0^{++}\rangle$ and assume the coupling to be

$$\langle 0|J_1^{0^{++}}|X_1; 0^{++}\rangle = f_{X_1}, \tag{36}$$

with f_{X_1} the decay constant. As summarized in Table I, its mass has been calculated in Ref. [120] through the QCD sum rule method to be 18.45 ± 0.15 GeV.

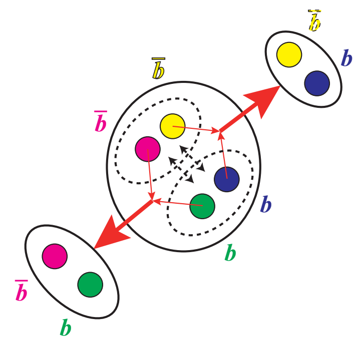

FIG. 2. The fall-apart decay of a compact diquark-antidiquark $[bb][\bar{b}\bar{b}]$ state into two bottomonium states. Quarks are shown in red, green, and blue, and antiquarks are shown in cyan, magenta, and yellow.

As depicted in Fig. 2, when the magenta $\bar{b}$ antiquark and the green b quark meet each other, and the yellow $\bar{b}$ antiquark and the blue b quark meet each other at the same time, $|X_1; 0^{++}\rangle$ can decay into two bottomonium states:

$$\begin{aligned}[b_a(x)b_b(x)][\bar{b}_a(x)\bar{b}_b(x)] &\xrightarrow{\text{Fierz}} [\bar{b}_a(x)b_a(x)][\bar{b}_b(x)b_b(x)] \\ &\xrightarrow{\text{decay}} [\bar{b}_a(y)b_a(y)][\bar{b}_b(z)b_b(z)].\end{aligned} \tag{37}$$

This decay process can be described by the Fierz rearrangement given in Eq. (16), i.e.,

$$\begin{aligned}J_1^{0^{++}} = b_a^T C\gamma_5 b_b \bar{b}_a \gamma_5 C \bar{b}_b^T \rightarrow &-\frac{1}{4}\bar{b}_a b_a \bar{b}_b b_b - \frac{1}{4}\bar{b}_a\gamma_5 b_a \bar{b}_b \gamma_5 b_b \\ &-\frac{1}{4}\bar{b}_a\gamma_\mu\gamma_5 b_a \bar{b}_b\gamma^\mu\gamma_5 b_b - \frac{1}{4}\bar{b}_a\gamma_\mu b_a \bar{b}_b\gamma^\mu b_b \\ &+\frac{1}{8}\bar{b}_a\sigma_{\mu\nu} b_a \bar{b}_b\sigma^{\mu\nu} b_b.\end{aligned} \tag{38}$$

In principle, we need the decay constant f_{X_1} as an input to calculate the partial decay widths, but it is not necessary any more if we calculate only the relative branching ratios. Moreover, because the couplings of meson operators to meson states have been well studied in the literature, but the couplings of tetraquark currents to tetraquark states have not, the decay constant f_{X_1} is not so well determined compared to the bottomonium decay constants listed in Table II. Therefore, we can calculate the relative branching ratios more reliably than the partial decay widths.

To do this, we apply Eq. (38) to derive the couplings of the current $J_1^{0^{++}}$ to both the $\Upsilon(1S)\Upsilon(1S)$ and $\Upsilon(1S)h_b(1P)$ channels to be

TABLE II. Couplings of the charmonium and bottomonium operators to the charmonium and bottomonium states.

Operators	J^{PC}	Mesons	J^{PC}	Couplings	Decay constants
$I^S = \bar{c}c$	0^{++}	$\chi_{c0}(1P)$	0^{++}	$\langle 0\|I^S\|\chi_{c0}(1P)\rangle = m_{\chi_{c0}(1P)} f_{\chi_{c0}(1P)}$	$f_{\chi_{c0}(1P)} = 343$ MeV [140]
$I^P = \bar{c}i\gamma_5 c$	0^{-+}	$\eta_c(1S)$	0^{-+}	$\langle 0\|I^P\|\eta_c(1S)\rangle = \lambda_{\eta_c(1S)}$	$\lambda_{\eta_c(1S)} = \frac{f_{\eta_c(1S)} m^2_{\eta_c(1S)}}{2m_c}$
		$\eta_c(2S)$	0^{-+}	$\langle 0\|I^P\|\eta_c(2S)\rangle = \lambda_{\eta_c(2S)}$	$\lambda_{\eta_c(2S)} = \lambda_{\eta_c(1S)} \times \frac{f_{\psi(2S)}}{f_{J/\psi}}$
$I^V_\mu = \bar{c}\gamma_\mu c$	1^{--}	J/ψ	1^{--}	$\langle 0\|I^V_\mu\|J/\psi\rangle = m_{J/\psi} f_{J/\psi} \epsilon_\mu$	$f_{J/\psi} = 418$ MeV [141]
		$\psi(2S)$	1^{--}	$\langle 0\|I^V_\mu\|\psi(2S)\rangle = m_{\psi(2S)} f_{\psi(2S)} \epsilon_\mu$	$f_{\psi(2S)} = 294.5$ MeV [142]
$I^A_\mu = \bar{c}\gamma_\mu\gamma_5 c$	1^{++}	$\eta_c(1S)$	0^{-+}	$\langle 0\|I^A_\mu\|\eta_c(1S)\rangle = i p_\mu f_{\eta_c(1S)}$	$f_{\eta_c(1S)} = 387$ MeV [141]
		$\eta_c(2S)$	0^{-+}	$\langle 0\|I^A_\mu\|\eta_c(2S)\rangle = i p_\mu f_{\eta_c(2S)}$	$f_{\eta_c(2S)} = f_{\eta_c(1S)} \times \frac{f_{\psi(2S)}}{f_{J/\psi}}$
		$\chi_{c1}(1P)$	1^{++}	$\langle 0\|I^A_\mu\|\chi_{c1}(1P)\rangle = m_{\chi_{c1}(1P)} f_{\chi_{c1}(1P)} \epsilon_\mu$	$f_{\chi_{c1}(1P)} = 335$ MeV [143]
$I^T_{\mu\nu} = \bar{c}\sigma_{\mu\nu} c$	$1^{\pm-}$	J/ψ	1^{--}	$\langle 0\|I^T_{\mu\nu}\|J/\psi\rangle = i f^T_{J/\psi}(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$	$f^T_{J/\psi} = 410$ MeV [141]
		$\psi(2S)$	1^{--}	$\langle 0\|I^T_{\mu\nu}\|\psi(2S)\rangle = i f^T_{\psi(2S)}(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$	$f^T_{\psi(2S)} = f^T_{J/\psi} \times \frac{f_{\psi(2S)}}{f_{J/\psi}}$
		$h_c(1P)$	1^{+-}	$\langle 0\|I^T_{\mu\nu}\|h_c(1P)\rangle = i f^T_{h_c(1P)} \epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha p^\beta$	$f^T_{h_c(1P)} = 235$ MeV [141]
$J^S = \bar{b}b$	0^{++}	$\chi_{b0}(1P)$	0^{++}	$\langle 0\|J^S\|\chi_{b0}(1P)\rangle = m_{\chi_{b0}(1P)} f_{\chi_{b0}(1P)}$	$f_{\chi_{b0}(1P)} = 175$ MeV [140]
$J^P = \bar{b}i\gamma_5 b$	0^{-+}	$\eta_b(1S)$	0^{-+}	$\langle 0\|J^P\|\eta_b(1S)\rangle = \lambda_{\eta_b(1S)}$	$\lambda_{\eta_b(1S)} = \frac{f_{\eta_b(1S)} m^2_{\eta_b(1S)}}{2m_b}$
		$\eta_b(2S)$	0^{-+}	$\langle 0\|J^P\|\eta_b(2S)\rangle = \lambda_{\eta_b(2S)}$	$\lambda_{\eta_b(2S)} = \lambda_{\eta_b(1S)} \times \frac{f_{\Upsilon(2S)}}{f_{\Upsilon(1S)}}$
$J^V_\mu = \bar{b}\gamma_\mu b$	1^{--}	$\Upsilon(1S)$	1^{--}	$\langle 0\|J^V_\mu\|\Upsilon(1S)\rangle = m_{\Upsilon(1S)} f_{\Upsilon(1S)} \epsilon_\mu$	$f_{\Upsilon(1S)} = 715$ MeV [142]
		$\Upsilon(2S)$	1^{--}	$\langle 0\|J^V_\mu\|\Upsilon(2S)\rangle = m_{\Upsilon(2S)} f_{\Upsilon(2S)} \epsilon_\mu$	$f_{\Upsilon(2S)} = 497.5$ MeV [142]
$J^A_\mu = \bar{b}\gamma_\mu\gamma_5 b$	1^{++}	$\eta_b(1S)$	0^{-+}	$\langle 0\|J^A_\mu\|\eta_b(1S)\rangle = i p_\mu f_{\eta_b(1S)}$	$f_{\eta_b(1S)} = 801$ MeV [117]
		$\eta_b(2S)$	0^{-+}	$\langle 0\|J^A_\mu\|\eta_b(2S)\rangle = i p_\mu f_{\eta_b(2S)}$	$f_{\eta_b(2S)} = f_{\eta_b(1S)} \times \frac{f_{\Upsilon(2S)}}{f_{\Upsilon(1S)}}$
		$\chi_{b1}(1P)$	1^{++}	$\langle 0\|J^A_\mu\|\chi_{b1}(1P)\rangle = m_{\chi_{b1}(1P)} f_{\chi_{b1}(1P)} \epsilon_\mu$	$f_{\chi_{b1}(1P)} = f_{\chi_{b0}(1P)} \times \frac{f_{\chi_{c1}(1P)}}{f_{\chi_{c0}(1P)}}$
$J^T_{\mu\nu} = \bar{b}\sigma_{\mu\nu} b$	$1^{\pm-}$	$\Upsilon(1S)$	1^{--}	$\langle 0\|J^T_{\mu\nu}\|\Upsilon(1S)\rangle = i f^T_{\Upsilon(1S)}(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$	$f^T_{\Upsilon(1S)} = f^T_{J/\psi} \times \frac{f_{\Upsilon(1S)}}{f_{J/\psi}}$
		$\Upsilon(2S)$	1^{--}	$\langle 0\|J^T_{\mu\nu}\|\Upsilon(2S)\rangle = i f^T_{\Upsilon(2S)}(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$	$f^T_{\Upsilon(2S)} = f^T_{\Upsilon(1S)} \times \frac{f_{\Upsilon(2S)}}{f_{\Upsilon(1S)}}$
		$h_b(1P)$	1^{+-}	$\langle 0\|J^T_{\mu\nu}\|h_b(1P)\rangle = i f^T_{h_b(1P)} \epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha p^\beta$	$f^T_{h_b(1P)} = f^T_{h_c(1P)} \times \frac{f_{\Upsilon(1S)}}{f_{J/\psi}}$

$$\langle 0|J_1^{0^{++}}|\Upsilon(p_1,\epsilon_1)\Upsilon(p_2,\epsilon_2)\rangle = \epsilon_1^\mu\epsilon_2^\nu\left(-\frac{1}{2}m_\Upsilon^2 f_\Upsilon^2 g_{\mu\nu} - \frac{1}{2}(f_\Upsilon^T)^2 p_1\cdot p_2 g_{\mu\nu} + \frac{1}{2}(f_\Upsilon^T)^2 p_{1\nu}p_{2\mu}\right), \tag{39}$$

$$\langle 0|J_1^{0^{++}}|\Upsilon(p_1,\epsilon_1)h_b(p_2,\epsilon_2)\rangle = -\frac{1}{2}\times\epsilon_1^\mu\epsilon_2^\nu f_\Upsilon^T f_{h_b}^T \epsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma, \tag{40}$$

from which we further extract the couplings of $|X_1;0^{++}\rangle$ to both the $\Upsilon(1S)\Upsilon(1S)$ and $\Upsilon(1S)h_b(1P)$ channels to be

$$\langle X_1(p);0^{++}|\Upsilon(p_1,\epsilon_1)\Upsilon(p_2,\epsilon_2)\rangle = c\times\epsilon_1^\mu\epsilon_2^\nu\left(-\frac{1}{2}m_\Upsilon^2 f_\Upsilon^2 g_{\mu\nu} - \frac{1}{2}(f_\Upsilon^T)^2 p_1\cdot p_2 g_{\mu\nu} + \frac{1}{2}(f_\Upsilon^T)^2 p_{1\nu}p_{2\mu}\right), \tag{41}$$

$$\langle X_1(p);0^{++}|\Upsilon(p_1,\epsilon_1)h_b(p_2,\epsilon_2)\rangle = -\frac{c}{2}\times\epsilon_1^\mu\epsilon_2^\nu f_\Upsilon^T f_{h_b}^T \epsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma. \tag{42}$$

The decay constants $f_\Upsilon \equiv f_{\Upsilon(1S)}$, $f_\Upsilon^T \equiv f_{\Upsilon(1S)}^T$, and $f_{h_b}^T \equiv f_{h_b(1P)}^T$ of the $\Upsilon(1S)$ and $h_b(1P)$ mesons are given in Table II. The overall factor c is related to the decay constant f_{X_1}, which will be eliminated when calculating the relative branching ratios.

Based on Eq. (41), we can write the decay amplitude of the three-body decay process $|X_1;0^{++}\rangle \to \Upsilon(1S)\Upsilon(1S) \to \Upsilon(1S)\mu^+\mu^-$ as

$$\mathcal{M}(X_1(p) \to \Upsilon(p_1,\epsilon_1)\Upsilon(q,\epsilon_2) \to \Upsilon(p_1,\epsilon_1)\mu^-(p_2)\mu^+(p_3))$$
$$= cc'e \times \epsilon_1^\mu \left(-\frac{1}{2}m_\Upsilon^2 f_\Upsilon^2 g_{\mu\nu} - \frac{1}{2}(f_\Upsilon^T)^2 p_1\cdot q g_{\mu\nu} + \frac{1}{2}(f_\Upsilon^T)^2 p_{1\nu}q_\mu\right) \frac{\bar{u}(p_2)\gamma_\alpha v(p_3)}{q^2(q^2-m_\Upsilon^2+\mathrm{i}m_\Upsilon\Gamma_\Upsilon)}\left(g^{\alpha\nu} - \frac{q^\alpha q^\nu}{m_\Upsilon^2}\right), \quad (43)$$

where $u(p_2)$ and $v(p_3)$ are the Dirac spinors of the μ^- and μ^+, respectively. The overall factor c' is related to the coupling of $\Upsilon(1S)$ to the photon, which will also be eliminated when calculating the relative branching ratios.

We use Eq. (43) to further evaluate the partial decay width to be

$$\Gamma(X_1(p) \to \Upsilon(p_1,\epsilon_1)\mu^+(p_2)\mu^-(p_3)) = \frac{1}{(2\pi)^3}\frac{c^2c'^2e^2}{32m_{X_1}^3}\int \mathrm{d}m_{12}^2\mathrm{d}m_{23}^2\left|\frac{1}{q^2-m_\Upsilon^2+\mathrm{i}m_\Upsilon\Gamma_\Upsilon}\right|^2\left|\frac{1}{q^2}\right|^2$$
$$\times \mathrm{Tr}[(\not p_2+m_{\mu^-})\gamma_\alpha(\not p_3-m_{\mu^+})\gamma_{\alpha'}]\left(-\frac{1}{2}m_\Upsilon^2 f_\Upsilon^2 g_{\mu\nu} - \frac{1}{2}(f_\Upsilon^T)^2 p_1\cdot q g_{\mu\nu}\right.$$
$$\left.+\frac{1}{2}(f_\Upsilon^T)^2 p_{1\nu}q_\mu\right)\left(-\frac{1}{2}m_\Upsilon^2 f_\Upsilon^2 g_{\mu'\nu'} - \frac{1}{2}(f_\Upsilon^T)^2 p_1\cdot q g_{\mu'\nu'} + \frac{1}{2}(f_\Upsilon^T)^2 p_{1\nu'}q_{\mu'}\right)$$
$$\times\left(g^{\alpha\nu} - \frac{q^\alpha q^\nu}{m_\Upsilon^2}\right)\left(g^{\alpha'\nu'} - \frac{q^{\alpha'} q^{\nu'}}{m_\Upsilon^2}\right)\left(g^{\mu\mu'} - \frac{p_1^\mu p_1^{\mu'}}{m_\Upsilon^2}\right). \quad (44)$$

Similarly, we study the three-body decay process $|X_1;0^{++}\rangle \to h_b(1P)\Upsilon(1S) \to h_b(1P)\mu^+\mu^-$ and calculate its partial decay width. After eliminating the overall factors c and c', we obtain

$$\frac{\mathcal{B}(|X_1;0^{++}\rangle \to h_b(1P)\Upsilon(1S) \to h_b(1P)\mu^+\mu^-)}{\mathcal{B}(|X_1;0^{++}\rangle \to \Upsilon(1S)\Upsilon(1S) \to \Upsilon(1S)\mu^+\mu^-)} = 0.002. \quad (45)$$

The above procedures are applied to investigate the process with the intermediate $\Upsilon(1S)$ meson. We can apply the same procedures to investigate the process with the intermediate $\Upsilon(2S)/\Upsilon(3S)/\ldots$ mesons, and the obtained results are approximately the same, while we do not consider other intermediate bottomonium mesons in the present study, such as the $\Upsilon(1D)$ meson, etc. Assuming that the $|X_1;0^{++}\rangle \to \Upsilon(1S)\mu^+\mu^-$ and $|X_1;0^{++}\rangle \to h_b(1P)\mu^+\mu^-$ decays are dominated by these processes, we finally obtain

$$\frac{\mathcal{B}(|X_1;0^{++}\rangle \to h_b(1P)\mu^+\mu^-)}{\mathcal{B}(|X_1;0^{++}\rangle \to \Upsilon(1S)\mu^+\mu^-)} \approx 0.002. \quad (46)$$

After considering several relevant channels, we obtain

$$\mathcal{B}(|X_1;0^{++}\rangle \to \Upsilon(1S)\mu\mu : \Upsilon(2S)\mu\mu : h_b(1P)\mu\mu) \approx 1:0.42:0.002. \quad (47)$$

Similarly, we apply the above procedures to investigate the S- and P-wave fully bottom tetraquark states $|X_{2\ldots11};J^{PC}\rangle$ through the currents $J_{2\ldots11}^{\cdots}$. The obtained results are summarized in Table III, which we shall use to draw conclusions in the next section. It is interesting to notice that some relative branching ratios are significantly larger or smaller than the others, which is partly due to that these ratios are proportional to the square of the Fierz coefficients given in Eqs. (16)–(19) and (28)–(34). For example, the relative branching ratio of $|X_{11};1^{--}\rangle$ decaying into the $\mu^+\mu^-\chi_{b0}(1P)$ channel is significantly larger than that of the $\mu^+\mu^-\eta_b(1S)$ channel:

$$\frac{\mathcal{B}(|X_{11};1^{--}\rangle \to \chi_{b0}(1P)\mu^+\mu^-)}{\mathcal{B}(|X_{11};1^{--}\rangle \to \eta_b(1S)\mu^+\mu^-)} \approx 12, \quad (48)$$

where the factor contributed by the Fierz coefficients is 9. Besides, the relative branching ratios are also contributed by the decay constants as well as the kinematics. For example, the relative branching ratio of $|X_8;1^{-+}\rangle$ decaying into the $\mu^+\mu^-h_b(1P)$ channel is not far from that of the $\mu^+\mu^-\Upsilon(1S)$ channel:

$$\frac{\mathcal{B}(|X_8;1^{-+}\rangle \to h_b(1P)\mu^+\mu^-)}{\mathcal{B}(|X_8;1^{-+}\rangle \to \Upsilon(1S)\mu^+\mu^-)} \approx 0.27. \quad (49)$$

The Fierz coefficients do not contribute to this ratio, while the factors contributed by the decay constants and the kinematics are about 0.33 and 0.81, respectively.

In our previous study [120], we have studied the decays of the fully charm tetraquark states into the $1S$ and $1P$ double-charmonium channels $J/\psi J/\psi$, $J/\psi\eta_c(1S)$, and $\eta_c(1S)\eta_c(1S)$, etc. In the present study, we further take into account the $2S$ double-charmonium channels $J/\psi\psi(2S)$,

TABLE III. Relative branching ratios of the S- and P-wave fully bottom tetraquark states $|X_{1\ldots11}; J^{PC}\rangle$ corresponding to the currents $J_{1\ldots11}^{\cdots}$. In the 3rd–5th columns we show the branching ratios relative to the $\mu^+\mu^-\Upsilon(1S)$ channel, and in the 6th–9th columns we show the branching ratios relative to the $\mu^+\mu^-\eta_b(1S)$ channel.

		Decay channels						
J^{PC}	Current	$\mu^+\mu^-\Upsilon(1S)$	$\mu^+\mu^-\Upsilon(2S)$	$\mu^+\mu^-h_b(1P)$	$\mu^+\mu^-\eta_b(1S)$	$\mu^+\mu^-\eta_b(2S)$	$\mu^+\mu^-\chi_{b0}(1P)$	$\mu^+\mu^-\chi_{b1}(1P)$
0^{++}	$J_1^{0^{++}}$	1	0.42	0.002	…	…	…	…
	$J_2^{0^{++}}$	1	0.42	…	…	…	…	…
1^{+-}	$J_{3\alpha}^{1^{+-}}$	…	…	…	1	0.42	…	1×10^{-4}
2^{++}	$J_{4\alpha\beta}^{2^{++}}$	1	0.42	0.002	…	…	…	…
0^{-+}	$J_5^{0^{-+}}$	1	0.39	0.090	…	…	…	…
	$J_6^{0^{-+}}$	1	0.39	0.090	…	…	…	…
0^{--}	$J_7^{0^{--}}$	…	…	…	1	0.42	…	0.041
1^{-+}	$J_{8\alpha}^{1^{-+}}$	1	0.43	0.27	…	…	…	…
	$J_{9\alpha}^{1^{-+}}$	1	0.43	0.27	…	…	…	…
1^{--}	$J_{10\alpha}^{1^{--}}$	…	…	…	1	0.38	1.3	0.070
	$J_{11\alpha}^{1^{--}}$	…	…	…	1	0.38	12	0.070

$\eta_c(1S)\eta_c(2S)$, $J/\psi\eta_c(2S)$, and $\eta_c(1S)\psi(2S)$. The obtained results are summarized in Table IV, which we shall also use to draw conclusions in the next section.

IV. SUMMARY AND DISCUSSIONS

In this paper, we systematically study the decay behaviors of the fully bottom and fully charm tetraquark states through their corresponding interpolating currents without derivatives. We work within the diquark-antidiquark picture and apply the Fierz rearrangement of the Dirac and color indices to transform the diquark-antidiquark currents into the meson-meson currents. The obtained Fierz identities are given in Eqs. (16)–(19) and (28)–(34).

Based on these Fierz identities, we study the decay mechanism depicted in Fig. 1, where a fully bottom tetraquark state decays into one bottomonium meson and one intermediate $\Upsilon(1S)/\Upsilon(2S)/\ldots$ meson, with the intermediate $\Upsilon(1S)/\Upsilon(2S)/\ldots$ meson annihilating to be a photon and then transferring into a muon-antimuon pair. We consider several possible decay channels and calculate their relative branching ratios. The obtained results are summarized in Table III, where the masses of the S- and P-wave fully bottom tetraquark states are assumed to be 18.5 and 18.8 GeV, respectively [121]. In the calculations, we work within the naive factorization scheme, so the uncertainty of our results is significantly larger than the well-developed QCD factorization scheme (about 5% when

TABLE IV. Relative branching ratios of the S- and P-wave fully charm tetraquark states, calculated through the fully charm tetraquark currents $J_{1\ldots11}^{\cdots}|_{b/\bar{b}\to c/\bar{c}}$. In the 3rd–9th columns we show the branching ratios relative to the $J/\psi J/\psi$ channel, and in the 10th–15th columns we show the branching ratios relative to the $J/\psi\eta_c$ channel. The notations $\psi' \equiv \psi(2S)$ and $\eta_c' \equiv \eta_c(2S)$ are used here.

		Decay channels												
J^{PC}	Current	$J/\psi J/\psi$	$J/\psi\psi'$	$\eta_c\eta_c$	$\eta_c\eta_c'$	$J/\psi h_c$	$\eta_c\chi_{c0}$	$\eta_c\chi_{c1}$	$J/\psi\eta_c$	$J/\psi\eta_c'$	$\psi'\eta_c$	$J/\psi\chi_{c0}$	$J/\psi\chi_{c1}$	$\eta_c h_c$
0^{++}	$J_1^{0^{++}}$	1	…	0.45	…	…	…	2×10^{-5}	…	…	…	…	…	…
	$J_2^{0^{++}}$	1	…	4.1	…	…	…	9×10^{-5}	…	…	…	…	…	…
1^{+-}	$J_{3\alpha}^{1^{+-}}$	…	…	…	…	…	…	…	1	…	…	…	…	…
2^{++}	$J_{4\alpha\beta}^{2^{++}}$	1	…	0.036	…	…	…	0.003	…	…	…	…	…	…
0^{-+}	$J_5^{0^{-+}}$	1	0.071	…	…	0.21	0.69	…	…	…	…	…	…	…
	$J_6^{0^{-+}}$	1	0.071	…	…	0.21	6.2	…	…	…	…	…	…	…
0^{--}	$J_7^{0^{--}}$	…	…	…	…	…	…	…	1	0.048	0.078	…	1.4	…
1^{-+}	$J_{8\alpha}^{1^{-+}}$	1	0.071	…	…	0.78	…	0.94	…	…	…	…	…	…
	$J_{9\alpha}^{1^{-+}}$	1	0.071	…	…	0.78	…	8.4	…	…	…	…	…	…
1^{--}	$J_{10\alpha}^{1^{--}}$	…	…	…	…	…	…	…	1	0.048	0.078	0.79	1.5	0.43
	$J_{11\alpha}^{1^{--}}$	…	…	…	…	…	…	…	1	0.048	0.078	7.1	1.5	0.43

studying the weak and radiative decays of the conventional hadrons) [144–147]. However, we calculate the relative branching ratios after eliminating several ambiguous overall factors, such as the decay constant f_{X_1} and the coupling of the $\Upsilon(1S)$ to the photon. This largely reduces our uncertainty; e.g., we roughly estimate the uncertainty of Eq. (48) to be

$$\frac{\mathcal{B}(|X_{11};1^{--}\rangle \to \chi_{b0}(1P)\mu^+\mu^-)}{\mathcal{B}(|X_{11};1^{--}\rangle \to \eta_b(1S)\mu^+\mu^-)} \approx 12^{+24}_{-8}, \tag{50}$$

based on our previous systematical QCD sum rule studies on the decay properties of the excited heavy baryons [148,149].

Our results suggest that the fully bottom tetraquark states of $J^{PC} = 0^{++}/2^{++}/0^{-+}/1^{-+}$ can be searched for in the $\mu^+\mu^-\Upsilon(1S)$ channel, and they can also be searched for in the $\mu^+\mu^-\Upsilon(2S)$ channel, with the relative branching ratio $\mathcal{B}(X \to \mu\mu\Upsilon(2S))/\mathcal{B}(X \to \mu\mu\Upsilon(1S)) \approx 0.4$. Our results also suggest that the fully bottom tetraquark states of $J^{PC} = 1^{+-}/0^{--}/1^{--}$ can be searched for in the $\mu^+\mu^-\eta_b(1S)$ channel, and they can also be searched for in the $\mu^+\mu^-\eta_b(2S)$ channel, with the similar relative branching ratio $\mathcal{B}(X \to \mu\mu\eta_b(2S))/\mathcal{B}(X \to \mu\mu\eta_b(1S)) \approx 0.4$. We propose to examine these decay channels to search for the fully bottom tetraquark states in future CMS experiments.

In this paper, we also update our previous study of Ref. [120] and reanalyze the fall-apart two-body decays of the fully charm tetraquark states. The obtained results are summarized in Table IV, where the masses of the S- and P-wave fully charm tetraquark states are assumed to be 6.5 and 6.9 GeV, respectively [121]. These states were used in Ref. [120] to explain the broad structure at around 6.2–6.8 GeV and the narrow structure at around 6.9 GeV observed by LHCb in the di-J/ψ invariant mass spectrum [1]. Based on the results of the present study, we calculate the $X(6900)$ decay into the $J/\psi\psi(2S)$ channel and obtain the relative branching ratio $\mathcal{B}(X \to J/\psi\psi(2S))/\mathcal{B}(X \to J/\psi J/\psi) \approx 0.1$.

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