

Novel counterexample to the Nelson-Seiberg theorem

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We present a new type of counterexample to the Nelson-Seiberg theorem. It is a generic R -symmetric Wess-Zumino model with nine chiral superfields, including one field of R -charge 2 and no R -charge 0 field. As in previous counterexamples, the model gives a set of degenerate supersymmetric vacua with a nonzero expectation value for a pair of oppositely R -charged fields. However, one of these fields appears quadratically in the superpotential, and many other fields with nonzero R charges gain nonzero expectation values at the vacuum, and so this model escapes the sufficient condition for counterexamples established in previous literature. Thus there are still open problems in the relation of R symmetries to supersymmetry breaking in generic models.

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I. INTRODUCTION

The Nelson-Seiberg theorem relates R symmetries to supersymmetry (SUSY) breaking in generic $\mathcal{N} = 1$ Wess-Zumino models. The original result [1] states that the presence of an R symmetry is a necessary condition, and a broken R symmetry is a sufficient condition, for SUSY breaking at the stable vacuum of a generic model. A refinement of this result [2,3] relates the existence of a SUSY vacuum to the numbers of fields with certain R charges in a model with a polynomial superpotential. However, exceptions [4] to both of these results have been found, in which a model with generic coefficients breaks the R symmetry at the SUSY vacuum. The source of these exceptions has been identified [5] as pairs of fields with opposite R charges obtaining vacuum expectation values (VEVs). Features of these exceptions can be summarized into a sufficient condition [6]. To summarize, a sufficient condition for the existence of a SUSY vacuum in a generic R -symmetric Wess-Zumino model is that the number of R -charge 2 fields is less than or equal to the sum of the number of R -charge 0 fields and the number of independent products of oppositely R -charged fields, which appear only linearly in cubic terms of a renormalizable superpotential.

In this note, we demonstrate that this sufficient condition is not also necessary by constructing a generic R -symmetric

superpotential that does not satisfy the above condition. The model nonetheless possesses a set of SUSY vacua where many fields with nonzero R charges gain nonzero VEVs. Therefore, this model is a counterexample to the Nelson-Seiberg theorem and escapes the sufficient condition established in previous literature.

The rest of this paper is arranged as follows. Section II reviews the sufficient condition for SUSY vacua in R -symmetric Wess-Zumino models, which covers all previous counterexamples. Section III presents the new counterexample and its vacuum structure, showing that it is a counterexample escaping the previous sufficient condition. Section IV discusses properties of the SUSY vacuum and implications of the result.

II. THE SUFFICIENT CONDITION FOR SUSY VACUA

Here, we briefly summarize the results of [2,6]; for details, we refer readers to those papers.

Under a continuous $U(1)$ R symmetry, where the R charge for Grassmann numbers θ^α is set to 1, the superpotential $W(\phi_i)$, built from scalar fields ϕ_i or their corresponding chiral superfields, must have R -charge 2 to make the SUSY action R invariant. Thus only R -charge 2 fields may appear as linear terms in the superpotential. Following the convention of [6], we call such fields X_i . The terms linear in X_i , which may appear in a renormalizable superpotential, are

$$W_X = a_i X_i + b_{ij} X_i Y_j + c_{ijk} X_i Y_j Y_k + d_{(r)ijk} X_i P_{(r)j} Q_{(-r)k}, \quad (1)$$

where a_i , b_{ij} , c_{ijk} , and $d_{(r)ijk}$ are coefficients, Y_j are R -charge 0 fields, and the fields $P_{(r)i}$ and $Q_{(-r)i}$ have opposite

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R -charges $\pm r$ so that their product is R neutral. In addition, the assumption is made that the P and Q fields appear only linearly in cubic terms. Thus, in addition to W_X , other terms that may appear in a renormalizable superpotential are

$$\begin{aligned}
 W_A = & \underbrace{\xi_{ijk} X_i X_j A_k}_{r_k=-2} + \underbrace{\rho_{ijk} X_i A_j A_k}_{r_j+r_k=0} + \underbrace{\sigma_{(r)ijk} P_{(r)i} A_j A_k}_{r_j+r_k=2-r} \\
 & + \underbrace{\tau_{(r)ijk} Q_{(-r)i} A_j A_k}_{r_j+r_k=2+r} + \underbrace{(\mu_{ij} + \nu_{ijk} Y_k) A_i A_j}_{r_i+r_j=2} \\
 & + \underbrace{\lambda_{ijk} A_i A_j A_k}_{r_i+r_j+r_k=2}, \quad (2)
 \end{aligned}$$

where ξ_{ijk} , ρ_{ijk} , $\sigma_{(r)ijk}$, $\tau_{(r)ijk}$, μ_{ij} , ν_{ijk} , and λ_{ijk} are coefficients, and A_i are fields that have R charges not equal to 2 or 0 and cannot be identified as P or Q fields. The full superpotential

$$W = W_X + W_A \quad (3)$$

contains all possible R -charge 2 terms built from all fields in our classification according to their R charges.

When seeking SUSY vacua, that is, solutions to the F -term equations,

$$\partial_i W = \frac{\partial W}{\partial \phi_i} = 0, \quad (4)$$

one can satisfy all the F -term equations coming from derivatives with respect to Y , P , Q , and A fields, by assuming that only Y , P , and Q fields obtain nonzero VEVs. The number of F -term equations coming from derivatives with respect to X fields is equal to N_X , the number of X fields, while the number of independent variables in these equations is equal to the sum of N_Y , the number of Y fields, and N_{PQ} , the number of independent P - Q pair products, which can be expressed as

$$N_{PQ} = \sum_r (N_{P(r)} + N_{Q(-r)} - 1), \quad (5)$$

where $N_{P(r)}$ and $N_{Q(-r)}$ are the numbers of P and Q fields with R -charges $\pm r$, and the sum is taken only over values of r for which $N_{P(r)}$ and $N_{Q(-r)}$ are nonzero. These equations are always solvable [7] for generic superpotential coefficients if the number of equations is less than or equal to the number of variables, and so a sufficient condition for the existence of SUSY vacua is

$$N_X \leq N_Y + N_{PQ}. \quad (6)$$

This condition includes the case $N_X \leq N_Y$, under which the revised Nelson-Seiberg theorem predicts the existence of SUSY vacua [8], and the case $N_Y < N_X \leq N_Y + N_{PQ}$,

which is satisfied by all previous counterexample models [4–6]. In the latter case, the facts $N_X > N_Y$ and that P and Q fields get nonzero VEVs for generic superpotential coefficients indicate that models in this case are counterexamples to both the original Nelson-Seiberg theorem [1] and its revision [2].

In the following section, we shall demonstrate a counterexample that does not satisfy the sufficient condition (6). The model gives a set of SUSY vacua where many fields other than Y , P , and Q fields get VEVs. The existence of such a new counterexample means that the sufficient condition presented here is not also a necessary condition for SUSY vacua in R -symmetric Wess-Zumino models.

III. THE NEW COUNTEREXAMPLE

Consider a Wess-Zumino model with nine fields: X , B , C , Ξ_1 , Ξ_2 , Ξ_3 , A_1 , A_2 , and A_3 . The superpotential is given as

$$\begin{aligned}
 W = & X(a + bBC) + \Xi_1(\alpha_1 A_1 + \beta_1 B^2) + \Xi_2(\alpha_2 A_2 + \beta_2 A_3^2) \\
 & + \Xi_3(\alpha_3 B + \beta_3 A_2^2 + \gamma_3 A_1 C) + \gamma_1 \Xi_1^2 A_3, \quad (7)
 \end{aligned}$$

where a , b , α_i , β_i , γ_i are coefficients. This superpotential possesses a $U(1)$ R symmetry, under which the fields have the R -charge assignment:

$$\begin{aligned}
 & \{r_X, r_B, r_C, r_{\Xi_1}, r_{\Xi_2}, r_{\Xi_3}, r_{A_1}, r_{A_2}, r_{A_3}\} \\
 & = \left\{ 2, \frac{8}{15}, -\frac{8}{15}, \frac{14}{15}, \frac{26}{15}, \frac{22}{15}, \frac{16}{15}, \frac{4}{15}, \frac{2}{15} \right\}. \quad (8)
 \end{aligned}$$

This assignment is unique, or equivalently [9], there is no other continuous symmetry of the model. The superpotential above contains all renormalizable terms permitted by this R symmetry, so it is the form of a generic superpotential given the fields and their R charges.

For generic values of the coefficients, we have a set of SUSY vacua at

$$\begin{aligned}
 X = \Xi_1 = \Xi_2 = \Xi_3 = 0, \quad BC = \frac{-a}{b}, \quad A_1 = \frac{-\beta_1}{\alpha_1} B^2, \\
 A_2 = \left(-\left(\frac{\alpha_3}{\beta_3} + \gamma_3 \frac{a\beta_1}{b\alpha_1\beta_3} \right) B \right)^{\frac{1}{2}}, \\
 A_3 = \left(-\frac{\alpha_2^2}{\beta_2^2} \left(\frac{\alpha_3}{\beta_3} + \gamma_3 \frac{a\beta_1}{b\alpha_1\beta_3} \right) B \right)^{\frac{1}{4}}, \quad (9)
 \end{aligned}$$

with a one complex dimensional degeneracy parametrized by the nonzero VEV of B . Like any SUSY vacuum in generic R -symmetric models, the vacua have the property that the superpotential vanishes term by term [10] and satisfies the bound found in [11]. The R symmetry is spontaneously broken everywhere on the degeneracy by all the nonzero VEVs of B , C , and A_i . Thus, this model is a counterexample to the Nelson-Seiberg theorem. The model has $N_X = 1$, $N_Y = 0$. Although B and C have opposite R charges, they cannot be identified as P and Q fields because

B appears quadratically in $\beta_1 \Xi_1 B^2$ and in the quadratic term $\alpha_3 \Xi_3 B$. Therefore, we have $N_{PQ} = 0$, and the model escapes the previous sufficient condition (6).

We may see the full vacuum structure of the model from the scalar potential

$$V = (\partial^i W)^* \partial_i W, \quad (10)$$

where a minimal Kähler potential is assumed. Like any R -symmetric polynomial superpotential that does not contain at least one field of charge 2 and at least one field of charge 0, the scalar potential has a stationary point at the origin of field space.¹ In this case, this point is a saddle. Numerical searches also indicate that there are several metastable local minima with $|V| > 0$, thus SUSY breaking.

Finally, we note that other than $\partial_X W$, which is unchanged, all the F -terms $\partial_i W$ have a positive R charge. This means that [12–15] under a complexified R symmetry,

$$\phi \rightarrow e^{-R_\phi t} \phi, \quad t \in \mathbb{R}, \quad (11)$$

all the non- X F -terms will tend to zero as $t \rightarrow +\infty$. We thus might have a runaway direction as $C \rightarrow \infty, B \rightarrow 0$. However, as the complexified R symmetry also takes all other fields to zero in this limit, this coincides with the large- Q limit of the SUSY solution we found above.

IV. DISCUSSIONS

As we have shown, the model presented in this work has a field count satisfying $N_X > N_Y + N_{PQ}$, which is outside

¹At a stationary point, we have $\partial_j V = (\partial^i W)^* \partial_i \partial_j W = 0$. For a polynomial superpotential, these equations may contain a constant term only if the superpotential contains terms of the form $aX + bXY$, where X necessarily has an R charge of 2, and Y an R charge of 0. This property is independent of any considerations of genericity, assuming $a, b \neq 0$.

of the previous classes of both the R -symmetric SUSY vacua [8] and the R -symmetry breaking SUSY vacua covered by the sufficient condition [6]. That the SUSY vacua are R -symmetry breaking also indicates that the model is a counterexample to the original Nelson-Seiberg theorem. The existence of such a new counterexample suggests that there are still some unexplored corners in the classification of R -symmetric Wess-Zumino models.

Just like any SUSY vacuum in R -symmetric models, the SUSY vacua in the new counterexample give $W = 0$ at the SUSY vacuum [10,11,16], and the supergravity version of the model also gives SUSY vacua with zero vacuum energy. One may hope use the supergravity model as a low energy effective description for flux compactification of type IIB string theory [17–20], and such string constructions of $W = 0$ SUSY vacua [21–27] serve as the first step toward vacua with small superpotentials [28]. However, the R -symmetry breaking feature of the vacua means that some complex structure moduli obtain nonzero VEVs, which send the Calabi-Yau manifold away from the R -symmetric point in its moduli space. It is then unnatural to turn on only R -symmetric fluxes and obtain an R -symmetric effective superpotential from the start. Thus, similarly to previous counterexample models, the new counterexample here does not contribute to the string landscape of $W = 0$ SUSY vacua if we only consider R -symmetric SUSY vacua [23] or string vacua with enhanced symmetries [21,22]. It is still an open question whether these counterexamples could be low energy effective models for other string constructions.

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- [1] A. E. Nelson and N. Seiberg, R symmetry breaking versus supersymmetry breaking, *Nucl. Phys.* **B416**, 46 (1994).
 - [2] Z. Kang, T. Li, and Z. Sun, The Nelson-Seiberg theorem revised, *J. High Energy Phys.* **12** (2013) 093.
 - [3] Z. Li and Z. Sun, The Nelson-Seiberg theorem generalized with nonpolynomial superpotentials, *Adv. High Energy Phys.* **2020**, 3701943 (2020).
 - [4] Z. Sun, Z. Tan, and L. Yang, A counterexample to the Nelson-Seiberg theorem, *J. High Energy Phys.* **10** (2020) 072.
 - [5] A. Amariti and D. Sauro, On the Nelson-Seiberg theorem: Generalizations and counter-examples, [arXiv:2005.02076](https://arxiv.org/abs/2005.02076).
 - [6] Z. Sun, Z. Tan, and L. Yang, A sufficient condition for counterexamples to the Nelson-Seiberg theorem, *J. High Energy Phys.* **07** (2021) 175.
 - [7] Z. Li and Z. Sun, Nonexistence of supersymmetry breaking counterexamples to the Nelson-Seiberg theorem, *J. High Energy Phys.* **10** (2021) 170.
 - [8] Z. Sun, Low energy supersymmetry from R symmetries, *Phys. Lett. B* **712**, 442 (2012).
 - [9] Z. Komargodski and D. Shih, Notes on SUSY and R symmetry breaking in Wess-Zumino models, *J. High Energy Phys.* **04** (2009) 093.
 - [10] J. Brister, Z. Sun, and G. Yang, A formal notion of genericity and term-by-term vanishing superpotentials at supersymmetric vacua from R -symmetric Wess-Zumino models, *J. High Energy Phys.* **12** (2021) 199.
 - [11] M. Dine, G. Festuccia, and Z. Komargodski, A bound on the superpotential, *J. High Energy Phys.* **03** (2010) 011.

- [12] L. Ferretti, R symmetry breaking, runaway directions and global symmetries in O’Raifeartaigh models, *J. High Energy Phys.* **12** (2007) 064.
- [13] L. Ferretti, O’Raifeartaigh models with spontaneous R symmetry breaking, *AIP Conf. Proc.* **957**, 221 (2007).
- [14] T. Azeyanagi, T. Kobayashi, A. Ogasahara, and K. Yoshioka, Runaway, D term and R symmetry breaking, *Phys. Rev. D* **86**, 095026 (2012).
- [15] Z. Sun and X. Wei, Runaway directions in O’Raifeartaigh models, *Commun. Theor. Phys.* **70**, 677 (2018).
- [16] R. Kappl, H. P. Nilles, S. Ramos-Sanchez, M. Ratz, K. Schmidt-Hoberg, and P. K. S. Vaudrevange, Large Hierarchies from Approximate R Symmetries, *Phys. Rev. Lett.* **102**, 121602 (2009).
- [17] M. Grana, Flux compactifications in string theory: A comprehensive review, *Phys. Rep.* **423**, 91 (2006).
- [18] M. R. Douglas and S. Kachru, Flux compactification, *Rev. Mod. Phys.* **79**, 733 (2007).
- [19] R. Blumenhagen, B. Kors, D. Lüst, and S. Stieberger, Four-dimensional string compactifications with D-branes, orientifolds and fluxes, *Phys. Rep.* **445**, 1 (2007).
- [20] L. E. Ibanez and A. M. Uranga, *String Theory and Particle Physics: An Introduction to String Phenomenology* (Cambridge University Press, Cambridge, England, 2012).
- [21] O. DeWolfe, A. Giryavets, S. Kachru, and W. Taylor, Enumerating flux vacua with enhanced symmetries, *J. High Energy Phys.* **02** (2005) 037.
- [22] O. DeWolfe, Enhanced symmetries in multiparameter flux vacua, *J. High Energy Phys.* **10** (2005) 066.
- [23] M. Dine and Z. Sun, R symmetries in the landscape, *J. High Energy Phys.* **01** (2006) 129.
- [24] E. Palti, Low energy supersymmetry from non-geometry, *J. High Energy Phys.* **10** (2007) 011.
- [25] K. Kanno and T. Watari, Revisiting arithmetic solutions to the $W = 0$ condition, *Phys. Rev. D* **96**, 106001 (2017).
- [26] E. Palti, C. Vafa, and T. Weigand, Supersymmetric protection and the Swampland, *J. High Energy Phys.* **06** (2020) 168.
- [27] K. Kanno and T. Watari, $W = 0$ complex structure moduli stabilization on CM-type $K3 \times K3$ orbifolds:—arithmetic, geometry and particle physics—, [arXiv:2012.01111](https://arxiv.org/abs/2012.01111).
- [28] M. Demirtas, M. Kim, L. McAllister, and J. Moritz, Vacua with Small Flux Superpotential, *Phys. Rev. Lett.* **124**, 211603 (2020).