Six-loop anomalous dimension of the ϕ^Q operator in the $O(N)$ symmetric model

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A technique of large-charge expansion provides a novel opportunity for calculation of critical dimensions of operators ϕ^Q with fixed charge Q. In the small-coupling regime the polynomial structure of the anomalous dimensions can be fixed from a number of direct perturbative calculations for a fixed Q. At the six-loop level, one needs to include new diagrams that correspond to operators with five or more legs. The latter never appeared before in scalar-theory calculations. Here we show how to compute the anomalous dimension of the operator $\phi^{Q=5}$ at the six-loop order. In combination with results for operators with $Q < 5$, which are extracted from the six-loop beta functions for general scalar theory, and with predictions from the large-charge expansion, our calculation allows us to derive the answer for general-Q anomalous dimensions. At the critical point resummation in three dimensions enables us to compare the critical exponents with results of Monte Carlo simulations and large-N predictions.

I. INTRODUCTION

The renormalization group (RG) method allows one to systematically improve the accuracy of calculations in perturbation theory. The key objects of the method are the renormalization group functions, which specify the response of various quantities to a scale variation.

Among many applications of the field-theoretical RG are studies of universal critical behavior of different physical systems near second-order phase transitions.

The well-known examples are three-dimensional $O(N)$ universality classes. The corresponding critical indices can be derived by considering different operators within the ϕ^4 model in $d = 4 - 2\varepsilon$ dimensions. In particular, the exponents are computed from the RG functions evaluated at a nontrivial Wilson-Fisher [1] (WF) fixed point, for which the quartic coupling $g = g^* = \mathcal{O}(\varepsilon)$. Recent six-loop [2] and even seven-loop [3] results together with modern resummation techniques (see [2] and references therein) give very precise critical exponents, which can be compared both to experimental measurements and to values obtained by other methods.

While the expansion in small couplings seems natural and convenient, there exist several nonperturbative

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approaches to the calculation of critical indices. For example, in $O(N)$ theory, one can consider the limit of large-N and systematically obtain corrections for scaling exponents as series in $1/N$ [4,5]. The latter are valid for any space dimensions and effectively resum an infinite number of terms in the ε expansion. The case of ϕ^Q operators with total charge Q was considered in Ref. [6].

Recently, a different method was proposed [7,8] allowing one to study the anomalous dimensions of operators ϕ^Q semiclassically via operator-state correspondence.

In Ref. [7] the $U(1) = O(2)$ model is considered and a new 't Hooft-like coupling [9] is introduced g^*Q. The scaling dimension of ϕ^Q-type operators is written as

$$\Delta_Q = \sum_{i=-1}^{\infty} (g^*)^i \Delta_i(g^*Q) = \frac{\Delta_{-1}(g^*Q)}{g^*} + \Delta_0(g^*Q) + \cdots \quad (1)$$

and the first two terms of the expansion in small g^* for a fixed g^*Q are computed. The approach was generalized to the case of the $O(N)$ model [8] together with $U(N) \times U(N)$ [10] and $U(N) \times U(M)$ [11] theories.

Expanding Δ_{-1} and Δ_0 in small g^*Q, one can predict leading and subleading terms in large Q at an arbitrary high loop. The latter can be compared with existing perturbative calculations (see, e.g., Refs. [12,13]), which for the case of the $O(N)$ model are available (for arbitrary Q) up to five loops from a recent paper [14].

In this work, we use the results of Ref. [8] as an input together with explicit computations for $Q = 1\ldots5$, to deduce the six-loop terms in Δ_Q for general Q in the $O(N)$ model. While the cases up to $Q = 4$ can easily be

treated given our general formulas for beta functions valid in arbitrary ϕ^4 theory [15], the computation of the anomalous dimension for $Q = 5$ constitutes the main technical challenge of the current study.

The paper is organized as follows. In Sec. II we briefly review the $O(N)$ model and operators of our interest. In Sec. III we discuss the details of calculation. Section IV is devoted to our main results: six-loop expressions for the anomalous and scaling dimension of ϕ^Q-type operators in the $O(N)$ model. Discussion and the conclusion can be found in Sec. V.

II. FIXED-CHARGE OPERATORS IN THE $O(N)$ MODEL

The Euclidean Lagrangian that describes the $O(N)$-symmetric model is given by

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi}\right) + \frac{g}{4!}\left(\vec{\phi} \cdot \vec{\phi}\right)^2, \qquad (2)$$

where $\vec{\phi} = \{\phi_a\}$, $a = 1\ldots N$ is an N-component scalar field.

Following Ref. [8], we consider lowest-lying $O(N)$ operators of fixed total charge Q, which can be represented as

$$\phi^Q = d_{i_1\ldots i_Q}\phi_{i_1}\cdots\phi_{i_Q}, \qquad d_{\ldots i\ldots i\ldots} = 0 \qquad (3)$$

with $d_{i_1\ldots i_Q}$ being the symmetric traceless tensor.

To compute the anomalous dimension γ_Q of (3) perturbatively, one can consider the insertion of the ϕ^Q operator in the one-particle irreducible (1PI) Green function with Q external legs (see., e.g., [13,14] for four- and five-loop results). Using dimensional regularization [16] with $d = 4 - 2\varepsilon$ and modified minimal subtraction scheme ($\overline{\text{MS}}$), one extracts the renormalization constants Z_{ϕ^Q}, which relate bare fields ϕ_B to the renormalized operator $[\phi^Q]$:

$$\phi_B^Q = Z_{\phi^Q}[\phi^Q]. \qquad (4)$$

The anomalous dimension can be cast in the following general form:

$$\gamma_Q(g) \equiv \frac{\partial \log Z_{\phi^Q}}{\partial g}(-2\epsilon g + \beta_g) = Q\sum_{l=1}^{\infty}g^l\gamma_Q^{(l)},$$

$$\gamma_Q^{(l)} = \sum_{r=0}^{l}\sum_{s=0}^{l-r}Q^r N^s\gamma_{r,s}^{(l)}, \qquad \gamma_{0,l}^{(l)} = 0. \qquad (5)$$

Here β_g is the 4d part of the well-known beta function of the self-coupling g known up to six loops from Ref. [2]. In Eq. (5) we sum over l-loop contributions $\gamma_Q^{(l)}$. The latter are polynomials in Q up to degree l, and for each monomial Q^r the coefficient is a polynomial in N up to degree $l - r$. The coefficients $\gamma_{r,s}^{(l)}$ are just numbers.

Let us mention here that the $Q = 1$ case corresponds to the field anomalous dimension [2], while, due to tensor nature of the operator, $Q = 2$ is related to the crossover exponent [17].

Evaluating γ_Q at the WF fixed point $g = g^* \simeq \frac{6\varepsilon}{N+8}$, we compute the scaling dimensions of the operators in the form of ε expansion:

$$\Delta_Q = Q(1 - \varepsilon) + \gamma_Q(g^*)$$

$$= Q + Q\sum_{l=1}^{\infty}\sum_{r=0}^{l}\sum_{p=r}^{2l-r}(2\varepsilon)^l\frac{Q^r}{(N+8)^p}P_{r,p}^{(l)},$$

$$P_{0,0}^{(l)} = 0, \qquad P_{0,2l}^{(l)} = 0. \qquad (6)$$

For convenience, in Eq. (6) we introduce numerical coefficients $P_{r,p}^{(l)}$ and make the dependence on ε and N explicit.

The ultimate goal of this paper is to compute six-loop contributions $\gamma_{r,s}^{(6)}$ and $P_{r,p}^{(6)}$ to Eqs. (5) and (6), respectively. In what follows, we briefly review our approach to the calculation.

III. CALCULATION DETAILS

According to Eq. (5), the six-loop contribution to the anomalous dimension is a polynomial in Q:

$$\gamma_Q^{(6)} = Q^0(\gamma_{0,0}^{(6)} + N\gamma_{0,1}^{(6)} + N^2\gamma_{0,2}^{(6)} + N^3\gamma_{0,3}^{(6)} + N^4\gamma_{0,4}^{(6)} + N^5\gamma_{0,5}^{(6)})$$
$$+ Q^1(\gamma_{1,0}^{(6)} + N\gamma_{1,1}^{(6)} + N^2\gamma_{1,2}^{(6)} + N^3\gamma_{1,3}^{(6)} + N^4\gamma_{1,4}^{(6)} + N^5\gamma_{1,5}^{(6)})$$
$$+ Q^2(\gamma_{2,0}^{(6)} + N\gamma_{2,1}^{(6)} + N^2\gamma_{2,2}^{(6)} + N^3\gamma_{2,3}^{(6)} + N^4\gamma_{2,4}^{(6)})$$
$$+ Q^3(\gamma_{3,0}^{(6)} + N\gamma_{3,1}^{(6)} + N^2\gamma_{3,2}^{(6)} + N^3\gamma_{3,3}^{(6)})$$
$$+ Q^4(\gamma_{4,0}^{(6)} + N\gamma_{4,1}^{(6)} + N^2\gamma_{4,2}^{(6)})$$
$$+ Q^5(\gamma_{5,0}^{(6)} + N\gamma_{5,1}^{(6)})$$
$$+ Q^6\gamma_{6,0}^{(6)}, \qquad (7)$$

which has seven N-dependent coefficients. The latter can, in principle, be determined from explicit perturbative results for seven anomalous dimensions corresponding, e.g., to $Q = 1\ldots7$.

In this paper, we compute the anomalous dimensions for all operators up to $Q = 5$, giving five independent constraints on (7). The remaining two constraints are derived from the semiclassical result (1) of Ref. [8] expanded in g^*Q up to relevant order[1]:

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[1]We fix a small misprint $2/279 \rightarrow 2/729$ in the published version of Ref. [8].
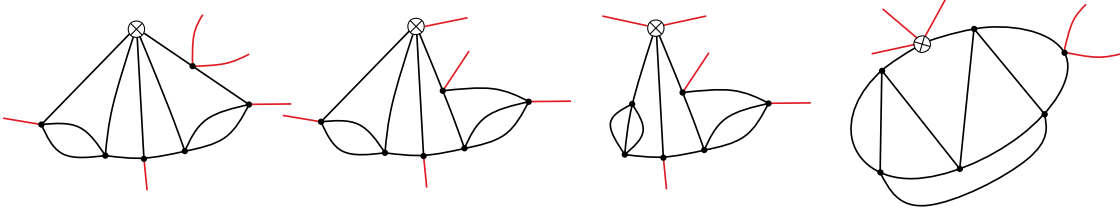
FIG. 1.   Examples of nonfactorizable six-loop diagrams contributing to the 1PI five-point function with an operator insertion (denoted by cross). Only integrals similar to the first one require special treatment. All the others are known from the ϕ^4 renormalization [2].

six-loop: $\left(-\dfrac{572}{243}Q + \dfrac{2}{729}[10191 - 64N \right.$

$$- 2\zeta_3(1327 + 160N) - 2\zeta_5(1441 + 80N)$$

$$\left. - 70\zeta_7(46 + N) - 21\zeta_9(126 + N)] \right)(g^*Q)^6. \quad (8)$$

As it was noted in Ref. [13], the contribution (8) to the anomalous dimension $\gamma_Q(g^*)$ derived initially under the assumption $g = g^*$ is also valid away from the fixed point, so we can immediately read off the coefficients $\gamma_{6,0}^{(6)}$, $\gamma_{5,0}^{(6)}$, and $\gamma_{5,1}^{(6)}$ from Eq. (8).

It turns out that the calculation of the operators with charge up to $Q = 4$ is trivial. One can use our six-loop result [15] for the RG functions in the most general renormalizable ϕ^4 theory. For example, to compute $\gamma_{Q=4}$ we introduce a coupling λ_4 for the $\phi^{Q=4}$ operator. To extract the corresponding anomalous dimension from the beta function β_{abcd} of general self-coupling λ_{abcd}, we substitute

$$\lambda_{abcd} \rightarrow \frac{g}{3}(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + \lambda_4 d_{abcd} \quad (9)$$

and keep only terms that are linear in λ_4. The beta function of λ_4 is related to $\gamma_{Q=4}$ defined in (5) via

$$\beta_{\lambda_4}(g) = \lambda_4 \gamma_{Q=4}(g). \quad (10)$$

The cases $Q = 2$ and $Q = 3$ are treated in a similar fashion, i.e., we replace the mass parameter and the trilinear coupling by traceless symmetric tensors with two and three indices,

$$m_{ab}^2 \rightarrow \lambda_2 d_{ab}, \qquad h_{abc} \rightarrow \lambda_3 d_{abc}. \quad (11)$$

It is worth mentioning that we routinely utilize FORM [18] to implement the traceless condition on $d_{i_1 \ldots i_Q}$ (3) and to contract dummy indices.

The case $Q = 5$ deserves special attention. We use DIANA [19] to generate five-point 1PI Green functions with a $\phi^{Q=5}$ insertion. The corresponding Feynman rule again involves the traceless symmetric tensor d_{abcde}. After carrying out $O(N)$ algebra and factoring d_{abcde}, we are left with scalar integrals multiplied by polynomials in N. It is

obvious that some of the indices entering $d_{i_1 \ldots i_5}$ can be external (see, e.g., Fig. 1). In this case we effectively have a 1PI loop diagram with a reduced number of external legs.

To extract the renormalization constant Z_5 from the ultraviolet (UV) divergences, we apply the $\mathcal{K}\mathcal{R}'$ operation to each logarithmically divergent diagram G_i:

$$Z_\phi^5 Z_5 = 1 - \sum_i \mathcal{K}\mathcal{R}'G_i, \quad (12)$$

where Z_ϕ is a field renormalization constant $\phi_B = Z_\phi\phi$.

The $\mathcal{K}\mathcal{R}'$ operation can be written recursively as

$$\mathcal{K}\mathcal{R}'G = \mathcal{K}G + \sum_{\{\gamma\}} \mathcal{K}\left[\prod_{\gamma_i \in \{\gamma\}} (-\mathcal{K}\mathcal{R}'\gamma_i) * G/\{\gamma\} \right], \quad (13)$$

where G is the original diagram, and $\mathcal{K}G$ extracts its singular $\mathcal{O}(1/\varepsilon)$ part. The sum goes over all sets of disjoint UV-divergent 1PI subgraphs $\{\gamma\} = \cup_i \gamma_i$ (with G itself excluded), and $G/\{\gamma\}$ is a cograph obtained from G after shrinking all γ_i belonging to $\{\gamma\}$. To implement (13) at the six-loop order, one needs to compute all lower-loop counterterms $\mathcal{K}\mathcal{R}'\gamma_i$. Some of the diagrams contain cut vertices (see, e.g., Fig. 2). In this case $\mathcal{K}\mathcal{R}'$ factorizes. In spite of the fact that these diagrams do not contribute to the anomalous dimension of the operator, we keep them for further cross-checks (see below).

Drastic simplification comes from the application of the infrared (IR) rearrangement trick [20] to the logarithmically divergent diagrams. One can set all but one of the external momenta to zero and reroute the momentum flow in a way to avoid (as much as possible) the appearance of spurious
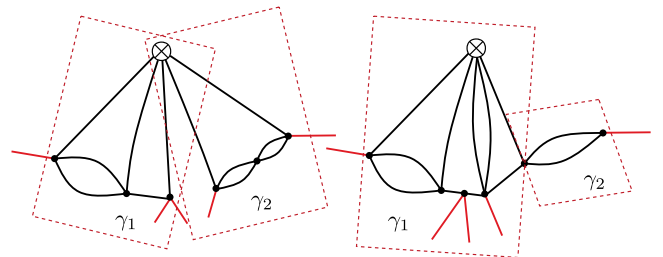

FIG. 2.   Examples of factorizable diagrams for which $\mathcal{K}\mathcal{R}'(G) = \mathcal{K}\mathcal{R}'(\gamma_1) \cdot \mathcal{K}\mathcal{R}'(\gamma_2)$.

IR divergences. The choice of the IR-safe routing together with the UV-subgraph identification was automated by means of the private computer code.

It turns out that all integrals but one entering $\mathcal{K}G$ and $G/\{\gamma\}$ can be calculated with IR-safe nonexceptional external momentum routing in terms of graphical functions 3,21,22]] implemented in the HYPERLOGPROCEDURES package.[2]

There remains a single diagram for which the IR-safe routing leads to an integral not calculable with this approach. Because of this, the external momentum routing was changed at the price of introduction of IR divergent subgraphs. The latter were treated manually via infrared $\mathcal{K}\mathcal{R}^*$ operation [23–26]. Further details are provided in the Appendix.

Given Z_5, the required anomalous dimension is derived via

$$\gamma_{Q=5} = -\frac{\partial \log Z_5}{\partial \log \mu} = (2\epsilon g - \beta_g)\frac{\partial \log Z_5}{\partial g}. \quad (14)$$

As usual only single poles in ϵ contribute to $\gamma_{Q=5}$. However, the crucial cross-check of the obtained expression is the cancellations of ϵ poles in the final formula for (14).

IV. RESULTS

Given large-charge prediction (1) and explicit results for the anomalous dimensions of the first five operators ϕ^Q, we can fix all the coefficients in $\gamma_Q^{(6)}$:

$$\gamma_Q^{(6)} = \sum_{r=0}^{6}\sum_{s=0}^{6-r} Q^r N^s \gamma_{r,s}^{(6)}, \qquad \gamma_{0,6}^{(6)} = 0, \quad (15)$$

as [27]

$$\gamma_{6,0}^{(6)} = -\frac{572}{243}, \quad (16)$$

$$\gamma_{5,1}^{(6)} = -\frac{640\zeta_3}{729} - \frac{320\zeta_5}{729} - \frac{140\zeta_7}{729} - \frac{14\zeta_9}{243} - \frac{128}{729}, \quad (17)$$

$$\gamma_{5,0}^{(6)} = -\frac{5308\zeta_3}{729} - \frac{5764\zeta_5}{729} - \frac{6440\zeta_7}{729} - \frac{196\zeta_9}{27} + \frac{6794}{243}, \quad (18)$$

$$\gamma_{4,2}^{(6)} = -\frac{70\zeta_3^2}{729} + \zeta_3\left(-\frac{46\zeta_5}{729} - \frac{14}{81}\right) - \frac{100\zeta_5}{729} - \frac{49\zeta_7}{729} + \frac{14}{243} + \frac{7\pi^4}{7290} + \frac{10\pi^6}{137781} + \frac{7\pi^8}{1968300}, \quad (19)$$

$$\gamma_{4,1}^{(6)} = -\frac{236\zeta_3^2}{243} - \frac{176\zeta_3^3}{729} + \zeta_3\left(\frac{1564}{243} - \frac{808\zeta_5}{729}\right) - \frac{490\zeta_5}{243} - \frac{11935\zeta_7}{1458} - \frac{42412\zeta_9}{6561} + \frac{932}{729} + \frac{182\pi^4}{10935} + \frac{100\pi^6}{45927} + \frac{49\pi^8}{196830}, \quad (20)$$

$$\gamma_{4,0}^{(6)} = \frac{2368\zeta_3^2}{729} - \frac{2720\zeta_3^3}{729} + \zeta_3\left(\frac{39340}{729} - \frac{9208\zeta_5}{729}\right) + \frac{26564\zeta_5}{729} + \frac{10451\zeta_7}{729} - \frac{81448\zeta_9}{6561} - \frac{102694}{729} + \frac{784\pi^4}{10935}$$
$$+ \frac{1760\pi^6}{137781} + \frac{868\pi^8}{492075}, \quad (21)$$

$$\gamma_{3,3}^{(6)} = \pi^4\left(\frac{13\zeta_3}{87480} + \frac{1}{4860}\right) + \frac{2\zeta_3}{243} - \frac{65\zeta_3^2}{2916} - \frac{59\zeta_5}{2916} - \frac{25\zeta_7}{2916} - \frac{1}{243} + \frac{25\pi^6}{1102248}, \quad (22)$$

$$\gamma_{3,2}^{(6)} = \pi^4\left(\frac{131\zeta_3}{43740} - \frac{133}{87480}\right) + \frac{607\zeta_3^2}{1458} - \frac{8\zeta_3^3}{81} + \zeta_3\left(\frac{2537}{2916} - \frac{218\zeta_5}{729}\right) - \frac{37\zeta_5}{54} - \frac{20143\zeta_7}{5832} - \frac{1063\zeta_9}{729} - \frac{\zeta_{5,3}}{3} - \frac{335}{729}$$
$$+ \frac{160\pi^6}{137781} + \frac{5417\pi^8}{22044960}, \quad (23)$$

$$\gamma_{3,1}^{(6)} = \pi^4\left(\frac{296\zeta_3}{10935} - \frac{49}{729}\right) - \frac{788\zeta_3^2}{729} - \frac{128\zeta_3^3}{81} + \zeta_3\left(-\frac{12296\zeta_5}{729} - \frac{35633}{1458}\right) + \frac{512\zeta_5}{243} + \frac{11\zeta_7}{2} - \frac{5738\zeta_9}{729} - \frac{2(7191\zeta_{5,3} + 3275)}{1215}$$
$$+ \frac{860\pi^6}{137781} + \frac{1213171\pi^8}{275562000}, \quad (24)$$

$$\gamma_{3,0}^{(6)} = \pi^4\left(\frac{368\zeta_3}{3645} - \frac{7151}{21870}\right) - \frac{84580\zeta_3^2}{729} + \frac{5152\zeta_3^3}{729} + \zeta_3\left(-\frac{59144\zeta_5}{729} - \frac{44186}{243}\right) - \frac{59306\zeta_5}{729} - \frac{12529\zeta_7}{486} + \frac{46256\zeta_9}{6561}$$
$$- \frac{10768\zeta_{5,3}}{135} + \frac{91750}{243} - \frac{2585\pi^6}{137781} + \frac{727847\pi^8}{34445250}, \tag{25}$$

$$\gamma_{2,4}^{(6)} = \frac{\zeta_3}{972} + \frac{\zeta_3^2}{729} - \frac{\zeta_5}{486} + \frac{1}{5832} - \frac{\pi^4}{174960} + \frac{\pi^6}{1102248}, \tag{26}$$

$$\gamma_{2,3}^{(6)} = \pi^4\left(-\frac{53\zeta_3}{43740} - \frac{1}{9720}\right) + \frac{7\zeta_3^2}{36} + \zeta_3\left(\frac{5\zeta_5}{729} - \frac{631}{5832}\right) - \frac{35\zeta_5}{1944} - \frac{7735\zeta_7}{11664} - \frac{\zeta_{5,3}}{45} + \frac{1619}{46656} + \frac{263\pi^6}{1102248} + \frac{2063\pi^8}{61236000}, \tag{27}$$

$$\gamma_{2,2}^{(6)} = \pi^4\left(\frac{641}{43740} - \frac{449\zeta_3}{21870}\right) - \frac{163\zeta_3^2}{729} + \frac{32\zeta_3^3}{729} - \frac{283\zeta_5}{162} + \zeta_3\left(\frac{448\zeta_5}{243} - \frac{3535}{972}\right) + \frac{553\zeta_7}{5832} - \frac{22964\zeta_9}{6561} + \frac{4\zeta_{5,3}}{45} + \frac{3541}{3888}$$
$$+ \frac{241\pi^6}{137781} + \frac{29453\pi^8}{137781000}, \tag{28}$$

$$\gamma_{2,1}^{(6)} = \pi^4\left(\frac{5599}{21870} - \frac{1426\zeta_3}{10935}\right) + \frac{12799\zeta_3^2}{729} + \frac{2992\zeta_3^3}{729} - \frac{43871\zeta_5}{1458} + \zeta_3\left(\frac{42052\zeta_5}{729} + \frac{14243}{486}\right) - \frac{46361\zeta_7}{972} - \frac{142852\zeta_9}{6561}$$
$$+ \frac{14428\zeta_{5,3}}{405} + \frac{41047}{5832} + \frac{185\pi^6}{275562} - \frac{1274101\pi^8}{137781000}, \tag{29}$$

$$\gamma_{2,0}^{(6)} = \pi^4\left(\frac{3449}{3645} - \frac{3824\zeta_3}{10935}\right) + \frac{380672\zeta_3^2}{729} + \frac{32\zeta_3^3}{729} - \frac{5050\zeta_5}{81} + \zeta_3\left(\frac{100720\zeta_5}{243} + \frac{22307}{81}\right) - \frac{320719\zeta_7}{1458} - \frac{596264\zeta_9}{6561}$$
$$+ \frac{26944\zeta_{5,3}}{81} - \frac{3367853}{5832} + \frac{8146\pi^6}{137781} - \frac{299533\pi^8}{3444525}, \tag{30}$$

$$\gamma_{1,5}^{(6)} = -\frac{\zeta_3}{11664} + \frac{\zeta_5}{3888} - \frac{1}{23328} - \frac{\pi^4}{699840}, \tag{31}$$

$$\gamma_{1,4}^{(6)} = \frac{169\zeta_3}{29160} - \frac{\zeta_3^2}{243} + \frac{13\zeta_5}{1458} - \frac{299}{103680} - \frac{37\pi^4}{874800} - \frac{\pi^6}{367416}, \tag{32}$$

$$\gamma_{1,3}^{(6)} = \pi^4\left(\frac{1091\zeta_3}{437400} - \frac{493}{437400}\right) - \frac{659\zeta_3^2}{1620} + \zeta_3\left(\frac{12001}{58320} - \frac{5\zeta_5}{243}\right) + \frac{3389\zeta_5}{14580} + \frac{1636\zeta_7}{729} + \frac{\zeta_{5,3}}{15} - \frac{10403}{155520}$$
$$- \frac{137\pi^6}{122472} - \frac{2063\pi^8}{20412000}, \tag{33}$$

$$\gamma_{1,2}^{(6)} = \pi^4\left(\frac{7837\zeta_3}{218700} - \frac{539}{9720}\right) - \frac{4834\zeta_3^2}{3645} + \frac{136\zeta_3^3}{243} + \zeta_3\left(\frac{96511}{29160} - \frac{340\zeta_5}{729}\right) + \frac{48371\zeta_5}{3645} + \frac{171947\zeta_7}{5832} + \frac{45287\zeta_9}{2187} + \frac{521\zeta_{5,3}}{225}$$
$$- \frac{10499}{2160} - \frac{53213\pi^6}{2755620} - \frac{7693807\pi^8}{2755620000}, \tag{34}$$

$$\gamma_{1,1}^{(6)} = \pi^4\left(\frac{8158\zeta_3}{54675} - \frac{6853}{10935}\right) - \frac{135304\zeta_3^2}{3645} + \frac{64\zeta_3^3}{27} + \zeta_3\left(-\frac{8956\zeta_5}{729} - \frac{256211}{14580}\right) + \frac{424234\zeta_5}{3645} + \frac{14111\zeta_7}{54} + \frac{159772\zeta_9}{729}$$
$$- \frac{13942\zeta_{5,3}}{675} - \frac{4451899}{116640} - \frac{77492\pi^6}{688905} - \frac{16401281\pi^8}{1377810000}, \tag{35}$$

$$\gamma_{1,0}^{(6)} = \pi^4\left(\frac{3484\zeta_3}{18225} - \frac{97517}{54675}\right) - \frac{3006466\zeta_3^2}{3645} + \frac{4640\zeta_3^3}{729} + \zeta_3\left(-\frac{302656\zeta_5}{729} - \frac{1567481}{7290}\right) + \frac{314518\zeta_5}{729} + \frac{6962111\zeta_7}{7290}$$
$$+ \frac{4163188\zeta_9}{6561} - \frac{290408\zeta_{5,3}}{675} + \frac{7820977}{19440} - \frac{215048\pi^6}{688905} + \frac{24913489\pi^8}{344452500}, \tag{36}$$

$$\gamma_{0,5}^{(6)} = \frac{\zeta_3}{15552} - \frac{\zeta_5}{3888} + \frac{1}{248832} + \frac{\pi^4}{466560}, \tag{37}$$

$$\gamma_{0,4}^{(6)} = -\frac{89\zeta_3}{12960} + \frac{2\zeta_3^2}{729} - \frac{5\zeta_5}{729} + \frac{733}{466560} + \frac{89\pi^4}{1749600} + \frac{\pi^6}{551124}, \tag{38}$$

$$\gamma_{0,3}^{(6)} = \pi^4 \left(\frac{2207}{1749600} - \frac{313\zeta_3}{218700} \right) + \frac{3401\zeta_3^2}{14580} + \zeta_3 \left(\frac{10\zeta_5}{729} - \frac{1667}{19440} \right) - \frac{61\zeta_5}{270} - \frac{18341\zeta_7}{11664} - \frac{2\zeta_{5,3}}{45} + \frac{69623}{933120} + \frac{965\pi^6}{1102248}$$
$$+ \frac{2063\pi^8}{30618000}, \tag{39}$$

$$\gamma_{0,2}^{(6)} = \pi^4 \left(\frac{197}{4374} - \frac{667\zeta_3}{36450} \right) + \frac{4334\zeta_3^2}{3645} - \frac{368\zeta_3^3}{729} + \zeta_3 \left(-\frac{740\zeta_5}{729} - \frac{2297}{9720} \right) - \frac{4519\zeta_5}{405} - \frac{16885\zeta_7}{648} - \frac{103330\zeta_9}{6561} - \frac{466\zeta_{5,3}}{225}$$
$$+ \frac{100471}{19440} + \frac{11617\pi^6}{688905} + \frac{1069637\pi^8}{459270000}, \tag{40}$$

$$\gamma_{0,1}^{(6)} = \pi^4 \left(\frac{38221}{87480} - \frac{836\zeta_3}{18225} \right) + \frac{78049\zeta_3^2}{3645} - \frac{3392\zeta_3^3}{729} + \zeta_3 \left(\frac{112751}{14580} - \frac{6664\zeta_5}{243} \right) - \frac{320224\zeta_5}{3645} - \frac{614519\zeta_7}{2916} - \frac{1200664\zeta_9}{6561}$$
$$- \frac{6344\zeta_{5,3}}{2025} + \frac{9046223}{233280} + \frac{146159\pi^6}{1377810} + \frac{1894453\pi^8}{114817500}, \tag{41}$$

$$\gamma_{0,0}^{(6)} = \pi^4 \left(\frac{3148\zeta_3}{54675} + \frac{242993}{218700} \right) + \frac{504412\zeta_3^2}{1215} - \frac{2368\zeta_3^3}{243} - \frac{232190\zeta_5}{729} + \zeta_3 \left(\frac{68848\zeta_5}{729} + \frac{550921}{7290} \right) - \frac{1736897\zeta_7}{2430} - \frac{1161368\zeta_9}{2187}$$
$$+ \frac{359144\zeta_{5,3}}{2025} - \frac{9723527}{116640} + \frac{8608\pi^6}{32805} - \frac{11713\pi^8}{1417500}. \tag{42}$$

Evaluating the anomalous dimension at the fixed point, we obtain the ε expansion of the scaling dimension Δ_Q (6). We reproduce the five-loop results [14]. The new six-loop coefficients are given by

$$P_{6,6}^{(6)} = -1716, \tag{43}$$

$$P_{5,7}^{(6)} = -38400, \tag{44}$$

$$P_{5,6}^{(6)} = -2(94\zeta_3 + 1602\zeta_5 + 2660\zeta_7 + 2478\zeta_9 - 16463), \tag{45}$$

$$P_{5,5}^{(6)} = -2(320\zeta_3 + 160\zeta_5 + 70\zeta_7 + 21\zeta_9 + 64), \tag{46}$$

$$P_{4,8}^{(6)} = -529200, \tag{47}$$

$$P_{4,7}^{(6)} = -24(2212\zeta_3 + 3500\zeta_5 + 4725\zeta_7 - 27264), \tag{48}$$

$$P_{4,6}^{(6)} = 3552\zeta_3^2 - 1312\zeta_3^3 + 51124\zeta_5 - 4\zeta_3(1422\zeta_5 + 935)$$
$$+ 86975\zeta_7 + \frac{257848\zeta_9}{9} - 265526, \tag{49}$$

$$P_{4,5}^{(6)} = 412\zeta_3^2 - 176\zeta_3^3 + 1930\zeta_5 - 24\zeta_3(3\zeta_5 - 437)$$
$$- \frac{9107\zeta_7}{2} - \frac{42412\zeta_9}{9} + 1898 + \frac{14\pi^4}{15} + \frac{20\pi^6}{27} + \frac{7\pi^8}{50}, \tag{50}$$

$$P_{4,4}^{(6)} = -70\zeta_3^2 - 100\zeta_5 - 2\zeta_3(23\zeta_5 + 63) - 49\zeta_7 + 42$$
$$+ \frac{7\pi^4}{10} + \frac{10\pi^6}{189} + \frac{7\pi^8}{2700}, \tag{51}$$

$$P_{3,9}^{(6)} = -6048000, \tag{52}$$

$$P_{3,8}^{(6)} = -5040(301\zeta_3 + 275\zeta_5 - 1602), \tag{53}$$

$$P_{3,7}^{(6)} = -79776\zeta_3^2 + \zeta_3(951996 - 95040\zeta_5) + 1306920\zeta_5$$
$$+ 704025\zeta_7 - 4540356, \tag{54}$$

$$P_{3,6}^{(6)} = -33870\zeta_3^2 + 9760\zeta_3^3 - 425010\zeta_5$$
$$+ 24\zeta_3(1973\zeta_5 + 856) - \frac{1029567\zeta_7}{2} - \frac{152896\zeta_9}{9}$$
$$- \frac{23328\zeta_{5,3}}{5} + 1148614 + \frac{386\pi^4}{5} + \frac{1100\pi^6}{63}$$
$$+ \frac{1044\pi^8}{875}, \tag{55}$$

$$P_{3,5}^{(6)} = -7984\zeta_3^2 + \frac{1}{60}\pi^4(336\zeta_3 - 493) + 17146\zeta_5$$
$$- \frac{3}{2}\zeta_3(5072\zeta_5 + 44331) + \frac{286097\zeta_7}{4} + 11270\zeta_9$$
$$- \frac{23706\zeta_{5,3}}{5} - 10492 - \frac{2150\pi^6}{189} + \frac{14419\pi^8}{42000}, \tag{56}$$

$$P_{3,4}^{(6)} = 986\zeta_3^2 - 72\zeta_3^3 - \frac{1}{24}\pi^4(10\zeta_3 + 221)$$
$$+ \zeta_3\left(\frac{5993}{4} - 218\zeta_5\right) + \frac{999\zeta_5}{2} - \frac{18943\zeta_7}{8}$$
$$- 1063\zeta_9 - 243\zeta_{5,3} - \frac{939}{2} + \frac{115\pi^6}{756} + \frac{5417\pi^8}{30240}, \tag{57}$$

$$P_{3,3}^{(6)} = \frac{1}{120}\pi^4(13\zeta_3 + 18) + \frac{1}{4}(24\zeta_3 - 65\zeta_3^2 - 59\zeta_5$$
$$- 25\zeta_7 - 12) + \frac{25\pi^6}{1512}, \tag{58}$$

$$P_{2,10}^{(6)} = -68040000, \tag{59}$$

$$P_{2,9}^{(6)} = -604800(51\zeta_3 - 142), \tag{60}$$

$$P_{2,8}^{(6)} = -252(-114928\zeta_3 + 9072\zeta_3^2 - 18200\zeta_5 + 195133), \tag{61}$$

$$P_{2,7}^{(6)} = 6(219240\zeta_3^2 + 21\zeta_3(3360\zeta_5 - 72017) + 351\pi^4)$$
$$- 30(187650\zeta_5 + 33957\zeta_7 - 538628), \tag{62}$$

$$P_{2,6}^{(6)} = 74436\zeta_3^2 - 21856\zeta_3^3 + \frac{1}{5}\pi^4(648\zeta_3 - 6619)$$
$$+ \zeta_3(447674 - 176400\zeta_5) - \frac{760\pi^6}{9} + 2091276\zeta_5$$
$$+ \frac{1646491\zeta_7}{2} - \frac{923144\zeta_9}{9} + \frac{235872\zeta_{5,3}}{5}$$
$$- 2793137 - \frac{1508\pi^8}{125}, \tag{63}$$

$$P_{2,5}^{(6)} = 27831\zeta_3^2 + 2480\zeta_3^3 - \frac{2}{15}\pi^4(297\zeta_3 - 844)$$
$$- \frac{476097\zeta_5}{2} + \zeta_3\left(46708\zeta_5 + \frac{361793}{2}\right)$$
$$- \frac{790853\zeta_7}{4} + \frac{224572\zeta_9}{9} + \frac{109116\zeta_{5,3}}{5} + \frac{52581}{2}$$
$$+ \frac{24205\pi^6}{378} - \frac{40673\pi^8}{9000}, \tag{64}$$

$$P_{2,4}^{(6)} = -4162\zeta_3^2 + 32\zeta_3^3 + \frac{7}{60}\pi^4(2\zeta_3 + 351) + 6263\zeta_5$$
$$+ \zeta_3\left(1704\zeta_5 - \frac{11473}{2}\right) + \frac{133063\zeta_7}{8} - \frac{22964\zeta_9}{9}$$
$$+ \frac{2268\zeta_{5,3}}{5} + \frac{14669}{8} - \frac{385\pi^6}{54} - \frac{11707\pi^8}{27000}, \tag{65}$$

$$P_{2,3}^{(6)} = \frac{1}{80}(8780\zeta_3^2 + 6390\zeta_5 + 100\zeta_3(4\zeta_5 - 63)$$
$$- 38675\zeta_7 - 1296\zeta_{5,3} + 2175)$$
$$- \frac{1}{120}\pi^4(106\zeta_3 + 161) + \frac{11\pi^6}{72} + \frac{2063\pi^8}{84000}, \tag{66}$$

$$P_{2,2}^{(6)} = \frac{3\zeta_3}{4} + \zeta_3^2 - \frac{3\zeta_5}{2} + \frac{1}{8} - \frac{\pi^4}{240} + \frac{\pi^6}{1512}, \tag{67}$$

$$P_{1,11}^{(6)} = -1020600000, \tag{68}$$

$$P_{1,10}^{(6)} = -27216000(18\zeta_3 - 41), \tag{69}$$

$$P_{1,9}^{(6)} = -7560(-77856\zeta_3 + 5184\zeta_3^2 + 16800\zeta_5 + 71741), \tag{70}$$

$$P_{1,8}^{(6)} = 126(324144\zeta_3^2 + 610860\zeta_5 + 1250399 + 270\pi^4)$$
$$- 126(30\zeta_3(2016\zeta_5 + 73393) + 66150\zeta_7), \tag{71}$$

$$P_{1,7}^{(6)} = -13852800\zeta_3^2 + 50688\zeta_3^3 + \frac{1296}{5}\pi^4(9\zeta_3 - 133)$$
$$- 6666360\zeta_5 + 24\zeta_3(110880\zeta_5 + 2632463)$$
$$- 2469033\zeta_7 + 966080\zeta_9 + \frac{497664\zeta_{5,3}}{5}$$
$$- \frac{121888917}{4} + 1200\pi^6 - \frac{22272\pi^8}{875}, \tag{72}$$

$$P_{1,6}^{(6)} = \pi^4\left(\frac{24247}{2} - \frac{8172\zeta_3}{5}\right) + 1369356\zeta_3^2 - 41696\zeta_3^3$$
$$- 3917542\zeta_5 - \frac{3}{2}\zeta_3(155136\zeta_5 + 4083031)$$
$$+ \frac{5677959\zeta_7}{2} - \frac{8721100\zeta_9}{9} - \frac{1313064\zeta_{5,3}}{5}$$
$$+ \frac{28571673}{8} - \frac{24965\pi^6}{63} + \frac{782339\pi^8}{10500}, \tag{73}$$

$$P_{1,5}^{(6)} = -122928\zeta_3^2 + 5696\zeta_3^3 + \frac{4}{15}\pi^4(951\zeta_3 - 5767)$$
$$+ \zeta_3\left(\frac{249699}{4} - 44604\zeta_5\right) + 878664\zeta_5$$
$$- \frac{1581769\zeta_7}{4} + \frac{997516\zeta_9}{9} - \frac{24138\zeta_{5,3}}{5} - \frac{59181}{2}$$
$$- \frac{1660\pi^6}{21} + \frac{68489\pi^8}{14000}, \tag{74}$$

$$P_{1,4}^{(6)} = \pi^4\left(\frac{61}{4} - \frac{35\zeta_3}{12}\right) + \frac{21937\zeta_3^2}{2} + 664\zeta_3^3$$
$$+ \zeta_3\left(\frac{831}{8} - 2188\zeta_5\right) - \frac{101553\zeta_5}{2} - \frac{31659\zeta_7}{8}$$
$$+ \frac{194069\zeta_9}{9} + \frac{6237\zeta_{5,3}}{5} - \frac{77953}{32} + \frac{9467\pi^6}{378}$$
$$- \frac{147677\pi^8}{108000}, \tag{75}$$

$$P_{1,3}^{(6)} = -\frac{1415\zeta_3^2}{4} + \frac{1}{120}\pi^4(247\zeta_3 - 5) + \zeta_3\left(\frac{6225}{16} - 15\zeta_5\right)$$
$$+ \frac{2645\zeta_5}{4} + 1724\zeta_7 + \frac{243\zeta_{5,3}}{5} - \frac{10273}{128}$$
$$- \frac{1733\pi^6}{1512} - \frac{2063\pi^8}{28000}, \tag{76}$$

$$P_{1,2}^{(6)} = -4\zeta_3 - 3\zeta_3^2 - \zeta_5 - \frac{383}{256} + \frac{13\pi^4}{120} - \frac{\pi^6}{504}, \tag{77}$$

$$P_{1,1}^{(6)} = \frac{1}{960}(-30(2\zeta_3 - 6\zeta_5 + 1) - \pi^4), \tag{78}$$

$$P_{0,11}^{(6)} = 1020600000, \tag{79}$$

$$P_{0,10}^{(6)} = 3402000(144\zeta_3 - 323), \tag{80}$$

$$P_{0,9}^{(6)} = 15120(-37968\zeta_3 + 2592\zeta_3^2 + 8400\zeta_5 + 33763), \tag{81}$$

$$P_{0,8}^{(6)} = -126(309888\zeta_3^2 + 661460\zeta_5 + 1065945 + 270\pi^4)$$
$$+ 126(2\zeta_3(30240\zeta_5 + 1055743) + 66150\zeta_7), \tag{82}$$

$$P_{0,7}^{(6)} = 12961872\zeta_3^2 - 50688\zeta_3^3 - \frac{486\pi^4}{5}(24\zeta_3 - 343)$$
$$+ 12986880\zeta_5 - 18\zeta_3(166080\zeta_5 + 3400493)$$
$$+ 2778048\zeta_7 - 966080\zeta_9 - \frac{497664\zeta_{5,3}}{5}$$
$$+ \frac{87424317}{4} - 1200\pi^6 + \frac{22272\pi^8}{875}, \tag{83}$$

$$P_{0,6}^{(6)} = -1493358\zeta_3^2 + 55104\zeta_3^3 + \frac{1}{10}\pi^4(15048\zeta_3 - 116131)$$
$$+ 1861686\zeta_5 + \zeta_3(367440\zeta_5 + 6850142)$$
$$- \frac{6511113\zeta_7}{2} + \frac{3194632\zeta_9}{3} + 220104\zeta_{5,3}$$
$$- \frac{32994733}{16} + \frac{30445\pi^6}{63} - \frac{133639\pi^8}{2100}, \tag{84}$$

$$P_{0,5}^{(6)} = 109509\zeta_3^2 - 8000\zeta_3^3 - \frac{1}{30}\pi^4(6588\zeta_3 - 48821)$$
$$- 655038\zeta_5 + \zeta_3\left(5576\zeta_5 - \frac{608795}{2}\right) + \frac{2210257\zeta_7}{4}$$
$$- \frac{1280728\zeta_9}{9} - \frac{61272\zeta_{5,3}}{5} + \frac{892443}{32} + \frac{7055\pi^6}{378}$$
$$- \frac{26969\pi^8}{31500}, \tag{85}$$

$$P_{0,4}^{(6)} = -\frac{15819\zeta_3^2}{2} - 624\zeta_3^3 + \frac{1}{80}\pi^4(248\zeta_3 - 5521)$$
$$+ \frac{95231\zeta_5}{2} + \zeta_3\left(748\zeta_5 + \frac{80693}{8}\right) - \frac{106765\zeta_7}{8}$$
$$- \frac{53846\zeta_9}{3} - 1458\zeta_{5,3} + \frac{34005}{32} - \frac{4493\pi^6}{252}$$
$$+ \frac{8161\pi^8}{5040}, \tag{86}$$

$$P_{0,3}^{(6)} = \frac{1073\zeta_3^2}{4} - \frac{7}{240}\pi^4(44\zeta_3 - 73) - 917\zeta_5$$
$$+ \zeta_3\left(10\zeta_5 - \frac{3807}{8}\right) - \frac{19749\zeta_7}{16} - \frac{162\zeta_{5,3}}{5}$$
$$+ \frac{1693}{32} + \frac{173\pi^6}{168} + \frac{2063\pi^8}{42000}, \tag{87}$$

$$P_{0,2}^{(6)} = \frac{111\zeta_3}{16} + 2\zeta_3^2 + \frac{5\zeta_5}{2} + \frac{343}{512} - \frac{59\pi^4}{480} + \frac{\pi^6}{756}, \tag{88}$$

$$P_{0,1}^{(6)} = \frac{240\zeta_3 - 960\zeta_5 + 15 + 8\pi^4}{5120}. \tag{89}$$

Provided results for coefficients $\gamma_{r,s}^{(l)}$ and $P_{r,p}^{(l)}$ in addition to the Riemann zeta functions $\zeta_n = \sum_{i=1}^{\infty} 1/i^n$ contain multiple zeta value $\zeta_{5,3} = \sum_{i=1}^{\infty}\sum_{j=1}^{i-1} 1/(i^5 j^3) \approx 0.0377077$.

TABLE I. Scaling dimensions of ϕ^Q ($d=3$) obtained by resumming the six-loop result for $N=2,4,6,8$.

					Q					
N	1	2	3	4	5	6	7	8	9	10
2	0.5187(5)	1.2351(6)	2.1085(7)	3.112(2)	4.230(3)	5.45(1)	6.75(2)	8.15(2)	9.62(3)	11.17(3)
4	0.5181(5)	1.1885(3)	1.9856(14)	2.892(3)	3.896(7)	4.99(2)	6.16(3)	7.40(3)	8.72(4)	10.10(4)
6	0.5163(5)	1.1537(13)	1.895(3)	2.728(5)	3.643(11)	4.63(2)	5.70(3)	6.82(3)	8.01(4)	9.26(4)
8	0.5146(4)	1.127(2)	1.827(5)	2.604(8)	3.45(2)	4.36(2)	5.34(3)	6.37(3)	7.46(4)	8.60(4)

To verify our expressions we consider various limits of Eq. (5). For example, defining $J = Q/N$ we get an expansion in J. Neglecting terms further suppressed by large N, we obtain (for $g = g^*$)

$$\frac{\Delta_Q}{Q} \simeq 1 - \varepsilon + \sum_{l=1}^{\infty}(6\varepsilon)^l \sum_{k=1}^{l} J^k \gamma_{k,l-k}^{(l)} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$= 1 - \varepsilon + \sum_{l=1}^{\infty}(2\varepsilon)^l \sum_{k=1}^{l} J^k P_{k,k}^{(l)} + \mathcal{O}\left(\frac{1}{N}\right) \quad (90)$$

with

$$P_{k,k}^{(l)} = 3^l \gamma_{k,l-k}^{(l)}. \quad (91)$$

It turns out that the expansion of Δ_Q in small J for large N can be found for a general dimension d [28]:

$$\frac{\Delta Q}{Q} = \frac{d}{2} - 1 + h_2(d)j + h_3(d)j^2 + \cdots, \quad (92)$$

where, e.g.,

$$h_2(d) = -\frac{2^{d-3}d\sin(\frac{\pi d}{2})\Gamma(\frac{d-1}{2})}{\pi^{3/2}\Gamma(\frac{d}{2}+1)}. \quad (93)$$

We follow [28] to compute the ε-expansion of $h_i(d)$, for $i = 2\ldots7$, and find perfect agreement with our perturbative result (90). In this way, we check $\gamma_{r,6-r}^{(6)}$ for $r = 1\ldots6$.

In addition, we also compare our result with the first two nontrivial orders of large-N expansion [6], which begins as

$$\frac{\Delta Q}{Q} = \frac{d}{2} - 1 + \frac{h_2(d)}{N}\left(Q - 2 + \frac{4}{d}\right) + \cdots, \quad (94)$$

At six loops only five coefficients of Eq. (5) contribute to large-N expansion of our result (6) up to $\mathcal{O}(1/N^2)$. Two of them $\gamma_{1,5}^{(6)}$ and $\gamma_{2,4}^{(6)}$ also enter (92), while the comparison with ε expansion of (94) provides additional checks for $\gamma_{0,4}^{(6)}$, $\gamma_{1,4}^{(6)}$, and $\gamma_{0,5}^{(6)}$.

V. DISCUSSION AND CONCLUSION

In this paper we derived the six-loop anomalous dimension of the charged ϕ^Q operator in the $O(N)$ model. Our computation was based on the combination of semiclassical results and explicit diagram calculations. Our expression up to five loops coincides with that obtained recently in Refs. [14,29].

The utilized approach relies on diagram-by-diagram computation of Feynman graphs with an ϕ^5 insertion, and, thus, facilitates further six-loop studies of the charged operators in models with other symmetries once semiclassical results are available.

Given our result, one can apply various resummation techniques to the ε expansion of Δ_Q and obtain numerical values of scaling dimension in a bunch of three-dimensional $O(N)$ models.

Based on the fact that we are working at the same order of perturbation theory and in the same theory, we use in straightforward manner the advanced technique of Ref. [2] to compute numerical values and uncertainties of Δ_Q for $Q = 1\ldots10$ and $N = 2, 4, 6, 8$ (see Table I). The latter can be compared to the Monte Carlo results [30–32].

It is interesting to study how our computation matches the large Q expansion [33]

$$\Delta_Q = c_{3/2}Q^{3/2} + c_{1/2}Q^{1/2} + c_0 + \mathcal{O}(Q^{-1/2}). \quad (95)$$

Here $c_{3/2}$, $c_{1/2}$ are N-dependent constants, while $c_0 \approx -0.0937254$ originates from Casimir energy and is universal. In Ref. [34] the large-N limit was considered and the following predictions were obtained:

TABLE II. Fit from resummation results using $Q = 1\ldots6$ expressions with $c_0 = -0.0937$ fixed and an additional free parameter $c_{-1/2}$ in the $c_{-1/2}Q^{-1/2}$ term appended to the ansatz (95).

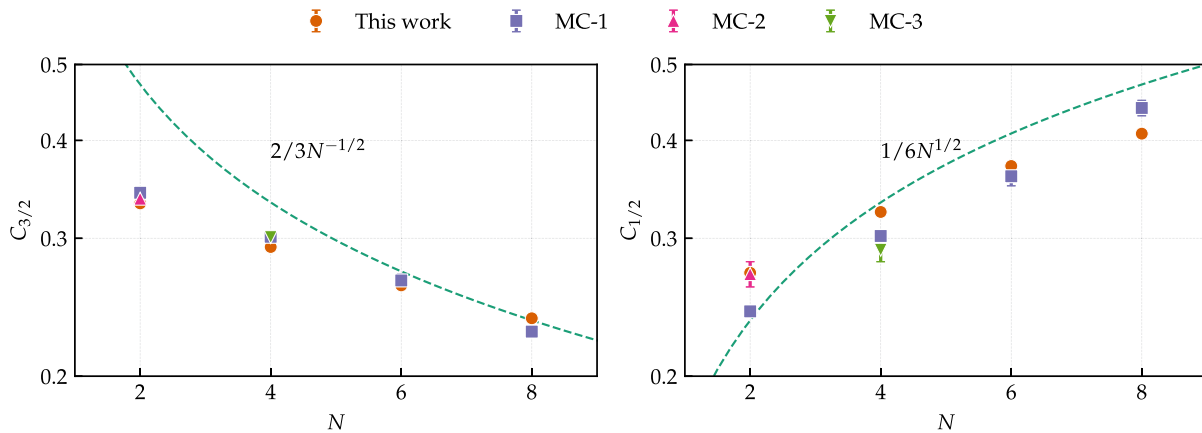| N	$c_{3/2}$	$c_{1/2}$	$c_{-1/2}$
2	0.3324(6)	0.271(3)	0.009(3)
4	0.2925(4)	0.3241(13)	-0.0048(11)
6	0.2612(6)	0.371(3)	-0.022(2)
8	0.2371(9)	0.408(3)	-0.036(3)

FIG. 3. Leading coefficients in large-charge expansion $c_{3/2}$ and $c_{1/2}$ as functions of N. We also add results of Monte Carlo simulation for $N = 2, 4, 6, 8$ from Ref. [32] (MC-1), for $N = 2$ from Ref. [30] (MC-2) and for $N = 4$ from Ref. [31] (MC-3).

$$c_{3/2} = \frac{2}{3}N^{-1/2}, \qquad c_{1/2} = \frac{1}{6}N^{1/2} \qquad (96)$$

which are valid for $1 \ll Q \ll N \ll Q^2$. We follow [30,32] and fit our numerical data to (95). The extracted coefficients for a fixed c_0 are given in Table II. One can see satisfactory agreement between our values and Monte Carlo results [30–32] (see Fig. 3). While our fits for $c_{3/2}$ are close to the Monte Carlo values and approaches the large-N limit (96), we see rather big discrepancies in $c_{1/2}$. We expect that our error estimates according to Ref. [2] may be too optimistic and require further studies. Nevertheless, we see that according to Eq. (96) the coefficient $c_{3/2}$ decreases with N, while $c_{1/2}$ increases.

We believe our result contributes important information to conformal field theory data (see, e.g., [29] for review). While we rely on nonperturbative large Q expressions to fix part of the coefficients in the ansatz (7), it would be interesting to reproduce them independently, e.g., by considering Green functions with $\phi^{Q=6}$ and $\phi^{Q=7}$ insertions. We postpone this verification for the future.

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APPENDIX: DETAILS OF CALCULATION WITH $\mathcal{KR}^*$ OPERATION

Here, we present the expression used for manual computation of a single diagram by means of $\mathcal{KR}^*$ operation. We closely follow the notation of Refs. [25,26]. One can see that there is only one nontrivial IR-divergent counterterm,

$$\gamma_{\mathrm{IR}} = \left(\begin{array}{c} \circ \\ \bullet \\ \circ \end{array} \right) = \frac{1}{\varepsilon}, \qquad (A1)$$

which cancels spurious IR divergences appearing in $\Gamma_i = G/\gamma_i$ for a UV subgraph γ_i. The graph $\tilde{\Gamma}_i = \Gamma_i \backslash \gamma_{\mathrm{IR}}$ is obtained from Γ_i by deleting lines and internal vertices (denoted by filled dots) of γ_{IR}. One can see that even if Γ_i can be zero in dimensional regularization the corresponding IR counterterm can give a nontrivial contribution to the final result. All $\mathcal{KR}'(\gamma_i)$ are known from lower-loop calculations, while G, $\tilde{G}$, Γ_i, and $\tilde{\Gamma}_i$ are calculated with HYPERLOGPROCEDURES:

$$\text{(A2)}$$

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