Electroweak production of χ_{Q1} states in e^+e^- collisions: A brief review

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A brief review of the available experimental and theoretical results on the production of the χ_{Q1} states in e^+e^- annihilation and photon-photon $\gamma\gamma^*$ interactions is presented. Future data on the production of the $\chi_{c1}(1P), \chi_{c1}(3872), \chi_{c1}(4140), \chi_{c1}(4274), \chi_{c1}(4685), \chi_{b1}(1P), \chi_{b1}(2P)$, and $\chi_{b1}(3P)$ resonances in e^+e^- annihilation and $\gamma\gamma^*$ interactions will help the development and unification of theoretical predictions related to the electroweak decays of heavy quarkonia, the reduction of their spread and model uncertainties.

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I. INTRODUCTION

In the report made at the *PhiPsi 2019* conference in Novosibirsk [1] (see also references therein), a point of view was presented on the nature of the $X(3872)$ state [2] as the first excited state in the axial-vector charmonium family, $X(3872) = \chi_{c1}(3872) = \chi_{c1}(2P)$, and discussed as probes to study the internal structure of the χ_{c1} and χ_{b1} states electroweak reactions $e^+e^- \to \chi_{c1}/\chi_{b1}$. Recently, important progress has been made in this direction. The BESIII and Belle collaborations performed the first measurements of the direct production of the $\chi_{c1}(1P)$ in e^+e^- annihilation [3] and the $X(3872)$ formation in photon-photon interactions [4]. In addition, the BESIII collaboration [5] has improved the upper limit for the electronic width of the $X(3872)$ resonance by a factor of about 13 compared to the previous limit. There is a huge theoretical discussion about the nature of the $X(3872)$ in the literature, for a review see Refs. [6–14] and references therein. Various scenarios are tested for it, from the resonance $c\bar{c}$ state to a loosely bound $D\bar{D}^*$ molecule. However, understanding the nature of the $X(3872)$ is still an open challenge. Let us take a few interesting statements as an example. From Ref. [9]: "Although the technology for describing the $X(3872)$ as a primarily $D^0\bar{D}^{*0}$ molecule is quite mature, solid reasons exist for questioning this interpretation." From Ref. [12]: "In this review we only state that at present there is no full understanding of the production rates of shallow molecular states. Implications of the molecular scenario are contrasted with different quark model approaches in Ref. [540]." From Ref. [13]: "Reasonable cross sections can be obtained in either $c\bar{c}$ or molecular $D\bar{D}^*$ scenarios for $X(3872)$. Also a hybrid scenario is not excluded." The situation is developing dynamically and each new evidence about the nature of the $X(3872)$ state is highly important.

This work gives a brief overview of the current situation related to the electroweak production of χ_{Q1} states in e^+e^- collisions. Section II presents the available experimental data. Theoretical estimates for the widths of the $\chi_{Q1} \to e^+e^-$ and $\chi_{c1} \to \gamma\gamma^*$ decays are discussed in Secs. III and IV, respectively. The conclusions are presented in Sec. V.

II. EXPERIMENTAL DATA

The first estimates of the decay width into e^+e^- for the P-wave states of heavy quarkonia χ_{Q1} ($Q = c, b$) were obtained as far back as the late 1970s [15–17]. It was assumed that the direct creation of resonances with $J^{PC} = 1^{++}$ in e^+e^- annihilation can occur due to two different mechanisms: via Z^0 exchange, $e^+e^- \to Z^0 \to \chi_{Q1}$, and through two virtual photons, $e^+e^- \to \gamma^*\gamma^* \to \chi_{Q1}$. Calculations of the electronic decay width, $\Gamma(\chi_{Q1} \to e^+e^-)$, carried out in Refs. [15–23] have demonstrated a significant dependence of the result on the assumptions made and on the choice of parameters. For example, for $\Gamma(\chi_{c1}(1P) \to e^+e^-)$ (i.e., for electronic width of the ground axial-vector state of charmonium) the values were predicted in the range from 0.044 to 0.41 eV [16,19–22], and for the excited state $\chi_{c1}(3872)$ it was found that $\Gamma(\chi_{c1}(3872) \to e^+e^-) \gtrsim 0.03$ eV [20]. More details about the theoretical estimates of $\Gamma(\chi_{Q1} \to e^+e^-)$ will be discussed in Sec. III.

An upper limit for the electronic width of the $\chi_{c1}(3872)$ on the $\mathcal{O}$(eV) level (namely, < 4.3 eV) was first obtained in 2015 by the BESIII collaboration [24] while studying the

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reaction $e^+e^- \to \gamma_{\rm ISR}\chi_{c1}(3872) \to \gamma_{\rm ISR}\pi^+\pi^- J/\psi$ with $\gamma_{\rm ISR}$ photon emission from the initial state. More recently, the BESIII collaboration lowered an upper limit on $\Gamma(\chi_{c1}(3872) \to e^+e^-)$ up to 0.32 eV [5]. As for the ground state $\chi_{c1}(1P)$, its direct production in e^+e^- annihilation with a significance of 5.1σ was first reported by the BESIII collaboration most recently [3]. For the electron width of the $\chi_{c1}(1P)$ the following value was obtained [3]:

$$\Gamma(\chi_{c1}(1P) \to e^+e^-) = (0.12^{+0.13}_{-0.08})\ \text{eV}. \qquad (1)$$

It was the result of processing data on the interference between the signal process $e^+e^- \to \chi_{c1}(1P) \to \gamma J/\psi \to \gamma\mu^+\mu^-$ and coherent background processes $e^+e^- \to \gamma_{\rm ISR}J/\psi \to \gamma_{\rm ISR}\mu^+\mu^-$ and $e^+e^- \to \gamma_{\rm ISR}\mu^+\mu^-$ [3,22,23].

As is known, for the axial-vector state 1^{++} there is a selection rule [25,26] that excludes its decay into two real photons. However, such a state can be produced in two-photon collisions, when one or both photons are virtual [virtual photon $\gamma^* \equiv \gamma^*(Q^2)$, where $-Q^2$ is the square of its invariant mass]. The result of the first experiment recently performed by the Belle collaboration [4] to search for $\chi_{c1}(3872)$ production in $\gamma\gamma^*$ collisions is the following estimate based on the observation of three $\chi_{c1}(3872)$ candidates with a significance of 3.2σ and assuming the Q^2 dependence predicted by a $c\bar{c}$ meson model:

$$\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)\mathcal{B}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi) = (5.5 \pm^{4.1}_{3.8}\ (\text{stat}) \pm 0.7(\text{syst}))\ \text{eV}. \qquad (2)$$

Here $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)$ is so-called the reduced $\gamma\gamma$ decay width defined (with use of the short notation $\tilde{\Gamma}_{\gamma\gamma}$) as [4,27–31]

$$\tilde{\Gamma}_{\gamma\gamma} \equiv \lim_{Q^2\to 0} \frac{M^2}{Q^2}\Gamma^{TL}_{\gamma\gamma^*}(Q^2), \qquad (3)$$

where M is the mass of the resonance and $\Gamma^{TL}_{\gamma\gamma^*}(Q^2)$ is the $\gamma\gamma^*$ decay width corresponding to a formation of the resonance by a transverse (real) photon γ and a longitudinal (virtual) photon γ^*. For the reduced width $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)$ in Ref. [4] a possible range of values from 20 to 500 eV is indicated. If one utilizes the updated branching ratio $\mathcal{B}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi) = (3.8 \pm 1.2)\%$ [32], then this interval, while remaining wide, moves slightly towards larger values:

$$24\ \text{eV} < \tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma) < 615\ \text{eV}. \qquad (4)$$

Investigations of the $\chi_{c1}(3872)$ production in $\gamma\gamma^*$ collisions will be continued at higher luminosities at the Belle II facility [4]. Theoretical estimates of the width $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)$ we will discuss in Sec. IV.

III. DIRECT PRODUCTION OF χ_{Q1} STATES IN e^+e^- ANNIHILATION

The width of the decay $\chi_{Q1} \to Z^0 \to e^+e^-$ due to the weak neutral current (Z^0 exchange) at $m^2_{\chi_{Q1}}/m^2_{Z^0} \ll 1$ and $m_e = 0$ has the form [15,17–19,22]

$$\Gamma(\chi_{Q1} \to Z^0 \to e^+e^-) = \frac{3G_F^2}{4\pi^2}|R'_{\chi_{Q1}}(0)|^2(g_a^{e2} + g_v^{e2}), \qquad (5)$$

where the Fermi constant $G_F = 1.116 \times 10^{-5}\ \text{GeV}^{-2}$, $g_a^e = 1$, $g_v^e = -1 + 4\sin^2\theta_W$, and $R'_{\chi_{Q1}}(0)$ is the derivative of the χ_{Q1} radial wave function at the origin [of course, this function is the same for all states $\chi_{QJ=0,1,2}(nP)$ at given n ($n = 1, 2, \ldots$)]. For estimates, we put $\sin^2\theta_W = 1/4$, and also $|R'_{\chi_{c1}}(0)|^2 \approx 0.1\ \text{GeV}^5$ for the $\chi_{c1}(1P)$ state and $|R'_{\chi_{b1}}(0)|^2 \approx 2\ \text{GeV}^5$ for the more compact $\chi_{b1}(1P)$ state (tabulated values of the quarkonium radial wave functions at the origin can be found, for example, in Refs. [33,34]). From this we have

$$\Gamma(\chi_{c1}(1P) \to Z^0 \to e^+e^-) \approx 10^{-3}\ \text{eV} \quad \text{and}$$
$$\Gamma(\chi_{b1}(1P) \to Z^0 \to e^+e^-) \approx 2 \times 10^{-2}\ \text{eV}. \qquad (6)$$

As already noted in Sec. II, the direct production of an axial-vector resonance with $J^{PC} = 1^{++}$ in e^+e^- annihilation can occur due to two different mechanisms: via Z^0 exchange [see Eqs. (5) and (6)] and through two virtual photons. The decay width $\chi_{Q1} \to e^+e^-$ corresponding to these two mechanisms and their interference can be represented in the so-called logarithmic approximation in the following form (details of calculations, in particular, in the vector dominance model, discussions about the meaning and method of regularizing the logarithmic singularity, as well as a number of modifications of the simple formula below, taken from [19], can be found in Refs. [16–23,35–37]):

$$\Gamma(\chi_{Q1} \to e^+e^-) = \frac{3}{4\pi^2}|R'_{\chi_{Q1}}(0)|^2\left[G_F^2(g_a^{e2} + g_v^{e2}) \mp \frac{2\sqrt{2}e_Q^2\alpha^2 G_F}{m_Q^2} \times g_a^e \text{Re}[f_1] + \frac{2e_Q^4\alpha^4}{m_Q^4}|f_1|^2\right]. \qquad (7)$$

Here "−" corresponds to $Q = c$ and "+" to $Q = b$; $e_c = 2/3$ and $e_b = -1/3$; the coefficient $f_1 = 4\ln(m_Q/\omega)$. The parameter ω has no unambiguous interpretation. On an intuitive level, ω is defined, for example, as the binding energy of χ_{Q1}, $\omega = 2m_Q - M_{\chi_{Q1}}$ [16,23,35,36], or as the binding energy, but with the opposite sign $\omega = M_{\chi_{Q1}} - 2m_Q$ [19] [in both cases, the appearance in the decay amplitude of the imaginary part due to $\ln(m_Q/\omega)$ at $\omega < 0$ is rather obscure], or one supposes that $\omega \sim 1/R \simeq (300\text{–}500)$ MeV, where R is the quarkonium radius [37], or $\omega \simeq$ (300–500) MeV is considered to be the characteristic virtuality scale of one of the intermediate virtual photons, i.e., of the soft virtual photon in the decay $\chi_{Q1} \to \gamma^*\gamma^* \to e^+e^-$ [21]. Here we do not dwell on the consideration of contributions from intermediate states like $\gamma^* J/\psi$, $\gamma^*\psi(2S)$, $\gamma^*\Upsilon(1S)$, etc. These contributions are discussed in detail in Refs. [16,17,19–23], where various values are predicted for them (in this connection see also Ref. [38]). In order to have concrete numerical estimates before our eyes, we set, following [19], $\omega = M_{\chi_{Q1}} - 2m_Q$, $M_{\chi_{c1}(1P)} = 3.51$ GeV, $m_c = 1.65$ GeV, $M_{\chi_{b1}(1P)} = 9.89$ GeV, and $m_b = 4.67$ GeV. Then according to Eq. (7) we have

$$\Gamma(\chi_{c1}(1P) \to e^+e^-) \approx (0.00095 - 0.01720 + 0.07817)\ \text{eV} \approx 0.06192\ \text{eV}, \tag{8}$$

$$\Gamma(\chi_{b1}(1P) \to e^+e^-) \approx (0.0189 + 0.0111 + 0.0016)\ \text{eV} \approx 0.0316\ \text{eV}, \tag{9}$$

where in parentheses are the contributions of individual terms. The estimate in Eq. (8) does not contradict the BESIII data, taking into account their errors, see Eq. (1). Although the electron widths in Eqs. (8) and (9) formally turn out to be of the same order, the mechanism of their formation upon going from $\chi_{c1}(1P)$ to $\chi_{b1}(1P)$ is undergoing a fundamental changing. Really, the contribution of the $\chi_{b1}(1P) \to \gamma^*\gamma^* \to e^+e^-$ mechanism [see the third term in Eq. (9)] decreases compared to the contribution of the $\chi_{c1}(1P) \to \gamma^*\gamma^* \to e^+e^-$ mechanism [see the third term in Eq. (8)] by about 50 times owing to the factor $|R'_{\chi_{Q1}}(0)|^2(e_Q/m_Q)^4$ [in so doing the factor $(e_Q/m_Q)^4$ decreases by about a thousand times]. Thus, it is natural to expect that the contribution of the weak neutral current dominates in the decay $\chi_{b1}(1P) \to e^+e^-$, while the decay $\chi_{c1}(1P) \to e^+e^-$ is dominated by the two-photon transition.

What can be at least qualitatively said about the electronic widths of excited states $\chi_{c1}(3872)$, $\chi_{c1}(4140)$, $\chi_{c1}(4274)$, $\chi_{c1}(4685)$, and $\chi_{b1}(2P)$, $\chi_{b1}(3P)$ [32]? For all states $\chi_{b1}(nP)$ $(n = 1, 2, \ldots)$ it is reasonable to assume that their electronic widths are determined mainly by the Z^0 exchange mechanism and therefore must be controlled by the corresponding values of $|R'_{\chi_{b1}(nP)}(0)|^2$. Calculations in potential models [33,34] show that $|R'_{\chi_{b1}(nP)}(0)|^2$ either grows weakly with n or remains virtually constant. Therefore, the widths $\Gamma(\chi_{b1}(nP) \to e^+e^-)$ should be expected to be approximately the same. For the states $\chi_{c1}(nP)$, $|R'_{\chi_{c1}(nP)}(0)|^2$ behave with n increasing by a similar way [33,34]. However, the dominant two-photon transition mechanism $\chi_{c1}(nP) \to \gamma^*\gamma^* \to e^+e^-$ can significantly depend on n due to the different dependence on n of the contributions of the intermediate states $\gamma^* J/\psi$, $\gamma^*\psi(2S)$, etc. The measurement of the widths $\Gamma(\chi_{Q1}(nP) \to e^+e^-)$ is a very difficult task [see the description of the BESIII experiment on measuring $\Gamma(\chi_{c1}(1P) \to e^+e^-)$ [3]]. But there is no doubt that each such measurement is an important step towards understanding internal structure of heavy quarkonia.

The total cross section for the χ_{Q1} resonance production with unpolarized e^+e^- beams is given by

$$\sigma(e^+e^- \to \chi_{Q1}; E) = \frac{3\pi}{E^2} \frac{\Gamma(\chi_{Q1} \to e^+e^-)\Gamma_{\chi_{Q1}}}{(M_{\chi_{Q1}} - E)^2 + \Gamma^2_{\chi_{Q1}}/4}, \tag{10}$$

where E is the energy in the e^+e^- center-of-mass system and $\Gamma_{\chi_{Q1}}$ is the total width of the χ_{Q1} state. If we put as an example $\Gamma(\chi_{Q1} \to e^+e^-) = 0.1$ eV and $\Gamma_{\chi_{Q1}} = 1$ MeV, then for the $R_{\chi_{Q1}}$ value in the peak of the χ_{Q1} resonance we find

$$R_{\chi_{Q1}} = \frac{\sigma(e^+e^- \to \chi_{Q1}; E)}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-; E)} = \frac{9}{\alpha^2} \frac{\Gamma(\chi_{Q1} \to e^+e^-)}{\Gamma_{\chi_{Q1}}} \approx 0.017. \tag{11}$$

Presently upgrading the SuperKEKB electron-positron collider with polarized electron beams is planned that opens a new physics program owing to precision neutral current measurements [39]; see also Ref. [40]. Dependence of the amplitude of χ_{Q1} production, $\mathcal{M}(e^+e^- \to \chi_{Q1})$, on the sign of the electron helicity λ (or on the direction of the polarization of the electron beam) is determined by the vector part of the weak neutral current. The corresponding contribution is proportional to $g_v^e = -1 + 4\sin^2\theta_W$, i.e., deviation of $4\sin^2\theta_W$ from 1. Setting $\sin^2\theta_W = 0.231$ [32], we get

$$\mathcal{M}_{\lambda=\pm1}(e^+e^- \to \chi_{Q1}) \propto [(\pm g_v^e + g_a^e)A_{Z^0}^{\chi_{Q1}} + A_{\gamma^*\gamma^*}^{\chi_{Q1}}] = [(\mp 0.076 + 1)A_{Z^0}^{\chi_{Q1}} + A_{\gamma^*\gamma^*}^{\chi_{Q1}}], \tag{12}$$

where $A_{\gamma^*\gamma^*}^{\chi_{Q1}}$ and $A_{Z^0}^{\chi_{Q1}}$ are the amplitudes of the $\chi_{Q1} \to e^+e^-$ transitions via $\gamma^*\gamma^*$ and Z^0 mechanisms respectively. If the contribution of the amplitude $A_{\gamma^*\gamma^*}^{\chi_{Q1}}$ dominates, then the effect associated with the polarization is very small. If the contribution of the amplitude $A_{Z^0}^{\chi_{Q1}}$ dominates, which is very likely for the χ_{b1} states, then due to the interference phenomena, the relative effect of polarization (the effect of parity violation) can be up to 15%.

IV. DIRECT ANNIHILATION TRANSITION $\chi_{c1} \to \gamma\gamma^*$

According to Ref. [29], the decay width of the $^3P_1$ nonrelativistic bound state of charmonium with mass M into $\gamma\gamma^*(Q^2)$ for small Q^2 can be represented as

$$\Gamma(^3P_1 \to \gamma\gamma^*(Q^2)) = 192\alpha^2 e_c^4 \frac{|R'(0)|^2}{M^4}\frac{Q^2}{M^2} \equiv \tilde{\Gamma}(^3P_1 \to \gamma\gamma)\frac{Q^2}{M^2}, \quad (13)$$

where $\tilde{\Gamma}(^3P_1 \to \gamma\gamma)$ is the so-called reduced $\gamma\gamma$ decay width [4,27–31], see Eq. (4), and $R'(0)$ is the derivative of the radial wave function at the origin for the corresponding P-wave state of charmonium. The decay widths of the $^3P_0$ and $^3P_2$ states into $\gamma\gamma$ have the form [41]

$$\Gamma(^3P_0 \to \gamma\gamma) = 432\alpha^2 e_c^4 \frac{|R'(0)|^2}{M^4}, \quad (14)$$

$$\Gamma(^3P_2 \to \gamma\gamma) = \frac{576}{5}\alpha^2 e_c^4 \frac{|R'(0)|^2}{M^4}. \quad (15)$$

A discussion of α_s corrections to these relations can be found, for example, in Refs. [42,43]. The value of the ratio [41]

$$\frac{\Gamma(^3P_2 \to \gamma\gamma)}{\Gamma(^3P_0 \to \gamma\gamma)} = \frac{4}{15} \simeq 0.27 \quad (16)$$

for the case of the states $\chi_{c2}(1P)$ and $\chi_{c0}(1P)$ is in good agreement with the available data [32,42,44]. From Eqs. (13) and (15) we have

$$\frac{\tilde{\Gamma}(^3P_1 \to \gamma\gamma)}{\Gamma(^3P_2 \to \gamma\gamma)} = \frac{5}{3}. \quad (17)$$

According the Particle Data Group [32] $\Gamma(\chi_{c2}(1P) \to \gamma\gamma) \simeq 0.56$ keV. Hence for $\tilde{\Gamma}(\chi_{c1}(1P) \to \gamma\gamma)$ we obtain the estimate

$$\tilde{\Gamma}(\chi_{c1}(1P) \to \gamma\gamma) \approx \frac{5}{3} \times 0.56 \text{ keV} \approx 0.93 \text{ keV}. \quad (18)$$

For the excited 2^3P_2 state $\chi_{c2}(3930)$ it is known [32] that

$$\Gamma(\chi_{c2}(3930) \to \gamma\gamma)\mathcal{B}(\chi_{c2}(3930) \to D\bar{D}) = (0.21 \pm 0.04) \text{ keV}. \quad (19)$$

If $\mathcal{B}(\chi_{c2}(2P) \to D\bar{D}) \approx 1$ [see however Refs. [45,46]; future experiments will help refine the value of $\mathcal{B}(\chi_{c2}(2P) \to D\bar{D})$], then for its 2^3P_1 partner $\chi_{c1}(3872)$ from Eqs. (17) and (19) we obtain the following estimate for the corresponding reduced $\gamma\gamma$ decay width:

$$\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma) \approx \frac{5}{3} \times 0.21 \text{ keV} \approx 350 \text{ eV}. \quad (20)$$

This estimate falls within the range of possible values for $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)$ specified in Eq. (4) based on the data from Belle [4].

Recently, in the light-front approach, the following values for the reduced two-photon widths of the $\chi_{c1}(1P)$ and $\chi_{c1}(3872)$ states have been predicted [47]: $\tilde{\Gamma}(\chi_{c1}(1P) \to \gamma\gamma) = (3 \pm 0.5)$ keV and $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma) = (3 \pm 1)$ keV. Both these values significantly exceed the above theoretical and experimental estimates.

The $\chi_{c1}(3872)$ state has a finite width ≈ 1 MeV, which is due to strong and radiative decays, among which the dominant decay is $\chi_{c1}(3872) \to (D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ [32]. Therefore, the estimates of the contributions to the width $\tilde{\Gamma}(\chi_{c1}(3872) \to \gamma\gamma)$ corresponding to rescattering mechanisms like $\chi_{c1}(3872) \to (D^*\bar{D} + \bar{D}^*D) \to \gamma\gamma^*$ are of quite natural interest. So far, there are no such estimates.

V. CONCLUSION

The recent data on the $\chi_{c1}(1P) \to e^+e^-$ and $\chi_{c1}(3872) \to \gamma\gamma^*$ decays are related to subtle questions of the electroweak interactions of heavy quarkonia. Their refinement and extension to other χ_{Q1} states will certainly lead to a new wave of theoretical research and predictions within the framework of various potential nonrelativistic QCD models currently in use, as well as to the development of new ideas and methods of studying the nature of heavy quarkonia. The restrictions imposed by experiment will serve as an effective tool for selecting theoretical models.

The BESIII experiment [3] demonstrates that with the current generation of electron-positron colliders, the observation of the direct production of the χ_{c1} in e^+e^- annihilation is possible with the use of the interference phenomena. It is to be hoped that this method will also be successfully used in subsequent measurements of the $\chi_{Q1} \to e^+e^-$ decay widths at the BESIII and Belle II installations and at the future Super Charm-Tau factory [48].

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