Maximally symmetric nonlinear extension of electrodynamics with Galilean conformal symmetries

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A maximally symmetric nonlinear extension of Maxwell's theory in four dimensions called ModMax has been recently introduced in the literature. This theory preserves both electromagnetic duality and conformal invariance of the linear theory. In this short paper, we introduce a Galilean cousin of the ModMax theory, written in a covariant formalism, that is explicitly shown to be invariant under Galilean conformal symmetries. We discuss the construction of such a theory involving Galilean electromagnetic invariants and show how the classical structure of the theory is invariant under the action of Galilean conformal algebra.

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I. INTRODUCTION

Maxwell's electrodynamics in $4d$ is special, in the sense it has two very important symmetries, namely, the four-dimensional conformal symmetry and electric-magnetic duality. Maxwell theory is certainly the most successful and well-known gauge theory of $U(1)$ fields since its introduction one and half centuries ago. Most notably, Maxwell's equations derived from this theory are linear in the field strength $F_{\mu\nu}$. It has been widely accepted that a generic quantum theory of electrodynamics should have higher-order corrections to the linear terms in $F_{\mu\nu}$ in the Lagrangian, arising from loop contributions [1], which reduces to the pure Maxwell term in the low-energy limit. This gave rise to the question of whether there exists other classical Lagrangians for $U(1)$ gauge fields which are already nonlinear in the field strength and gives rise to Maxwell theory in an effective description. These theories may capture new physics at different energy scales where the full nonlinear theory has to be taken care of, leading to corrections to known results. Thus materialized the studies of nonlinear electrodynamics (NLED), which has been going strong for more than a century already.

Notable examples of NLEDs include the famous Born-Infeld theory [2], a crucial component of string theory in the study of D-branes [3], that makes sure to keep self-energy of point particles finite. Other well-studied examples include actions involving various functionals of the field tensor, and a nice review for these constructions can be found in Ref. [4] and references therein. But the main caveat lies in the problem that a generic NLED in $4d$ is not conformally invariant; neither is it invariant under Hodge duality rotations. So the question people have been asking for decades, reads: *is it possible to write down a Lorentz invariant nonlinear theory of electrodynamics that preserves the symmetries of Maxwell Lagrangian?* As far as conformal invariance is concerned, it has been shown that as long as the Lagrangian is a homogeneous function of degree one of Maxwell Lorentz invariants, the theory remains invariant. However, the requirement of electromagnetic duality invariance takes a more involved form, as shown first by Bialynicki-Birula [5]. Reconciling the two conditions seemed to be an involved problem for decades; for example, Born-Infeld theory gives rise to manifestly duality invariant equations, but it is not conformal invariant.

Only recently, the question we posed has been completely answered by Bandos, Lechner, Sorokin, and Townsend [6] (see also [7]), who proposed a simultaneously duality-invariant and conformal theory of $U(1)$ fields, that reduces to Maxwell theory in a zero coupling limit. This theory has been generically called modified Maxwell theory or by the nickname "ModMax." This has generated considerable interest in the past couple years and has been shown to have many interesting properties as seen in the classical solutions [8], Hamiltonian formulation

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[9,10], coupling to charged particles [11], supersymmetric and other generalizations [12–15], and connection to black hole solutions [16–19]. It has also been shown that ModMax theories can be generated by a $T\bar{T}$-like (or $\sqrt{T\bar{T}}$-like) deformation of Maxwell theories [20–23], and a string theory context for ModMax has been introduced in Ref. [24]. This list, of course, does not do justice to the literature, and readers are directed to references and citations of these papers. Certainly, the general symmetry structure of ModMax is intriguing on its own and will be part of various studies in the near future.

Our goal in this note, however, is to meander from the well-traversed paths and try something very new. We would like to focus on the conformal nature of the ModMax theory and would like to see what happens when one looks at these symmetries going away from the relativistic situation. Specifically, we set out trying to write an analog *Galilean covariant* nonlinear Lagrangian which is invariant under the $4d$ Galilean conformal algebra (GCA). These symmetries arise when the we take a nonrelativistic (speed of light going to infinity) limit on the d-dimensional conformal algebra. At each and every dimension, this limit results in an infinite-dimensional Galilean conformal algebra [25], in contrast to their relativistic cousins which are only infinite dimensional in $2d$ [26]. Galilean electrodynamics has been studied for a long time, starting as early as with Le Ballac and Levy-Leblond [27]. In recent years, theories of Galilean electrodynamics have generated newfound interest due to the larger and rich symmetry structures associated to it, and in Ref. [28] a reformulation of Galilean conformal electrodynamics in various dimensions was introduced via taking nonrelativistic (NR) limits on the equations of motion of the Maxwell theory and has subsequently been developed in a bunch of works [29–35]. A caveat for taking such a limit is loss of manifest electric-magnetic duality, as the physics splits in two subsectors, where either the electric or the magnetic components of the gauge field A_μ dominate. The other problem is that the procedure of taking NR limits does not work accurately on the action formalism, and, hence, a proper covariant Galilean electrodynamic action is hard to write down.

The search for a covariant Galilean electrodynamics theory needs to be addressed by putting the gauge fields explicitly on a non-Lorentzian manifold, in this case a Newton-Cartan manifold. Newton-Cartan structures arise when we take the speed of light to infinity and the usual Riemannian notion of a manifold degenerates, paving the way for Galilean relativity [29,36,37]. These manifolds, in general, have a fiber bundle structure that keeps temporal and spatial diffeomorphisms separate from each other. An attempt to write down a Galilean covariant electrodynamics Lagrangian was recently made in Ref. [38], one which simultaneously describes electric- and magnetic-dominated realms of the theory. This Galilean Maxwell Lagrangian will be the building block of our current work, and, since that Lagrangian is manifestly invariant under the $4d$ GCA, we will try to introduce an explicit nonlinear covariant Lagrangian with a ModMax-like form, consequently showing the GCA invariance for the same as well. This will be the first instance of a nonlinear covariant Galilean conformal electrodynamics Lagrangian in the literature, to the best of our knowledge.

The rest of this short paper is organized in the following way: In Sec. II, we will briefly review the structure of ModMax electrodynamics. In Sec. III, we will revisit Newton-Cartan structures and construction of a gauge field Lagrangian on such a structure. Here, we will slightly differ from the approach of Ref. [38] and, instead, focus on the transformation of components of gauge fields under Galilean conformal symmetries. We will also discuss in detail the structure of Galilean invariant field bilinear that will be important to our construction. Then, in Sec. IV, we will go ahead and present our Galilean ModMax-like Lagrangian and show its explicit invariance under GCA symmetries, giving the generic form of ModMax Lagrangians (with square roots) an aura of universality when it comes to conformal invariance both in and beyond Lorentzian cases. In Sec. V, we will have further discussions and talk about probable future extensions.

II. MODMAX AS CONFORMAL NONLINEAR ELECTRODYNAMICS

As mentioned earlier, source-free ModMax theory in $4d$ Minkowski spacetime is both conformally invariant and E-M duality invariant. In this section we will briefly revisit some aspects of the ModMax Lagrangian, which will be later crucial for the discussion of the analogous Galilean theory.

A. Lagrangian and symmetries of the relativistic ModMax

In $4d$, generic electrodynamics Lagrangians can only be functions of the field strength tensor and do not include the derivatives of those. For $4d$ Maxwell electrodynamics, there are just two Lorentz invariant quantities, i.e.,

$$S=-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}=\frac{1}{2}(\mathbf{E^2}-\mathbf{B^2}),\qquad P=-\frac{1}{4}F^{\mu\nu}\tilde{F}_{\mu\nu}=\mathbf{E}\cdot\mathbf{B}. \tag{1}$$

Here, the first one is a Lorentz scalar and the second one is pseudoscalar, where the field strength and the Hodge dual field strength are defined as, respectively,

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\qquad \tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}, \tag{2}$$

where A^μ is an $U(1)$ gauge field. A generic theory of electrodynamics in this dimension will be an analytic function of these Lorentz invariants and will reduce to the pure Maxwell term in some (often weak field) limit.

As shown in Ref. [6], the unique one-parameter deformed family of Lorentz invariant modifications of the Maxwell Lagrangian which stays conformal invariant and produces E-M duality invariant equations of motion is the ModMax theory. The Lagrangian density for this theory can be written down as

$$\mathcal{L}_{MM} = \cosh\gamma S + \sinh\gamma\sqrt{S^2+P^2} = -\frac{\cosh\gamma}{4}[F^{\mu\nu}F_{\mu\nu}] + \frac{\sinh\gamma}{4}\sqrt{(F^{\mu\nu}F_{\mu\nu})^2 + (\tilde{F}^{\mu\nu}F_{\mu\nu})^2}. \tag{3}$$

Here, γ is the dimensionless parameter that controls the deformation, and at $\gamma = 0$ the theory reduces down to pure Maxwell, with $\gamma > 0$ having a well-defined solution space. The conditions posed by causality and unitarity demand the coupling constant to be non-negative $\gamma \geq 0$ [6]. The square root term can be thought of as a deformation to the Maxwell theory. The term under square root can also be rewritten as $\mathcal{F}^{\mu\nu}\bar{\mathcal{F}}_{\mu\nu}$, where $\mathcal{F}^{\mu\nu} = F^{\mu\nu} + i\tilde{F}^{\mu\nu}$ and the bar denotes complex conjugation. This structure of the deformation turns out to be crucial to show invariance of the equations of motion under conventional electromagnetic duality as well.

The unique nature of the ModMax Lagrangian makes sure of conformal invariance, as it is a homogeneous function of degree one under scale transformations [10]. One can see this symmetry via explicitly computing the stress-energy tensors of this theory, which turn out to be traceless and explicitly proportional to their Maxwell cousins.

B. Equations of motion

The equation of motion of the Lagrangian (3) is given by

$$\partial_\mu\left[(\cosh\gamma)F^{\mu\nu} - \sinh\gamma \times\left(\frac{(F^{\alpha\beta}F_{\alpha\beta})F^{\mu\nu} + (\tilde{F}^{\alpha\beta}F_{\alpha\beta})\tilde{F}^{\mu\nu}}{\sqrt{(F^{\alpha\beta}F_{\alpha\beta})^2 + (\tilde{F}^{\alpha\beta}F_{\alpha\beta})^2}}\right)\right] = 0, \tag{4}$$

which again boils down to the standard equation $dF = 0$ when we put $\gamma = 0$. One can note that these equations of motion for generic γ are ill defined for solutions having null electromagnetic fields, i.e., $FF = F\tilde{F} = 0$, like in the case of electromagnetic waves. It has been shown that these pathologies can be cured if one instead works in the Hamiltonian formalism [6].

The duality invariance associated to these equations, although not so important for our discussion here, is given by the so-called Galliard-Zumino duality conditions [39] for nonlinear electrodynamics:

$$\tilde{F}^{\mu\nu}F_{\mu\nu} = \tilde{E}^{\mu\nu}E_{\mu\nu}, \tag{5}$$

which replaces the usual rotations of field strengths well known for the source-free Maxwell equations. Here, the excitation field strength is given by $E_{\mu\nu} = \frac{\partial\mathcal{L}_{MM}}{\partial F^{\mu\nu}}$, and the Hodge dual of that is defined in the usual way. Notice that $E^{\mu\nu}$ turns out to be just $F^{\mu\nu}$ for a pure Maxwell theory. So it is clear that equations of motion deduced from general nonlinear theories would demand a symmetry under $U(1)$ rotations of $E^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$.[1] One can notice that our equation of motion (4) can be rewritten using the tensor $E^{\mu\nu}$ in Eq. (5) as

$$\partial_\mu E^{\mu\nu} = 0, \tag{6}$$

which makes sure this and the usual Bianchi identity $d\tilde{F} = 0$ are a set of duality invariant nonlinear equations of motion for the ModMax theory.

III. COVARIANT GALILEAN CONFORMAL ELECTRODYNAMICS

In this section, we will discuss the formalism associated to a covariant formulation of Galilean electrodynamics. As we have introduced before, it is essential to start with a geometric formulation of tensors on a Newton-Cartan manifold and put gauge fields explicitly on it to understand the true nature of Galilean conformal symmetries.

A. Galilean geometry

In Galilean sense, the usual Riemannian metrics are of no use, since they are degenerate and cannot be used to raise or lower indices on objects. At the limit of $c \to \infty$ the Poincaré group is replaced by the Galilei group, and the kinematical structure of the group allows one to define a manifold, called the Newton-Cartan manifold. The main ingredients of an intrinsically Galilean (or Newton-Cartan) manifold is the degenerate spatial metric $h^{\mu\nu}$ and a choice of null time direction τ_μ that gives rise to another two-index object $\tau_{\mu\nu} = -\tau_\mu\tau_\nu$ [32,40–42]. In $4d$, the simplest choices to represent these tensors are

$$h^{\mu\nu} = \begin{bmatrix} 0 & 0 \\ 0 & 1_{3\times3} \end{bmatrix}, \qquad \tau_{\mu\nu} = \begin{bmatrix} -1 & 0 \\ 0 & 0_{3\times3} \end{bmatrix}. \tag{7}$$

These two noninvertible Galilean tensors are used to define contravariant and covariant Galilean vectors, and the nowhere vanishing timelike vector field τ_μ is given by

[1] One can, however, write down an action principle for generic nonlinear electrodynamics that also manifestly shows electromagnetic duality symmetry beyond the equations of motion. See Ref. [15] for such a democratic formulation.

$$\tau_\mu = [1 \quad 0 \quad 0 \quad 0], \qquad \tau_\mu \tau^\mu = 1. \tag{8}$$

These two geometric ingredients are orthogonal in the sense $h^{\mu\nu}\tau_\nu = 0$. Both these objects remain invariant under Galilean boosts and rotation. We can remind the reader that time is absolute in Galilean relativity, which is inherent in these invariant structures. For the (h, τ) duo, which defines a gauge choice [i.e., the form of the tensors in Eq. (7)] for a Newton-Cartan spacetime, the respective covariant and contravariant objects are given by projective inverses of $(h^{\mu\nu}, \tau_\nu)$ and are given by

$$h_{\mu\nu} = \begin{bmatrix} a & b_i \\ b_i & 1_{3\times 3} \end{bmatrix}, \qquad \tau^{\mu\nu} = \tau^\mu \tau^\nu,$$
$$\text{where } \tau^\mu = (1, c_1, c_2, c_3). \tag{9}$$

These expressions follow from the definition of the Galilean tensors using projective inverse definitions given by $\tau^\mu \tau_\mu = 1$ and by $h^{\mu\alpha} h_{\alpha\beta} h^{\beta\nu} = h^{\mu\nu}$. One should note that these projective inverses are not generally Galilean invariants for all choices of the constants (a, b_i, c_i).

These tensors are crucial in defining Galilean objects in the theory; i.e., a covariant vector $\tilde{K}_\mu$ will be defined from the knowledge of a contravariant vector K^μ via the operation $\tilde{K}_\mu = \tau_{\mu\nu} K^\nu$, and an opposite operation $(K_\mu \to \tilde{K}^\mu)$ will be done via $h^{\mu\nu}$. Contrary to relativistic tensors, these operations are not invertible, since temporal and spatial components are split from each other due to the structure of $(h^{\mu\nu}, \tau_\nu)$. Similarly, covariant derivatives are defined as $\partial_\mu = (\partial_t, \partial_x, \partial_y, \partial_z)$, while the associated contravariant object reads $\partial^\mu = h^{\mu\nu}\partial_\nu = (0, \partial_x, \partial_y, \partial_z)$. We will be using these Galilean derivatives throughout the rest of this work, and they are not to be confused with usual derivatives used in Sec. II, for example.

B. Galilean isometries

As mentioned in the introduction, GCA is a Galilean or nonrelativistic contraction of the relativistic conformal algebra. Equivalently to the intrinsic description in the last section, Galilean conformal objects can be realized by taking the following contraction of coordinates on associated conformal theories [25]:

$$x_i \to \epsilon x_i, \qquad t \to t, \qquad \epsilon \to 0. \tag{10}$$

The above turns out to be equivalent to taking a $c \to \infty$ scaling. Remember that in $4d$ the conformal algebra is a finite dimensional algebra, and, hence, to start with we get the finite part of the GCA only when the above mentioned contraction is acted upon. This finite algebra is generated by rotations (J_{ij}), spacetime translations (H and P_i), boosts (B_i), scaling (D), and special conformal transformations (K and K_i). The vector fields associated to these generators are given by

$$J_{ij} = -(x_i\partial_j - x_j\partial_i), \quad P_i = \partial_i, \quad H = -\partial_t, \quad B_i = t\partial_i, \tag{11}$$

$$D = -(t\partial_t + x^i\partial_i), \qquad K = -(2tx^i\partial_i + t^2\partial_t), \qquad K_i = t^2\partial_i. \tag{12}$$

The i, j indices all correspond to purely spatial components in the above. Consider an extension of the generators in an n-dependent form:

$$L^{(n)} = -(n+1)t^n x^i \partial_i - t^{n+1}\partial_t, \qquad M_i^{(n)} = t^{n+1}\partial_i, \tag{13}$$

where, for $n = 0, \pm 1$, the generators $L^{(n)}$ and $M_i^{(n)}$ denote the set of finite GCA generators

$$L^{(-1,0,1)} = \{H, D, K\}, \qquad M_i^{(-1,0,1)} = \{P_i, B_i, K_i\}, \tag{14}$$

but, in principle, any value of n is admissible, hence giving rise to generators spanning an infinite-dimensional vector space. The rotation generators could also be given an infinite-dimensional lift as follows:

$$J_{ij}^{(n)} = -t^n(x_i\partial_j - x_j\partial_i). \tag{15}$$

Armed with these new generators, the full infinite-dimensional extended GCA can be written in the following form:

$$\begin{aligned}
&[L^{(n)}, L^{(m)}] = (n-m)L^{(n+m)}, \qquad [L^{(n)}, M_i^{(m)}] = (n-m)M_i^{(n+m)}, \qquad [M_i^{(n)}, M_j^{(m)}] = 0,\\
&[L^{(n)}, J_{ij}^{(m)}] = -mJ_{ij}^{(n+m)}, \qquad [J_{ij}^{(n)}, M_r^{(m)}] = -(M_i^{(n+m)}\delta_{jr} - M_j^{(n+m)}\delta_{ir}),\\
&[J_{ij}^{(n)}, J_{rs}^{(m)}] = \delta_{is}J_{rj}^{(n+m)} + \delta_{jr}J_{si}^{(n+m)} + \delta_{ir}J_{js}^{(n+m)} + \delta_{js}J_{ir}^{(n+m)}.
\end{aligned} \tag{16}$$

In the rest of the paper, we will be talking about theories which are manifestly invariant under transformations induced by this algebra.

C. Covariant Lagrangian and transformation laws

Conventionally, Galilean electrodynamics, and more specifically the conformal cousin of it, has been studied in the literature from an equation of motion point of view [27,28]. This hinges on the fact that there can be two distinct limits of relativistic electrodynamics, known as the electric and magnetic ones, that may correspond to a theory of Galilean electrodynamics. In the electric limit, the timelike components of the gauge field A^μ dominate (i.e., $E_i \gg B_i$), while in the magnetic case the spacelike components of the same dominate (i.e., $B_i \gg E_i$). For a theory with sources, one could take the same limits on the currents to write electric and magnetic equations of motion. To this effect, in Ref. [38] a composite albeit covariant Lagrangian was introduced, which consistently reproduces both electric and magnetic equations of motion.

The source-free action for the Galilean covariant Lagrangian can be written as

$$S(a_\mu, a^\mu, \partial_\mu a_\nu, \partial^\mu a^\nu) = \int d^3x dt \left[-\frac{1}{4} f_{\mu\nu} f^{\mu\nu}\right], \tag{17}$$

where the contravariant and covariant field strengths are given in terms of Galilean gauge fields a^μ and a_μ. These fields here are distinct objects due to the structure of Galilean tensors, and the respective field strengths read

$$f^{\mu\nu} = (\partial^\mu a^\nu - \partial^\nu a^\mu), \qquad f_{\mu\nu} = (\partial_\mu a_\nu - \partial_\nu a_\mu). \tag{18}$$

So there are two distinct equations of motion, obtained by varying the above action with respect to one of the two kinds of gauge fields, which do not depend on each other. One can see while variation with respect to a^μ gives rise to

$$\partial_\nu f^{\mu\nu} = 0, \tag{19}$$

which are the equations of motion in the electric limit; on the other hand, variation with respect to a_μ leads one to

$$\partial^\nu f_{\mu\nu} = 0, \tag{20}$$

generating the magnetic equations of motion[2] (see [28] for details). So, evidently, the contravariant gauge fields are responsible for the electric limit of the theory, while covariant ones are responsible for the magnetic limit of the theory.[3]

We will now write down the transformation of both kinds of the gauge fields under GCA,[4] which will be important in the two separate limits. The covariant formalism used here ensures that the theory is invariant under Galilean boosts, rotations, and translations. For gauge fields in the magnetic limit, the transformation laws for the covariant fields under rotations, Galilean boost, scale transformations, and the special conformal transformations (SCT) take the following form:

Rotations.—

$$\begin{aligned} \delta_J a_0 &= (x^i\partial^j - x^j\partial^i)a_0, \\ \delta_J a_k &= (x^i\partial^j - x^j\partial^i)a_k + (\delta^i_k a_j - \delta^j_k a_i). \end{aligned} \tag{22}$$

Galilean boosts.—

$$\delta_{B_m} a_0 = -t\partial_m a_0 - a_m, \qquad \delta_{B_m} a_i = -t\partial_m a_i. \tag{23}$$

Scale transformations.—

$$\delta_D(a_0, a_i) = (t\partial_t + x^l\partial_l + 1)(a_0, a_i). \tag{24}$$

SCT.—

$$\begin{aligned} \delta_K a_0 &= (t^2\partial_t + 2tx^l\partial_l + 2t)a_0 + 2x^l a_l, \\ \delta_K a_i &= (t^2\partial_t + 2tx^l\partial_l + 2t)a_i, \end{aligned} \tag{25a}$$

$$\delta_{K_m} a_0 = -t^2\partial_m a_0 - 2t a_m, \qquad \delta_{K_m} a_i = -t^2\partial_m a_i. \tag{25b}$$

Finally, we will write the variation of gauge fields in the magnetic limit under infinite-dimensional GCA generators $(L^{(n)}, M^{(n)}_m)$. They are given by

$$\begin{aligned} \delta_{L^{(n)}} a_0 &= (t^{n+1}\partial_t + (n+1)t^n x^l\partial_l + (n+1)t^n)a_0 \\ &\quad + n(n+1)t^{n-1}x^l a_l, \end{aligned} \tag{26a}$$

$$\delta_{L^{(n)}} a_i = (t^{n+1}\partial_t + (n+1)t^n x^l\partial_l + (n+1)t^n)a_i, \tag{26b}$$

$$\delta_{M^{(n)}_m} a_0 = -t^{n+1}\partial_m a_0 - (n+1)t^n a_m, \tag{26c}$$

$$\delta_{M^{(n)}_m} a_i = -t^{n+1}\partial_m a_i. \tag{26d}$$

Similarly, in the electric limit, we have transformation laws for contravariant gauge fields.

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[2]In component form, the electric and magnetic equations can be written as

$$\begin{aligned} &\partial_i\partial^i a^0 = 0, \qquad \partial^j(\partial_t a^0 + \partial_i a^i) = (\partial_i\partial^i)a^j \quad \text{(electric)} \\ &\partial_t\partial_i a_i = \partial_i\partial^i a_0, \qquad \partial^j\partial_i a_i = \partial_i\partial^i a_j \quad \text{(magnetic)}. \end{aligned} \tag{21}$$

[3]Covariant gauge fields are defined by $a^\mu\tau_\mu = 0$ and contravariant ones are defined by $a_\mu h^{\mu\nu} = 0$; these belong to invariant vector spaces under the action of the Galilean group.

[4]To get a better understanding of the representation theory, we urge the reader to look at Refs. [28,38].

Rotations.—

$$\delta_J a^0 = (x^i\partial^j - x^j\partial^i)a^0,$$
$$\delta_J a^k = (x^i\partial^j - x^j\partial^i)a^k + (\delta^{ki}a^j - \delta^{kj}a^i). \quad (27)$$

Galilean boosts.—

$$\delta_{B_m} a^0 = -t\partial_m a^0, \qquad \delta_{B_m} a^i = -t\partial_m a^i + \delta^i_m a^0. \quad (28)$$

Scale transformations.—

$$\delta_D(a^0, a^i) = (t\partial_t + x^l\partial_l + 1)(a^0, a^i). \quad (29)$$

SCT.—

$$\delta_K a^0 = (t^2\partial_t + 2tx^l\partial_l + 2t)a^0,$$
$$\delta_K a^i = (t^2\partial_t + 2tx^l\partial_l + 2t)a^i - 2x^i a^0, \quad (30a)$$

$$\delta_{K_m} a^0 = -t^2\partial_m a^0, \qquad \delta_{K_m} a^i = -t^2\partial_m a^i + 2t\delta^i_m a^0. \quad (30b)$$

Under $(L^{(n)}, M^{(n)}_m)$, the transformation laws are given by

$$\delta_{L^{(n)}} a^0 = (t^{n+1}\partial_t + (n+1)t^n x^l\partial_l + (n+1)t^n)a^0, \quad (31a)$$

$$\delta_{L^{(n)}} a^i = (t^{n+1}\partial_t + (n+1)t^n x^l\partial_l + (n+1)t^n)a^i - n(n+1)t^{n-1}x^i a^0, \quad (31b)$$

$$\delta_{M^{(n)}_m} a^0 = -t^{n+1}\partial_m a^0, \quad (31c)$$

$$\delta_{M^{(n)}_m} a^i = -t^{n+1}\partial_m a^i + (n+1)t^n\delta^i_m a^0. \quad (31d)$$

One can easily see the sheer asymmetry between the transformations of covariant and contravariant objects in this case, and, of course, the same shows up between temporal and spatial components. Using these above transformations, one could deduce the relevant transformation laws for the electric and magnetic field strengths as well and explicitly check the invariance of Eq. (17) under the same.

D. Electric and magnetic invariants

We have seen earlier that a relativistic ModMax Lagrangian depends on both Lorentz invariants in electrodynamics. Hence, for the purpose of this work, defining electromagnetic invariants under Galilean transformations is very important. Now, for example, $f^{\mu\nu}$ is clearly an electric object, since the gauge fields are contravariant here; similarly, $f_{\mu\nu}$ is a magnetic object for similar reasons. To mark their properties, we call them $f^{\mu\nu}_{(E)}$ and $f^{(M)}_{\mu\nu}$ from now and hereon.

The obvious invariant quantity is the covariant Lagrangian for Galilean Maxwell theory, which is a composite of electric and magnetic objects:

$$\mathcal{L} = -\frac{1}{4} f^{\mu\nu}_{(E)} f^{(M)}_{\mu\nu}; \quad (32)$$

i.e., a "true" Lagrangian is one composed of both electric and magnetic tensors with contracted indices and gives the right electric or magnetic equation of motion when varied with respect to one or the other gauge fields. But this is not the Lagrangian when one takes the relativistic Lagrangian and performs an electric or a magnetic limit. In that case, both field strength components of the Lagrangian change; i.e., we get either of

$$\mathcal{L}^{(E)} = -\frac{1}{4} f^{\mu\nu}_{(E)} f^{(E)}_{\mu\nu}, \qquad \mathcal{L}^{(M)} = -\frac{1}{4} f^{\mu\nu}_{(M)} f^{(M)}_{\mu\nu}, \quad (33)$$

which are useful in only one or the other limit. Here, the inverse field strengths $f^{(E)}_{\mu\nu}$ and $f^{\mu\nu}_{(M)}$ are not electric or magnetic objects, respectively, not at least by contra- or covariance of the associated gauge field. But they are *electric inverse of the electric field strength* and *magnetic inverse of the magnetic field strength*, in the same vein as defining the projective inverses for our Galilean tensors. To connect to the notation of Ref. [38], these are actually defined by the tilde conjugation, which acts via a Galilean contraction of gauge fields. For example, in the electric case,

$$f^{(E)}_{\mu\nu} = \tilde{f}_{\mu\nu} = -f^{\alpha\beta}T_{\mu\alpha\beta\nu} = (\partial_\mu\tilde{a}_\nu - \partial_\nu\tilde{a}_\mu), \quad \tilde{a}_\mu = a^\nu\tau_{\mu\nu}, \quad (34)$$

where the tensor T is defined as a combination of τ and h to achieve this:

$$T_{\alpha\beta\mu\nu} \coloneqq 4\tau_{[\alpha}h_{\beta][\mu}\tau_{\nu]} = (\tau_\alpha h_{\beta\mu}\tau_\nu - \tau_\beta h_{\alpha\mu}\tau_\nu - \tau_\alpha h_{\beta\nu}\tau_\mu + \tau_\beta h_{\alpha\nu}\tau_\mu). \quad (35)$$

It is easy to see that $T_{\alpha\beta\mu\nu}$ is symmetric under exchange of α and ν and of β and μ, i.e., $T_{\alpha\beta\mu\nu} = T_{\nu\mu\beta\alpha}$. Notice that $T_{\alpha\beta\mu\nu}$ is antisymmetric if we exchange α with β or μ with ν:

$$T_{\alpha\beta\mu\nu} = -T_{\beta\alpha\mu\nu} = -T_{\alpha\beta\nu\mu} = T_{\beta\alpha\nu\mu}. \quad (36)$$

From these, it also follows that $T_{\alpha\beta\mu\nu} - T_{\alpha\mu\beta\nu} = T_{\alpha\nu\mu\beta}$. Similarly, the magnetic inverse is given by contraction with only h's:

$$f^{\mu\nu}_{(M)} = \tilde{f}^{\mu\nu} = h^{\alpha\mu}f_{\alpha\beta}h^{\beta\nu} = (\partial^\mu\tilde{a}^\nu - \partial^\nu\tilde{a}^\mu), \quad \tilde{a}^\mu = h^{\mu\nu}a_\nu. \quad (37)$$

Note that one of these field strengths is dualized by $\tau_{\mu\nu}$ and the other by $h^{\mu\nu}$, thereby giving them the notion of an electric (temporal) or a magnetic (spatial) contraction. Hence, the Lagrangians (33) $\mathcal{L}^{(E/M)}$ are not "true" Lagrangians but limits of relativistic Lagrangians in the respective regimes where only notions of electric terms or magnetic terms survive. By definition, $\mathcal{L}^{(E/M)}$ both are Galilean invariants as one can explicitly show, and we will go ahead to define an additional GCA invariant other than that of $\mathcal{L}$ in Eq. (32):

$$\mathcal{M} = \frac{1}{\sqrt{2}}\sqrt{\mathcal{L}^{(E)2} + \mathcal{L}^{(M)2}}. \tag{38}$$

This particular quantity will be crucial in our later discussions.

Although we have defined a notion of "dual" field strengths for Galilean theories using the tilde conjugation, a real Hodge dual in this case is ill defined as the associated metric degenerates. Evidently, the notion of EM duality is lost, as the two regimes are not simply connected in a Galilean theory. To the best of our knowledge, the notion of Hodge duals on a Newton-Cartan manifold is not discussed in the literature as well. However, we can always go ahead and define the Hodge-dual-like field strength tensor for the Galilean case in accordance with its relativistic counterpart: $\star f_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}f^{\rho\sigma}$, assuming the definition of Levi-Civita will not change under NR limits. In this sense, it relates true electric and magnetic representations on either side of the equality

$$\star f^{(M)}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}f^{\rho\sigma}_{(E)} \quad \text{and} \quad \star f^{\mu\nu}_{(E)} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}f^{(M)}_{\rho\sigma}. \tag{39}$$

As we discussed before, a true contracted Galilean object will be a combination of purely electric and magnetic tensors. And based on the definitions we provided earlier, we can show that

$$\mathcal{L}^{*(E)} = -\frac{1}{4}f^{\mu\nu}_{(E)}\star f^{(E)}_{\mu\nu} \quad \text{or} \quad \mathcal{L}^{*(M)} = -\frac{1}{4}f^{(M)}_{\mu\nu}\star f^{\mu\nu}_{(M)} \tag{40}$$

are both invariant under the GCA transformations as well. Here, we have gone further to define the *electric and magnetic inverses of the Hodge dual tensor*:

$$\star f^{(E)}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}f^{\rho\sigma}_{(M)} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\tilde{f}^{\rho\sigma},$$
$$\star f^{\mu\nu}_{(M)} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}f^{(E)}_{\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\tilde{f}_{\rho\sigma}. \tag{41}$$

We should again remind the reader that the objects defined under this star conjugation are not real Galilean objects and are defined only in an *ad hoc* basis. We will comment on these invariants later in the paper.

IV. GALILEAN CONFORMAL MODMAX-LIKE LAGRANGIAN

A. Symmetries of the Lagrangian

Let us now come to the crux of this paper, that is to construct a nonlinear ModMax-like, albeit GCA invariant, Galilean electrodynamics theory. As in the case of the relativistic ModMax theory, we can define a Lagrangian in terms of the two GCA invariants $\mathcal{L}$ and $\mathcal{M}$ we described in the last section [see (32) and (38)], with the same general distinctive structure:

$$\mathcal{L}_{GMM} = -\frac{\cosh\gamma}{4}\mathcal{L} + \frac{\sinh\gamma}{4}\sqrt{\mathcal{L}^2 + \mathcal{M}^2}$$
$$= -\frac{\cosh\gamma}{4}[f^{\mu\nu}f_{\mu\nu}] + \frac{\sinh\gamma}{4}\sqrt{(f^{\mu\nu}f_{\mu\nu})^2 + \frac{1}{2}(\tilde{f}^{\mu\nu}f_{\mu\nu})^2 + \frac{1}{2}(\tilde{f}_{\mu\nu}f^{\mu\nu})^2}. \tag{42}$$

This simply becomes Eq. (17) when we choose $\gamma = 0$. In component form, the Lagrangian can be written down as

$$\mathcal{L}_{GMM} = -\frac{\cosh\gamma}{4}E + \frac{\sinh\gamma}{4}\sqrt{C}, \tag{43}$$

where, written in component form, the quantities read $C = [E^2 + \frac{1}{2}(\tilde{f}^{ij}f_{ij})^2 + \frac{1}{2}(2\tilde{f}_{i0}(\partial^i a^0))^2]$ and $E = [2f_{i0}(\partial^i a^0) + f^{ij}f_{ij}]$. Let us now move on to the invariance of this Lagrangian (43) under GCA. The Lagrangian is trivially invariant under translations and rotations. We will show only the invariance under the boost, scale transformations, and SCT. The change of the Lagrangian under action of the Boost generator is given by

$$\delta_B\mathcal{L}_{GMM} = \frac{\cosh\gamma}{4}\partial_m[t(2f_{i0}(\partial^i a^0) + f^{ij}f_{ij})]$$
$$- \frac{\sinh\gamma}{4}\frac{1}{2\sqrt{C}}\partial_m(tC) = \partial_m(-t\mathcal{L}_{GMM}). \tag{44}$$

We will next check the variation under scale transformation. It is given by

$$\delta_D\mathcal{L}_{GMM} = \frac{\cosh\gamma}{4}[(t\partial_t + x^m\partial_m + 4)E] + \frac{\sinh\gamma}{4}[(t\partial_t + x^m\partial_m + 4)\sqrt{C}]$$
$$= (t\partial_t + x^m\partial_m + 4)\mathcal{L}_{GMM} = \partial_t(t\mathcal{L}_{GMM}) + \partial_m(x^m\mathcal{L}_{GMM}). \tag{45}$$

Finally, looking at the change of (43) under special conformal transformations K, we get

$$\delta_K\mathcal{L}_{GMM} = (t^2\partial_t + 2tx^m\partial_m + 8t)\left[-\frac{\cosh\gamma}{4}E + \frac{\sinh\gamma}{4}\sqrt{C}\right]$$
$$= \partial_t(t^2\mathcal{L}_{GMM}) + \partial_m(2tx^m\mathcal{L}_{GMM}). \tag{46}$$

We have looked at the invariance of (43) under the finite part of GCA. The next step will be to move on to the infinite extension of GCA. Under the generators $M_m^{(n)}$, we get the variation

$$\delta_M\mathcal{L}_{GMM} = \frac{\cosh\gamma}{4}\partial_m(t^{n+1}E) - \frac{\sinh\gamma}{4}(t^{n+1}\partial_m\sqrt{C}) = \partial_m(-t^{n+1}\mathcal{L}_{GMM}). \tag{47}$$

Similarly, under $L^{(n)}$, we have

$$\delta_L\mathcal{L}_{GMM} = (t^{n+1}\partial_t + (n+1)t^n x^m\partial_m + 4(n+1)t^n)\left[-\frac{\cosh\gamma}{4}E + \frac{\sinh\gamma}{4}\sqrt{C}\right]$$
$$= \partial_t(t^{n+1}\mathcal{L}_{GMM}) + (n+1)\partial_m(t^n x^m\mathcal{L}_{GMM}). \tag{48}$$

In all of these cases, the transformations change the Lagrangian via a total derivative term, and, hence, we see that the theory is fully invariant under the extended part of GCA.

Now a few comments are in order at this point. Clearly, the structure of Eq. (42), like the relativistic counterpart, depends on the use of electromagnetic invariants, which occur directly in the Lagrangian. In the relativistic case, choices of these invariants are straightforward; however, it is evidently not so simple in the Galilean counterpart, as we have discussed before. It actually turns out that instead of $\mathcal{M}$ one could choose some other Galilean invariant as mentioned in Eq. (53). Consequently, one may go ahead and write down a test Lagrangian of the form:

$$\bar{\mathcal{L}}_{GMM} = -\frac{\cosh\gamma}{4}[f^{\mu\nu}_{(E)}f^{(M)}_{\mu\nu}] + \frac{\sinh\gamma}{4}\sqrt{(f^{\mu\nu}_{(E)}f^{(M)}_{\mu\nu})^2 + (f^{\mu\nu}_{(E)}\star f^{(E)}_{\mu\nu})^2} \tag{49}$$

or an equivalent one with the $f\star f$ term replaced by the magnetic version. It can be explicitly shown using methods we discussed in this section that the above Lagrangian is invariant under GCA symmetries as well. However as we mentioned earlier, these $f\star f$ terms in the Galilean case are defined in an *ad hoc* way; hence, we do not delve into detailed discussions on them. For us, Eq. (42) will be the master Lagrangian to follow through.

B. Equations of motion and gauge invariance

The equation of motion from Eq. (42) comes out to be twofold, as in the case of its Maxwellian cousin. Varying the action with respect to the contravariant fields a^μ, we get the electric-like equation of motion:

$$\partial_\mu\left[(\cosh\gamma)f^{\mu\nu} - \sinh\gamma\left(\frac{(f^{\alpha\beta}f_{\alpha\beta})f^{\mu\nu} + (\tilde{f}^{\alpha\beta}f_{\alpha\beta})\tilde{f}^{\mu\nu}}{\sqrt{(f^{\alpha\beta}f_{\alpha\beta})^2 + \frac{1}{2}(\tilde{f}^{\alpha\beta}f_{\alpha\beta})^2 + \frac{1}{2}(\tilde{f}_{\alpha\beta}f^{\alpha\beta})^2}}\right)\right] = 0, \tag{50}$$

while varying with respect to covariant gauge fields a_μ gives us the magnetic-like equation of motion

$$\partial^\mu\left[(\cosh\gamma)f_{\mu\nu} - \sinh\gamma\left(\frac{(f^{\alpha\beta}f_{\alpha\beta})f_{\mu\nu} + (\tilde{f}_{\alpha\beta}f^{\alpha\beta})\tilde{f}_{\mu\nu}}{\sqrt{(f^{\alpha\beta}f_{\alpha\beta})^2 + \frac{1}{2}(\tilde{f}^{\alpha\beta}f_{\alpha\beta})^2 + \frac{1}{2}(\tilde{f}_{\alpha\beta}f^{\alpha\beta})^2}}\right)\right] = 0. \tag{51}$$

Observe that both equations reduce down to the Galilean Maxwell electric and magnetic equations of motion when we put in $\gamma = 0$. Also, these are in the same footing as the equations of motion from the relativistic version of the ModMax theory. One can look at these equations and note that the electric and the magnetic sectors are interchanged via the exchange of field tensors $f^{\mu\nu} \leftrightarrow f_{\mu\nu}$ (which is equivalent to doing $E_i \to B_i$ and $B_i \to -E_i$ in the relativistic case). One can think of this as a Galilean remnant of the original electromagnetic duality.

We will now look at the gauge transformations (GT) for this theory. The gauge transformations for a^μ and a_μ, that keep the structure of Lagrangian unchanged, are given by

$$a^\mu \to a^\mu + \partial^\mu \Lambda_1 \Rightarrow f^{\mu\nu} \to f^{\mu\nu}, \tag{52a}$$

$$a_\mu \to a_\mu + \partial_\mu \Lambda_2 \Rightarrow f_{\mu\nu} \to f_{\mu\nu}, \tag{52b}$$

whereas $(\Lambda_{1,2})$ are two different gauge potentials corresponding to symmetries in either limit.[5] Similarly, the transformation of the conjugate field strengths $\tilde{f}^{\mu\nu}$ and $\tilde{f}_{\mu\nu}$ under Eqs. (52) are given by

$$\begin{aligned}\delta_{GT}\tilde{f}^{\alpha\beta} &= h^{\mu\alpha}(\delta_{GT} f_{\mu\nu})h^{\nu\beta} = 0,\\ \delta_{GT}\tilde{f}_{\alpha\beta} &= -(\delta_{GT} f^{\mu\nu})T_{\alpha\mu\nu\beta} = 0.\end{aligned} \tag{53}$$

We thus conclude that the Lagrangian and equations of motion are invariant under gauge transformations (52).

C. Energy-momentum tensors

Let us now write down the energy-momentum (EM) tensors in the electric and magnetic limit of Galilean ModMax theory (42). We will use the Noether charge methodology followed in Ref. [38] to deduce purely electric or magnetic stress tensors in either of those limits. In the electric limit, it is given by

$$T_{E\nu}^{\ \mu} = \left(f^{\mu\alpha}\tilde{f}_{\alpha\nu} + \frac{1}{4}\delta^\mu_\nu f^{\alpha\beta}\tilde{f}_{\alpha\beta}\right) \times \left(\cosh\gamma - \sinh\gamma \frac{\mathcal{L}^{(E)}}{\sqrt{\mathcal{L}^2 + \mathcal{M}^2}}\right), \tag{54}$$

whereas in the magnetic limit the stress tensor reads

$$T_{M\nu}^{\ \mu} = \left(\tilde{f}^{\mu\alpha} f_{\alpha\nu} + \frac{1}{4}\delta^\mu_\nu \tilde{f}^{\alpha\beta} f_{\alpha\beta}\right) \times \left(\cosh\gamma - \sinh\gamma \frac{\mathcal{L}^{(M)}}{\sqrt{\mathcal{L}^2 + \mathcal{M}^2}}\right). \tag{55}$$

[5] One would use an analog of the Lorenz gauge in either limit, i.e., $\partial_\mu a^\mu = 0$ or $\partial^\mu a_\mu = 0$, and this would imply that the gauge potentials satisfy a Laplace equation $\nabla^2 \Lambda_{1,2} = 0$.

This is again reminiscent of the relativistic ModMax case, as our stress tensors in either limit are explicitly proportional to the Galilean Maxwell ones of Ref. [38], with a multiplied contracted term. As we know for generic Galilean conformal theories, the stress tensor needs to be traceless, i.e., $T^\mu_\mu = 0$, and the condition on the component $T^0_i = 0$ has to be satisfied, since there is no momentum flux in nonrelativistic theories [43]. In case of the electric limit, it is easy to check these conditions:

$$T_{E\mu}^{\ \mu} = (-f^{\alpha\mu}\tilde{f}_{\alpha\mu} + f^{\alpha\beta}\tilde{f}_{\alpha\beta}) \times \left(\cosh\gamma - \sinh\gamma \frac{\mathcal{L}^{(E)}}{\sqrt{\mathcal{L}^2 + \mathcal{M}^2}}\right) = 0, \tag{56}$$

$$T_{Ei}^{\ 0} = (f^{0\alpha}\tilde{f}_{\alpha i})\left(\cosh\gamma - \sinh\gamma \frac{\mathcal{L}^{(E)}}{\sqrt{\mathcal{L}^2 + \mathcal{M}^2}}\right) = 0. \tag{57}$$

Similarly, in the magnetic limit, the stress tensor satisfies

$$T_{M\mu}^{\ \mu} = 0, \qquad T_{Mi}^{\ 0} = 0. \tag{58}$$

This shows both electric and magnetic sectors of our nonlinear theory are explicitly Galilean invariant as well.

V. DISCUSSIONS AND CONCLUSIONS

In this short paper, we described a nonlinear Galilean covariant Lagrangian that is invariant under Galilean conformal symmetries by construction. Interestingly, the Lagrangian was written in the same vein of the ModMax Lagrangian and, hence, reaffirms the conformal nature of the ModMax construction beyond the relativistic case. We focused on the invariants of the Galilean Maxwell theory and used them as building blocks to build our Lagrangian, with an explicit proof of invariance under GCA transformations. We also discussed the Galilean equations of motion and stress tensors, in both the electric and magnetic limits of the theory. The nonlinear equations in all of the cases reduce to the known Galilean equations in the $\gamma = 0$ limit.

As we mentioned earlier, our calculation here introduces the first example of a nonlinear Galilean covariant electrodynamics theory. It remains to explore whether the usual NLED physics, like classical solutions, carry forward to these sort of Galilean theories as well. As we mentioned earlier, it is useful to go into the Hamiltonian formalism for the relativistic ModMax theory in order to make sense of null electromagnetic fields. It would be interesting to explore the canonical structure of the Galilean theory as well along this route, starting from Eq. (17) and proceeding to find the same for the total Galilean ModMax theory. Another very straightforward extension would be to discuss Galilean superconformal extension of this theory in the vein of Ref. [13]. Super-GCA algebras have already been described well in the literature [44–46], and one would hope to find a nonlinear realization of that as well in the super-ModMax-like theory. We hope to come back to these issues in future work.

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