# Family nonuniversal  $Z'$  effects on  $B_{d,s}\to K^{*0}\bar{K}^{*0}$  decays in perturbative QCD approach

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The nonleptonic decays  $B_{d,s} \to K^{*0} \bar{K}^{*0}$  are reanalyzed in perturbative QCD approach, which is based on the  $k<sub>T</sub>$  factorization. In the Standard Model, the calculated branching fraction and longitudinal polarization fraction of  $B_d \to K^{*0} \bar{K}^{*0}$  are in agreement with experimental measurements, while the predictions of  $B_s \to K^{*0} \bar{K}^{*0}$  cannot agree with data simultaneously. The parameter that combines longitudinal polarization fractions and branching fractions is calculated to be  $L_{K}^{PQCD} = 12.7^{+5.6}_{-3.2}$ , which is also larger than that extracted from experimental measurements. We then study all observables by introducing a family nonuniversal Z' boson in  $b \rightarrow s q \bar{q}$  transitions. In order to reduce the number of new parameters, we simplify the model as possible. It is found that with the fixed value  $\omega_{B_s} = 0.55$ , these exists parameter space where all measurements, including the branching fraction, longitudinal polarization fraction, and  $L_{K^*\bar{K}^{*0}}$ -parameter, could be accommodated simultaneously. All our results and the small parameter space could be further tested in the running LHC experiment, Belle-II, and future high-energy colliders.

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### I. INTRODUCTION

It is well known that  $B$  meson rare decays provide us an abundant source of information on QCD, CP violation, and new physics (NP) beyond the Standard Model (SM). In recent years, the anomalies such as  $R(D^{(*)})$  and  $R_{K^{(*)}}$ <br>observed in semilentonic R meson rare decays at large observed in semileptonic  $B$  meson rare decays at large hadron collider (LHC) and *B* factories imply that the lepton flavor universality may be violated, which, in particular, are viewed as the signals of the effects of NP (for recent reviews, see, e.g., Refs. [[1](#page-9-0)–[4\]](#page-9-1)). Unlike the semileptonic decays, the hadronic  $B$  decays suffer from larger uncertainties and are therefore more difficult to calculate with a high accuracy because the hadronic matrix elements cannot be calculated from the first principle directly. In the past 20 years, based on the factorization hypothesis [\[5](#page-10-0)], some QCD based approaches to handle such kinds of problems are usually discussed in the heavy quark limit and implemented by the heavy quark expansion, such as the lightcone sum rule (LCSR) [[6](#page-10-1)], the QCD factorization (QCDF) [\[7,](#page-10-2)[8](#page-10-3)], the soft-collinear effective theory (SCET) [\[9](#page-10-4),[10](#page-10-5)], and

<span id="page-0-1"></span><span id="page-0-0"></span>[\\*](#page-0-2) liying@ytu.edu.cn [†](#page-0-2) sunyanjun@nwnu.edu.cn the perturbative QCD (PQCD) factorization approach [\[11](#page-10-6)–[13\]](#page-10-7). However, the observables such as the branching fractions, CP asymmetries, polarization fractions, and angular distributions might suffer from large uncertainties from higher-order and higher-power contributions. In this sense, in hadronic B decays, a deviation with respect to the SM prediction requires one to be much more conservative regarding these uncertainties than in the case of semileptonic B decays. For this reason, in order to search for the signals of NP in the hadronic heavy flavor particle decays, on the one hand, we should reduce the theoretical uncertainties as possible by preforming the higher-order and higher-power corrections with the developments of QCD technique, but on the other, we are encouraged to search for new observables that are insensitive to the theoretical uncertainties.

Among the two-body  $B$  meson hadronic decays, it is of great interest to us that the decays  $B_d \to K^{*0} \bar{K}^{*0}$  and  $B_s \to$  $K^{*0} \bar{K}^{*0}$  have same final states and are related by U spin. Both two decays are induced by the flavor-changing neutral-current (FCNC) transitions, in which new particles of NP could affect the observables by entering the loops. In addition,  $B_s \to K^{*0} \bar{K}^{*0}$  decay is also regarded as a golden channel for a precision measurement of the CKM phase  $\beta_s$ [\[14\]](#page-10-8). On the experimental side, both the branching fractions and the longitudinal polarization fractions have been measured in two  $B$  factories [[15](#page-10-9)–[17](#page-10-10)] and LHCb experiment [\[18](#page-10-11)–[21\]](#page-10-12). For the decay  $B_d \to K^{*0} \bar{K}^{*0}$ , the theoretical predictions of the branching fraction and polarization

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fractions based on QCDF [\[22\]](#page-10-13) and PQCD [\[23,](#page-10-14)[24\]](#page-10-15) are all in agreement with the averaged experimental results  $[25] B(B_d \to K^{*0} \bar{K}^{*0}) = (8.3 \pm 2.4) \times 10^{-7}$  $[25] B(B_d \to K^{*0} \bar{K}^{*0}) = (8.3 \pm 2.4) \times 10^{-7}$  and  $f_L(B_d \to K^{*0} \bar{K}^{*0}) = 0.74 \pm 0.05$  within the large uncertainties  $K^{*0} \overline{K}^{*0} = 0.74 \pm 0.05$  within the large uncertainties.<br>Furthermore, the measurement of  $f_*(R) \rightarrow K^{*0} \overline{K}^{*0}$ . Furthermore, the measurement of  $f_L(B_d \to K^{*0} \bar{K}^{*0})$ <br>agrees with the naïve hypothesis based on the quar- $\frac{1}{2}$  agrees with the naïve hypothesis, based on the quark helicity conservation and the  $(V - A)$  nature of the weak interaction. For the decay  $B_s \to K^{*0} \bar{K}^{*0}$ , the latest averaged experimental results [\[25\]](#page-10-16) are  $B(B_s \to K^{*0} \bar{K}^{*0}) = (11.1 \pm 2.7) \times 10^{-6}$  f,  $(B_s \to K^{*0} \bar{K}^{*0}) = 0.240 + 0.031 + 0.025$  $2.7 \times 10^{-6}$ ,  $f_L(B_s \to K^{*0} \bar{K}^{*0}) = 0.240 \pm 0.031 \pm 0.025$ ,<br>and  $f_L(B_s \to K^{*0} \bar{K}^{*0}) = 0.38 \pm 0.11 \pm 0.04$  It is found and  $f_{\perp}(B_s \to K^{*0} \bar{K}^{*0}) = 0.38 \pm 0.11 \pm 0.04$ . It is found that the prediction of branching fraction  $B(B_s \rightarrow$  $K^{*0}\bar{K}^{*0}$  =  $(9.1^{+0.5+11.3}_{-0.4-6.8}) \times 10^{-6}$  in QCDF [\[22\]](#page-10-13) agrees well with the data, but the longitudinal polarization fraction  $f_L(B_s \to K^{*0} \bar{K}^{*0}) = 0.63_{-0.29}^{+0.42}$  is much larger than the data.<br>On the another side, based on POCD approach [23], the On the another side, based on PQCD approach [[23](#page-10-14)], the predicted branching fraction and longitudinal polarization fraction are  $B(B_s \to K^{*0} \bar{K}^{*0}) = (5.4^{+3.0}_{-2.4}) \times 10^{-6}$  and<br>f  $(B_s \to K^{*0} \bar{K}^{*0}) = 0.38^{+0.12}$  respectively It is seen that  $f_L(B_s \to K^{*0} \bar{K}^{*0}) = 0.38_{-0.10}^{+0.12}$ , respectively. It is seen that although the longitudinal polarization fraction f, is conalthough the longitudinal polarization fraction  $f<sub>L</sub>$  is consistent with the data, its center value is a bit smaller than the experimental measurement. Altogether, the theoretical predictions with large uncertainties from two approaches cannot explain all available data convincingly. In order to explain the current data simultaneously, the theoretical predictions with high precision in both approaches are called, and we are also encouraged to explore the contributions of NP.

Following [\[26](#page-10-17)], the authors in Ref. [\[27\]](#page-10-18) defined an observable that is sensitive to the U-spin asymmetry but with a cleaner theoretical prediction as

$$
L_{K^{*0}\bar{K}^{*0}} = \frac{B(B_s \to K^{*0}\bar{K}^{*0})g(B_s \to K^{*0}\bar{K}^{*0})f_L(B_s \to K^{*0}\bar{K}^{*0})}{B(B_d \to K^{*0}\bar{K}^{*0})g(B_d \to K^{*0}\bar{K}^{*0})f_L(B_d \to K^{*0}\bar{K}^{*0})},
$$
\n(1)

where the phase space factors  $g(B_Q \to K^{*0} \bar{K}^{*0})$  involved in<br>the corresponding branching fractions are given as the corresponding branching fractions are given as

$$
g(B_Q \to K^{*0} \bar{K}^{*0}) = \frac{\tau_{B_Q}}{16\pi M_{B_Q}^2} \sqrt{M_{B_Q}^2 - 4M_{K^{*0}}^2}.
$$
 (2)

<span id="page-1-1"></span>In such a ratio, the experimental uncertainties are reduced, as the uncertainties in the denominator and numerator can be canceled out by each other. In [[21](#page-10-12)], LHCb collaboration released the measurements of the ratio between two branching fractions and the longitudinal polarization fraction of  $B_s \to K^{*0} \bar{K}^{*0}$ . With the latest results and the longitudinal polarization fraction of  $B_d \to K^{*0} \bar{K}^{*0}$  from PDG [[25](#page-10-16)], we could obtain this new observable as

$$
L_{K^*\bar{K}^*}^{\text{Exp}} = 4.43 \pm 0.92,\tag{3}
$$

where the effect of  $B_s$  meson mixing in the measurement of the branching fraction is included. In QCDF, the prediction based on the results from [[22](#page-10-13)] is given as [[27](#page-10-18)]

$$
L_{K^*\bar{K}^*}^{\text{QCDF}} = 19.5^{+9.3}_{-6.8},\tag{4}
$$

which implies a  $2.6\sigma$  tension with respect to the experimental data. This new "anomaly" discrepancy is viewed as a new signal of NP [\[27\]](#page-10-18). However,  $L_{K^*\bar{K}^*}$  of PQCD is not available yet till now. Motivated by this, we shall exploit this observable in PQCD in this work and try to check whether the  $L_{K^*\bar{K}^*}$  is still lager than the experimental data. Moreover, the branching fractions and polarizations of both two decays will also be recalculated with the new fitted distribution amplitudes of  $K^*$  [\[28\]](#page-10-19).

As aforementioned, in order to interpret the called  $R_K$ and  $R_{K^*}$  anomalies, a large number of NP models have been proposed. One of the most popular NP explanations are models with an extra heavy vector  $Z'$  boson [[29](#page-10-20),[30](#page-10-21)], where the new introduced  $Z'$  boson has couplings to quarks, as well as to either electrons or muons with nonuniversal parameters. In order to test these models, besides searching  $Z<sup>′</sup>$  at the higher energy colliders directly, the signals in other observables involving the similar transitions are also expected. A straightforward place to explore the possible existence of these signals are hadronic B decays induced by the FCNC transitions  $b \rightarrow (d, s)q\bar{q}$ . In SM, such kind of decays are forbidden at tree level and only occur by loops. The comparable contributions from  $Z<sup>'</sup>$  at tree level may change the observables remarkably. Hence, another purpose of this work is to explore whether the contributions of an extra  $Z'$  boson can be used to explain all measured observables in some certain spaces of parameters.

This paper is organized as follows. We will first present the calculations of  $B_d \to K^{*0} \bar{K}^{*0}$  and  $B_s \to K^{*0} \bar{K}^{*0}$  decays in SM within the PQCD approach, and more attentions are mainly paid not only on the branching fractions and the longitudinal polarization fractions but also on the new observable  $L_{K^*\bar{K}^*}$ . In Sec. [III,](#page-5-0) we will study contributions from the nonuniversal  $Z'$  boson, which could change the observables in the suitable parameters space. Lastly, we shall summarize this work in Sec. [IV.](#page-9-2)

### II. CALCULATION IN SM

<span id="page-1-0"></span>In SM, the decay amplitudes of  $B_{d,s} \to K^{*0} \bar{K}^{*0}$  decays follow from the matrix elements  $\langle V_2 V_3 | H_{\text{eff}} | B \rangle$  of the effective Hamiltonian

$$
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left\{ C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,...10} C_i Q_i^p \right\} + \text{H.c},\tag{5}
$$

with  $D \in \{d, s\}$  and  $\lambda_p^{(D)} = V_{pb}^* V_{pD}$ . The functions  $C_i(\mu)$ are Wilson coefficients, and  $O_i(\mu)$  ( $i = 1, 2, 3 \cdots, 10$ ) are the corresponding four-quark effective operators, whose specific forms refer to [[31](#page-10-22)].

In PQCD, the  $B$  meson amplitude can be expressed as [[11](#page-10-6)]

$$
\langle V_2 V_3 | H_{\text{eff}} | B \rangle \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3
$$
  
× Tr[*C*(*t*) $\Phi_B(x_1, b_1) \Phi_{V_2}(x_2, b_2)$   
×  $\Phi_{V_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}].$  (6)

The meson wave functions  $\Phi_i$  ( $i = B, V_2, V_3$ ) include the dynamical information of how the quarks are combined into a hadron. They are nonperturbative but universal. Tr is the sum of degrees of freedom in the spin and color space.  $b_i$  is the conjugate variable of the quark transverse momentum  $k_{iT}$ , and  $x_i$  is the longitudinal momentum fraction carried by the light quark in each meson.  $H(x_i, b_i, t)$  describes the four quark operators and the spectator quark connected by a hard gluon and can be calculated perturbatively. The jet function  $S_t(x_i)$  coming from the threshold resummation of the double logarithms  $\ln^2 x_i$  smears the end-point singularities in  $x_i$  [\[32\]](#page-10-23). The Sudakov form factor  $e^{-S(t)}$  arising from the resummation of the double logarithms suppresses the soft dynamics effectively, i.e., the long distance contributions in the large-b region [\[33](#page-10-24)[,34\]](#page-10-25). The main advantage of this approach is that it preserves the transverse momenta of quarks and avoids the problem of end-point divergence.

Because there are three kinds of polarizations for a vector meson, namely longitudinal  $(L)$ , normal  $(N)$ , and transverse  $(T)$ , the amplitudes for a B meson decay to two vector mesons are generally characterized by the polarization states of two vector mesons. Thus, the amplitude  $A^{(\sigma)}$ for the decay  $B(P_B) \to V_2(P_2, \varepsilon_{2\mu}^*) V_3(P_3, \varepsilon_{3\mu}^*)$  can be decomposed as follows: decomposed as follows:

$$
A^{(\sigma)} = \varepsilon_{2\mu}^{*}(\sigma)\varepsilon_{3\nu}^{*}(\sigma)\left[ag^{\mu\nu} + \frac{b}{M_{2}M_{3}}P_{B}^{\mu}P_{B}^{\nu}\right] + i\frac{c}{M_{2}M_{3}}\varepsilon^{\mu\nu\alpha\beta}P_{2\alpha}P_{3\beta}\right] = A_{L} + A_{N}\varepsilon_{2}^{*}(\sigma = T) \cdot \varepsilon_{3}^{*}(\sigma = T) + i\frac{A_{T}}{M_{B}^{2}}\varepsilon^{\mu\nu\gamma\rho}\varepsilon_{2\mu}^{*}(\sigma)\varepsilon_{3\nu}^{*}(\sigma)P_{2\gamma}P_{3\rho}, \tag{7}
$$

where  $M_2$  and  $M_3$  are the masses of the vector mesons  $V_2$ and  $V_3$ , respectively. The definitions of the amplitudes  $A_i$  $(i = L, N, T)$  in terms of the Lorentz-invariant amplitudes  $a, b,$  and  $c$  could be written as

$$
A_L = a\varepsilon_2^*(L) \cdot \varepsilon_3^*(L) + \frac{b}{M_2M_3} \varepsilon_2^*(L) \cdot P_3 \varepsilon_3^*(L) \cdot P_2, \qquad (8)
$$

$$
A_N = a,\t\t(9)
$$

$$
A_T = \frac{c}{r_2 r_3},\tag{10}
$$

with  $r_{2,3} = M_{V_{2,3}}/M_B$ . The amplitudes  $A_i$  ( $i = L, N, T$ ) could be calculated in PQCD approach directly.

Alternatively, we can also define the polarization amplitudes of three directions and their relationships with  $A_L$ ,  $A_N$ , and  $A_T$  are given as follows:

$$
A_0 = -A_L, \quad A_{\parallel} = \sqrt{2}A_N, \quad A_{\perp} = r_2 r_3 \sqrt{2(\kappa^2 - 1)}A_T,
$$
\n(11)

with the ratio  $\kappa = \frac{P_2 \cdot P_3}{M_{K^{*0}}}$ . Then, the branching fraction of  $B \to V_2V_3$  is expressed as

$$
B(B \to VV) = \tau_B \frac{|p_c|}{8\pi M_B^2} [|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2], \quad (12)
$$

where  $\tau_B$  is the lifetime of the B meson, and  $p_c$  is the threedimension momentum of the vector meson. Three polarization fractions  $f_i(i = L, \parallel, \perp)$  are also defined as

$$
f_i = \frac{|A_i|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}.
$$
 (13)

In PQCD approach, the most important inputs are the wave functions of hadrons. For the initial state  $B$  meson, its wave function is of the form [\[13](#page-10-7)[,23,](#page-10-14)[35](#page-10-26)[,36\]](#page-10-27)

$$
\Phi_B(x,b) = \frac{i}{\sqrt{2N_c}} [\mathbf{P}_B \gamma_5 + M_B \gamma_5] \phi_B(x,b), \quad (14)
$$

where  $b$  is the conjugate space coordinate of the transverse momentum  $k_{\perp}$ , and  $N_c = 3$  is the number of color. The distribution amplitude  $\phi_B$  is in the form of

$$
\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp\left[-\frac{1}{2} \left(\frac{x m_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right],\tag{15}
$$

where  $N_B$  is the normalization factor and satisfies

$$
\int_0^1 dx \phi_B(x, b = 0) = \frac{f_B}{2\sqrt{2N_c}},
$$
\n(16)

 $f_B$  being the decay constant of B meson. The shape parameters  $\omega_B = 0.30$  and  $\omega_{B_s} = 0.50$  are determined by experimental data or calculated from the first principle [[37](#page-10-28)].

Unlike the pseudoscalar particle, the vector meson has the longitudinal polarization vector  $\varepsilon_L$  and the transverse polarization one  $\varepsilon_T$ . For a special final state  $K^{*0}$  moving in the plus direction  $(n_+)$  with momentum P, two wave functions of the  $K^{*0}$  up to twist three are given as [[38](#page-10-29)]

$$
\Phi_{K^*}^{\parallel} = \frac{1}{\sqrt{2N_c}} [M_{K^*} \notin_L \phi_{K^*}(x) + \notin_L \mathcal{P} \phi_{K^*}^t(x) + M_{K^*} \phi_{K^*}^s(x)],
$$
\n(17)

$$
\Phi_{K^*}^{\perp} = \frac{1}{\sqrt{2N_c}} [M_{K^*} \phi_{T}^* \phi_{K^*}^* (x) + \phi_{T}^* \phi_{K^*}^T (x) + i M_{K^*} \varepsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu} \varepsilon_{T}^{*} n_{+}^{\rho} n_{-}^{\sigma} \phi_{K^*}^a (x)], \tag{18}
$$

where  $n_+ = (1, 0, 0_T)$  and  $n_- = (0, 1, 0_T)$ . Two polarizations are defined as

$$
\varepsilon(L) = \frac{P}{M_{K^*}} - \frac{M_{K^*}}{P \cdot n_+} n_+, \qquad \varepsilon(T) = (0, 0, 1_T). \tag{19}
$$

The light-cone distribution amplitudes in the wave function have been calculated within the QCD sum rules [\[39,](#page-10-30)[40](#page-10-31)],

$$
\phi_{K^*}(x) = \frac{3f_{K^*}}{\sqrt{2N_c}} x(1-x)[1 + a_{1K^*}^{\parallel} C_1^{3/2}(t) + a_{2K^*}^{\parallel} C_2^{3/2}(t)],
$$
\n(20)

$$
\phi_{K^*}^T(x) = \frac{3f_{K^*}}{\sqrt{2N_c}} x(1-x)[1 + a_{1K^*}^{\perp} C_1^{3/2}(t) + a_{2K^*}^{\perp} C_2^{3/2}(t)],
$$
\n(21)

$$
\phi_{K^*}^t(x) = \frac{3f_{K^*}^T}{2\sqrt{2N_c}}t^2,\tag{22}
$$

$$
\phi_{K^*}^s(x) = \frac{3f_{K^*}^T}{2\sqrt{2N_c}}(-t),\tag{23}
$$

$$
\phi_{K^*}^v(x) = \frac{3f_{K^*}}{8\sqrt{2N_c}}(1+t^2),\tag{24}
$$

$$
\phi_{K^*}^a(x) = \frac{3f_{K^*}}{4\sqrt{2N_c}}(-t). \tag{25}
$$

The Gegenbauer polynomials in the distribution amplitude are given as

$$
C_1^{3/2}(t) = 3t
$$
,  $C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1)$ , (26)

where  $t = 2x - 1$ , and x is the momentum fraction of the light quark.

<span id="page-3-0"></span>

FIG. 1. The leading order Feynman diagrams for  $B_s \to K^{*0} \bar{K}^{*0}$ .

According to the effective Hamiltonian Eq. [\(5\)](#page-1-0), we could draw the lowest-order diagrams contributing to  $B_{d,s} \to K^{*0} \bar{K}^{*0}$ . For example, the Feynman diagrams of  $B_s \to K^{*0} \bar{K}^{*0}$  are shown in Fig. [1,](#page-3-0) where the symbols " $\otimes$ " are the effective operators. Figures  $1(a)$  and  $1(b)$  are factorizable emission diagrams, while [1\(c\)](#page-3-0) and [1\(d\)](#page-3-0) are nonfactorizable emission ones. Similarly, Figs. [1\(e\)](#page-3-0) and [1](#page-3-0) [\(f\)](#page-3-0) are factorizable annihilation diagrams, and  $1(g)$  and  $1(h)$ are nonfactorizable annihilation ones. We also note that in  $B_s \to K^{*0} \bar{K}^{*0}$  decay, the final vector meson  $\bar{K}^{*0}$  takes the spectator strange quark, while in  $B_d \to K^{*0} \bar{K}^{*0}$  decay, the spectator down quark enters  $K^{*0}$  meson.

After calculating the amplitudes of each diagram with different operators, we obtain the amplitudes of  $B^0 \rightarrow$  $K^{*0}\bar{K}^{*0}$  and  $B_s^0 \to K^{*0}\bar{K}^{*0}$ , which are given as

<span id="page-4-0"></span>
$$
A^{i}(B^{0} \to K^{*0}\bar{K}^{*0}) = -\frac{G_{F}}{\sqrt{2}}V_{tb}^{*}V_{td}\left\{M_{\text{fh}}^{\text{LL},i}\left[a_{4}-\frac{1}{2}a_{10}\right]+M_{\text{nfh}}^{\text{LL},i}\left[C_{3}-\frac{1}{2}C_{9}\right]+M_{\text{nfh}}^{\text{LR},i}\left[C_{5}-\frac{1}{2}C_{7}\right]\right.+M_{\text{fa}}^{\text{LL},i}\left[\frac{4}{3}a_{3}+\frac{4}{3}a_{4}-\frac{2}{3}a_{9}-\frac{2}{3}a_{10}\right]+M_{\text{fa}}^{\text{LR},i}\left[a_{5}-\frac{1}{2}a_{7}\right]+M_{\text{fa}}^{\text{SP},i}\left[a_{6}-\frac{1}{2}a_{8}\right]+M_{\text{nfa}}^{\text{LL},i}\left[C_{3}-\frac{1}{2}C_{9}+C_{4}-\frac{1}{2}C_{10}\right]+M_{\text{nfa}}^{\text{LR},i}\left[C_{5}-\frac{1}{2}C_{7}\right]+M_{\text{nfa}}^{\text{SP},i}\left[C_{6}-\frac{1}{2}C_{8}\right]+M_{\text{fa}}^{\text{LL},i}\left[a_{3}-\frac{1}{2}a_{9}\right]+M_{\text{fa}}^{\text{LR},i}\left[a_{5}-\frac{1}{2}a_{7}\right]\right)_{K^{*0}\leftrightarrow\bar{K}^{*0}}+M_{\text{nfa}}^{\text{LL},i}\left[C_{4}-\frac{1}{2}C_{10}\right]+M_{\text{nfa}}^{\text{SP},i}\left[C_{6}-\frac{1}{2}C_{8}\right]\right)_{K^{*0}\leftrightarrow\bar{K}^{*0}},\tag{27}
$$

<span id="page-4-1"></span>
$$
A^{i}(B_{s}^{0} \to K^{*0}\bar{K}^{*0}) = -\frac{G_{F}}{\sqrt{2}}V_{tb}^{*}V_{ts}\left\{M_{\text{fh}}^{\text{LL},i}\left[a_{4} - \frac{1}{2}a_{10}\right] + M_{\text{nfh}}^{\text{LL},i}\left[C_{3} - \frac{1}{2}C_{9}\right] + M_{\text{nfh}}^{\text{LR},i}\left[C_{5} - \frac{1}{2}C_{7}\right] \right.+ M_{\text{fa}}^{\text{LL},i}\left[\frac{4}{3}a_{3} + \frac{4}{3}a_{4} - \frac{2}{3}a_{9} - \frac{2}{3}a_{10}\right] + M_{\text{fa}}^{\text{LR},i}\left[a_{5} - \frac{1}{2}a_{7}\right] + M_{\text{fa}}^{\text{SP},i}\left[a_{6} - \frac{1}{2}a_{8}\right] \right.+ M_{\text{nfa}}^{\text{LL},i}\left[C_{3} - \frac{1}{2}C_{9} + C_{4} - \frac{1}{2}C_{10}\right] + M_{\text{nfa}}^{\text{LR},i}\left[C_{5} - \frac{1}{2}C_{7}\right] + M_{\text{nfa}}^{\text{SP},i}\left[C_{6} - \frac{1}{2}C_{8}\right] \right.+ \left(M_{\text{fa}}^{\text{LL},i}\left[a_{3} - \frac{1}{2}a_{9}\right] + M_{\text{fa}}^{\text{LR},i}\left[a_{5} - \frac{1}{2}a_{7}\right]\right)_{K^{*0} \leftrightarrow \bar{K}^{*0}} \left.+ M_{\text{nfa}}^{\text{LL},i}\left[C_{4} - \frac{1}{2}C_{10}\right] + M_{\text{nfa}}^{\text{SP},i}\left[C_{6} - \frac{1}{2}C_{8}\right]\right)_{K^{*0} \leftrightarrow \bar{K}^{*0}}\right\},\tag{28}
$$

with

$$
a_1 = C_2 + C_1/3, \t a_2 = C_1 + C_2/3, \t a_3 = C_3 + C_4/3, \t a_4 = C_4 + C_3/3,
$$
  
\n
$$
a_5 = C_5 + C_6/3, \t a_6 = C_6 + C_5/3, \t a_7 = C_7 + C_8/3, \t a_8 = C_8 + C_7/3,
$$
  
\n
$$
a_9 = C_9 + C_{10}/3, \t a_{10} = C_{10} + C_9/3,
$$
\n(29)

where  $i = L, N, T$  denote the longitudinal polarization and the two transverse polarizations. In above two formulae, the superscripts LL, LR, and SP indicate the operators  $(V - A)(V - A), (V - A)(V + A), \text{ and } (S - P)(S + P),$ respectively. The subscript "fh" in  $M_{\text{fh}}$  meas factorizable emission diagrams  $(a)$  and  $(b)$ , while "nfh" means nonfactorizable ones  $(c)$  and  $(d)$ . Similarly, "fa" and "nfa" are the factorizable and nonfactorizable annihilation diagrams, respectively. Due to the limit of space, we will not list the above amplitudes for each  $M$ , and the explicit expressions can be found in Refs. [\[13,](#page-10-7)[23](#page-10-14)]. It should be stressed that all amplitudes "M" are mode dependent, as the spectator quarks are different in these two decays, though the Eqs. [\(27\)](#page-4-0) and [\(28\)](#page-4-1) are very similar.

With above formulae, we then calculate the observables in SM. The branching fractions and longitudinal polarization fractions of both decays are given in Table [I](#page-4-2), together with predictions of QCDF and the available experimental

<span id="page-4-2"></span>TABLE I. Numerical results for observables in  $B_{d,s} \to K^{*0} \bar{K}^{*0}$  decays in SM, together with results of QCDF and experimental results.

Decay mode	$BF(10^{-6})$	$f_L(\%)$	$f_{\parallel}(\%)$	$f_+(\%)$
$B^0 \to K^{*0} \bar{K}^{*0}$ <b>QCDF</b> [22] Exp. [25]	$0.5^{+0.2+0.2}_{-0.1-0.1}$ $0.6^{+0.1+0.5}_{-0.1-0.3}$ $0.8 \pm 0.09 \pm 0.04$	$67.1_{-5.7-0.4}^{+5.1+0.3}$ $69^{+1+34}_{-1-27}$ $72.4 \pm 5.1 \pm 1.6$	$17.4^{+3.6+0.1}_{-3.4-0.0}$ $11.6 \pm 3.3 \pm 1.2$	$15.5^{+2.7+0.1}_{-2.5-0.2}$ $16 + 4.4 + 1.2$
$B_s^0 \to K^{*0} \bar{K}^{*0}$ <b>QCDF</b> [22] Exp. [25]	$7.8^{+1.9+2.3}_{-1.4-1.5}$ $9.1_{-0.4-6.8}^{+0.5+11.3}$ $11.1 \pm 2.2 \pm 1.2$	$51.1_{-6.8-0.3}^{+7.3+0.6}$ $63^{+0+42}_{-0-29}$ $24 + 3.1 + 2.5$	$25.6^{+3.7}_{-4.2}{}_{-0.3}^{+0.1}$	$23.3^{+3.3+0.3}_{-3.5-0.2}$ $38^{+11+4}_{-11-4}$

data. In our numerical calculations, the updated distribution amplitudes [[28](#page-10-19)] of  $K^*$  are adopted. We acknowledge that there are still some uncertainties in our calculations, and we here only discuss two main uncertainties. In the table, the first errors arise from the wave functions of heavy  $B$ mesons, in which the shape parameters  $\omega_{B_d}$  and  $\omega_{B_s}$  are the only inputs, and we make them change 30%. The second ones are from the next-leading power (order) corrections characterized by the hard scale  $t$ , which changes from  $0.8t$ to 1.2t. It can be seen that the branching fractions are affected by both parameters, while the polarization fractions are only sensitive to the shape parameter  $\omega_{B_d}$  or  $\omega_{B_s}$ . In PQCD, both  $B_d^0 \rightarrow K^{*0} \bar{K}^{*0}$  and  $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$  are induced only by the penguin operators so that the direct CP asymmetries of two decays are zero in PQCD. However, including the contributions from charm penguins, the direct CP asymmetries from QCDF are nonzero. Thus, the measurements of direct CP asymmetries in future could discriminate two approaches.

From Table [I,](#page-4-2) we find that for the decay  $B^0 \to K^{*0} \bar{K}^{*0}$ , the predictions of branching fractions and polarization fractions from PQCD and QCDF are in agreement with the experimental results, though the theoretical center values of branching fraction are smaller than the experimental data. In fact, the longitudinal contribution is dominant, which is roughly proportional to the form factor  $A_0^{B\to K^*}$ . In QCDF,  $A_0^{B\to K^*}(0) = 0.39 \pm 0.06$  calculated from light-cone sum<br>rules [411 was adopted while  $A^{B\to K^*}(0) = 0.36 \pm 0.05$  is rules [\[41](#page-10-32)] was adopted, while  $A_0^{B\to K^*}(0) = 0.36 \pm 0.05$  is<br>obtained in POCD. In addition, the form factors  $A_{B\to K^*}(0)$ obtained in PQCD. In addition, the form factors  $A_1^{B \to K^*}(0)$ <br>and  $V^{B \to K^*}(0)$  that are related to transverse applitudes are and  $V^{B\to K^*}(0)$  that are related to transverse amplitudes are<br>almost same in POCD and OCDE For the decay almost same in PQCD and QCDF. For the decay  $B_s^0 \to K^{*0} \bar{K}^{*0}$ , the theoretical predictions are in agreement with each other with uncertainties, with  $A_0^{B_s \to K^*}(0) = 0.33 \pm 0.05$  in DOCD 0.05 in QCDF and  $A_0^{B_s \to K^*}(0) = 0.30 \pm 0.05$  in PQCD.<br>However in comparison to the experimental results both However, in comparison to the experimental results, both branching fractions are smaller than the data, and both theoretical longitudinal polarization fractions are much larger than data; even the predictions of QCDF have large uncertainties arising from annihilation diagrams. In our previous study [[23](#page-10-14)], with the large suppression from threshold resummation, the predicted longitudinal polarization fraction  $f_L = (38.3^{+12.1}_{-10.5})\%$  could be comparable to data, but the corresponding branching fraction  $(5.4_{-2.4}^{+3.0}) \times 10^{-6}$  is smaller than the current data. Although there are many uncertainties in the theoretical calculations, this discrepancy could be a hint of NP beyond SM.

In recent years, the width effects of the  $K^*$  have been also discussed but with  $K_{\pi}$  distribution amplitudes instead of ones of  $K^*$ . It is shown that for the quasi-two-body decay  $B \to K^*P_3 \to K\pi P_3$ , the width effects is less than 10% [\[42\]](#page-10-33). Within the light-cone sum rules and the narrow-width limit, the authors in Ref. [[43](#page-10-34)] also estimated the effect of a nonvanishing  $K^*$  width in  $B \to K^*$  transitions and found that this effect is universal and increases the factorizable

part of the rate of  $B \to K^*X$  decays by a factor of 20%. Meanwhile, in Ref. [\[44\]](#page-10-35), with the P-wave  $K\pi$  distribution amplitudes, the four-body decays  $B(d, s) \rightarrow (K\pi)_P(K\pi)_P$ in the  $K\pi$  invariant mass region around the  $K^*$  resonance have been investigated in PQCD approach, and our results are in agreement with theirs with uncertainties.

Now, we calculate the  $L_{K^*\bar{K}^{*0}}$ -parameter and obtain

$$
L_{K^*\bar{K}^{*0}}^{\text{PQCD}} = 12.7^{+5.6}_{-3.2},\tag{30}
$$

where the uncertainty is mainly from the shape parameters in the distribution amplitudes of  $B_d^0$  and  $B_s^0$  mesons. The uncertainties taken by high-order corrections are almost canceled. In this sense, the more precise and reliable shape parameters of heavy mesons based on the nonperturbative approaches are needed. By comparison, we find our result is also larger than one from the current data, Eq. [\(3\)](#page-1-1), though it is smaller than that of QCDF.

## III. CALCULATION IN FAMILY NONUNIVERSAL Z' MODEL

<span id="page-5-0"></span>Now, we turn to study the contributions of the extra gauge boson Z' to the decays  $B_s^0 \to K^{*0} \bar{K}^{*0}$ , which is induced by the FCNC  $b \rightarrow s\bar{d}d$  transition. Supposing there is no mixing between  $Z$  and  $Z'$ , the  $Z'$  term of the neutral-current Lagrangian in the gauge basis can be written as [[45](#page-11-0),[46](#page-11-1)]

$$
L^{Z'} = -g'Z'^\mu \sum_{i} \bar{\psi}_i^I \gamma_\mu [(\varepsilon_{\psi_L})_i P_L + (\varepsilon_{\psi_R})_i P_R] \psi_j^I, \quad (31)
$$

where  $\psi_i^I$  means the *i*-th family fermion, and the superscript I refers to the gauge interaction eigenstate.  $g'$  is the gauge coupling constant at the electroweak scale  $M_W$ , and  $P_{L,R} = (1 \mp \gamma_5)/2$ . The parameter  $\varepsilon_{\psi_L}$  ( $\varepsilon_{\psi_R}$ ) denotes the left-handed (right-handed) chiral coupling. According to certain string constructions [[47](#page-11-2)] or GUT models [\[48\]](#page-11-3), the couplings can be family nonuniversal. When we change the weak basis to the physical one, FCNC's generally appear at tree level in both left-handed and right-handed sectors, explicitly, as

$$
B^{L} = V_{\psi_{L}} \varepsilon_{\psi_{L}} V_{\psi_{L}}^{\dagger}, \qquad B^{R} = V_{\psi_{R}} \varepsilon_{\psi_{R}} V_{\psi_{R}}^{\dagger}, \qquad (32)
$$

<span id="page-5-1"></span>where  $V_{\psi_{LR}}$  are unitary matrices. For simplicity, the righthanded couplings are supposed to be flavor diagonal. Therefore, the FCNC  $b \rightarrow s\bar{q}q$  (and  $q = u, d$ ) transition can also be mediated by the  $Z'$  at tree level, and the corresponding effective Hamiltonian has the form of

$$
H_{\text{eff}}^{Z'} = \frac{2G_F}{\sqrt{2}} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L(\bar{s}b)_{V-A} \sum_q (B_{qq}^L(\bar{q}q)_{V-A} + B_{qq}^R(\bar{q}q)_{V+A}) + \text{H.c.},
$$
\n(33)

where  $g_1 = e/(\sin \theta_W \cos \theta_W)$ , and  $M_{Z'}$  is the mass of the new Z' boson. The current structures  $(V - A)(V - A)$  and  $(V - A)(V + A)$ , are same as Eq. [\(5\)](#page-1-0) of SM, which allow us to translate Eq. [\(33\)](#page-5-1) as

$$
H_{\text{eff}}^{Z'} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_q (\Delta C_3 O_3^q + \Delta C_5 O_5^q + \Delta C_7 O_7^q + \Delta C_9 O_9^q) + \text{H.c.}
$$
\n(34)

In above Hamiltonian,  $\Delta C_i$  denote Z' corrections to the Wilson coefficients of the SM operators, which can be written as

$$
\Delta C_3 = -\frac{2}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^L + 2B_{dd}^L),
$$
  
\n
$$
\Delta C_5 = -\frac{2}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^R + 2B_{dd}^R),
$$
  
\n
$$
\Delta C_7 = -\frac{4}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^R - B_{dd}^R),
$$
  
\n
$$
\Delta C_9 = -\frac{4}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^L - B_{dd}^L).
$$
\n(35)

It is obvious that  $Z'$  contributes to the OCD penguins as well as to the EW penguins. For simplicity, we follow the assumptions in Refs. [[49](#page-11-4)–[55](#page-11-5)] and set  $B_{uu}^{L,R} = -2B_{dd}^{L,R}$  so that new physics is manifest in the EW penguing namely that new physics is manifest in the EW penguins, namely  $O<sub>7</sub>$  and  $O<sub>9</sub>$ . Furthermore, without loss of generality, the diagonal elements of the effective coupling matrices  $B_{qq}^{L,R}$ are supposed to be real due to the hermiticity of the effective Hamiltonian. However, there is no constraint that the off-diagonal  $B_{sb}^L$  should be a real, and a new weak phase  $\phi_{bs}$  can exist. Taking all these information together, we then have the new Wilson coefficients

$$
\Delta C_{3,5} \simeq 0,
$$
  
\n
$$
\Delta C_{9,7} = 4 \frac{|V_{tb} V_{ts}^*|}{V_{tb} V_{ts}^*} \xi^{L,R} e^{i\phi_{bs}},
$$
\n(36)

<span id="page-6-0"></span>with

$$
\xi^{L,R} = \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 \left|\frac{B_{sb}^L B_{dd}^{L,R}}{V_{tb} V_{ts}^*}\right|.
$$
 (37)

With the assumption that both  $U(1)_Y$  in the SM and  $U(1)$ introduced in new models originate from the grand unified theory, the gauge coupling constants for  $Z$  and  $Z'$  bosons are the same, implying that  $g/g_1 = 1$ . So far, the obvious<br>signal of the new  $Z'$  hoson has not been observed in the signal of the new  $Z'$  boson has not been observed in the current experiments such as CMS and ATLAS, which indicates that the mass of  $Z'$  would be larger than the Tev scale. Conservatively, we set  $M_Z/M_{Z'} \approx 0.1$ . In order to accommodate the mass difference between  $B_s^0$  and  $\bar{B}_s^0$ , which is one of the most strictest constraints to the models with Z' boson,  $|B_{sb}^L| \sim |V_{tb}V_{ts}^*|$  is theoretically required.<br>Meanwhile in order to evoluin the experimental data of Meanwhile, in order to explain the experimental data of  $B \to K\pi$ ,  $B \to K\phi$  and  $B \to K^*\pi$ , the diagonal elements should satisfy  $|B_{qq}^{L,R}| \sim 1$  [[52](#page-11-6),[56](#page-11-7),[57](#page-11-8)]. For the newly intro-<br>duced weak phase  $\phi$ . it is assumed to be a free parameter duced weak phase  $\phi_{bs}$ , it is assumed to be a free parameter without any restriction. In order to reduce the number of new parameters, we further assume  $\xi = \xi^{LL} = \xi^{LR}$ , which means that the left-handed couplings are same as righthanded ones. This simplification has not been adopted in previous studies. Therefore, in our following discussion, we have only two parameters  $\xi \in [0.001, 0.02]$  and  $\phi_{bs} \in [-180^\circ, 180^\circ]$ . Alteratively,  $\xi^{LL} = 0$  or  $\xi^{LR} = 0$  can also be assumed, which correspond to different scenarios in Refs. [[52](#page-11-6),[56](#page-11-7)[,57\]](#page-11-8). Our discussions can be generalized to other cases directly, and we shall not discuss these cases any more..

In Figs. [2](#page-7-0) and [3,](#page-7-1) we present the branching fraction and longitudinal polarization fraction of  $B_s \to K^{*0} \bar{K}^{*0}$  as functions of the new weak phase  $\phi_{bs}$ , for a fixed value  $\xi = 0.01$ with  $\omega_{B_s} = 0.45, 0.50,$  and 0.55 in the left panels, and for a fixed  $\omega_{B_s} = 0.50$  with  $\xi = 0.02, 0.01$ , and 0.005 in the right panels. The experimental data and the SM predictions are also shown in the figures for comparisons. As aforementioned, the experimental result and theoretical prediction of SM on the branching fraction have some overlaps, but there is no overlap on the longitudinal polarization fraction. From Table [I](#page-4-2), we could see that in the SM the uncertainty of the branching fraction arising from the  $\omega_{B_s}$  is about 20%. With the fixed parameter  $\xi = 0.01$ , for each  $\omega_{B_s}$ , the uncertainties coming from the unknown phase  $\phi_{bs}$ are also around 20%, as shown in the left panel of Fig. [2](#page-7-0). Comparing the theoretical results with the data, a small  $\omega_{B_s}$ is preferred by the experimental data. Given  $\omega_{B_s} = 0.50$ , it is found from the right panel of Fig. [2](#page-7-0) that if  $\xi$  < 0.01, the contributions of the new particle would be plagued by the large theoretical uncertainties. However, when we set  $\xi = 0.02$ , the effect from Z' boson becomes more remarkable, and the branching fraction could be as large as  $11.2 \times 10^{-6}$  when  $\phi_{bs} = 0^{\circ}$ . Specifically, for  $\xi = 0.02$ and  $\omega_{B_s} = 0.50$ , the new weak phase  $\phi_{bs}$  is constrained in the range  $[-100^\circ, 100^\circ]$  by the current data, and the range decreases as ξ becomes smaller.

In contrast to the branching fraction, the measured longitudinal polarization fraction is smaller than the theoretical prediction, which allows us to find out some mechanisms to suppress the longitudinal contribution or enhance the transverse contributions. It can be seen from the left panel of Fig. [3](#page-7-1) that for the fixed value  $\xi = 0.01$ , most results are larger than the data, and only few results approach the upper limit of experimental data when  $\omega_{B_s} =$ 0.55 and  $\phi_{bs} \approx 50^{\circ}$ . Therefore, a larger  $\omega_{B_s}$  is favored, which is different from the result from the well-measured branching fraction. It is shown in the right panel that, for

<span id="page-7-0"></span>

FIG. 2. The dependence of the branching fraction of  $B_s \to K^{*0} \bar{K}^{*0}$  on the weak phase  $\phi_{bs}$ , for a fixed value  $\xi = 0.01$  with  $\omega_{B_s} = 0.45$ <br>(dotted blue line) 0.50 (solid black line) and 0.55 (dashed red line) in (dotted blue line), 0.50 (solid black line), and 0.55 (dashed red line) in the left panels; and for a fixed  $\omega_{B_s} = 0.50$  with  $\xi = 0.005$  (dotdashed blue line), 0.01 (dashed purple line), and 0.02 (dotted red line) in the right panels. The blue and yellow regions represent the experimental data and SM prediction, respectively.

the fixed  $\omega_{B_s} = 0.50$ , the theoretical predictions of longitudinal polarization fractions  $f<sub>L</sub>$  are larger than the data, for both  $\xi = 0.01$  and  $\xi = 0.001$ . When  $\xi = 0.02$ ,  $f<sub>L</sub>$  changes in a wide range with the changes of  $\phi_{bs}$  and could fall into the experimental range within  $\phi_{bs} \in [8^\circ, 93^\circ]$ . When  $\phi_{bs} \approx 50^{\circ}$ ,  $f_L$  could be as small as 22%.

From the above analysis, the branching fraction prefers a smaller  $\omega_{B_s}$ , while the longitudinal polarization fraction prefers a larger one. Also, we found that once  $\xi = 0.02$  is adopted, both the branching fraction and the longitudinal polarization fraction vary in a large region with the change of  $\phi_{bs}$ . Thus, with  $\xi = 0.02$ , we plot all possible regions for  $\omega_{B_s} = 0.50 \pm 0.05$  in Fig. [4.](#page-8-0) These two figures illustrate that for the fixed  $\xi = 0.02$ , both of the two observables could be consistent with the experimental data well, even  $\omega_{B_s} = 0.45$  is adopted. In addition, a positive weak phase  $\phi_{bs}$  is preferred, as implied in Fig. [4](#page-8-0).

Now, we shall discuss the effect of the new introduced Z' boson on the new defined parameter  $L_{K^*\bar{K}^{*0}}$ . As aforementioned, we suppose that  $Z'$  only participates in the  $b \rightarrow s$  transitions, and its contribution to the FCNC  $b \rightarrow d$  transitions is suppressed by small  $|B_{db}|$  and negligible. In this respect,  $L_{K^*\bar{K}^{*0}}$  does in fact reflect the contribution of longitudinal amplitude of decay  $B_s^0 \to K^{*0} \bar{K}^{*0}$ . In the left panel of Fig. [5,](#page-8-1) we adopt  $\xi = 0.01$  again and show the variant of  $L_{\text{max}}$  with changes of 0.01 again and show the variant of  $L_{K^*\bar{K}^{*0}}$  with changes of  $\phi_{bs}$  for  $\omega_{B_s} = 0.45, 0.50,$  and 0.55. The SM prediction and the latest measurement are also shown. By comparison, we find that if  $\xi = 0.01$ , the theoretical predictions cannot agree with experimental data, even if  $\omega_{B_s} = 0.55$  is adopted. By setting  $\omega_{B_s} = 0.45, 0.50,$  and 0.55, we also calculated  $L_{K^*\bar{K}^{*0}}$ . The numerical results show that if  $\xi$  < 0.02, the values of  $\omega_{B_s}$  = 0.45, 0.50 are not preferred by the experimental data. Thus, we adopt  $\omega_{B_s} = 0.55$  and plot  $L_{K^*\bar{K}^{*0}}$  dependence on the phase for  $\xi = 0.001, 0.01$ , and 0.02 in the right panel. It can be clearly seen that  $L_{K^*\bar{K}^{*0}}$ and 0.02 in the right panel. It can be clearly seen that  $L_{K^*\bar{K}^{*0}}$ changes in a wide range for  $\xi = 0.02$ , and it could be 4.61 as  $\phi_{bs} \approx 75^{\circ}$ . Combining Figs. [4](#page-8-0) and [5,](#page-8-1) we find that in such

<span id="page-7-1"></span>

FIG. 3. The dependence of the longitudinal polarization fraction  $(f_L)$  of  $B_s \to K^{*0} \bar{K}^{*0}$  on the weak phase  $\phi_{bs}$ , for a fixed value  $\xi = 0.01$  with  $\omega_{B_s} = 0.45$  (dotted blue line), 0.50 (solid black line), and 0.55 (dashed red line) in the left panel; and for a fixed  $\omega_{B_s} = 0.50$  with  $\xi = 0.005$  (dot-dashed blue line), 0.01 (dashed purple line), and 0.02 (dotted red line) in the right panel. The blue and yellow regions represent the experimental data and SM prediction, respectively.

<span id="page-8-0"></span>

FIG. 4. The dependence of the branching fraction (left panel) and longitudinal polarization fraction  $(f_L)$  (right panel) of  $B_s \to K^{*0} \bar{K}^{*0}$ on the weak phase  $\phi_{bs}$ , for a fixed value  $\xi = 0.02$  with  $\omega_{B_s} = 0.50 \pm 0.05$ . The blue bands represent the experimental data.

a family nonuniversal  $Z'$  model, there might exist a certain parameter space, where all observables can be achieved. In order to obtain the parameter space, we show the combined result in the  $(\phi_{bs}, \xi)$  two-dimensional plane for the fixed value  $\omega_{B_s} = 0.55$ , as shown Fig. [6.](#page-9-3) The green and yellow bands represent the regions fitting the branching fraction and the longitudinal polarization fraction, respectively, while the region of the parameter space corresponding to a viable fit of  $L_{K^*\bar{K}^{*0}}$  has been marked in blue. Evidently, the experimental data of  $L_{K^*\bar{K}^{*0}}$  gives the most stringent constraint. As was expected, these three bands overlap in a very small region,  $\xi \in [0.017, 0.018]$  and  $\phi_{bs} \in [50^{\circ}, 65^{\circ}]$ . Within this small parameter space, we then have

$$
B(B_s^0 \to K^{*0} \bar{K}^{*0}) = (8.6 \pm 0.4) \times 10^{-6}, \qquad (38)
$$

$$
f_L(B_s^0 \to K^{*0} \bar{K}^{*0}) = (19.5 \pm 0.7)\% \tag{39}
$$

<span id="page-8-1"></span>25

 $20$ 

 $\circ$ 

 $-150$ 

 $-100$ 

 $-50$ 

 $\Omega$ 

 $\phi_{\rm bs}$ 

50

100

150

 $L_{K^{*0},\tilde{K}^{*0}}$ 

$$
L_{K^*\bar{K}^{*0}}^{\text{PQCD}} = 5.3 \pm 0.3. \tag{40}
$$

These results with few uncertainties could be further tested with high precision in the current LHCb experiment or the Belle-II experiment.

Finally, we present some comments on the direct searches of  $Z'$  boson. At LHC, the main way to search directly for a  $Z'$  is via a resonance peak in the invariantmass distribution of its decay products. This experimental analysis is usually performed by the ATLAS and CMS collaborations for  $Z'$  production in the s channel in a rather model-independent way but assuming that the observed new resonance is narrow, such that any interference of SM and NP contributions can be neglected. Under these assumptions, the  $Z'$  Drell-Yan cross section at a hadron machine can be approximated as [[25](#page-10-16),[58](#page-11-9)[,59\]](#page-11-10)

$$
\sigma(p \, p \to Z'X \to f \bar{f}X) \simeq \frac{\pi}{6s} \sum_{q} c_q^f w_q(s, M_{Z'}^2), \qquad (41)
$$

where  $q = u, d, s, c, b$ . Here, the hadronic structure functions  $w_q(s, M_{Z'}^2)$  are independent of the Z' model<br>and contain all information on parton distribution functions and contain all information on parton distribution functions and QCD corrections. On the other hand, the coefficients  $c_q^f$ 



FIG. 5. The dependence of  $L_{K^*\bar{K}^{*0}}$ -parameter on the weak phase  $\phi_{bs}$ , for a fixed value  $\xi = 0.01$  with  $\omega_{B_s} = 0.45$  (dotted blue line), 0.01<br>0.50 (solid black line), and 0.55 (dashed red line) in the left pane 0.50 (solid black line), and 0.55 (dashed red line) in the left panel, and for a fixed  $\omega_{B_s} = 0.50$  with  $\xi = 0.005$  (dot-dashed blue line), 0.01 (dashed purple line), and 0.02 (dotted red line) in the right panel. The blue and yellow regions represent the experimental data and SM prediction, respectively.

<span id="page-9-3"></span>

FIG. 6. Combined constraints on the  $(\phi_{bs}, \xi)$  two-dimensional plane for the fixed value  $\omega_{B_s} = 0.55$ . The green, yellow and blue regions represent the constraints from the branching fraction and the longitudinal polarization fraction of  $B_s \to K^{*0} \bar{K}^{*0}$  decay, and  $L_{K^*\bar{K}^{*0}}$ -parameter, respectively.

contain all model-dependent information. Recently, ATLAS and CMS collaborations published the limits on  $M_{Z'}$  as a function of  $c_{u,d}^e$  where  $e' = e, \mu$  [\[60](#page-11-11)[,61\]](#page-11-12). The lower mass limits of 5.15(4.56) TeV are set based on the sequential standard model (superstring-inspired model) [\[61\]](#page-11-12), and the lower limits could reach 4.5 TeV for the  $E_6$ -motivated Z' boson [\[60\]](#page-11-11). However, our results are challenged by above measurements, because the combined parameter  $\xi \in [0.017, 0.018]$  implies that the large g' or small  $M_{Z'}$  are needed, as shown in Eq. [\(37\).](#page-6-0) We also note for high values of  $g'$ , the ratio  $g'/M_{Z'}$  can be quite large, which could spoil the narrow-width approximation. Besides, the current limits are all model dependent, and the model-independent analyses are not available yet. Therefore, the models with  $M_{Z'} \leq 3-4$  TeV required by flavor physics cannot be excluded totally by current data. We look forward to further searches of  $Z'$  in the current LHC experiment or future high-energy colliders.

#### IV. SUMMARY

<span id="page-9-2"></span>In this work, we studied the nonleptonic decays  $B_d \rightarrow$  $K^{*0} \bar{K}^{*0}$  and  $B_s \to K^{*0} \bar{K}^{*0}$  within the perturbative QCD approach, which is based on the  $k<sub>T</sub>$  factorization. With the new fitted distribution amplitudes of  $K^*$ , both the branching fractions and the polarization fractions are recalculated. Numerical results show that the theoretical results of  $B_d \rightarrow$  $K^{*0} \bar{K}^{*0}$  are in agreement with experimental measurements, while for the decay  $B_s \to K^{*0} \bar{K}^{*0}$ , the branching fraction and the longitudinal polarization fraction cannot agree with data simultaneously. We also explored the  $L_{K^*\bar{K}^{*0}}$ -parameter that is a combination of polarization fractions and branching fractions in order to reduce the theoretical uncertainties. In SM,  $L_{\text{R}}^{\text{PQCD}} = 12.7^{+5.6}_{-3.2}$  is obtained based<br>on POCD, which is still leves than the experimental data on PQCD, which is still larger than the experimental data. In order to identify whether the deviations come from the contribution of new physics, the accuracy of theoretical calculations should be further improved in the future, for example, by exploring the wave function of heavy B meson. On the other side, we are also encouraged to search for the effects of NP beyond SM. Then, we interpreted these deviations by introducing a family nonuniversal Z' boson in  $b \rightarrow s q \bar{q}$  transition. In order to reduce the number of new parameters, we simplified the model as possible. With the large shape parameter  $\omega_{B_s} = 0.55$  in the distribution amplitude of  $B_s$  meson, it is in a small parameter space  $\xi \in [0.017, 0.018]$  and  $\phi_{bs} \in [50^{\circ}, 65^{\circ}]$ that these three measurements (branching fraction, longitudinal polarization fraction, and  $L_{K^*\bar{K}^{*0}}$ -parameter) could be accommodated simultaneously. In such small parameter space, the theoretical uncertainties could be reduced remarkably. All our results are hopefully tested in LHCb experiment, Belle-II and future high-energy colliders.

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- <span id="page-9-0"></span>[1] Y. Li and C.-D. Lu, Recent anomalies in B physics, [Sci.](https://doi.org/10.1016/j.scib.2018.02.003) Bull. 63[, 267 \(2018\).](https://doi.org/10.1016/j.scib.2018.02.003)
- [2] S. Bifani, S. Descotes-Genon, A. Romero Vidal, and M.-H. Schune, Review of lepton universality tests in B decays, [J.](https://doi.org/10.1088/1361-6471/aaf5de) Phys. G 46[, 023001 \(2019\)](https://doi.org/10.1088/1361-6471/aaf5de).
- [3] D. London and J. Matias, B flavor anomalies: 2021 theoretical status report, [arXiv:2110.13270](https://arXiv.org/abs/2110.13270).
- <span id="page-9-1"></span>[4] W. Altmannshofer and P. Stangl, New physics in rare B decays after Moriond 2021, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-021-09725-1) 81, 952 [\(2021\).](https://doi.org/10.1140/epjc/s10052-021-09725-1)
- <span id="page-10-0"></span>[5] M. Bauer, B. Stech, and M. Wirbel, Exclusive nonleptonic decays of D, D(s), and B mesons, Z. Phys. C 34[, 103 \(1987\).](https://doi.org/10.1007/BF01561122)
- <span id="page-10-1"></span>[6] A. Khodjamirian,  $B \to \pi \pi$  decay in QCD, [Nucl. Phys.](https://doi.org/10.1016/S0550-3213(01)00194-8) B605[, 558 \(2001\).](https://doi.org/10.1016/S0550-3213(01)00194-8)
- <span id="page-10-2"></span>[7] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD Factorization for  $B \to \pi \pi$  Decays: Strong Phases and CP Violation in the Heavy Quark Limit, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.83.1914) 83, [1914 \(1999\)](https://doi.org/10.1103/PhysRevLett.83.1914).
- <span id="page-10-3"></span>[8] M. Beneke and M. Neubert, QCD factorization for  $B \to PP$ and  $B \rightarrow PV$  decays, Nucl. Phys. **B675**[, 333 \(2003\).](https://doi.org/10.1016/j.nuclphysb.2003.09.026)
- <span id="page-10-4"></span>[9] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, An effective field theory for collinear and soft gluons: Heavy to light decays, Phys. Rev. D 63[, 114020 \(2001\)](https://doi.org/10.1103/PhysRevD.63.114020).
- <span id="page-10-5"></span>[10] C. W. Bauer, D. Pirjol, and I. W. Stewart, A Proof of Factorization for  $B \to D\pi$ , [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.87.201806) 87, 201806 [\(2001\).](https://doi.org/10.1103/PhysRevLett.87.201806)
- <span id="page-10-6"></span>[11] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Fat penguins and imaginary penguins in perturbative QCD, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(01)00247-7) 504, [6 \(2001\).](https://doi.org/10.1016/S0370-2693(01)00247-7)
- [12] C.-D. Lu, K. Ukai, and M.-Z. Yang, Branching ratio and CP violation of  $B \to \pi \pi$  decays in perturbative QCD approach, Phys. Rev. D 63[, 074009 \(2001\)](https://doi.org/10.1103/PhysRevD.63.074009).
- <span id="page-10-7"></span>[13] A. Ali, G. Kramer, Y. Li, C.-D. Lu, Y.-L. Shen, W. Wang, and Y.-M. Wang, Charmless non-leptonic  $B_s$  decays to  $PP$ , PV and VV final states in the pQCD approach, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.76.074018) 76[, 074018 \(2007\).](https://doi.org/10.1103/PhysRevD.76.074018)
- <span id="page-10-8"></span>[14] M. Ciuchini, M. Pierini, and L. Silvestrini,  $B_s \to K^{(*)0} \bar{K}^{(*)0}$ Decays: The Golden Channels for New Physics Searches, Phys. Rev. Lett. 100[, 031802 \(2008\).](https://doi.org/10.1103/PhysRevLett.100.031802)
- <span id="page-10-9"></span>[15] B. Aubert et al. (BABAR Collaboration), Observation of  $B^0 \to K^{*0} \bar{K}^{*0}$  and Search for  $B^0 \to K^{*0} K^{*0}$ , [Phys. Rev.](https://doi.org/10.1103/PhysRevLett.100.081801) Lett. 100[, 081801 \(2008\)](https://doi.org/10.1103/PhysRevLett.100.081801).
- [16] B. Aubert et al. (BABAR Collaboration), Observation of  $B^0 \to K^{*0} \bar{K}^{*0}$  and Search for  $B^0 \to K^{*0} K^{*0}$ , [Phys. Rev.](https://doi.org/10.1103/PhysRevLett.100.081801) Lett. 100[, 081801 \(2008\)](https://doi.org/10.1103/PhysRevLett.100.081801).
- <span id="page-10-10"></span>[17] C. C. Chiang et al. (Belle Collaboration), Search for  $B^0 \to K^{*0} \bar{K}^{*0}, B^0 \to K^{*0} K^{*0}$  and  $B^0 \to K^+ \pi^- K^+ \pi^{\pm}$  Decays, Phys. Rev. D 81[, 071101 \(2010\).](https://doi.org/10.1103/PhysRevD.81.071101)
- <span id="page-10-11"></span>[18] R. Aaij et al. (LHCb Collaboration), First observation of the decay  $B_s^0 \to K^{*0} \bar{K}^{*0}$ , [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2012.02.001) **709**, 50 (2012).
- [19] R. Aaij et al. (LHCb Collaboration), Measurement of CP asymmetries and polarisation fractions in  $B_s^0 \to K^{*0} \bar{K}^{*0}$ decays, [J. High Energy Phys. 07 \(2015\) 166.](https://doi.org/10.1007/JHEP07(2015)166)
- [20] R. Aaij et al. (LHCb Collaboration), First measurement of the CP-violating phase  $\phi_s^{d\bar{d}}$  in  $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$  de-<br>cave I High Energy Phys. 03. (2018) 140 cays, [J. High Energy Phys. 03 \(2018\) 140.](https://doi.org/10.1007/JHEP03(2018)140)
- <span id="page-10-12"></span>[21] R. Aaij et al. (LHCb Collaboration), Amplitude analysis of the  $B_{(s)}^0 \rightarrow K^{*0} \bar{K}^{*0}$  decays and measurement of the branch-<br>ing fraction of the  $B^0 \rightarrow K^{*0} \bar{K}^{*0}$  decay. I High Energy ing fraction of the  $B^0 \to K^{*0} \bar{K}^{*0}$  decay, [J. High Energy](https://doi.org/10.1007/JHEP07(2019)032) [Phys. 07 \(2019\) 032.](https://doi.org/10.1007/JHEP07(2019)032)
- <span id="page-10-13"></span>[22] M. Beneke, J. Rohrer, and D. Yang, Branching fractions, polarisation and asymmetries of  $B \to VV$  decays, [Nucl.](https://doi.org/10.1016/j.nuclphysb.2007.03.020) Phys. B774[, 64 \(2007\)](https://doi.org/10.1016/j.nuclphysb.2007.03.020).
- <span id="page-10-14"></span>[23] Z.-T. Zou, A. Ali, C.-D. Lu, X. Liu, and Y. Li, Improved estimates of The  $B_{(s)} \rightarrow VV$  decays in perturbative QCD approach, Phys. Rev. D 91[, 054033 \(2015\).](https://doi.org/10.1103/PhysRevD.91.054033)
- <span id="page-10-15"></span>[24] J. Chai, S. Cheng, Y.-h. Ju, D.-C. Yan, C.-D. Lü, and Z.-J. Xiao, Charmless two-body B meson decays in perturbative QCD factorization approach, [arXiv:2207.04190](https://arXiv.org/abs/2207.04190).
- <span id="page-10-16"></span>[25] R. L. Workman et al. (Particle Data Group), Review of particle physics, [Prog. Theor. Exp. Phys.](https://doi.org/10.1093/ptep/ptac097) 2022, 083C01 [\(2022\).](https://doi.org/10.1093/ptep/ptac097)
- <span id="page-10-17"></span>[26] S. Descotes-Genon, J. Matias, and J. Virto, An analysis of  $B_{d,s}$  mixing angles in presence of new physics and an update of  $B_s \to \bar{K}^{0*}$  anti –  $K^{0*}$ , Phys. Rev. D 85[, 034010 \(2012\).](https://doi.org/10.1103/PhysRevD.85.034010)
- <span id="page-10-18"></span>[27] M. Algueró, A. Crivellin, S. Descotes-Genon, J. Matias, and M. Novoa-Brunet, A new B-flavor anomaly in  $B_{d,s} \to K^{*0} \bar{K}^{*0}$ : Anatomy and interpretation, [J. High](https://doi.org/10.1007/JHEP04(2021)066) [Energy Phys. 04 \(2021\) 066.](https://doi.org/10.1007/JHEP04(2021)066)
- <span id="page-10-19"></span>[28] J. Hua, H.-n. Li, C.-D. Lu, W. Wang, and Z.-P. Xing, Global analysis of hadronic two-body B decays in the perturbative QCD approach, Phys. Rev. D 104[, 016025 \(2021\).](https://doi.org/10.1103/PhysRevD.104.016025)
- <span id="page-10-20"></span>[29] J. Albrecht, D. van Dyk, and C. Langenbruch, flavor anomalies in heavy quark decays, [Prog. Part. Nucl. Phys.](https://doi.org/10.1016/j.ppnp.2021.103885) 120[, 103885 \(2021\).](https://doi.org/10.1016/j.ppnp.2021.103885)
- <span id="page-10-21"></span>[30] L.-S. Geng, B. Grinstein, S. Jäger, S.-Y. Li, J. Martin Camalich, and R.-X. Shi, Implications of new evidence for lepton-universality violation in  $b \rightarrow s \ell^+ \ell^-$  decays, [Phys.](https://doi.org/10.1103/PhysRevD.104.035029) Rev. D 104[, 035029 \(2021\)](https://doi.org/10.1103/PhysRevD.104.035029).
- <span id="page-10-22"></span>[31] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Weak decays beyond leading logarithms, [Rev. Mod. Phys.](https://doi.org/10.1103/RevModPhys.68.1125) 68, [1125 \(1996\)](https://doi.org/10.1103/RevModPhys.68.1125).
- <span id="page-10-23"></span>[32] H.-n. Li, Threshold resummation for exclusive B meson decays, Phys. Rev. D 66[, 094010 \(2002\).](https://doi.org/10.1103/PhysRevD.66.094010)
- <span id="page-10-24"></span>[33] H.-n. Li and H.-L. Yu, Perturbative QCD analysis of B meson decays, Phys. Rev. D 53[, 2480 \(1996\)](https://doi.org/10.1103/PhysRevD.53.2480).
- <span id="page-10-25"></span>[34] Y. Y. Keum, H.-N. Li, and A. I. Sanda, Penguin enhancement and  $B \to K\pi$  decays in perturbative QCD, [Phys. Rev.](https://doi.org/10.1103/PhysRevD.63.054008) D 63[, 054008 \(2001\)](https://doi.org/10.1103/PhysRevD.63.054008).
- <span id="page-10-26"></span>[35] Z.-j. Xiao, X.-f. Chen, and D.-q. Guo, Branching ratio and CP asymmetry of  $B_s \to \rho(\omega)K$  decays in the perturbative QCD approach, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-007-0209-7) 50, 363 (2007).
- <span id="page-10-27"></span>[36] Y. Li, C.-D. Lu, Z.-J. Xiao, and X.-Q. Yu, Branching ratio and CP asymmetry of  $B_s \to \pi^+\pi^-$  decays in the perturbative QCD approach, Phys. Rev. D 70[, 034009 \(2004\)](https://doi.org/10.1103/PhysRevD.70.034009).
- <span id="page-10-28"></span>[37] W. Wang, Y.-M. Wang, J. Xu, and S. Zhao, B-meson lightcone distribution amplitude from Euclidean quantities, Phys. Rev. D 102[, 011502 \(2020\)](https://doi.org/10.1103/PhysRevD.102.011502).
- <span id="page-10-30"></span><span id="page-10-29"></span>[38] P. Ball and G. W. Jones, Twist-3 distribution amplitudes of  $K^*$  and phi mesons, [J. High Energy Phys. 03 \(2007\) 069.](https://doi.org/10.1088/1126-6708/2007/03/069)
- [39] P. Ball and R. Zwicky,  $SU(3)$  breaking of leading-twist K and  $K^*$  distribution amplitudes: A reprise, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2005.11.068) 633, [289 \(2006\)](https://doi.org/10.1016/j.physletb.2005.11.068).
- <span id="page-10-31"></span>[40] P. Ball, V. M. Braun, and A. Lenz, Twist-4 distribution amplitudes of the K\* and phi mesons in QCD, [J. High](https://doi.org/10.1088/1126-6708/2007/08/090) [Energy Phys. 08 \(2007\) 090.](https://doi.org/10.1088/1126-6708/2007/08/090)
- <span id="page-10-32"></span>[41] P. Ball and R. Zwicky,  $B_{d,s} \to \rho, \omega, K^*, \phi$  decay formfactors from light-cone sum rules revisited, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.71.014029) 71[, 014029 \(2005\).](https://doi.org/10.1103/PhysRevD.71.014029)
- <span id="page-10-33"></span>[42] H.-Y. Cheng, C.-W. Chiang, and C.-K. Chua, Width effects in resonant three-body decays:  $B$  decay as an example, Phys. Lett. B 813[, 136058 \(2021\)](https://doi.org/10.1016/j.physletb.2020.136058).
- <span id="page-10-34"></span>[43] S. Descotes-Genon, A. Khodjamirian, and J. Virto, Lightcone sum rules for  $B \to K\pi$  form factors and applications to rare decays, [J. High Energy Phys. 12 \(2019\) 083.](https://doi.org/10.1007/JHEP12(2019)083)
- <span id="page-10-35"></span>[44] Z. Rui, Y. Li, and H.-n. Li, Four-body decays  $B_{(s)} \rightarrow$  $(K\pi)_{S/P}(K\pi)_{S/P}$  in the perturbative QCD approach, [J. High](https://doi.org/10.1007/JHEP05(2021)082) [Energy Phys. 05 \(2021\) 082.](https://doi.org/10.1007/JHEP05(2021)082)
- <span id="page-11-0"></span>[45] P. Langacker and M. Plumacher, Flavor changing effects in theories with a heavy  $Z'$  boson with family nonuniversal couplings, Phys. Rev. D 62[, 013006 \(2000\).](https://doi.org/10.1103/PhysRevD.62.013006)
- <span id="page-11-1"></span>[46] P. Langacker, The physics of heavy  $Z'$  gauge bosons, [Rev.](https://doi.org/10.1103/RevModPhys.81.1199) Mod. Phys. 81[, 1199 \(2009\)](https://doi.org/10.1103/RevModPhys.81.1199).
- <span id="page-11-2"></span>[47] S. Chaudhuri, S. W. Chung, G. Hockney, and J. D. Lykken, String consistency for unified model building, [Nucl. Phys.](https://doi.org/10.1016/0550-3213(95)00147-7) B456[, 89 \(1995\)](https://doi.org/10.1016/0550-3213(95)00147-7).
- <span id="page-11-3"></span>[48] V. D. Barger, N. G. Deshpande, T. Kuo, A. Bagneid, S. Pakvasa, and K. Whisnant, Discovery limits of new gauge bosons of  $Sp<sub>L</sub>(6) \times U(1)$ , [Int. J. Mod. Phys. A](https://doi.org/10.1142/S0217751X87000697) 02, 1327 [\(1987\).](https://doi.org/10.1142/S0217751X87000697)
- <span id="page-11-4"></span>[49] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab,  $B \to \pi \pi$ , new physics in  $B \to \pi K$  and implications for rare K and B decays, Phys. Rev. Lett. 92[, 101804 \(2004\)](https://doi.org/10.1103/PhysRevLett.92.101804).
- [50] V. Barger, L. Everett, J. Jiang, P. Langacker, T. Liu, and C. Wagner, Family nonuniversal U(1)-prime gauge symmetries and  $b \rightarrow s$  Transitions, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.80.055008) 80, 055008 [\(2009\).](https://doi.org/10.1103/PhysRevD.80.055008)
- [51] J. Hua, C. S. Kim, and Y. Li, Testing the nonuniversal Z' model in  $B_s \rightarrow \phi \pi^0$  decay, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2010.06.004) 690, 508 [\(2010\).](https://doi.org/10.1016/j.physletb.2010.06.004)
- <span id="page-11-6"></span>[52] Q. Chang, X.-Q. Li, and Y.-D. Yang, A comprehensive analysis of hadronic  $b \rightarrow s$  transitions in a family nonuniversal Z-prime model, J. Phys. G 41[, 105002 \(2014\).](https://doi.org/10.1088/0954-3899/41/10/105002)
- [53] Y. Li, W.-L. Wang, D.-S. Du, Z.-H. Li, and H.-X. Xu, Impact of family-nonuniversal  $Z'$  boson on pure annihilation

 $B_s \to \pi^+\pi^-$  and  $B_d \to K^+K^-$  decays, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-015-3552-0) 75, [328 \(2015\)](https://doi.org/10.1140/epjc/s10052-015-3552-0).

- [54] Q. Chang, X.-Q. Li, and Y.-D. Yang, Constraints on the nonuniversal Z' couplings from  $B \to \pi K$ ,  $\pi K^*$  and  $\rho K$ Decays, [J. High Energy Phys. 05 \(2009\) 056.](https://doi.org/10.1088/1126-6708/2009/05/056)
- <span id="page-11-5"></span>[55] A. Celis, J. Fuentes-Martin, M. Jung, and H. Serodio, Family nonuniversal Z' models with protected flavorchanging interactions, Phys. Rev. D 92[, 015007 \(2015\).](https://doi.org/10.1103/PhysRevD.92.015007)
- <span id="page-11-7"></span>[56] V. Barger, C.-W. Chiang, P. Langacker, and H.-S. Lee, Solution to the  $B \to \pi K$  puzzle in a flavor-changing Z' model, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2004.07.057) 598, 218 (2004).
- <span id="page-11-8"></span>[57] R. Mohanta and A. K. Giri, Explaining  $B \to K\pi$  anomaly with nonuniversal  $Z'$  boson, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.79.057902) 79, 057902 [\(2009\).](https://doi.org/10.1103/PhysRevD.79.057902)
- <span id="page-11-9"></span>[58] E. Accomando, A. Belyaev, L. Fedeli, S. F. King, and C. Shepherd-Themistocleous, Z' physics with early LHC data, Phys. Rev. D 83[, 075012 \(2011\)](https://doi.org/10.1103/PhysRevD.83.075012).
- <span id="page-11-10"></span>[59] G. Paz and J. Roy, Remarks on the Z' Drell-Yan cross section, Phys. Rev. D 97[, 075025 \(2018\).](https://doi.org/10.1103/PhysRevD.97.075025)
- <span id="page-11-11"></span>[60] G. Aad et al. (ATLAS Collaboration), Search for high-mass dilepton resonances using 139 fb<sup>-1</sup> of  $pp$  collision data collected at  $\sqrt{s}$  = 13 TeV with the ATLAS detector, [Phys.](https://doi.org/10.1016/j.physletb.2019.07.016)<br>Lett B **796** 68 (2019) Lett. B 796[, 68 \(2019\)](https://doi.org/10.1016/j.physletb.2019.07.016).
- <span id="page-11-12"></span>[61] A. M. Sirunyan et al. (CMS Collaboration), Search for resonant and nonresonant new phenomena in high-mass dilepton final states at  $\sqrt{s} = 13$  TeV, [J. High Energy Phys.](https://doi.org/10.1007/JHEP07(2021)208)<br>07 (2021) 208 [07 \(2021\) 208.](https://doi.org/10.1007/JHEP07(2021)208)