Pion superfluid phase transition under an external magnetic field including the inverse magnetic catalysis effect

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Pion superfluid phase transition under external magnetic field including the inverse magnetic catalysis (IMC) effect is investigated by the Pauli-Villars regularized Nambu-Jona-Lasinio model. Based on Goldstone's theorem, we apply the massless Goldstone boson (π^+ meson) to determine the onset of the pion superfluid phase. The inverse magnetic catalysis effect is introduced by the magnetic field dependent coupling $G(eB)$, which is a decreasing function of magnetic field. At fixed temperature and baryon chemical potential, the critical isospin chemical potential for the pion superfluid phase transition, including the IMC effect, increases as the magnetic field grows, which is similar to the case without the IMC effect. This demonstrates that magnetic field disfavors the pion superfluid phase when considering or ignoring the IMC effect. The critical isospin chemical potential at fixed magnetic field, temperature, and baryon chemical potential is shifted to a higher value by the IMC effect. Because it is more difficult to form a pion superfluid with weaker coupling.

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I. INTRODUCTION

Recently, the magnetic field effect on the QCD phase structure attracted much attention [1–6] due to its close relation to high-energy nuclear collisions and compact stars. For instance, the lattice QCD (LQCD) simulations performed with physical pion mass observe the inverse magnetic catalysis (IMC) phenomenon [7–14]; namely, the pseudocritical temperature T_{pc} of chiral symmetry restoration and the quark mass near T_{pc} drops down with increasing magnetic field. On the analytical side, many scenarios are proposed to understand this inverse magnetic catalysis phenomenon, but the physical mechanism is not clear [15–44].

QCD phase structure at finite isospin chemical potential contains the chiral symmetry restoration and pion superfluid phase transitions. With vanishing external magnetic field and temperature, when the isospin chemical potential is higher than the pion mass in vacuum, the u quark and $\bar{d}$ quark form coherent pairs and condensate. The system enters the pion superfluid phase and the charged pion becomes massless as the corresponding Goldstone mode [45–70]. With a finite magnetic field, the charged pion condensate breaks both isospin symmetry in the flavor space and translational invariance in the coordinate space, due to its direct interaction with the external magnetic field. Furthermore, when one introduces a magnetic field into a pion superfluid, either there is a superconductor or a magnetic vortex, both of which can change the magnetic field. LQCD simulations exhibit a sign problem at finite isospin chemical potential and magnetic field. By using a Taylor expansion in the magnetic field, it is reported that at vanishing temperature, the onset of pion condensate shifts to a higher isospin chemical potential under magnetic fields [71]. In the study of pion condensate in effective models, the interaction between the charged pion condensate and the magnetic field is simply neglected in Ref. [72–74]. To avoid this complication, we have investigated the magnetic field effect on pion superfluid phase transition through the Goldstone's theorem [75], where, starting from the normal phase without pion condensate, the phase boundary of pion superfluid is determined by its massless Goldstone mode (π^+ meson). Note that the chiral symmetry, which will be restored with increasing isospin chemical potential, controls the mass of quarks, and will influence the formation of pion (quark-antiquark pair) condensate and the pion superfluid phase transition. However, the previous work on pion superfluid phase transition under magnetic fields do not consider the IMC effect of chiral symmetry restoration.

This paper focuses on the IMC effect on pion superfluid phase transition under magnetic fields. Here we make use of a Pauli-Villars regularized Nambu-Jona-Lasinio (NJL) model, which is inspired by the Bardeen-Cooper-Shrieffer (BCS) theory and describes remarkably well the quark pairing mechanisms and the Goldstone mode [76–81]. Because the interaction between quarks determines the symmetry broken and restoration. In our calculations, the IMC effect is introduced by a magnetic field dependent coupling (see Fig. 1). As a straightforward extension of our previous work [75], we investigate pion superfluid phase

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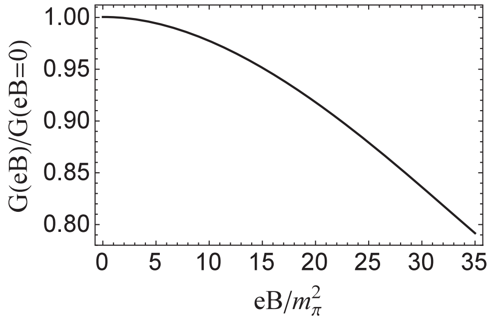

FIG. 1. Magnetic field dependent coupling $G(eB)$ fitted from LQCD reported decreasing pseudocritical temperature of chiral symmetry restoration. In this paper, we fix $m_\pi = 134$ MeV.

transition under magnetic fields through its Goldstone mode (π^+ meson).

The rest of paper is organized as follows. Section II describes our theoretical framework to study the pion superfluid phase transition including the IMC effect. The numerical results and discussions are presented in Sec. III, which focus on the comparison between the results with and without the IMC effect. Finally, we give the summary in Sec. IV.

II. FRAMEWORK

The two-flavor NJL model is defined through the Lagrangian density in terms of quark fields ψ [76–81]

$$\mathcal{L} = \bar{\psi}(i\gamma_\nu D^\nu - m_0 + \gamma_0\mu)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]. \quad (1)$$

Here the covariant derivative $D_\nu = \partial_\nu + iQA_\nu$ couples quarks with electric charge $Q = \mathrm{diag}(Q_u, Q_d) = \mathrm{diag}(2e/3, -e/3)$ to the external magnetic field $\mathbf{B} = (0,0,B)$ in z-direction through the potential $A_\nu = (0,0,Bx_1,0)$. m_0 is the current quark mass. The quark chemical potential $\mu = \mathrm{diag}(\mu_u, \mu_d) = \mathrm{diag}(\mu_B/3 + \mu_I/2, \mu_B/3 - \mu_I/2)$ is a matrix in the flavor space, with μ_u and μ_d being the u- and d-quark chemical potentials and μ_B and μ_I being the baryon and isospin chemical potentials.

G is the coupling constant in the scalar and pseudoscalar channels. In a vacuum, the chiral symmetry $U(1)_L \otimes U(1)_R \simeq U(1)_A \otimes U(1)_I$ is spontaneously broken into the isospin symmetry $U(1)_I$. In the medium with finite isospin chemical potential, the broken chiral symmetry will be (partially) restored, which leads to the chiral restoration phase transition. Meanwhile, the isospin symmetry will be broken, which leads to the pion superfluid phase transition. Corresponding to the symmetries and their spontaneous breaking, we have two order parameters; a neutral chiral condensate $\langle\bar{\psi}\psi\rangle$ for chiral restoration phase transition and a charged pion condensate $\langle\bar{\psi}\gamma_5\tau^1\psi\rangle$ for pion superfluid phase transition. According to the Goldstone's theorem, the pseudo-Goldstone mode of chiral symmetry breaking is the neutral pion π^0, and the Goldstone mode of isospin symmetry breaking is the charged pion π^+. Physically, it is equivalent to define the phase transition by the order parameter and Goldstone mode [82,83].

As a straightforward extension of our previous paper [75], we use the Goldstone mode (massless π^+ meson) to determine the pion superfluid phase transition at finite temperature, chemical potential, and magnetic field,

$$m_{\pi^+}(eB, T, \mu_B, \mu_I) = 0. \quad (2)$$

The inverse magnetic catalysis phenomenon can be characterized either by the chiral condensate or the critical temperature of chiral symmetry restoration from LQCD simulations [7–14]. Therefore, to include the inverse magnetic catalysis effect in the NJL model, one approach is to fit the LQCD results of chiral condensate [84–87], and another approach is to fit the LQCD result of critical temperature [27,28,84,88]. In our calculations, following Refs. [27,28,84,88], we use our two-flavor NJL model with a magnetic field dependent coupling $G(eB)$, derive the normalized critical temperature $T_c(eB)/T_c(0)$, and fit the LQCD reported decreasing pseudocritical temperature of chiral symmetry restoration [7]. As plotted in Fig. 1, the magnetic field dependent coupling $G(eB)/G(eB=0)$ is a decreasing function of magnetic field, and it reduces 20% at $eB/m_\pi^2 = 35$ (with $m_\pi = 134$ MeV). As we have checked, with our fitted coupling constant $G(eB)$, the magnetic catalysis phenomena of chiral condensates at low temperatures and the inverse magnetic catalysis phenomena at high temperatures can be reproduced.

In the NJL model, mesons are constructed through quark bubble summations in the frame of random phase approximation [77–81]. Taking into account of the interaction between charged mesons and magnetic fields, the meson propagator D_{π^+} can be expressed in terms of the polarization function Π_{π^+} [75,89–92],

$$D_{\pi^+}(\bar{k}) = \frac{2G(eB)}{1 - 2G(eB)\Pi_{\pi^+}(\bar{k})}. \quad (3)$$

where $\bar{k} = (k_0, 0, -\sqrt{(2l+1)eB}, k_3)$ is the conserved Ritus momentum of π^+ meson under magnetic fields.

The meson pole mass m_{π^+} is defined through the pole of the propagator at zero momentum $(l = 0, k_3 = 0)$,

$$1 - 2G(eB)\Pi_{\pi^+}(k_0 = m_{\pi^+}) = 0, \quad (4)$$

and

$$\Pi_{\pi^+}(k_0) = J_1(m_q) + J_2(k_0), \quad (5)$$

$$J_1(m_q) = 3\sum_{f,n}\alpha_n \frac{|Q_f B|}{2\pi}\int \frac{dp_3}{2\pi}\frac{1}{E_f} \times [1 - f(E_f + \mu_f) - f(E_f - \mu_f)], \quad (6)$$

$$J_2(k_0) = \sum_{n,n'}\int \frac{dp_3}{2\pi}\frac{j_{n,n'}(k_0)}{4E_n E_{n'}}\left[\frac{f(-E_{n'} - \mu_u) - f(E_n - \mu_d)}{k_0 + \mu_I + E_{n'} + E_n} + \frac{f(E_{n'} - \mu_u) - f(-E_n - \mu_d)}{k_0 + \mu_I - E_{n'} - E_n}\right], \quad (7)$$

$$j_{n,n'}(k_0) = [(k_0 + \mu_I)^2/2 - n'|Q_u B| - n|Q_d B|]j^+_{n,n'} - 2\sqrt{n'|Q_u B| n |Q_d B|}j^-_{n,n'}, \quad (8)$$

with flavors $f = u,\ d$, spin factor $\alpha_n = 2 - \delta_{n0}$, quark energy $E_f = \sqrt{p_3^2 + 2n|Q_f B| + m_q^2}$, quark Landau level $n = 0, 1, 2\ldots$, (dynamical) quark mass $m_q = m_0 - 2G(eB)\langle\bar\psi\psi\rangle$, and Fermi-Dirac distribution function $f(x) = 1/(e^{x/T} + 1)$ in $J_1(m_q)$, and the u-quark energy $E_{n'} = \sqrt{p_3^2 + 2n'|Q_u B| + m_q^2}$ and d-quark energy $E_n = \sqrt{p_3^2 + 2n|Q_d B| + m_q^2}$ in $J_2(k_0)$.

The (dynamical) quark mass m_q is determined by the gap equation,

$$1 - 2G(eB)J_1(m_q) = \frac{m_0}{m_q}. \quad (9)$$

Because of the four-fermion interaction, the NJL model is not a renormalizable theory and needs regularization. In this work, we make use of the gauge invariant Pauli-Villars regularization scheme [16,75–80,92,93], where the quark momentum runs formally from zero to infinity. The three parameters in the Pauli-Villars regularized NJL model, namely the current quark mass $m_0 = 5$ MeV, the coupling constant $G(eB = 0) = 3.44\ \text{GeV}^{-2}$ and the Pauli-Villars mass parameter $\Lambda = 1127$ MeV are fixed by fitting the chiral condensate $\langle\bar\psi\psi\rangle = -(250\ \text{MeV})^3$, pion mass $m_\pi = 134$ MeV and pion decay constant $f_\pi = 93$ MeV in vacuum with $T = \mu_B = \mu_I = 0$ and $eB = 0$.

III. RESULTS

Figure 2 depicts the critical isospin chemical potential μ_I^π for pion superfluid phase transition at $T = \mu_B = 0$ (in red), critical isospin chemical potential μ_I^c for chiral symmetry restoration at $T = \mu_B = 0$ (in blue) and π^+ meson mass M_{π^+} at $T = \mu_B = \mu_I = 0$ (in black) as functions of magnetic field with constant coupling $G(eB = 0)$ (dashed lines) and magnetic field dependent coupling $G(eB)$ (solid lines). Here, μ_I^π is determined by condition $m_{\pi^+}(eB, T = 0, \mu_B = 0, \mu_I = \mu_I^\pi) = 0$, and μ_I^c is defined through the quark mass jump. When including the inverse magnetic catalysis effect by $G(eB)$, the critical isospin chemical potential μ_I^c for chiral symmetry restoration changes from MC phenomenon (increasing with magnetic field) to IMC phenomenon (decreasing with magnetic field). However, the critical isospin chemical potential μ_I^π for pion superfluid phase transition with magnetic field dependent coupling $G(eB)$ is similar as the case with constant coupling $G(eB = 0)$. It is an increasing function of magnetic field, which means that magnetic field disfavors the pion superfluid phase, even including the IMC effect. There exists some numerical differences. In the regions $eB/m_\pi^2 < 4.5$ and $eB/m_\pi^2 > 5$, the μ_I^π with the IMC effect (red solid line) is higher than that without the IMC effect (red dashed line), and at strong magnetic field, for instance, $eB/m_\pi^2 = 35$, the difference increases up to 10%. Physically, it is harder to form the pion (quark-antiquark pair) condensate with weaker interaction. Therefore, it is expected to obtain higher μ_I^π when including IMC effect. However, with $4.5 < eB/m_\pi^2 < 5$, the μ_I^π with IMC effect (red solid line) is lower than that without IMC effect (red dashed line). This might be related to the first-order chiral restoration phase transition, associated with an abrupt jump of quark mass, which happens at very similar isospin chemical potential. Note that the crossing point of μ_I^c and μ_I^π located at $eB/m_\pi^2 = 4.5$ with IMC effect and $eB/m_\pi^2 = 4.75$ without IMC effect, respectively.

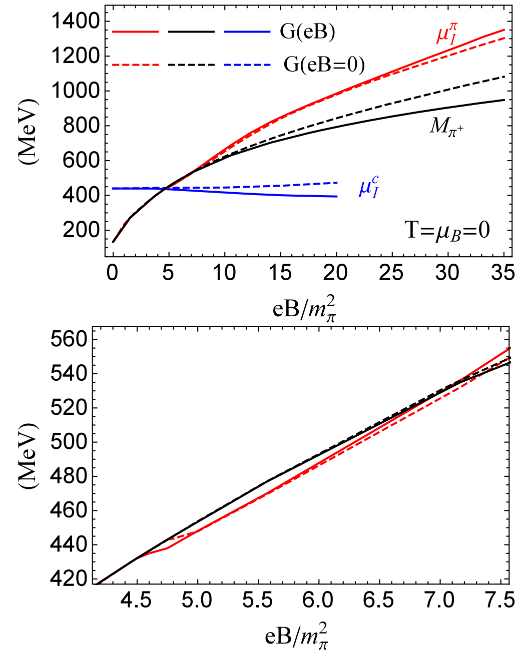

FIG. 2. (Upper panel) Critical isospin chemical potential μ_I^π for pion superfluid phase transition at $T = \mu_B = 0$ (red lines), π^+ meson mass M_{π^+} at $T = \mu_B = \mu_I = 0$ (black lines) and critical isospin chemical potential μ_I^c for chiral restoration phase transition at $T = \mu_B = 0$ (blue lines) as functions of magnetic field with constant coupling $G(eB = 0)$ (dashed lines) and magnetic field dependent coupling $G(eB)$ (solid lines). (Lower panel) The zoom-in figure in the region $4 < eB/m_\pi^2 < 7.5$. In this paper, we fix $m_\pi = 134$ MeV.

In Fig. 2, we also make comparison between critical isospin chemical potential μ_I^π for pion superfluid phase transition and π^+ meson mass in vacuum $M_{\pi^+} = m_{\pi^+}(eB, T = 0, \mu_B = 0, \mu_I = 0)$. Under weak-magnetic field $eB/m_\pi^2 < 4.5$, the μ_I^π is equal to the π^+ meson mass in vacuum M_{π^+}, which can be analytically proved by directly comparing the pole equation (4) and gap equation (9) [56,75]. With $4.5 < eB/m_\pi^2 < 7$, we obtain $\mu_I^\pi < M_{\pi^+}$, and with stronger magnetic field $eB/m_\pi^2 > 7$, we have $\mu_I^\pi > M_{\pi^+}$, which are obtained numerically. Without the IMC effect, the turning points are located at $eB/m_\pi^2 = 4.75$ and $eB/m_\pi^2 = 7.5$. The deviation between μ_I^π and M_{π^+} at strong magnetic field is enhanced by the IMC effect.

What is the situation when turning on the temperature and baryon chemical potential? Figure 3 is the phase diagram of pion superfluid in $\mu_I - T$ (with $\mu_B = 0$) and $\mu_I - \mu_B$ (with $T = 0$) planes at $eB/m_\pi^2 = 10$ and $eB/m_\pi^2 = 20$, where the solid (dashed) lines correspond to the case with (without) IMC effect. Fixing temperature (upper panel) or baryon chemical potential (lower panel), the phase transition from normal phase to pion superfluid phase happens with increasing isospin chemical potential. On the left side of the phase transition line, it is the normal phase, and on the right side, it is the pion superfluid phase. With higher temperature, the thermal motion of quarks are stronger. Hence, it is more difficult to form pion condensate, and the critical isospin chemical potential becomes higher. With fixed temperature and vanishing baryon chemical potential, the critical isospin chemical potential increases with magnetic fields. With higher baryon chemical potential, the mismatch between the Fermi surface of quark and antiquark is larger. This also prohibits the pion condensate, and leads to higher critical isospin chemical potential. With fixed baryon chemical potential and vanishing temperature, the critical isospin chemical potential also increases with magnetic fields. Including the IMC effect, at finite magnetic field, temperature and baryon chemical potential, the pion superfluid phase transition happens at higher isospin chemical potential, which is caused by the weaker coupling between quark and antiquark. The difference of critical isospin chemical potential is $\delta\mu_I^T \simeq (8 \sim 11)$ MeV with fixed T and vanishing μ_B, and $\delta\mu_I^{\mu_B} \simeq (10 \sim 19)$ MeV with fixed μ_B and vanishing T at $eB/m_\pi^2 = 10$, and $\delta\mu_I^T \simeq (14 \sim 25)$ MeV and $\delta\mu_I^{\mu_B} \simeq (14 \sim 48)$ MeV at $eB/m_\pi^2 = 20$. The deviation between the phase transition lines with and without IMC is enhanced by the magnetic field.

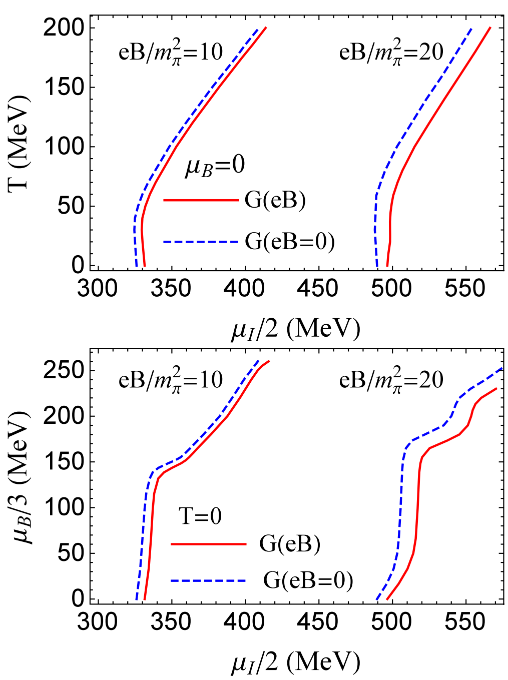

FIG. 3. (Upper panel) Pion superfluid phase diagram in $\mu_I - T$ plane with $\mu_B = 0$ and fixed magnetic field. (Lower panel) Pion superfluid phase diagram in $\mu_I - \mu_B$ plane with $T = 0$ and fixed magnetic field. The left lines are for $eB/m_\pi^2 = 10$, and right for $eB/m_\pi^2 = 20$. In this paper, we fix $m_\pi = 134$ MeV.

IV. SUMMARY

Pion superfluid phase transition under external magnetic field including the inverse magnetic catalysis effect is investigated through the Pauli-Villars regularized NJL model. Based on Goldstone's theorem, we apply the massless Goldstone boson (π^+ meson) to determine the phase boundary of pion superfluid. The inverse magnetic catalysis effect is introduced by the magnetic field dependent coupling $G(eB)$, which is a decreasing function of magnetic field.

At fixed temperature and baryon chemical potential, including IMC effect, the critical isospin chemical potential μ_I^π for pion superfluid phase transition increases with the magnetic field, which is qualitatively similar as the case without IMC effect. This indicates that magnetic field disfavors the pion superfluid phase with and without IMC effect. The deviation of μ_I^π with and without the IMC effect is enhanced by the magnetic field.

Comparing with the case ignoring the IMC effect, the critical isospin chemical potential for pion superfluid phase transition at fixed magnetic field, temperature and baryon chemical potential is shifted to higher value by the IMC effect. Due to the weakened coupling, it becomes harder to form pion condensate, and the critical isospin chemical potential for pion superfluid phase transition will become higher. This conclusion is independent on the specific formula for coupling $G(eB)$.

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