Spin evolution of massive fermion in QED plasma

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The dynamical evolution of spin of a massive probe fermion in a hot QED plasma at local equilibrium is investigated through the quantum kinetic theory. We consider the massive probe fermion undergoing 2-by-2 Coulomb scattering with the massless fermions in the medium. The axial kinetic equation is derived including the collision terms to the first order of gradients and leading logarithmic order of the coupling. The collision terms are vanishing at global equilibrium, around which the relaxation time can be extracted as an operator. We further decompose the axial kinetic equation into kinetic equations of axial-charge density as well as the transverse magnetic dipole moment. The polarization rate and diffusion rate are estimated in massless limit and nonrelativistic limit, between both limit, polarization and diffusion effects are illustrated through preliminary numerical analysis.

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I. INTRODUCTION

Recent STAR and ALICE experiments [1–5] have shed light on the spin polarization of hadrons in the rotating QCD plasma produced in off-central relativistic heavy-ion collisions. Such spin polarization of emitted hadrons [6–9] has motivated researches concerning the dynamical evolution of spin for particles in a finite temperature plasma. Part of the large initial orbital angular momentum characterized by the collective motion of the fluid is transferred to the spin of the particles through collisions. Whether the polarization survives after hadronization relies on the dynamics of spin in QGP phase and hadronic phase. The particles experience both polarization and relaxation processes that drive the spin polarization to equilibrium. The global polarization of Λ hyperons enslaved by the thermal vorticity [10] is a robust phenomenon, where model calculations [11–16] are in consistency with experiments. However, such satisfaction has not been achieved in local spin polarization. The measurement of azimuthal angle dependence of spin polarization in experiments [2] has not been fully understood in theoretical studies due to the opposite sign in the phenomenological studies assuming the global equilibrium of spin [17,18]. Such inconsistency is also known as the spin sign problem. Attempts to resolve this problem include modifying the understanding of vorticity [19], feed-down effect [20,21] and hyperon decay [22]. It is realized later that the inclusion of shear tensor in the polarization yields the qualitatively correct sign [23–26], indicating off-equilibrium effects of spin maybe essential in polarization phenomenon. It is also found that the numerical results could be sensitive to the parameters in numerical analysis [27–30]. This calls for more thorough investigations of the nonequilibrium effects and kinetic theory of spin.

Theoretical description of the dynamical evolution of spin polarization is mainly based on quantum kinetic theory [31–43] and spin hydrodynamics [44–53]. The chiral kinetic theory [54–64] was developed to describe the spin related anomalous transport phenomena, and has been applied to chiral magnetic effect [65] in heavy ion collisions. It is then extended to the quantum kinetic theory to describe the spin transport of massive fermions [31–34]. In recent years, the collision terms are also included to study the relaxation process of spin [37–42]. The general framework of quantum kinetic theory is based on the Wigner function and Keldysh formalism, which is able to keep the full power of quantum field theory in nonequilibrium system [66]. On the other hand, spin hydrodynamics extends the standard conservation laws to also include the conservation of angular momentum, describes the macroscopic evolution of spin density.

The polarization of Λ hyperons is dominated by the s-quark, which can not be approximated as massless fermion. In order to investigate the spin dynamics of s-quark in the quark gluon plasma, we in this work deal with a simplified scenario as a first step to the full problem. We consider the evolution of spin of a hard massive fermion $m \gg eT$ probing into a hot massless QED plasma at local equilibrium. As the Compton scattering is suppressed in case $m \gg eT$, and the evolution is dominated by Coulomb

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scattering. Two competing processes would contribute to the spin evolution, the diffusion process coming from the scattering with medium fermion drives the fluctuation of spin back to equilibrium, while the collective motion of the medium, characterized by the hydrodynamic gradients, acts as a source to polarize the spin of massive fermion. So as to self-consistently incorporate the two processes, we derive the collision terms to $\mathcal{O}(\partial)$ with all the first order hydrodynamic gradients included. Spin evolution of massive probe in similar physical system is also investigated in Refs. [36,37,67], where the author consider homogeneous QCD plasma and focus only on the diffusion term. In Ref. [42], kinetic equation of massless probe fermion traversing hot QED plasma is derived, the authors also extract diffusion and polarization rates from the collision terms. From the kinetic equations derived in this paper, the first order gradients of the plasma act as a source to axial charge and change the orientation of the probe spin. For massless probe fermion, the ratio between polarization rate and relaxation rate can be estimated to be $f_p|\partial|/p$, this is in general in consistency with Ref. [42]. In the nonrelativistic limit, the diffusion process dominates the polarization for axial charge, with the later suppressed by additional $(p/m)^3$; the changing in the orientation of spin get also suppressed by $(T/m)^2$. Between the both limit, we provide some preliminary numerical analysis to illustrate the polarization and diffusion process. For s-quark in the quark gluon plasma which is roughly $m \sim p \sim T$, ratio between polarization and diffusion rate is about $|\partial|/T$, indicating nonequilibrium effects important in the spin polarization in heavy ion collision.

This paper is organized as follows: in Sec. II, we briefly review the Wigner function and Kadanoff-Baym equations, as well as the power counting scheme. In Sec. III, we derive the general expression for the collision term and discuss contribution from the various part of the collision term. In Sec. IV, the result of collision term after integral over phase space momentum is presented, together with expression in massless and nonrelativistic limit. The relaxation rate near the global equilibrium is also extracted. In Sec. V, the axial kinetic equation is further decomposed into kinetic equation of axial-charge density and transverse dipole moment. A preliminary numerical analysis decorating the diffusion and polarization processes is presented. In Sec. VI, we provide conclusion and outlook. Calculation details are presented in Appendices A and B.

In this paper, we take the mostly negative convention of matrix $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and take the Dirac matrix in the Weyl basis with $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma^{\mu\nu} = i[\gamma^\mu,\gamma^\nu]/2$. The Levi-Civita symbol is chosen as $\epsilon^{0123} = -\epsilon_{0123} = +1$. We use a majuscule letter for four-dimension covariant momentum such as P^μ and use a minuscule letter for its component such as p^0 and its module such as $p = |\vec{p}|$. We use the projector $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ to project a vector onto direction perpendicular to the fluid velocity u^μ, such as $P^\mu_\perp = \Delta^{\mu\nu}P_\nu$ and define $\hat{P}^\mu_\perp = P^\mu_\perp/p$ with $p = (-P^\mu_\perp P_{\perp\mu})^{1/2}$. The projector $\Xi^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu + \hat{P}^\mu_\perp \hat{P}^\nu_\perp$ projects a vector onto direction perpendicular to both u^μ and $P^\mu_\perp$. We also use the following notations for the first order gradients: $\theta = \partial\cdot u$, $D = u\cdot\partial$, the fluid vorticity defined as $\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$ and shear tensor $\sigma^{\langle\alpha\beta\rangle}$ defined as the symmetric and traceless part of $\sigma^{\alpha\beta} = \frac{1}{2}(\partial^\alpha_\perp u^\beta + \partial^\beta_\perp u^\alpha) - \frac{1}{3}\Delta^{\alpha\beta}\theta$. The symmetrization and antisymmetrization of two symbols are defined through $X_{(\alpha}Y_{\beta)} = X_\alpha Y_\beta + X_\beta Y_\alpha$ and $X_{[\alpha}Y_{\beta]} = X_\alpha Y_\beta - X_\beta Y_\alpha$.

II. SPIN TRANSPORT EQUATION

In this section, we review the basic steps of deriving the axial kinetic equation with collision term. Starting from the Wigner transformation applied to contour Green's function [66]

$$S^{<(>)}_{\alpha\beta}(X,p) = \int d^4Y e^{ip\cdot Y}\tilde{S}^{<(>)}_{\alpha\beta}(x,y), \tag{1}$$

where $X = (x+y)/2$ and $Y = x-y$ are the center of mass coordinate and relative coordinate. Here, $\tilde{S}^<_{\alpha\beta}(x,y) = \langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$ and $\tilde{S}^>_{\alpha\beta}(x,y) = \langle\psi_\alpha(x)\bar{\psi}_\beta(y)\rangle$ are lessor and greater propagators, respectively. After the Wigner transformation, the lesser propagator obeys the Kadanoff-Baym equations derived from the Schwinger-Dyson equation,

$$(\gamma^\mu P_\mu - m)S^< + \frac{i}{2}\gamma^\mu\nabla_\mu S^< = \frac{i}{2}(\Sigma^<\star S^> - \Sigma^>\star S^<), \tag{2}$$

where $\Sigma^{>(<)}$ represents the lesser (greater) self-energy. The scattering process involves only $\Sigma^{<(>)}$, thus we have dropped the real parts of the retarded and advanced self-energies and of the retarded propagators. The electromagnetic fields decay quickly in the QGP, hence we neglect the background electromagnetic fields in the medium. The symbol $\star$ represents $A\star B = AB + \frac{i}{2}[AB]_{\mathrm{P.B.}} + \mathcal{O}(\partial^2)$, where the Poisson bracket is $[AB]_{\mathrm{P.B.}} \equiv (\partial^\mu_q A)(\partial_\mu B) - (\partial_\mu A)(\partial^\mu_q B)$. The commutators are defined as $\{F,G\} \equiv FG + GF$, $[F,G] \equiv FG - GF$, $\{F,G\}_\star \equiv F\star G + G\star F$ and $[F,G]_\star \equiv F\star G - G\star F$ with F and G being arbitrary matrix-valued functions. By using the complete basis for the Clifford algebra, the Wigner function is decomposed into $S^< = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{S}_{\mu\nu}\sigma^{\mu\nu}$ and $S^> = \bar{\mathcal{S}} + i\bar{\mathcal{P}}\gamma^5 + \bar{\mathcal{V}}_\mu\gamma^\mu + \bar{\mathcal{A}}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\bar{\mathcal{S}}_{\mu\nu}\sigma^{\mu\nu}$. Similarly, it is also useful to carry out the same spinor-basis decomposition for the self-energies, giving $\Sigma^< = \Sigma_S + i\Sigma_P\gamma^5 + \Sigma_{V\mu}\gamma^\mu + \Sigma_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\Sigma_{T\mu\nu}\sigma^{\mu\nu}$ and $\Sigma^> = \bar{\Sigma}_S + i\bar{\Sigma}_P\gamma^5 + \bar{\Sigma}_{V\mu}\gamma^\mu + \bar{\Sigma}_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\bar{\Sigma}_{T\mu\nu}\sigma^{\mu\nu}$. $\mathcal{V}$ and $\mathcal{A}$ give rise to the vector-charge and axial-charge currents through $J^\mu_V =$

$\int_q \mathcal{V}^\mu$ and $J_5^\mu = \int_q \mathcal{A}^\mu$. The axial-charge currents can be regarded as a spin current of fermion. Taking $\mathcal{V}$ and $\mathcal{A}$ as independent degrees of freedom, the scalar component $\mathcal{S}$, pseudoscalar component $\mathcal{P}$ and tensor component $\mathcal{S}_{\mu\nu}$ can be expressed in terms of $\mathcal{V}$ and $\mathcal{A}$.

We are going to investigate the relaxation of spin of a massive probe fermion traversing a hot massless QED plasma in local equilibrium. Before moving on to calculate the collision term, we first introduce the counting in gradients. In the heavy ion collision, the axial-vector currents are mostly induced by the electromagnetic field or the gradients of the fluid velocity. This motivates the counting of $\mathcal{A}_\mu \sim O(\partial)$. On the other hand, the vector charge current can be safely kept only to $\mathcal{O}(\partial^0)$, as it is dominated by classical process. The power counting of $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$ also leads to counting of the other components $\mathcal{S} \sim \mathcal{O}(\partial^0)$, $\mathcal{S}_{\mu\nu} \sim \mathcal{O}(\partial^1)$ and $\mathcal{P} \sim \mathcal{O}(\partial^2)$, as well as the components of the self-energy. In the Coulomb scattering we are going to investigate, the above counting leads to counting for the self-energy components, $\Sigma_S \sim \mathcal{O}(\partial^0)$, $\Sigma_{V\mu} \sim \mathcal{O}(\partial^0)$, $\Sigma_{A\mu} \sim \mathcal{O}(\partial^1)$, $\Sigma_{T\mu\nu} \sim \mathcal{O}(\partial^1)$ and $\Sigma_P \sim \mathcal{O}(\partial^2)$. The thermalization of the vector charge is dominated by the classical process, thus it is enough to keep only $\mathcal{O}(\partial^0)$ terms in the collision term. The thermalization of spin involves diffusion of the initial spin of the probe as well as polarization induced by gradients such as the vorticity and shear, it is required to evaluate the collision terms up to $\mathcal{O}(\partial)$. Then the collision terms for vector and axial-vector components can be obtained though comparing the Dirac structures on both sides of Kadanoff-Baym equation, giving the vector kinetic equation

$$\partial_\mu \mathcal{V}^\mu = -\frac{P_\mu}{m}\widehat{\Sigma_S \mathcal{V}^\mu} - \widehat{\Sigma_{V\mu}\mathcal{V}^\mu} + O(\partial), \tag{3}$$

where $\widehat{XY} = \bar{X}Y - X\bar{Y}$. And the axial kinetic equation

$$P\cdot\partial\mathcal{A}_\mu = -m\widehat{\Sigma_S\mathcal{A}_\mu} - P^\nu\widehat{\Sigma_{V\nu}\mathcal{A}_\mu} - P^\nu\widehat{\Sigma_{A\mu}\mathcal{V}_\nu} - \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_T^{\alpha\beta}\mathcal{V}^\lambda} + P_\mu\widehat{\Sigma_{A\nu}\mathcal{V}^\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\partial^\sigma\Sigma_V^\nu)\mathcal{V}^\rho} + O(\partial^2). \tag{4}$$

The power counting $\mathcal{A}_\mu \sim \mathcal{O}(\partial)$ guarantees the mass-shell condition of $\mathcal{A}_\mu$ [39], see also [37] for details of derivation. With the relations between various components of the Wigner function, the parametrization of the various components can be taken as

$$\begin{aligned}
\mathcal{S} &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)mf_V,\\
\mathcal{V}_\mu &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)P_\mu f_V,\\
\mathcal{A}_\mu &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)n_\mu,\\
\mathcal{S}_{\mu\nu} &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)S_{\mu\nu}.
\end{aligned} \tag{5}$$

We do not take any decomposition of n_μ at the moment, for now it is only constrained by $P^\mu n_\mu = 0$ coming directly from $P^\mu\mathcal{A}_\mu = 0$. With the relation $\mathcal{S}_{\mu\nu} = \frac{1}{2m}\partial_{[\mu}\mathcal{V}_{\nu]} - \frac{1}{m}\epsilon_{\mu\nu\rho\sigma}P^\rho\mathcal{A}^\sigma + \mathcal{O}(\partial^2)$ between the tensor and axial-vector component [39], $S_{\mu\nu}$ is expressed as

$$S_{\mu\nu}(P) = -\frac{1}{2m}P_{[\mu}\partial_{\nu]}f_V(P) - \frac{1}{m}\epsilon_{\mu\nu\rho\sigma}P^\rho n^\sigma(P). \tag{6}$$

For the greater components, one can just substitute f_V with $\bar{f}_V = 1 - f_V$ and substitute n_μ with $\bar{n}_\mu = -n_\mu$. Besides, within such power counting, one would have $\mathcal{P} \sim \mathcal{O}(\partial^2)$, and $\Sigma_P \sim \mathcal{O}(\partial^2)$, they are thus excluded from the current problem. For later convenience, the zeroth order and first order Wigner functions of massive fermion are given by

$$\begin{aligned}
S^{<(0)} &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)(m+\gamma^\mu P_\mu)f_V(P),\\
S^{<(1)} &= 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)\left(\gamma^5\gamma^\mu n_\mu(P) + \frac{\sigma^{\mu\nu}}{2}S_{\mu\nu}(P)\right).
\end{aligned} \tag{7}$$

For massless fermion $S^{<(0)} = 2\pi\epsilon(P\cdot u)\delta(P^2)\gamma^\mu P_\mu f_V(P)$ and $S^{<(1)} = 2\pi\epsilon(P\cdot u)\delta(P^2)\gamma^5\gamma^\mu n_\mu(P)$. In the following, we use the variable n_μ instead of $\mathcal{A}_\mu$ for the axial-vector component to avoid the coefficient $2\pi\epsilon(P\cdot u)\delta(P^2-m^2)$ on both sides of the transport equation.

III. COULOMB SCATTERING

In this section, we consider the scenario where the massive hard fermion probes into a hot QED plasma and undergoes a 2-by-2 scattering with hot medium at local equilibrium. The light fermions in the medium can be well approximated as massless. The mass of the probe fermion is assumed to be much greater than the thermal mass $m \gg eT$, in this case the Compton scattering does not contribute at the leading logarithmic order, thus only the Coulomb scattering is considered. This approximation can be understood as a toy model for the spin evolution of the s-quark in the quark gluon plasma. The collision terms will be grouped into two types. The diffusion terms are those terms linear in $\mathcal{A}_\mu$ of the probe fermion, such terms are dominated by classical dynamics of the interaction. The polarization terms are those terms containing first order gradients of the fluid, such terms delineate how the collective motion of the fluid acts as a source to polarize the spin of the probe fermion. Since the axial-vector component is counted as $\mathcal{A}_\mu \sim \mathcal{O}(\partial)$, both the spin diffusion and the first order gradients of the medium contribute at the same order and should be treated on the same basis. We calculate the collision term of axial kinetic equation keeping all the contributions of the first order gradient and work out the leading logarithmic order collision terms. The following Feynman diagram Fig. 1 describes the Coulomb scattering of the massive probe

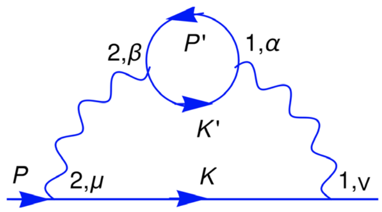

FIG. 1. Two-loop diagram for fermion self-energy containing propagator corrections [41]. The 12 labels are uniquely determined by the requirement that three propagators attached to a vertex cannot be simultaneously on shell.

fermion (P and K) with the massless medium fermion (P' and K'). The greater fermion self-energy is defined as

$$\Sigma^{>}(P) = e^4 \int_{Q,K} \gamma^\mu S^{>}(K)\gamma^\nu G^{<}_{\mu\nu}(Q), \tag{8}$$

where $\int_{Q,K} = \int d^4K d^4Q (2\pi)^{-8}\epsilon(K\cdot u)\delta(K^2 - m^2)(2\pi)^4\delta(P-K-Q)$. $G^{<}_{\mu\nu}(Q)$ is the photon propagator containing one fermion loop correction,

$$G^{(0,1)<}_{\mu\nu}(Q) = -D^{22}_{\mu\beta}(Q)D^{11}_{\alpha\nu}(Q)\Pi^{(0,1)<\alpha\beta}(Q). \tag{9}$$

For simplicity, we will choose Feynman gauge for $D^{22}_{\mu\beta}$ and $D^{11}_{\alpha\nu}$, namely $D^{22}_{\mu\beta} = \frac{ig_{\mu\beta}}{Q^2}$ and $D^{11}_{\alpha\nu} = \frac{-ig_{\alpha\nu}}{Q^2}$. As processes with the on-shell photon such as Compton scattering are suppressed, the collision term is gauge invariant. A short proof of gauge invariance is presented in the Appendix B. The photon self-energy get gradient correction from the fermion loop. It is also counted in ∂ with the leading order and first order photon self-energy are

$$\Pi^{(0)<\alpha\beta}(Q) = -e^2 \int_{K',P'} \mathrm{Tr}[\gamma^\alpha S^{(0)<}(P')\gamma^\beta S^{(0)>}(K')],$$
$$\Pi^{(1)<\alpha\beta}(Q) = -e^2 \int_{K',P'} \mathrm{Tr}[\gamma^\alpha S^{(1)<}(P')\gamma^\beta S^{(0)>}(K') + \gamma^\alpha S^{(0)<}(P')\gamma^\beta S^{(1)>}(K')], \tag{10}$$

where $\int_{K',P'} = \int d^4K'd^4P'(2\pi)^{-8}\epsilon(P'\cdot u)\epsilon(K'\cdot u)\delta(P'^2)\delta(K'^2)(2\pi)^4\delta(Q+P'-K')$. Fermions with momentum P and K are the massive probe fermion, while those with momentum P' and K' are the massless medium fermions. Substituting the Wigner function of the loop fermions, the photon propagator at zeroth order and first order are given by

$$G^{(0)<}_{\mu\nu}(Q) = 4e^2(2\pi)^2 \int_{K',P'} \frac{1}{(Q^2)^2}(P'_{\{\nu}K'_{\mu\}} - g_{\mu\nu}P'\cdot K') \times f_V(P')\bar{f}_V(K'),$$
$$G^{(1)<}_{\mu\nu}(Q) = 4e^2(2\pi)^2 \int_{K',P'} \frac{1}{(Q^2)^2} i\epsilon_{\mu\nu\rho\sigma}(K'^\sigma n^\rho(P') \times \bar{f}_V(K') - P'^\sigma \bar{n}^\rho(K')f_V(P')). \tag{11}$$

$G^{(0)<}_{\mu\nu}$ is symmetric in indices, while $G^{(1)<}_{\mu\nu}$ is antisymmetric. Instead of using the hard thermal loop (HTL) photon propagator and calculate the one loop fermion self-energy [36,37], here we use the loop-corrected photon propagator. The former captures the classical effects in the evolution of probe fermion, while quantum effects such as contributions from the gradients of the medium are not included. One loop fermion self-energy using the HTL photon propagator also assumes that the medium fermions are at equilibrium, and nonequilibrium effects are excluded. In comparison, the zeroth order $G^{(0)}_{\mu\nu}(Q)$ includes classical effects same as described by HTL photon propagator, and through the first order propagator $G^{(1)}_{\mu\nu}(Q)$, spin of the massless medium fermion could contribute to the spin evolution of the probe fermion. Besides, by calculating the two loop fermion self-energy, one can also investigate the evolution of probe fermion in a nonequilibrium medium. However, in this paper, as a first step, we restrict ourself to the scenario where the massless medium fermion is at local equilibrium.

Contracting $\chi^{>\mu\nu} = \gamma^\mu S^{>}\gamma^\nu$ with the photon propagator, the fermion self-energy can be decomposed to various Dirac components [37],

$$\chi^{>\mu\nu}G^{<}_{\mu\nu} = (\bar{\mathcal{S}}G^{<}_{\mu\nu}g^{\mu\nu} + i\bar{\mathcal{S}}^{\mu\nu}G^{<}_{\mu\nu}) + i\gamma^5\left(-\bar{\mathcal{P}}G^{<\mu}_{\mu} - i\bar{\mathcal{S}}_{\alpha\beta}G^{<}_{\mu\nu}\frac{\epsilon^{\mu\nu\alpha\beta}}{2}\right) + \gamma^\rho(\bar{\mathcal{V}}^\mu G^{<}_{(\mu\rho)} - \bar{\mathcal{V}}_\rho G^{<\mu}_{\mu} - i\epsilon_{\mu\nu\sigma\rho}\bar{\mathcal{A}}^\sigma G^{<\mu\nu})$$
$$+ \gamma^5\gamma^\rho(-\bar{\mathcal{A}}^\mu G^{<}_{(\mu\rho)} + \bar{\mathcal{A}}_\rho G^{<\mu}_{\mu} + i\epsilon_{\mu\nu\sigma\rho}\bar{\mathcal{V}}^\sigma G^{<\mu\nu}) + \frac{1}{2}\sigma^{\rho\sigma}(2\bar{\mathcal{S}}^\mu_\rho G^{<}_{(\mu\sigma)} + \bar{\mathcal{S}}_{\rho\sigma}G^{<\mu}_{\mu} - 2i\bar{\mathcal{S}}G^{<}_{\rho\sigma} - i\bar{\mathcal{P}}\epsilon_{\mu\nu\rho\sigma}G^{<\mu\nu}). \tag{12}$$

For the purpose of obtaining the collision terms to $\mathcal{O}(\partial)$, only terms linear in axial-vector component $\mathcal{A}^\mu$ appear while higher order of $\mathcal{A}$ are at least $\mathcal{O}(\partial^2)$ and are neglected. The self-energy components appearing in the transport equation (4) are Σ_S, $\Sigma_{V\mu}$, $\Sigma_{A\mu}$ and $\Sigma_{T\mu\nu}$; the greater components are expressed as

$$\bar{\Sigma}_S(P) = e^2 \int_{Q,K} \bar{\mathcal{S}}(K) G^{(0)<}_{\mu\nu}(Q) g^{\mu\nu},$$
$$\bar{\Sigma}_{V\mu}(P) = e^2 \int_{Q,K} 2\bar{\mathcal{V}}^\nu(K) G^{(0)<}_{\mu\nu}(Q) - \bar{\mathcal{V}}_\mu(K) G^{(0)<}_{\rho\nu}(Q) g^{\rho\nu},$$
$$\bar{\Sigma}_{A\mu}(P) = e^2 \int_{Q,K} -2\bar{\mathcal{A}}^\nu(K) G^{(0)<}_{\mu\nu}(Q) + \bar{\mathcal{A}}_\mu(K) G^{(0)<}_{\rho\nu}(Q) g^{\rho\nu} + i\epsilon_{\alpha\beta\nu\mu}\bar{\mathcal{V}}^\nu(K) G^{(1)<\alpha\beta}(Q),$$
$$\bar{\Sigma}_{T\mu\nu}(P) = e^2 \int_{Q,K} 2\bar{\mathcal{S}}^\rho_{[\mu}(K) G^{(0)<}_{\rho\nu]}(Q) + \bar{\mathcal{S}}_{\mu\nu}(K) G^{(0)<}_{\rho\lambda}(Q) g^{\rho\lambda} - 2i\bar{\mathcal{S}}(K) G^{(1)<}_{\mu\nu}(Q). \tag{13}$$

The lessor components of the self-energy can be obtained through replacing $S^>$ with $S^<$, and $G^<$ with $G^>$.

As a first step, we consider the case where the massive probe fermion has local equilibrium number distribution, so that we can use the local equilibrium of number distribution $\bar{f}_{K'}\bar{f}_K f_P f_{P'} - f_{K'} f_K \bar{f}_P \bar{f}_{P'} = 0$ in the derivation. For abbreviation, we denote $f_V(P)$ as a general nonequilibrium number distribution, and denote $f_P = f_V^{\mathrm{leq}}(P) = n_F(P)$ as the local equilibrium number distribution. Using local equilibrium of number distribution, one can convert $\partial^\mu(\bar{f}_{K'}\bar{f}_K f_{P'})$ into $\partial^\mu f_P$. This simplifies the last term $\partial^\sigma \Sigma_V^\nu(P)$ in (4), with $\Sigma_V^\nu(P)$ defined in (13). After straightforward but tedious algebra, the collision terms of axial kinetic equation can be casted into $P\cdot\partial n_\mu = C_\mu$,

$$\begin{aligned} C_\mu = -4e^4(2\pi)^3 \int_{Q,K}\int_{K',P'} \{ & M^{A1}_{\mu\nu}(\bar{f}_K f_{P'}\bar{f}_{K'} + f_K\bar{f}_{P'} f_{K'}) n^\nu(P) + M^{A2}_{\mu\nu}(f_P f_{P'}\bar{f}_{K'} + \bar{f}_P\bar{f}_{P'} f_{K'}) n^\nu(K) \\ & + M^{A3}_{\mu\nu}(\bar{f}_K\bar{f}_{K'} f_{P'} + f_K f_{K'}\bar{f}_{P'})\partial^\nu f_P + M^{A4}_{\mu\nu}(f_P f_{P'}\bar{f}_{K'} + \bar{f}_P\bar{f}_{P'} f_{K'})\partial^\nu f_K \\ & + M^{A5}_{\mu\nu}(\bar{f}_{K'} f_P \bar{f}_K + f_{K'}\bar{f}_P f_K) n^\nu_{\mathrm{leq}}(P') + M^{A6}_{\mu\nu}(f_{P'} f_P \bar{f}_K + \bar{f}_{P'}\bar{f}_P f_K) n^\nu_{\mathrm{leq}}(K') \}, \end{aligned} \tag{14}$$

where $\int_{Q,K}\int_{K',P'} = \int (2\pi)^{-16} d^4Q d^4K d^4K' d^4P' (2\pi)^8 \delta(P-K-Q)\delta(Q+P'-K')\epsilon(K\cdot u)\epsilon(P'\cdot u)\epsilon(K'\cdot u)\delta(K^2-m^2)$ $\delta(P'^2)\delta(K'^2)$. The various effective scattering amplitudes in (14) are given by

$$\begin{aligned} M^{A1}_{\mu\nu} &= \frac{1}{(Q^2)^2} g_{\mu\nu}(-m^2 P'\cdot K' + 2P\cdot P' K\cdot K') + \{P'\leftrightarrow K'\}, \\ M^{A2}_{\mu\nu} &= \frac{1}{(Q^2)^2}(g_{\mu\nu}(K\cdot P P'\cdot K' - 2K\cdot K' P\cdot P') - K'\cdot P' K_\mu P_\nu - 2P\cdot P' Q_\mu K'_\nu + 2P\cdot Q P'_\mu K'_\nu + 2K\cdot K' P'_\mu P_\nu) + \{P'\leftrightarrow K'\}, \\ M^{A3}_{\mu\nu} &= \frac{1}{(Q^2)^2}\epsilon_{\mu\nu\alpha\beta} K\cdot K' P^\beta P'^\alpha + \{P'\leftrightarrow K'\}, \\ M^{A4}_{\mu\nu} &= -\frac{1}{(Q^2)^2}\left(\epsilon_{\mu\nu\alpha\beta}\left(\frac{1}{2}P'\cdot K' P^\beta K^\alpha + P\cdot P' K^\beta K'^\alpha\right) + \epsilon_{\lambda\nu\alpha\beta} K'_\mu P^\alpha K^\beta P'^\lambda\right) + \{P'\leftrightarrow K'\}, \\ M^{A5}_{\mu\nu} &= \frac{2}{(Q^2)^2}(m^2 Q\cdot K' g_{\mu\nu} - m^2 K'_\mu Q_\nu + K\cdot K' P_\mu P_\nu - P\cdot K' P_\mu K_\nu), \\ M^{A6}_{\mu\nu} &= \frac{2}{(Q^2)^2}(m^2 Q\cdot P' g_{\mu\nu} - m^2 P'_\mu Q_\nu + K\cdot P' P_\mu P_\nu - P\cdot P' P_\mu K_\nu), \end{aligned} \tag{15}$$

where $\{P'\leftrightarrow K'\}$ denotes exchanging the two momentum in the terms before, namely $g(P',K') + \{P'\leftrightarrow K'\}$ means $g(P',K') + g(K',P')$, and g is an arbitrary function.

The first line in collision term (14) corresponds to the spin diffusion of the probe fermion, which are similar to classical spin relaxation processes considered in Refs. [36,37]. The last two lines are polarization of probe fermion resulting from the collection motion of the medium as well as spin of the medium fermion. The second line in (14) describes the polarization effect due to spacetime gradient of f_V of the probe fermion. The last line is the contribution from spin of the medium fermion. As $\mathcal{A}_\mu$ is $\mathcal{O}(\partial)$ in the power counting, in order to investigate its evolution, it is of key necessity to evaluate all the first order gradient in the collision term. Before going on to present the collision term after momentum integral, we first discuss each part of the collision terms.

A. Diffusion

Terms with $M^{A1}_{\mu\nu}$ and $M^{A2}_{\mu\nu}$ are the spin diffusion terms, these two terms have the same physical meaning as discussed in [36,37], describing the relaxation of probe spin by the QED dynamics. After simplifying the integral measure, the diffusion term can be recasted into

$$C^{\mathrm{diff}}_{A\mu} = -4e^4 \int_{q_0,q,k'} \{M^{A1}_{\mu\nu}(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'}) n^\nu(P) + M^{A2}_{\mu\nu}(f_P f_{P'} \bar{f}_{K'} + \bar{f}_P \bar{f}_{P'} f_{K'}) n^\nu(K)\}, \tag{16}$$

where $\int_{q_0,q,k'}$ is the abbreviation for $\frac{1}{(2\pi)^5}\int dq_0 d^3q d^3k' \frac{1}{2p'_0 2k'_0 2k_0}\delta(p_0-k_0-q_0)\delta(p'_0-k'_0+q_0)$. Instead of photon propagator in the HTL approximation, we use the photon propagator with one-loop correction (11). This leads to the same structure of the collision term as [37], while the coefficients will be different by a constant factor coming $SU(N)$ of the symmetry of color field in [37]. In the HTL approximation, the probe fermion and medium fermion are hard fermions $p, k, p', k' \sim T$, while the momentum transfer is soft $eT \ll q_0, q \ll T$. In order to complete the momentum integral and keep the result to leading logarithmic order, the axial-vector components of the outgoing probe fermion is expanded in terms of soft momentum Q as

$$n_\mu(K) = n_\mu(P-Q) \simeq n_\mu(P) - Q^\nu \partial_{P^\nu} n_\mu(P) + \frac{1}{2} Q^\rho \partial_{P^\rho} Q^\nu \partial_{P^\nu} n_\mu(P) + \mathcal{O}(Q^3). \tag{17}$$

And likewise in the expansion of f_K and $f_{P'}$ by soft momentum Q. The basic strategy is to expand the integrand to $\mathcal{O}(Q^{-2})$, together with the integral measure which is $\mathcal{O}(Q^2)$, then the momentum integral gives leading logarithmic result. Further details of momentum integral are presented in Appendix A 2.

B. Polarization

The second and third lines in (14) contain first order gradients through derivatives of number distribution of the probe $\partial^\nu f_P, \partial^\nu f_K$ as well as the axial-vector component of the medium fermion $n^\nu_{\mathrm{leq}}(P')$ and $n^\nu_{\mathrm{leq}}(K')$. For f_P and f_K at local equilibrium, $\partial^\nu f_P, \partial^\nu f_K$ can be decomposed in terms of gradients of the fluid. Similar to [62], derivative of the fluid velocity u can be decomposed into antisymmetric and symmetric part $\partial^\mu u^\nu = \omega^{\mu\nu} + \sigma^{\mu\nu}$, where $\omega^{\mu\nu} = (\partial^\mu u^\nu - \partial^\nu u^\mu)/2$ and $\sigma^{\mu\nu} = (\partial^\mu u^\nu + \partial^\nu u^\mu)/2$. With the vorticity defined as $\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu(\partial_\alpha u_\beta)$, the symmetric part is casted into $\omega_{\alpha\beta} = -\epsilon_{\alpha\beta\mu\nu}\omega^\mu u^\nu + \kappa_{\alpha\beta}$, with $\kappa_{\alpha\beta} = \frac{1}{2}(u_\alpha D u_\beta - u_\beta D u_\alpha)$. Defining $\tilde{E}^\sigma(P) = P_\lambda(\frac{1}{T}u^\lambda \partial^\sigma T - \sigma^{\sigma\lambda} - \kappa^{\sigma\lambda})$, we arrive at the following decomposition,

$$\partial^\nu f_P = -(\epsilon^{\nu\rho\alpha\beta} P_\rho \omega_\alpha u_\beta + \tilde{E}^\nu(P)) f'_P, \tag{18}$$

where $f'_P = \partial_{u\cdot P} f_P = (-\frac{1}{T}) f_P \bar{f}_P$. $\tilde{E}^\sigma(P)$ can be further casted in combination of shear tensor, acceleration and gradient of temperature

$$\tilde{E}^\sigma(P) = -P_\lambda\left(\partial^{\langle\sigma} u^{\lambda\rangle} + \frac{1}{3}\Delta^{\sigma\lambda}\theta + u^\sigma D u^\lambda\right) + P\cdot u[\partial^\sigma \ln T]. \tag{19}$$

The spin evolution of the massive probe fermion in a massless QED at local equilibrium also involves the exchanging of spin with the massless medium fermion, which are at local equilibrium. For the medium fermion, we take the local equilibrium distribution of spin [62], namely

$$\mathcal{A}^{\mathrm{leq}}_\mu(P) = 2\pi\epsilon(P\cdot u)\delta(P^2)\left(\frac{P\cdot\omega u_\mu}{2} - \frac{P\cdot u\omega_\mu}{2} - S^{(u)}_{\mu\sigma}(P)\tilde{E}^\sigma(P)\right) f'_V(P), \tag{20}$$

where

$$S^{(u)}_{\mu\nu}(P) = \frac{\epsilon_{\mu\nu\alpha\beta}P^\alpha u^\beta}{2P\cdot u}. \tag{21}$$

The polarization effect contains contribution from vorticity, shear tensor, acceleration and the gradient of temperature. With the decomposition (Sec. IV C) and (20), the vorticity related terms in the collision term can be collected into,

$$C^{\mathrm{vor}}_{A\mu} = -4e^4 \int_{q_0,q,k'} C^{\mathrm{vor}}_\mu(-\beta) \bar{f}_K \bar{f}_{K'} f_{P'} f_P, \tag{22}$$

with $\beta(x) = T(x)^{-1}$, and

$$C^{\mathrm{vor}}_\mu = -(M^{A3}_{\mu\nu} P_\rho + M^{A4}_{\mu\nu} K_\rho)\epsilon^{\nu\rho\alpha\beta}\omega_\alpha u_\beta + \frac{1}{2}(M^{A5}_{\mu\nu} P'_\rho + M^{A6}_{\mu\nu} K'_\rho)\omega^{[\rho} u^{\nu]}. \tag{23}$$

The effective amplitudes M^{Ai} are defined in (15). Equation (22) characterizes the polarization effect due the vorticity in the medium. Using the decomposition (Sec. IV C) and (20), the shear tensor related terms in the collision term can be casted into

$$C_{A\mu}^{\rm shear} = -4e^4 \int_{q_0,q,k'} C_{\mu\alpha\beta}^{\rm shear} \sigma^{\langle\alpha\beta\rangle}(-\beta)\bar{f}_K\bar{f}_{K'}f_{P'}f_P, \quad (24)$$

with $C_{\mu\alpha\beta}^{\rm shear}$ defined through

$$C_{\mu\alpha\beta}^{\rm shear} = M_{\mu\alpha}^{A3}P_\beta + M_{\mu\alpha}^{A4}K_\beta + \frac{1}{2P'\cdot u}(M^{A5})_\mu^\nu \epsilon_{\nu\alpha\rho\lambda}P'^\rho u^\lambda P'_\beta + \frac{1}{2K'\cdot u}(M^{A6})_\mu^\nu \epsilon_{\nu\alpha\rho\lambda}K'^\rho u^\lambda K'_\beta. \quad (25)$$

The shear tensor $\sigma^{\langle\alpha\beta\rangle}$ is the symmetric and traceless part of $\sigma^{\alpha\beta} = \frac{1}{2}(\partial_\perp^\alpha u^\beta + \partial_\perp^\beta u^\alpha) - \frac{1}{3}\Delta^{\alpha\beta}\theta$. In the local rest frame of the fluid, the shear tensor only spatial components $\sigma_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{1}{3}\delta_{ij}\vec{\partial}\cdot\vec{u}$. The efficient amplitudes are presented in (15). The remaining first order gradients are the temperature gradient and acceleration, the corresponding collision term is collected into

$$C_{A\mu}^{\rm Tgra+acc} = -4e^4 \int_{q_0,q,k'} (C_{\mu\lambda}^{\rm Tgra}\partial^\lambda \ln T + C_{\mu\lambda}^{\rm acc} Du^\lambda) \times (-\beta)\bar{f}_K\bar{f}_{K'}f_{P'}f_P, \quad (26)$$

with coefficients $C_{\mu\lambda}^{\rm Tgra}$ and $C_{\mu\lambda}^{\rm acc}$ defined as

$$C_{\mu\lambda}^{\rm Tgra} = -M_{\mu\lambda}^{A3}P\cdot u - M_{\mu\lambda}^{A4}K\cdot u - \frac{1}{2}(M^{A5})_\mu^\nu \epsilon_{\nu\lambda\alpha\beta}P'^\alpha u^\beta - \frac{1}{2}(M^{A6})_\mu^\nu \epsilon_{\nu\lambda\alpha\beta}K'^\alpha u^\beta,$$
$$C_{\mu\lambda}^{\rm acc} = M_{\mu\nu}^{A3}u^\nu P_\lambda + M_{\mu\nu}^{A4}u^\nu K_\lambda. \quad (27)$$

The final result of the collision term will be the sum of the all the parts above, namely $C_{A\mu} = C_{A\mu}^{\rm diff} + C_{A\mu}^{\rm vor} + C_{A\mu}^{\rm shear} + C_{A\mu}^{\rm Tgra+acc}$.

IV. RESULT

In this section, we explicitly present the result of the collision term. The leading logarithmic contribution comes from the soft $eT \ll q_0, q \ll T$ regime, the basic strategy to obtain the leading logarithmic contribution is to collect all the terms up to $\mathcal{O}(Q^{-2})$ in the integrand. Combined with the measure which is $\mathcal{O}(Q^2)$, both combined will give the leading logarithmic results. With the assumption that mass of the probe fermion is much larger than thermal mass $m \gg m_D \sim eT$, Compton scattering is subleading and only Coulomb scattering is considered. In the calculation, there is no more restriction for the mass of the probe fermion. In the following, we present the collision term after the momentum integral for arbitrary mass of the probe fermion, and also take massless and nonrelativistic limit for a comparison.

A. Collision terms

1. Arbitrary mass

For arbitrary nonzero mass of the probe fermion, the kinetic equation of the axial-vector component becomes

$$P\cdot\partial n^\mu(P) = -\kappa_{LL}\frac{T}{mv}\Big\{C^{(1)}n^\mu(P) + C^{(2)}u^\mu + C^{(3)}\hat{P}_\perp^\mu + C^{(4)}\hat{P}_\perp^\nu \partial_{P_{\perp\mu}} n_\nu(P) + C^{(5)}\hat{P}_\perp^\nu \partial_{P_\perp^\nu} n^\mu(P) + C^{(6)}g^{\nu\rho}\partial_{P_\perp^\nu}\partial_{P_\perp^\rho} n^\mu(P) + C^{(7)}\hat{P}_\perp^\nu \hat{P}_\perp^\rho \partial_{P_\perp^\nu}\partial_{P_\perp^\rho} n^\mu(P) + C^{(8)}(\omega^\mu + \hat{P}_\perp^\mu \hat{P}_\perp^\nu \omega_\nu) + C^{(9)}\frac{1}{2}(\epsilon^{\mu\nu\rho\alpha}u_\nu \hat{P}_{\perp\rho}\hat{P}_\perp^\beta + \epsilon^{\mu\nu\rho\beta}u_\nu \hat{P}_{\perp\rho}\hat{P}_\perp^\alpha)\sigma_{\langle\alpha\beta\rangle} + C^{(10)}\epsilon^{\mu\nu\alpha\beta}u_\alpha \hat{P}_{\perp\beta}Du_\nu + C^{(11)}\epsilon^{\mu\nu\alpha\beta}u_\alpha \hat{P}_{\perp\beta}\partial_\nu \ln T\Big\}, \quad (28)$$

note that both sides of the kinetic equation are on the mass shell $\delta(P^2 - m^2)$. κ_{LL} is the leading logarithmic coefficient given by

$$\kappa_{LL} = \frac{T^2}{8\pi}e^4 \ln\frac{1}{e}. \quad (29)$$

Similar to [37], we also introduce the four velocity $v^\mu \equiv P^\mu/m$ for simplicity. Which has the normalization $v^\mu v_\mu = 1$, and $v_0 = p_0/m$ with $v = |\vec{p}|/m$. The rapidity is $\eta_p \equiv \operatorname{arctanh}(p/p_0) \equiv 2^{-1}\ln[(p_0+p)/(p_0-p)]$. The shorthand notation $\theta_n \equiv v - v_0^n \eta_p$ is defined for simplicity. The coefficients in (28) are

$$
\begin{aligned}
C^{(1)} &= \frac{v}{3v_0} - \frac{m^2 v_0 \theta_{-1}}{3T^2}(1-f_p)f_p - \frac{mv^3}{3Tv_0^2}(1-2f_p), \\
C^{(2)} &= \left(\frac{1}{3} + \frac{\theta_{-1}}{3v} - \frac{mv_0\theta_{-1}}{6Tv}(1-2f_p)\right)\hat{P}^\nu_\perp n_\nu(P) + \frac{m(2v^3 - v_0^2\theta_{-1})}{6v^2}\partial_{P^\nu_\perp} n^\nu(P) + \frac{m(2v^3 - 3v_0^2\theta_{-1})}{6v^2}\hat{P}^\rho_\perp \hat{P}^\nu_\perp \partial_{P^\rho_\perp} n_\nu(P), \\
C^{(3)} &= \left(\frac{v}{3v_0^3} + \frac{\theta_{-1}}{3v_0} - \frac{m\theta_{-1}}{6T}(1-2f_p)\right)\hat{P}^\nu_\perp n_\nu(P) - \frac{mv_0\theta_{-1}}{6v}\partial_{P^\nu_\perp} n^\nu(P) + \frac{m(2v^3 - 3v_0^2\theta_{-1})}{6vv_0}\hat{P}^\rho_\perp \hat{P}^\nu_\perp \partial_{P^\rho_\perp} n_\nu(P), \\
C^{(4)} &= \frac{mv^2}{3v_0}, \\
C^{(5)} &= \frac{m}{3v_0} - \frac{m^2 v_0^2 \theta_{-1}}{6Tv}(1-2f_p), \\
C^{(6)} &= \frac{m^2(3v^3 v_0 - v_0^3\theta_{-3})}{12v^2}, \\
C^{(7)} &= -\frac{m^2 v_0(2\theta_1 + \theta_{-1})}{12v^2}, \\
C^{(8)} &= \left(-\frac{mv}{3v_0^2} - \frac{m^2 v_0\theta_{-1}}{6T}(1-2f_p)\right)\frac{(1-f_p)f_p}{2T}, \\
C^{(9)} &= \left(\frac{m(v^5 + 3v_0^2\theta_1)}{3v^2v_0^2} + \frac{m^2 v_0(2v^3 - (v^2 + 3v_0^2)\theta_{-1})}{6Tv^2}(1-2f_p)\right)\frac{(1-f_p)f_p}{2T}, \\
C^{(10)} &= \left(\frac{mv_0\theta_{-1}}{2v} + \frac{m^2(2v^3 - 3\theta_{-1}v_0^2)}{12Tv}(1-2f_p)\right)\frac{(1-f_p)f_p}{2T}, \\
C^{(11)} &= \left(\frac{m(2v - 3v_0^2\theta_{-1})}{6vv_0} + \frac{m^2(3v^3 + 5\theta_1)}{12Tv}(1-2f_p)\right)\frac{(1-f_p)f_p}{2T}.
\end{aligned} \tag{30}
$$

f_p is the local equilibrium number distribution of the probe, which is the Fermi Dirac distribution $f_p = 1/(e^{E_p/T} + 1)$, and $E_p = \sqrt{p^2 + m^2}$.

The first two lines on the rhs of (28) are dubbed as the spin-diffusion terms, since these terms are linear in $n_\mu(P)$ which is related to spin of the probe fermion. If the probe fermion is unpolarized in the initial state, the spin-diffusion term would be vanishing at the initial time. These terms have same structure as Refs. [36,37]. The last two lines on the rhs of (28) delineate the spin polarization effects induced by the first order hydrodynamic gradients of the medium including vorticity, shear tensor, acceleration and gradient of temperature. In previous works about the dynamical processes of spin evolution of massive fermion in QED and QCD plasma [36,67], the polarization effects are not included. For simplicity, only the spin diffusion is investigated. In Ref. [37], the polarization effect is discussed for a general collision term but not explicitly for a system with given interaction. In a recent paper [42], polarization effects are considered for a massless probe fermion. In this paper, the polarization effect in considered for the first time in the view of kinetic theory for massive spin carrier in gauge plasma. When traversing the QED plasma, the probe fermion experiences competing processes from diffusion and polarization, until the both balance each other, then the spin of the probe fermion achieves local equilibrium. It is worth emphasizing that we have taken the assumption that the vector distribution f_V of the probe fermion has achieved local equilibrium with the medium. In general, the relaxation of f_V of the probe is coupled with the relaxation of spin n^μ and should also be incorporated. These will be presented in further studies. With the collision terms (28) and the coefficients (30), the diffusion rate and polarization rate can be estimated. However, since n_μ still contains three degrees of freedom, the analysis of diffusion rate and polarization rate will be postponed until n_μ is decomposed in Sec. IV C. As a cross-check and for later convenience, we present first the massless limit and nonrelativistic limit of collision terms (28), (30).

2. Massless limit

The collision terms (28) arise from soft t-channel photon exchange of momentum Q in the scattering with background hard thermal fermions. If the probe fermion is light, the soft t-channel fermion exchange contribution is no more suppressed and contributes also at leading logarithmic order. This scattering process, also known as Compton scattering, allows for conversion of fermion to a photon. The discussion of Compton scattering to first order of gradients also requires inclusion of polarized photon which is beyond the scope of this paper. The massless limit is considered here to compare with other researches, and for the convenience of deriving the collision terms after decomposition Sec. IV C. In the massless limit, the structure of the collision term is unchanged, the only difference lies in the replacement in the coefficients, with $C^{(i)}$ replaced by $C^{(i)}_{\rm chi}$. With $C^{(7)}_{\rm chi}=0$, the nonvanishing coefficients are

$$
\begin{aligned}
C^{(1)}_{\rm chi} &= \frac{1}{3}-\frac{p^2}{3T^2}f_p(1-f_p)-\frac{p}{3T}(1-2f_p),\\
C^{(2)}_{\rm chi} &= \left(\frac{2}{3}-\frac{p}{6T}(1-2f_p)\right)\hat{P}^{\nu}_{\perp}n_\nu(P)+\frac{p}{6}\partial_{P^\nu_\perp}n^\nu(P)-\frac{p}{6}\hat{P}^{\rho}_{\perp}\hat{P}^{\nu}_{\perp}\partial_{P^\rho_\perp}n_\nu(P),\\
C^{(3)}_{\rm chi} &= \left(\frac{1}{3}-\frac{p}{6T}(1-2f_p)\right)\hat{P}^{\nu}_{\perp}n_\nu(P)-\frac{p}{6}\partial_{P^\nu_\perp}n^\nu(P)-\frac{p}{6}\hat{P}^{\rho}_{\perp}\hat{P}^{\nu}_{\perp}\partial_{P^\rho_\perp}n_\nu(P),\\
C^{(4)}_{\rm chi} &= \frac{p}{3},\qquad C^{(5)}_{\rm chi}=-\frac{p^2}{6T}(1-2f_p),\qquad C^{(6)}_{\rm chi}=\frac{p^2}{6},\\
C^{(8)}_{\rm chi} &= -\frac{p^2(1-2f_p)}{6T}\frac{f_p(1-f_p)}{2T},\qquad C^{(9)}_{\rm chi}=\left(-\frac{p}{3}+\frac{p^2}{3T}(1-2f_p)\right)\frac{f_p(1-f_p)}{2T},\\
C^{(10)}_{\rm chi} &= \left(\frac{p}{2}-\frac{p^2(1-2f_p)}{12T}\right)\frac{f_p(1-f_p)}{2T},\qquad C^{(11)}_{\rm chi}=\left(-\frac{p}{2}+\frac{p^2(1-2f_p)}{4T}\right)\frac{f_p(1-f_p)}{2T}.
\end{aligned}
\tag{31}
$$

With the above coefficients, one can derive the transport equation of f_A in the massless limit (48). As a consistency check, the diffusion terms in (48) agree with (4.28) in [37] and (4.7) in [36].

3. Nonrelativistic limit

To investigate the spin evolution of heavy quarks in the quark gluon plasma, the nonrelativistic limit of the kinetic equation is analyzed. In the nonrelativistic limit, it is assumed that $m\gg p\sim T$. We keep the collision term to $\mathcal{O}(m^{-2})$ for later convenience when checking the eliminating of the collision term in the global equilibrium.

$$
\begin{aligned}
\left(\partial_t+\frac{1}{m}P^{\nu}_{\perp}\partial_\nu\right)n^\mu(P) = -\kappa_{LL}\Big\{& C^{(1)}_{\rm non}n^\mu(P)+C^{(2)}_{\rm non}u^\mu+C^{(3)}_{\rm non}\hat{P}^{\mu}_{\perp}+C^{(4)}_{\rm non}\hat{P}^{\nu}_{\perp}\partial_{P_{\perp\mu}}n_\nu(P)+C^{(5)}_{\rm non}\hat{P}^{\nu}_{\perp}\partial_{P^\nu_\perp}n^\mu(P)\\
&+C^{(6)}_{\rm non}g^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}n^\mu(P)+C^{(8)}_{\rm non}(\omega^\mu+\hat{P}^{\mu}_{\perp}\hat{P}^{\nu}_{\perp}\omega_\nu)+C^{(9)}_{\rm non}\frac{1}{2}(\epsilon^{\mu\nu\rho\alpha}u_\nu\hat{P}_{\perp\rho}\hat{P}^{\beta}_{\perp}+\epsilon^{\mu\nu\rho\beta}u_\nu\hat{P}_{\perp\rho}\hat{P}^{\alpha}_{\perp})\sigma_{\langle\alpha\beta\rangle}\\
&+C^{(10)}_{\rm non}\epsilon^{\mu\nu\alpha\beta}u_\alpha\hat{P}_{\perp\beta}Du_\nu+C^{(11)}_{\rm non}\epsilon^{\mu\nu\alpha\beta}u_\alpha\hat{P}_{\perp\beta}\partial_\nu\ln T\Big\},
\end{aligned}
\tag{32}
$$

with the coefficients as

$$
\begin{aligned}
C^{(1)}_{\rm non} &= \frac{1}{m^2}\left(\frac{T}{3}-\frac{2p^2}{9T}f_p(1-f_p)\right),\qquad C^{(2)}_{\rm non}=\left(\frac{T}{3mp}-\frac{p}{9m^2}(1-2f_p)\right)\hat{P}^{\nu}_{\perp}n_\nu(P)+\frac{2T}{9m}\partial_{P^\nu_\perp}n^\nu(P),\\
C^{(3)}_{\rm non} &= \frac{T}{3m^2}\hat{P}^{\nu}_{\perp}n_\nu(P)-\frac{Tp}{9m^2}\partial_{P^\nu_\perp}n^\nu(P),\qquad C^{(4)}_{\rm non}=\frac{Tp}{3m^2},\\
C^{(5)}_{\rm non} &= \frac{T}{3p}-\frac{p}{9m}(1-2f_p)-\frac{pT}{6m^2},\qquad C^{(6)}_{\rm non}=\frac{T}{9}-\frac{p^2T}{12m^2},\\
C^{(8)}_{\rm non} &= \left(-\frac{T}{3m}-\frac{p^2}{9m^2}\right)\frac{f_p(1-f_p)}{2T},\qquad C^{(9)}_{\rm non}=\left(-\frac{T}{3m}-\frac{4p^2(1-2f_p)}{9m^2}\right)\frac{f_p(1-f_p)}{2T},\\
C^{(10)}_{\rm non} &= \frac{pT}{3m^2}\frac{f_p(1-f_p)}{2T},\qquad C^{(11)}_{\rm non}=\left(\frac{T}{3p}+\frac{p(1-2f_p)}{9m}-\frac{pT}{2m^2}\right)\frac{f_p(1-f_p)}{2T}.
\end{aligned}
\tag{33}
$$

Both diffusion and polarization get suppressed, guaranteeing that, in the heavy quark limit $m\to\infty$, the orientation of spin is fixed while the spin density still experience the diffusion process. The different behavior in spin orientation and spin density can be observed more clearly when further decomposing kinetic equation of n^μ to its three degrees of freedom, as presented in Sec. IV C.

B. Relaxation near global equilibrium

For quantum kinetic theory, the elimination of collision term in global equilibrium for massive fermion has been proved in Refs. [38,39]. The collision terms in chiral kinetic theory is also shown to be vanishing in local equilibrium in Refs. [42,62]. We here check the vanishing of collision term in global equilibrium as a guarantee of the correctness the above calculation, and also as a prerequisite for extracting the relaxation rate. In global equilibrium, $n_\mu(P)$ in a purely rotating fluid could be defined frame independently [39,62] as

$$n_\mu^{\rm geq}(P) = \left(\frac{P\cdot\omega u_\mu}{2} - \frac{P\cdot u\omega_\mu}{2}\right) f'_P. \tag{34}$$

Using the following derivatives of the equilibrium number distribution function, $f_p = n_F(E_p)$,

$$\begin{aligned}
\partial_{P_\perp^\mu} f_p &= f_p(1-f_p)\frac{P_{\perp\mu}}{p_0 T},\\
\partial_{P_\perp^\mu}\partial_{P_\perp^\nu} f_p &= f_p(1-f_p)(1-2f_p)\frac{P_{\perp\mu}P_{\perp\nu}}{p_0^2T^2} + f_p(1-f_p)\frac{1}{p_0T}\left(\frac{P_{\perp\mu}P_{\perp\nu}}{p_0^2} + \Delta_{\mu\nu}\right),
\end{aligned} \tag{35}$$

the momentum derivative of $n_\mu^{\rm geq}(P)$ in the collision terms can be explicitly obtaining with the help of the following tensors:

$$\begin{aligned}
\partial_{P_\perp^\nu} n_\mu^{\rm geq}(P) &= -\frac{f_p(1-f_p)}{2T}\left(\omega_\nu u_\mu + \frac{P_{\perp\nu}}{p_0}\omega_\mu\right) - \frac{f_p(1-f_p)(1-2f_p)}{2T}\frac{P_{\perp\nu}}{p_0T}(P_\perp^\sigma\omega_\sigma u_\mu - p_0\omega_\mu),\\
\partial_{P_\perp^\nu}\partial_{P_\perp^\rho} n_\mu^{\rm geq}(P) &= -\frac{f_p(1-f_p)}{2T}\frac{1}{p_0}\left(\Delta_{\nu\rho} + \frac{P_{\perp\nu}P_{\perp\rho}}{p_0^2}\right)\omega_\mu\\
&\quad - \frac{f_p(1-f_p)(1-2f_p)}{2T}\frac{1}{p_0T}\left(\left(\Delta_{\nu\rho} + \frac{P_{\perp\nu}P_{\perp\rho}}{p_0^2}\right)(P_\perp^\sigma\omega_\sigma u_\mu - p_0\omega_\mu) + \left(P_{\perp(\rho}\omega_{\nu)}u_\mu + \frac{2P_{\perp\nu}P_{\perp\rho}}{p_0}\omega_\mu\right)\right)\\
&\quad - \frac{f_p(1-f_p)((1-2f_p)^2 - 2f_p(1-f_p))}{2T}\frac{P_{\perp\nu}P_{\perp\rho}}{p_0^2T^2}(P_\perp^\sigma\omega_\sigma u_\mu - p_0\omega_\mu).
\end{aligned} \tag{36}$$

Substituting these derivatives back into the diffusion part, one can explicitly find that the diffusion part balances the vorticity part, and thus the collision terms are eliminated in global equilibrium. The vanishing of collision terms is also be proved for the massless limit and nonrelativistic limit when inserting the derivatives in corresponding limits.

Near the global equilibrium, the relaxation of the spin is dominated by the diffusion terms, leading to the relaxation rate near global equilibrium

$$P\cdot\partial n^\mu(P) = (\hat\tau^{-1})^{\mu\nu}\delta n_\nu(P), \tag{37}$$

where the relaxation time is now an operator,

$$\begin{aligned}
(\hat\tau^{-1})^{\mu\nu} = \kappa_{LL}\frac{T}{mv}\Bigg\{& g^{\mu\nu}\Bigg(\frac{v}{3v_0} - \frac{m^2v_0\theta_{-1}}{3T^2}(1-f_p)f_p - \frac{mv^3}{3Tv_0^2}(1-2f_p) + \left(\frac{m}{3v_0} - \frac{m^2v_0^2\theta_{-1}}{6Tv}(1-2f_p)\right)\hat P_\perp^\alpha\partial_{P_\perp^\alpha}\\
&+ \frac{m^2(3v^3v_0 - v_0^3\theta_{-3})}{12v^2}g^{\alpha\beta}\partial_{P_\perp^\alpha}\partial_{P_\perp^\beta} - \frac{m^2v_0(2\theta_1+\theta_{-1})}{12v^2}\hat P_\perp^\alpha\hat P_\perp^\beta\partial_{P_\perp^\alpha}\partial_{P_\perp^\beta}\Bigg) + \frac{mv^2}{3v_0}\hat P_\perp^\nu\partial_{P_{\perp\mu}}\\
&+ \left(\frac{1}{3} + \frac{\theta_{-1}}{3v} - \frac{mv_0\theta_{-1}}{6Tv}(1-2f_p)\right)u^\mu\hat P_\perp^\nu + \frac{m(2v^3 - v_0^2\theta_{-1})}{6v^2}u^\mu\partial_{P_\perp^\nu} + \frac{m(2v^3 - 3v_0^2\theta_{-1})}{6v^2}u^\mu\hat P_\perp^\nu\hat P_\perp^\rho\partial_{P_\perp^\rho}\\
&+ \left(\frac{v}{3v_0^3} + \frac{\theta_{-1}}{3v_0} - \frac{m\theta_{-1}}{6T}(1-2f_p)\right)\hat P_\perp^\mu\hat P_\perp^\nu - \frac{mv_0\theta_{-1}}{6v}\hat P_\perp^\mu\partial_{P_\perp^\nu} + \frac{m(2v^3 - 3v_0^2\theta_{-1})}{6vv_0}\hat P_\perp^\mu\hat P_\perp^\nu\hat P_\perp^\rho\partial_{P_\perp^\rho}\Bigg\}.
\end{aligned} \tag{38}$$

The assumption that the number distribution of the probe fermion has reached local equilibrium with the medium leads to the disappearance of gradient terms in the relaxation time. Otherwise, the kinetic equation of both spin and number distribution will couple with each other, the relaxation of the charge also contributes to spin evolution [40,42]. The full set of kinetic equation of both spin and charge to the first order of gradient will be included in an upcoming paper.

C. Diffusion and polarization rate

So far, the decomposition of $\mathcal{A}_\mu$ was not considered for the simplicity when deriving the transport equation. For massive fermion, the axial-vector component $\mathcal{A}_\mu$ has three degrees of freedom, constrained by the perpendicular relation $P^\mu \mathcal{A}_\mu = 0$. The further decomposition is necessary to obtain components with definite physical meanings, and to recover a smooth connection with the chiral limit. To get the transport equation for each of the three degrees of freedom of $\mathcal{A}_\mu$ and meanwhile keep the correct massless limit, we adopt the following decomposition of $\mathcal{A}_\mu = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)n_\mu$ [68],

$$n^\mu(P) = P^\mu f_A + \frac{P^2}{(u\cdot P)^2 - P^2}P^\mu_\perp f_A + \frac{P^2}{P\cdot u}\mathcal{M}^\mu_\perp + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha u_\beta}{2P\cdot u}\partial_\nu f_V. \tag{39}$$

$f_A = u\cdot n/u\cdot P$ is identified as the axial-charge density, $\mathcal{M}^\mu_\perp$ is the transverse magnetic dipole-moment $\mathcal{M}^\mu_\perp = \Xi^{\mu\nu}\mathcal{M}_\nu$, where $\mathcal{M}^\mu = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu S_{\alpha\beta}$ and $S_{\alpha\beta}$ is the dipole moment tensor defined in last equation in (5). In the massless limit, restricted by on-shell condition $\delta(P^2)$, the second and third term in (39) disappear. With such decomposition, the various components of n^μ have clear physical meanings and a smooth massless limit. See [68,69] for more details of the decomposition.

Consider that the charge distribution has achieved local equilibrium, the general f_V in (39) is replaced by the local equilibrium distribution function f_p. For convenience, we consider the scenario that the medium is in the global equilibrium, namely we keep only the vorticity and neglect all other first order gradients. The transport equations of the axial-charge density $f_A(P)$ and transverse magnetic dipole-moment $\mathcal{M}^\mu_\perp(P)$ become

$$\begin{aligned}
P\cdot\partial f_A &= -\kappa_{LL}\frac{T}{mv}\{C_A^{(1)}f_A + C_A^{(2)}\hat{P}^\nu_\perp\partial_{P^\nu_\perp}f_A + C_A^{(3)}g^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}f_A + C_A^{(4)}\hat{P}^\nu_\perp\hat{P}^\rho_\perp\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}f_A + C_A^{(5)}\partial_{P^\nu_\perp}\mathcal{M}^\nu_\perp + C_A^{(6)}\hat{P}^\nu_\perp\omega_\nu\},\\
P\cdot\partial\mathcal{M}^\mu_\perp &= -\kappa_{LL}\frac{T}{mv}\left\{C_M^{(1)}\mathcal{M}^\mu_\perp + C_M^{(2)}\hat{P}^\nu_\perp\partial_{P^\nu_\perp}\mathcal{M}^\mu_\perp + C_M^{(3)}\left(g^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}\mathcal{M}^\mu_\perp - \frac{2}{mv}\hat{P}^\mu_\perp\partial_{P^\nu_\perp}\mathcal{M}^\nu_\perp\right)\right.\\
&\quad\left.+ C_M^{(4)}\hat{P}^\nu_\perp\hat{P}^\rho_\perp\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}\mathcal{M}^\mu_\perp + C_M^{(5)}\Xi^{\mu\nu}\partial_{P^\nu_\perp}f_A + C_M^{(6)}\Xi^{\mu\nu}\omega_\nu\right\},
\end{aligned} \tag{40}$$

both equations are coupled, with the coefficient $C_A^{(i)}$ defined as

$$\begin{aligned}
C_A^{(1)} &= -\frac{m^2v_0\theta_{-1}}{3T^2}(1-f_p)f_p - \frac{m(2v^5 - v_0^2\theta_{-1})}{6Tv^2v_0^2}(1-2f_p) - \frac{v^3 + v_0^2(v_0^2+1)\theta_1}{3v^4v_0^3},\\
C_A^{(2)} &= \frac{(2f-1)\theta_{-1}m^2v_0^2}{6v} + \frac{m(\theta_1 + 2v^3)}{3v^3v_0},\\
C_A^{(5)} &= \frac{m(2v^3 - v_0^2\theta_{-1})}{6v^2v_0^2}, \qquad C_A^{(6)} = \frac{(2v^3 - v_0^2\theta_{-1})}{6vv_0^2}\frac{(1-f_p)f_p}{T},
\end{aligned} \tag{41}$$

with $C_{A,M}^{(3)} = C^{(6)}$, and $C_{A,M}^{(4)} = C^{(7)}$, with $C^{(6,7)}$ defined in (30). And the coefficients $C_M^{(i)}$ have the following expression,

$$\begin{aligned}
C_M^{(1)} &= -\frac{m^2v_0\theta_{-1}}{3T^2}(1-f_p)f_p - \frac{m(2v^3 - v_0^2\theta_{-1})}{6Tv_0^2}(1-2f_p) - \frac{\theta_1}{3v_0^3},\\
C_M^{(2)} &= -\frac{m^2v_0^2\theta_{-1}}{6Tv}(1-2f_p) + \frac{m(v_0^2\theta_{-1} + v)}{3vv_0},\\
C_M^{(5)} &= \frac{mv_0^2(v^3 - \theta_1)}{6v^4}, \qquad C_M^{(6)} = \left(\frac{m\theta_{-1}}{6T}(1-2f_p) - \frac{2v^5 + \theta_{-1}(1-v^2)v_0^2}{2v^2v_0^3}\right)\frac{(1-f_p)f_p}{2T}.
\end{aligned} \tag{42}$$

When the spin of the probe reaches global equilibrium, n^μ takes the solution (34). The global equilibrium expression for f_A and $\mathcal{M}^\mu_\perp$ can be solved accordingly, giving $f_A^{\mathrm{geq}} = -\hat{P}^\nu_\perp\omega_\nu(p/p_0)f_p(1-f_p)/(2T)$ and $\mathcal{M}^{\perp,\mu}_{\mathrm{geq}} = \Xi^{\mu\nu}\omega_\nu f_p$

$(1 - f_p)/(2T)$. Inserting back into the collision terms in (40), one will also find the elimination of collision term at global equilibrium.

The first four terms in both collision terms are the diffusion terms, the last terms characterize the polarization effect induced by the vorticity of the QED plasma. The projection of vorticity to direction of momentum $\omega_\parallel = \hat{P}^\nu_\perp \omega_\nu$ acts as a source of axial charge, with the polarization rate estimated by $\Gamma_{\text{Apol}} = \partial_0 f_A$

$$\Gamma_{\text{Apol}} = \frac{e^4 \ln(1/e)}{8\pi} \frac{T^3}{p_0 p} C_A^{(6)} \omega_\parallel, \tag{43}$$

where $C_A^{(6)}$ is a function of mass, momentum and temperature. On the other hand, the diffusion rate can be estimated as

$$\Gamma_{\text{Adif}} = \frac{e^4 \ln(1/e) T^3}{8\pi p_0 p} C_A^{(1)}. \tag{44}$$

The ratio of both rates serves as an estimation of whether the polarization effect is important during the thermalization,

$$\frac{\Gamma_{\text{Apol}}}{\Gamma_{\text{Adiff}}} = \frac{C_A^{(6)} T}{C_A^{(1)}} \frac{\omega_\parallel}{T}. \tag{45}$$

The dependence of $C_A^{(6)} T / C_A^{(1)}$ on momentum and mass is presented in left panel of Fig. 2. The ratio is obviously suppressed for large mass. For small momentum, the ratio approaches zero, indicating the polarization is suppressed in the nonrelativistic limit. For the large momentum side, the ratio is also suppressed, this is due to thermodynamical suppression coming from f_p in $C_A^{(6)}$. Consider s-quark in the quark gluon plasma, which is roughly $m \sim p \sim T$, the ratio is roughly $\Gamma_{\text{Apol}}/\Gamma_{\text{Adiff}} \sim \omega_\parallel / T$. $\mathcal{M}^\mu_\perp$ delineates the spin polarization in the plane transverse to momentum $\vec{p}$. It can be further decomposed into direction along $\omega^\mu_\perp = \Xi^{\mu\nu}\omega_\nu$ denoted by $\mathcal{M}_{\perp,\parallel\omega}$ and component perpendicular to this direction $\mathcal{M}_{\perp,\perp\omega}$. We assume that $\mathcal{M}^\mu_\perp$ has smooth dependence on momentum and that derivative terms can be discarded. The diffusion and polarization rates are estimated as

$$\Gamma_{\text{Mdif}} = \frac{e^4 \ln(1/e)}{8\pi} \frac{T^3}{p_0 p} C_M^{(1)},$$
$$\Gamma_{\text{Mpol}} = \frac{e^4 \ln(1/e)}{8\pi} \frac{T^3}{p_0 p} C_M^{(6)} \omega_\perp, \tag{46}$$

where $\omega_\perp = |\Xi^{\mu\nu}\omega_\nu|$. The ratio between the both is

$$\frac{\Gamma_{\text{Mpol}}}{\Gamma_{\text{Mdiff}}} = \frac{C_M^{(6)} T}{C_M^{(1)}} \frac{\omega_\perp}{T}. \tag{47}$$

The coefficient $C_M^{(6)} T / C_M^{(1)}$ at different momentum and mass is presented in the right panel in Fig. 2. The ratio is again suppressed when mass and momentum increase. Considering when $m \sim p \sim T$, the ratio is about $\Gamma_{\text{Apol}}/\Gamma_{\text{Adiff}} \sim \omega_\perp / T$. Thus for the s-quark in a quark gluon plasma, the ratio between polarization rate and diffusion rate is about $|\omega|/T$, which means that, compared to thermalization, it takes longer time for the spin to reach equilibrium, making the nonequilibrium effect important for the spin evolution.

The massless limit and nonrelativistic limit of the transport equations Eq. (40) are also considered, so as to have a better understanding of the momentum and mass dependence of the diffusion rate and polarization rate, and to compare with the previous works [42] for the massless case and [67] for the heavy quarks.

1. Massless limit

Limit restricted by $\delta(P^2)$ in the massless, the second and third term in (39) naturally returns to zero. f_A becomes the only degree of freedom, and its transport equation becomes

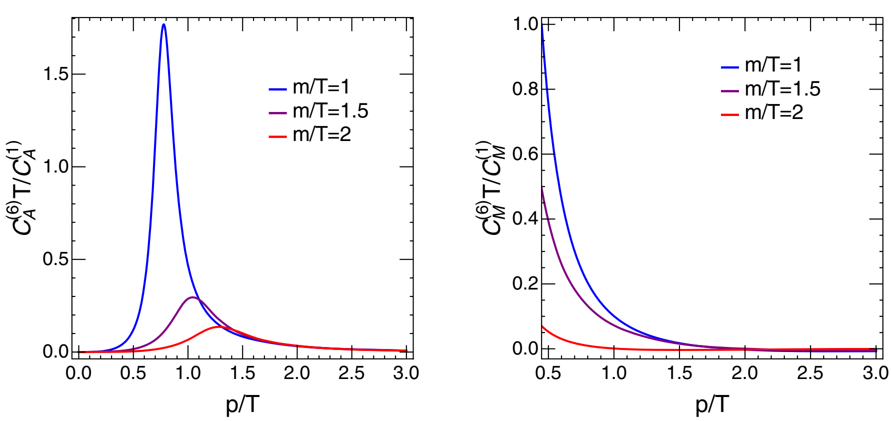

FIG. 2. Left: the coefficient $C_A^{(6)} T / C_A^{(1)}$ in the ratio between polarization and diffusion of f_A. Right: the coefficient $C_M^{(6)} T / C_M^{(1)}$ in the ratio between polarization and diffusion of $\mathcal{M}^\mu_\perp$.

$$P\cdot\partial f_A=\frac{\kappa_{LL}}{3}\left[\left(\frac{p(1-f_p)f_p}{T}+1-2f_p\right)f_A+\frac{p(1-2f_p)}{2}\hat{P}^\nu_\perp\partial_{P^\nu_\perp}f_A-\frac{pT}{2}g^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}f_A-\frac{f_p(1-f_p)}{2p}\hat{P}^\nu_\perp\omega_\nu\right], \tag{48}$$

which is consistent with [37], up to the overall constant $SU(N)$ coefficient. The first three terms delineate the diffusion of axial charge. Supposing that f_A has smooth dependence of momentum, the diffusion rate can be estimated to be

$$\Gamma^{\rm chi}_{\rm Adiff}\sim\frac{e^4\ln(1/e)T^2}{24\pi p}. \tag{49}$$

It agrees with that in [42]. On the other hand, the last term in (48) is the polarization effect. If no axial charge exists in the initial time, then there will be axial charge generated by the vorticity. The polarization rate is estimated through turning off the diffusion terms on the rhs and using $\Gamma_{\rm Apol}=\partial_0 f_A$

$$\Gamma^{\rm chi}_{\rm Apol}\sim\frac{e^4\ln(1/e)T^2}{48\pi p}\frac{\omega_\parallel}{p}f_p. \tag{50}$$

The polarization rate is further suppressed thermodynamically by the distribution function. The ratio of the both tells which effect dominates,

$$\frac{\Gamma^{\rm chi}_{\rm Apol}}{\Gamma^{\rm chi}_{\rm Adiff}}\sim\frac{\omega_\parallel}{p}f_p. \tag{51}$$

This agrees with $|\partial|/p$ observed in Ref. [42]. It is also suppressed thermodynamically for low temperature or particles with large momentum.

2. Nonrelativistic limit

In the nonrelativistic limit $m\gg p\sim T$, keeping leading order of each term, both transport equations become

$$\begin{aligned}\left(\partial_t+\frac{1}{m}P^\nu_\perp\partial_\nu\right)f_A&=\frac{\kappa_{LL}}{9}\left\{\frac{T}{p^2}f_A-\frac{5T}{p}\hat{P}^\nu_\perp\partial_{P^\nu_\perp}f_A-Tg^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}f_A-\frac{2T}{m}\partial_{P^\nu_\perp}\mathcal{M}^\nu_\perp-\frac{2p(1-f_p)f_p}{m^3}\hat{P}^\nu_\perp\omega_\nu\right\},\\ \left(\partial_t+\frac{1}{m}P^\nu_\perp\partial_\nu\right)\mathcal{M}^\mu_\perp&=\frac{\kappa_{LL}}{9}\left\{\frac{2p^2f_p(1-f_p)}{m^2T}\mathcal{M}^\mu_\perp+\frac{3T}{p}\hat{P}^\nu_\perp\partial_{P^\nu_\perp}\mathcal{M}^\mu_\perp+T\left(g^{\nu\rho}\partial_{P^\nu_\perp}\partial_{P^\rho_\perp}\mathcal{M}^\mu_\perp-\frac{2}{mv}\hat{P}^\mu_\perp\partial_{P^\nu_\perp}\mathcal{M}^\nu_\perp\right)\right.\\ &\quad\left.+\frac{2T}{m}\Xi^{\mu\nu}\partial_{P^\nu_\perp}f_A-\frac{(1-f_p)f_p}{2m^2}\Xi^{\mu\nu}\omega_\nu\right\}.\end{aligned} \tag{52}$$

The spin density characterized by f_A undergoes a diffusive process which is not suppressed, with a diffusion rate of about

$$\Gamma^{\rm non}_{\rm Adiff}\sim\frac{e^4\ln(1/e)T^3}{72\pi p^2}. \tag{53}$$

For the hard fermions $p\sim T$, it is about the same order as in the massless limit. On the other hand, the polarization process for f_A is strongly suppressed:

$$\Gamma^{\rm non}_{\rm Apol}\sim\frac{e^4\ln(1/e)T^2p\omega_\parallel}{36\pi m^3}f_p. \tag{54}$$

Compared to the polarization rate in the massless limit, $\Gamma^{\rm non}_{\rm Apol}$ is further suppressed by $(p/m)^3$. From the ratio between the both rates

$$\frac{\Gamma^{\rm non}_{\rm Apol}}{\Gamma^{\rm non}_{\rm Adiff}}\sim\frac{p^4}{m^3T}\frac{\omega_\parallel}{p}f_p, \tag{55}$$

one can tell that the diffusion process dominates in the nonrelativistic limit. As f_A dies off quickly, the orientation of the spin is mainly captured by the transverse dipole moment $\mathcal{M}^\mu_\perp$. Both $\mathcal{M}_{\perp,\parallel\omega}$ and $\mathcal{M}_{\perp,\perp\omega}$ have a diffusion rate

$$\Gamma^{\rm non}_{\rm Mdiff}\sim\frac{e^4\ln(1/e)Tp^2}{36\pi m^2}f_p. \tag{56}$$

Compared to f_A, diffusion of transverse dipole moment $\mathcal{M}^\mu_\perp$ is suppressed by $(p/m)^2$. This diffusion rate can be compared with Γ_s in Ref. [67], considering $p\sim T$ one can recover $g^4\ln(1/g)T(T/M)^2$ in Ref. [67]. The transverse dipole moment also experiences polarization in the direction along $\omega^\mu_\perp$, and the polarization rate is also suppressed by $(p/m)^2$

$$\Gamma^{\rm non}_{\rm Mpol}\sim\frac{e^4\ln(1/e)T^2\omega_\perp}{144\pi m^2}f_p. \tag{57}$$

Both the diffusion rate and polarization rate of transverse dipole moment are suppressed by $(T/m)^2$, explaining the lock of spin orientation in the nonrelativistic limit. The ratio between the both is

$$\frac{\Gamma^{\rm non}_{\rm Mpol}}{\Gamma^{\rm non}_{\rm Mdiff}} \sim \frac{T\omega_\perp}{p^2}. \qquad (58)$$

For hard fermion with $p \sim T$, although both rates are suppressed in the nonrelativistic limit, their ratio is still about $\omega_\perp/T$.

V. NUMERICAL ANALYSIS

In this section, we present preliminary numerical analysis to compare the evolution with and without polarization effect, and to show the suppression of diffusion and polarization by mass and momentum. In order to carry out numerical analysis, we assume for convenience that f_A and $\mathcal{M}^\mu_\perp$ are isotropic in momentum, then both transport equations in (40) decouple and can be further simplified. Besides, we ignore the spatial dependence and focus only on the time evolution. We here focus on the evolution of the transverse magnetic dipole moment $\mathcal{M}^\mu_\perp$, through some simple numerical process, its diffusion and polarization can be visualized. As in the last section, we define $\mathcal{M}_{\perp,\|\omega}$ to be the component parallel to $\omega^\mu_\perp$ as $\mathcal{M}_{\perp,\perp\omega}$ the perpendicular component. $\mathcal{M}_{\perp,\perp\omega}$ undergoes a purely diffusion process while $\mathcal{M}_{\perp,\|\omega}$ is affected by both diffusion and polarization processes.

We first compare the evolution for difference masses and momentum. Taking the Gaussian initial condition $\mathcal{M}_{\perp,\|\omega}(t=0,v) = 0.01e^{-v^2/10}$ and the same for $\mathcal{M}_{\perp,\perp\omega}$, with transverse component of vorticity $|\omega^\mu_\perp| = 0.2T$, we compare two different mass of the probe $m = 0.1T$ and $m = T$. To guarantee the stability of the evolution, we solve the transport equation from $t = 0$ to $t = 10\tau_m$, with τ_m characterizing the relaxation timescale $\tau_m = e^4 \ln(1/e) T^3/(8\pi^2 m^2)$, which depends on mass of the probe. The evolution of transverse dipole moment with $m = 0.1T$ is presented in the left panel of Fig. 3, with solid lines denoting $\mathcal{M}_{\perp,\|\omega}$ and dashed lines for $\mathcal{M}_{\perp,\perp\omega}$. The red line is the initial condition, from red to purple are early to later time in the evolution. One can directly observe that $\mathcal{M}_{\perp,\|\omega}$ gets polarized by $\omega^\mu_\perp$ while $\mathcal{M}_{\perp,\perp\omega}$ experiences only diffusion process. Evolution of the large momentum modes are suppressed compared to low momentum modes. This is consistent with the analysis in the last section, that both the polarization and diffusion are thermally suppressed by distribution function f_p.

The evolution trajectories of $\mathcal{M}_{\perp\mu}$ with different masses are presented in the middle panel of Fig. 3. The black solid dots are initial condition. The blue circles are $\mathcal{M}_{\perp\mu}$ with mass $m = 0.1T$ at $t = 10\tau_{m=0.1T}$, while the blue triangles are $\mathcal{M}_{\perp\mu}$ with mass $m = T$ at $t = 10\tau_{m=T}$. Dots connected with red trajectories are modes with low momentum $v = 1$, the trajectories are rainbow colored, with purple lines for modes with large momentum $v = 7$. The solid trajectories are for $m = 0.1T$ and dashed trajectories are for $m = T$. Comparing the final state for different mass, the trajectories for small masses rotate by a larger angle. This is in consistency with the analysis in (57) where the polarization effect is at least suppressed by $(1/m)^2$ when mass increases. The evolution trajectories are also suppressed for those modes with large momentum, in consistency with thermal suppression found in last section.

The middle panel shows the spin evolution driven by both diffusion and polarization effect, as a comparison, polarization effect is turned off in the right panel. In other word, in the right panel, only the diffusion process is included as was discussed in previous works [36,37,67]. With only the diffusion process, the transverse dipole moment would not rotate in the transverse plane, only shrink in its magnitude instead.

The diffusion and polarization effect can be obviously observed through the above simple numerical analysis. As is estimated in last section, both polarization and diffusion are strongly suppressed for modes with large momentum or large mass. When focusing on spin polarization of s-quark

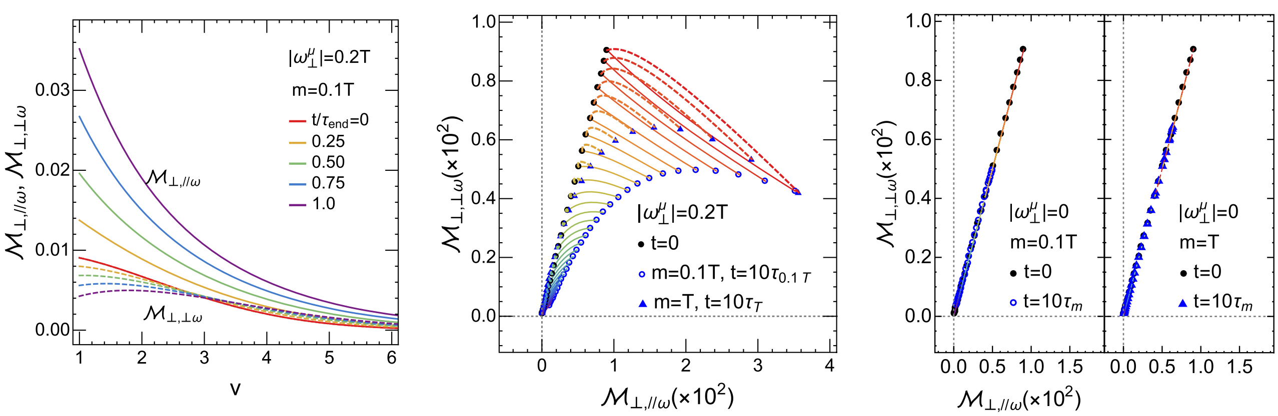

FIG. 3. Evolution of transverse dipole moment with initial condition $0.01e^{-v^2/10}$, transverse vorticity is $|\omega^\mu_\perp| = 0.2T$. Left: evolution of $\mathcal{M}_{\perp\mu}$ with $m = 0.1T$. Middle: compare the evolution of $\mathcal{M}_{\perp\mu}$ with difference mass $m = 0.1T$ and $m = T$. Right: evolution of $\mathcal{M}_{\perp\mu}$ with polarization effect turned off.

in heavy ion collision, where $m \sim p \sim T$, the ratio of polarization and diffusion can be roughly estimated by $|\partial|/T$, which means it takes a longer time for the spin to get polarized compared to thermalization, hence the nonequilibrium effect is important for spin polarization. In this sense, the full kinetic equation to first order of gradient serves as a necessary starting point for studying the nonequilibrium effect in the local spin polarization and also the starting point for deriving the spin hydrodynamics.

VI. CONCLUSION AND OUTLOOK

In this paper, we investigate the spin evolution of a hard massive probe fermion traversing in a massless hot QED plasma at local equilibrium in the framework of quantum kinetic equation. Mass of probe fermion is assumed to be $m \gg m_D \sim eT$ to exclude Compton from our discussion. As the first step, f_V of the probe fermion is assumed to be in local equilibrium. Under such assumptions, we derive the collision term of axial-kinetic equation to the leading logarithmic order with all the first order gradients included. The diffusion and polarization effects coexist in the collision term, where the former drives the spin fluctuation to damp out, and the later characterizes the spin getting polarized by the vorticity, shear tensor, acceleration and temperature gradients of the fluid. The effect of diffusion and polarization balance with each other, leading to the elimination of collision terms in the global equilibrium. Near the global equilibrium, the relaxation rate for the fluctuation is extracted. So as to illustrate the difference among the three degrees of freedom of $\mathcal{A}_\mu$, the axial kinetic equation is further decomposed into transport equation of axial charge and transverse magnetic dipole moment considering the purely rotational medium. The diffusion and polarization rates are extracted from the collision terms. The momentum and mass dependence of ratio between the both effects is analyzed. It is showed that, the ratio between polarization and diffusion is suppressed for large mass and momentum. When considering spin of s-quark in the quark gluon plasma, the ratio between polarization and diffusion can be estimated to be $|\partial|/T$. This indicates that compared with thermalization, spin take longer time to get polarized, indicating nonequilibrium effects important in spin polarization. Preliminary numerical analysis is carried out, showing that modes with small momentum and small mass get polarized easier, in consistency with the result of non-relativistic limit.

The physical settings in this paper can be viewed as a toy model for the evolution of spin of the s-quark in the quark gluon plasma. In a more self-consistent scenario, we will consider the scattering of massive quark with a QCD plasma, without assuming the local equilibrium of number distribution for the probe quark, the full set of vector kinetic equation and axial kinetic equation will be derived. The elimination of collision term of axial kinetic equation in global equation is considered in this paper, and local equilibrium for the massive fermion is still under discussion. These will be included in a future research.

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APPENDIX A: PHASE SPACE INTEGRAL

1. Simplification of integral measure

Assuming that the medium fermion and probe fermion are hard fermions with momentum comparable with temperature $p, k, p', k' \sim T$, while the momentum transfer is soft $q_0, q \ll T$, the phase space integral can be simplified using the small momentum transfer as well as the momentum conservation and on-shell condition

$$(2\pi)^3 \int \frac{d^4K d^4Q d^4P' d^4K'}{((2\pi)^4)^4} (2\pi)^8 \delta(P-K-Q)\delta(Q+P'-K')\epsilon(K\cdot u)\epsilon(P'\cdot u)\epsilon(K'\cdot u)\delta(K^2-m^2)\delta(P'^2)\delta(K'^2)$$
$$= \frac{1}{(2\pi)^5}\int dq_0 d^3q d^3k' \frac{1}{2p'_0 2k'_0 2k_0}\delta(p_0-k_0-q_0)\delta(p'_0-k'_0+q_0). \tag{A1}$$

The momentum integral is left with integral over Q and $\vec{k}'$. It is useful to decompose momentum $\vec{q}$ and $\vec{k}'$ into

$$\vec{k}' = k'\cos\theta_k \hat{p} + k'\sin\theta_k\cos\varphi_k\hat{x} + k'\sin\theta_k\sin\varphi_k\hat{y},$$
$$\vec{q} = q\cos\theta_q\hat{p} + q\sin\theta_q\cos\varphi_q\hat{x} + q\sin\theta_q\sin\varphi_q\hat{y}, \tag{A2}$$

where we have denoted $\hat{p}$ as $\hat{z}$ for now. And introduce Ω as the angle between $\vec{k}'$ and $\vec{q}$, namely $\cos\Omega = \cos\theta_k \cos\theta_q - \sin\theta_k\sin\theta_q\cos\Delta\varphi$, with $\Delta\varphi = \varphi_q - \varphi_k$. The measure can be parametrized as

$$\int d^3q d^3k' = \int q^2 dq d\cos\theta_q d\varphi_q k'^2 dk' d\cos\theta_k d\Delta\varphi. \tag{A3}$$

Considering that loop fermion are light fermions, which can be treated as massless. Using $\vec{p}' = \vec{k}' - \vec{q}$ and $\vec{k} = \vec{p} - \vec{q}$, we can use the on-shell condition to cast the δ-function into

$$\delta(p_0 - k_0 - q_0) \simeq \delta\left(q\frac{p}{p_0}\cos\theta_q - q^2\frac{(p^2\sin^2\theta_q + m^2)}{2p_0^3} - q_0\right),$$
$$\delta(p_0' - k_0' + q_0) \simeq \delta\left(q\cos\Omega - q^2\frac{\sin^2\Omega}{2k'} - q_0\right), \tag{A4}$$

with $p_0 = (p^2 + m^2)^{1/2}$. The angular integral over φ_q and φ_k can be performed to obtain

$$\int d\varphi_q d\varphi_k \delta(p_0' - k_0' + q_0) \simeq 4\pi\frac{1}{q(1+\frac{q_0}{k'})}\frac{1}{[\sin^2\theta_q\sin^2\theta_k - (\cos\Omega - \cos\theta_q\cos\theta_k)^2]^{1/2}}. \tag{A5}$$

Note that the above δ-function constrain the unique solution of $\cos\Delta\varphi$, yet $\sin\Delta\varphi$ can take both solutions $\pm(1-\cos^2\Delta\varphi)^{1/2}$. Thus integrals containing odd number of $\sin\Delta\varphi$ will be vanishing under the angular integral. The square root constrains the domain of $\cos\theta_k$ as $\cos(\theta_q - \Omega) < \cos\theta_k < \cos(\theta_q + \Omega)$. The other δ-function gives

$$\int d\cos\theta_q \delta(p_0 - k_0 - q_0) \simeq \frac{1}{\frac{pq}{p_0}(1+\frac{q_0}{p_0})}. \tag{A6}$$

From the δ-function, one can solve

$$\cos\Omega \simeq \frac{q_0}{q} + \frac{q}{2k'}\left(1 - \frac{q_0^2}{q^2}\right) + \mathcal{O}(q^2), \qquad \sin\Omega \simeq \left(1 - \frac{q_0^2}{q^2}\right)^{1/2}\left(1 - \frac{q_0}{2k'}\right),$$
$$\cos\theta_q \simeq \frac{p_0 q_0}{pq} + \frac{q}{2p}\left(1 - \frac{q_0^2}{q^2}\right) + \mathcal{O}(q^2), \qquad \sin\theta_q \simeq \left(1 - \frac{p_0^2}{p^2}\frac{q_0^2}{q^2}\right)^{1/2}\left(1 - \frac{q^2 - q_0^2}{p^2 - p_0^2 q_0^2}\frac{q_0 p_0}{2}\right), \tag{A7}$$

in obtaining the leading-log order result, it is enough to keep the above solution to the first order of q. Note that $-1 \leq \cos\Omega, \cos\theta_q \leq 1$ also set a limit to $x = q_0/q$ that $-\frac{p}{p_0} \leq \frac{q_0}{q} \leq \frac{p}{p_0}$. So that $\int dx dk'$ has the integration domain $\int dx dk' \to \int_0^\infty dk' \int_{-p/p_0}^{+p/p_0} dx$. With the above approximation of small momentum transfer, the collision term at leading logarithmic order can be explicitly calculated. The basic process is to collect all terms of integrad to Q^{-2}, after combining the measure, and integrate q ranges from $eT \ll q \ll T$, the log thus arises from $\int_{eT}^T dq/q = \ln(1/e)$.

To finish the remaining integral over k', the following expression are often utilized:

$$\int_0^\infty dk' k' n_F(k')(-1 + n_F(k')) = -T^2 \ln 2,$$
$$\int_0^\infty dk' k'^2 n_F(k')(-1 + n_F(k')) = -\frac{1}{6}\pi^2 T^3,$$
$$\int_0^\infty dk' k'^2 n_F^2(k')(-1 + n_F(k')) = -\frac{1}{6}\pi^2 T^3 + T^3 \ln 2, \tag{A8}$$

where $n_F(k') = 1/(e^{k'/T} + 1)$.

2. Diffusion

In this subsection, we show some details of calculation of diffusion term (16). The spin diffusion part defined in (16) is evaluated by first expanding the integrand in terms of Q. For this purpose, we use the following expansions

$$M_{A1}^{\mu\nu} n_\nu(P) = +\frac{2}{(Q^2)^2} T_{A1} n^\mu(P),$$
$$M_{A2}^{\mu\nu} n_\nu(K) = -\frac{2}{(Q^2)^2}(T_{A2} n^\mu(P) + T_{A2,1}^\nu \partial_{P_\perp^\nu} n^\mu(P) + T_{A2,1}^{\nu\rho} \partial_{P_\perp^\rho}\partial_{P_\perp^\nu} n^\mu(P) + T_{A2,2}^{\mu\rho} n_\rho(P) + T_{A2,2}^{\mu\rho\nu}\partial_{P_\perp^\nu} n_\rho(P)), \tag{A9}$$

where the coefficients T in the above expressions are kept to $\mathcal{O}(Q^2)$, giving

$$
\begin{aligned}
T_{A1} &= 2(P\cdot K')^2 + m^2 Q\cdot K' - 2Q\cdot K' P\cdot K' - 2P\cdot Q P\cdot K' + Q\cdot K' P\cdot Q + Q^2 P\cdot K' + \mathcal{O}(Q^3),\\
T_{A2} &= 2(P\cdot K')^2 + m^2 Q\cdot K' - 2Q\cdot K' P\cdot K' - 2P\cdot Q P\cdot K' + Q^2 P\cdot K' + \mathcal{O}(Q^3),\\
T^{\nu}_{A2,1} &= -Q^\nu(2(P\cdot K')^2 + m^2 Q\cdot K' - 2Q\cdot K' P\cdot K' - 2P\cdot Q P\cdot K') + \mathcal{O}(Q^3),\\
T^{\nu\rho}_{A2,1} &= Q^\nu Q^\rho (P\cdot K')^2 + \mathcal{O}(Q^3),\\
T^{\mu\rho}_{A2,2} &= -2K'^\mu K'^\rho P\cdot Q + 2Q^\mu K'^\rho P\cdot K' - P^\mu Q^\rho K'\cdot Q + 2K'^\mu Q^\rho(-P\cdot K' + P\cdot Q + Q\cdot K') + \mathcal{O}(Q^3),\\
T^{\mu\rho\nu}_{A2,2} &= 2P\cdot K'(K'^\mu Q^\rho Q^\nu - Q^\mu K'^\rho Q^\nu) + \mathcal{O}(Q^3).
\end{aligned}
\tag{A10}
$$

In obtaining the above expressions, we have used $P^\mu n_\mu(P) = 0$ to simplify the derivatives, thus $P^\mu Q^\nu \partial_{P_\nu} n_\mu(P) = -Q^\mu n_\mu(P)$ and $P^\mu Q^\rho \partial_{P_\rho} Q^\nu \partial_{P_\nu} n_\mu(P) = -2Q^\mu Q^\nu \partial_{P_\nu} n_\mu(P)$. The leading logarithmic order requires keeping the integrand to $\mathcal{O}(Q^{-2})$, thus it is sufficient to expand the distributions to $\mathcal{O}(Q)$, giving

$$
\begin{aligned}
\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} &= \bar{f}_{K'} f_{K'} - (f_P - \bar{f}_{K'}) f_{K'} \bar{f}_{K'} \frac{q\cos\Omega}{T} + \mathcal{O}(q^2),\\
f_P f_{P'} \bar{f}_{K'} + \bar{f}_P \bar{f}_{P'} f_{K'} &= \bar{f}_{K'} f_{K'} - (f_{K'} - f_P) f_{K'} \bar{f}_{K'} \frac{q\cos\Omega}{T} + \mathcal{O}(q^2).
\end{aligned}
\tag{A11}
$$

Taking $T^{\mu\rho\nu}_{A2,2}\partial_{P^\nu_\perp} n_\rho(P)$ for instance to illustrate the integral over such tensor structures. The basic strategy is to convert the integral over the tensor to scalars. After integral, $T^{\mu\rho\nu}_{A2,2}$ will be function of momentum p. Besides, as one can obverse, $T^{\mu\rho\nu}_{A2,2}$ is antisymmetric in exchanging $\mu\rho$, thus can be decomposed into

$$
T^{\mu\rho\nu}_{A2,2} = T^{(1)}_{A2,2} u^{[\mu} g^{\rho]\nu} + T^{(2)}_{A2,2} \hat{P}^{[\mu}_\perp g^{\rho]\nu} + T^{(3)}_{A2,2} \hat{P}^{[\mu}_\perp u^{\rho]} u^\nu + T^{(4)}_{A2,2} u^{[\mu} \hat{P}^{\rho]}_\perp \hat{P}^\nu_\perp,
\tag{A12}
$$

with other projectors vanishing in momentum integral. Using the relations between various projectors, $u_{[\mu} g_{\rho]\nu} T^{\mu\rho\nu}_{A2,2} = 6T^{(1)}_{A2,2} - 2T^{(4)}_{A2,2}$, $\hat{P}_{\perp[\mu} g_{\rho]\nu} T^{\mu\rho\nu}_{A2,2} = -6T^{(2)}_{A2,2} - 2T^{(3)}_{A2,2}$, $\hat{P}_{\perp[\mu} u_{\rho]} u_\nu T^{\mu\rho\nu}_{A2,2} = -2T^{(2)}_{A2,2} - 2T^{(3)}_{A2,2}$, $u_{[\mu}\hat{P}_{\perp\rho]}\hat{P}_{\perp\nu} T^{\mu\rho\nu}_{A2,2} = -2T^{(1)}_{A2,2} + 2T^{(4)}_{A2,2}$, then each coefficients can be obtained as combinations. The momentum integral of the various scalar functions then follows the processes described in Appendix A 1. Giving

$$
\begin{aligned}
T^{(1)}_{A2,2} &= \kappa_{LL} \frac{T}{mv} \frac{m(2v^3 - v_0^2\theta_{-1})}{6v^2},\\
T^{(2)}_{A2,2} &= -T^{(3)}_{A2,2} = \kappa_{LL} \frac{T}{mv} \frac{m(2v^3 - 3v_0^2\theta_{-1})}{6v^2},\\
T^{(4)}_{A2,2} &= -\kappa_{LL} \frac{T}{mv} \frac{mv_0\theta_{-1}}{6v}.
\end{aligned}
\tag{A13}
$$

With the coefficients, the original term becomes

$$
T^{\mu\rho\nu}_{A2,2}\partial_{P^\nu_\perp} n_\rho(P) = T^{(n)}_{A2,2} n_\mu(P) + T^{(u)}_{A2,2} u^\mu + T^{(p)}_{A2,2} \hat{P}^\mu_\perp + T^{(\partial)}_{A2,2} \hat{P}^\rho_\perp \partial_{P_{\perp\mu}} n_\rho(P),
\tag{A14}
$$

where

$$
\begin{aligned}
T^{(n)}_{A2,2} &= \frac{1}{p_0} T^{(1)}_{A2,2},\\
T^{(u)}_{A2,2} &= \frac{p}{p_0} T^{(1)}_{A2,2} \hat{P}^\rho_\perp n_\rho(P) + T^{(1)}_{A2,2} \partial_{P^\nu_\perp} n^\rho(P) + T^{(4)}_{A2,2} \hat{P}^\rho_\perp \hat{P}^\nu_\perp \partial_{P^\nu_\perp} n_\rho(P),\\
T^{(p)}_{A2,2} &= \left(\frac{p^2}{p_0^3} T^{(1)}_{A2,2} + \frac{m^2}{p_0^3} T^{(4)}_{A2,2}\right) \hat{P}^\rho_\perp n_\rho(P) + T^{(2)}_{A2,2} \partial_{P^\rho_\perp} n^\rho(P) + \frac{p}{p_0} T^{(4)}_{A2,2} \hat{P}^\rho_\perp \hat{P}^\nu_\perp \partial_{P^\nu_\perp} n_\rho(P),\\
T^{(\partial)}_{A2,2} &= \frac{p}{p_0} T^{(1)}_{A2,2} - T^{(2)}_{A2,2}.
\end{aligned}
\tag{A15}
$$

Other scalar function or tensors in (A10) are integrated in a similar way. After finishing the detailed calculation, one can arrive at the first two lines in (28) and coefficients $C^{(1)}$ to $C^{(7)}$ in (30).

3. First order gradients

To calculate the collision terms related to first order gradients including $C^{\rm vor}_{A\mu}, C^{\rm shear}_{A\mu}, C^{\rm Tgra+acc}_{A\mu}$ defined in (22), (24) and (26). The basic strategy is to converting the momentum integral over tensors into scalars, expanding the integrand to $\mathcal{O}(Q^{-2})$ and taking the momentum integral the same way as in Appendix A 1.

a. Vorticity

$C^{\rm vor}_{\mu}$ in the integrand of (22) can be further decomposed into

$$C^{\rm vor}_{\mu} = T^{\rm vor}_{\nu}\omega^{[\mu}u^{\nu]} + T^{\mu\rho\nu}_{\rm vor}\omega_{[\rho}u_{\nu]}. \tag{A16}$$

The momentum integral over the vorticity part also begins with expansion of the integrand over Q, the tensors above are expanded to $\mathcal{O}(Q^{-2})$ giving

$$\begin{aligned} T^{\rm vor}_{\nu} &= \frac{1}{(Q^2)^2}(2Q_\nu(P\cdot K')^2 + (m^2 - 2P\cdot K')Q\cdot K'Q_\nu + P\cdot Q(Q\cdot K'P_\nu - 2P\cdot K'Q_\nu) + \mathcal{O}(Q^3)),\\ T^{\mu\rho\nu}_{\rm vor} &= \frac{1}{(Q^2)^2}(2K'^\mu(P\cdot QK'^\rho(P-Q)^\nu + (P\cdot K' - P\cdot Q - K'\cdot Q)Q^\rho P^\nu)\\ &\quad + Q\cdot K'P^\mu Q^\rho P^\nu - 2P\cdot K'Q^\mu K'^\rho(P-Q)^\nu + \mathcal{O}(Q^3)), \end{aligned} \tag{A17}$$

since the above two tensors are at least $\mathcal{O}(Q^{-3})$, it is enough to keep $\mathcal{O}(Q)$ order of the distribution functions in order to get the leading logarithmic result,

$$\bar{f}_K\bar{f}_{K'}f_{P'}f_P = \bar{f}_{K'}f_{K'}\bar{f}_Pf_P\left(1 + \bar{f}_{K'}\frac{q\cos\Omega}{T} - f_P\frac{pq\cos\theta_q}{p_0T}\right) + \mathcal{O}(q^2). \tag{A18}$$

The momentum integral over tensors $T^{\rm vor}_{\nu}$ and $T^{\mu\rho\nu}_{\rm vor}$ are carried out after transforming the tensors to a series of scalar functions. Since after momentum integral, the vector $T^{\rm vor}_{\nu}$ will only be function of momentum P, it can be decomposed by

$$T^{\rm vor}_{\nu} = u_\nu T^{(1,1)}_{\rm vor} + \hat{P}_{\perp\nu}T^{(1,2)}_{\rm vor}, \tag{A19}$$

with $T^{(1,1)}_{\rm vor} = u^\nu T^{\rm vor}_{\nu}$ and $T^{(1,2)}_{\rm vor} = -\hat{P}^\nu_\perp T^{\rm vor}_{\nu}$. The scalar coefficients can be integrated according to the process in Appendix A 1. Then this part becomes $T^{\rm vor}_{\nu}\omega^{[\mu}u^{\nu]} = T^{(1,1)}_{\rm vor}\omega^\mu - T^{(1,2)}_{\rm vor}\hat{P}_{\perp\nu}\omega^\nu u^\mu$. In the other term $T^{\mu\rho\nu}_{\rm vor}\omega_{[\rho}u_{\nu]}$, $\omega_{[\rho}u_{\nu]}$ projects out the antisymmetric part of $T^{\mu\rho\nu}_{\rm vor}$, thus $T^{\mu\rho\nu}_{\rm vor}$ can be decomposed similar to (A12),

$$T^{\mu\rho\nu}_{\rm vor} = T^{(2,1)}_{\rm vor}u^{[\nu}g^{\rho]\mu} + T^{(2,2)}_{\rm vor}\hat{P}^{[\nu}_\perp g^{\rho]\mu} + T^{(2,3)}_{\rm vor}\hat{P}^{[\nu}_\perp u^{\rho]}u^\mu + T^{(2,4)}_{\rm vor}u^{[\nu}\hat{P}^{\rho]}_\perp\hat{P}^\mu_\perp, \tag{A20}$$

the momentum integral over the various scalar functions $T^{(2,i)}_{\rm vor}$ can be carried out according to Appendix A 1. After obtaining the coefficients, this part will be

$$T^{\mu\rho\nu}_{\rm vor}\omega_{[\rho}u_{\nu]} = 2T^{(2,1)}_{\rm vor}\omega^\mu - 2(T^{(2,2)}_{\rm vor} + T^{(2,3)}_{\rm vor})\hat{P}_{\perp\nu}\omega^\nu u^\mu + 2T^{(2,4)}_{\rm vor}\hat{P}_{\perp\nu}\omega^\nu\hat{P}^\mu_\perp. \tag{A21}$$

Together the above two parts, the vorticity term will be the $C^{(8)}$ term in (28), with $C^{(8)}$ defined in (30).

b. Shear

After momentum integral, the collision term $C^{\rm shear}_{A\mu}$ (24) will only be function of P, thus can in general be expressed in terms of a series of symmetric and traceless projectors as

$$C^{\rm shear}_{A\mu} = (u_\mu\hat{Q}_{\alpha\beta}C^{(1)}_{\rm shear} + P_{\perp\mu}\hat{Q}_{\alpha\beta}C^{(2)}_{\rm shear} + \hat{I}_{\mu\alpha\beta}C^{(3)}_{\rm shear} + \hat{T}_{\mu\alpha\beta}C^{(4)}_{\rm shear})\sigma^{\langle\alpha\beta\rangle}, \tag{A22}$$

where the projectors are defined through

$$\hat{Q}_{\alpha\beta} = \hat{P}_{\perp\alpha}\hat{P}_{\perp\beta} + \frac{1}{3}\Delta_{\alpha\beta},$$
$$\hat{I}_{\mu\alpha\beta} = \hat{P}_{\perp\alpha}\Delta_{\mu\beta} + \hat{P}_{\perp\beta}\Delta_{\mu\alpha} - \frac{2}{3}\hat{P}_{\perp\mu}\Delta_{\alpha\beta},$$
$$\hat{T}_{\mu\alpha\beta} = \frac{1}{2}(\epsilon_{\mu\nu\rho\alpha}u^\nu\hat{P}_\perp^\rho\hat{P}_{\perp\beta} + \epsilon_{\mu\nu\rho\beta}u^\nu\hat{P}_\perp^\rho\hat{P}_{\perp\alpha}), \tag{A23}$$

which are symmetric and traceless in the indices $\alpha\beta$. In the local rest frame of the fluid, the above projectors are $Q_{ij} = \hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}$, $I_{kij} = \hat{p}_j\delta_{ik} + \hat{p}_i\delta_{jk} - \frac{2}{3}\hat{p}_k\delta_{ij}$ and $T_{kij} = \frac{1}{2}(\epsilon_{kli}\hat{p}_l\hat{p}_j + \epsilon_{klj}\hat{p}_l\hat{p}_i)$, which are symmetric and traceless in ij. Each of the four coefficients $C^{(i)}_{\text{shear}}$ can be obtained by first projecting (24) onto the corresponding projectors and then taking the momentum integral. One will find that only $C^{(4)}_{\text{shear}}$ in nonvanishing. Hence after momentum integral, the shear tensor term in the collision term appears as

$$C^{\text{shear}}_{A\mu} = \hat{T}_{\mu\alpha\beta}C^{(4)}_{\text{shear}}\sigma^{\langle\alpha\beta\rangle}. \tag{A24}$$

Using the relation $\hat{T}^{\mu\alpha\beta}\hat{T}_{\mu\alpha\beta} = -1$, the evaluating of the contribution from the shear tensor is converted to calculating the scalar function

$$C^{(4)}_{\text{shear}} = -4e^4\frac{1}{(2\pi)^5}\int dq_0d^3qd^3k'\frac{1}{2p_0'2k_0'2k_0}\delta(p_0-k_0-q_0)\delta(p_0'-k_0'+q_0)\hat{T}^{\mu\alpha\beta}C^{\text{shear}}_{\mu\alpha\beta}(-\beta)\bar{f}_K\bar{f}_{K'}f_{P'}f_P \tag{A25}$$

and $C^{\text{shear}}_{\mu\alpha\beta}$ is defined in (25), which can be simplified into

$$C^{\text{shear}}_{\mu\alpha\beta} = +\epsilon_{\mu\alpha\sigma\lambda}\left(-\frac{P'\cdot K'}{2}K^\sigma P^\lambda K_\beta + K\cdot K'P'^\sigma P^\lambda P_\beta - P\cdot P'K'^\sigma K^\lambda K_\beta + \frac{m^2Q\cdot K'}{P'\cdot u}P'^\sigma u^\lambda P'_\beta\right)$$
$$-\epsilon_{\xi\alpha\sigma\lambda}\left(P^\sigma K^\lambda P'^\xi K_\beta + \frac{m^2}{P'\cdot u}Q^\xi P'^\sigma u^\lambda P'_\beta\right)K'_\mu + \{P'\leftrightarrow K'\}, \tag{A26}$$

where $\{P'\leftrightarrow K'\}$ part is to taking conversion accordingly in the above all terms. Using

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\alpha\beta} = -2\delta^{\rho\sigma}_{\alpha\beta} = -2(\delta^\rho_\alpha\delta^\sigma_\beta - \delta^\sigma_\alpha\delta^\rho_\beta),$$
$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\xi\alpha\beta} = -\delta^{\nu\rho\sigma}_{\xi\alpha\beta} = -[\delta^\nu_\xi(\delta^\rho_\alpha\delta^\sigma_\beta - \delta^\sigma_\alpha\delta^\rho_\beta) - \delta^\nu_\alpha(\delta^\rho_\xi\delta^\sigma_\beta - \delta^\sigma_\xi\delta^\rho_\beta) + \delta^\nu_\beta(\delta^\rho_\xi\delta^\sigma_\alpha - \delta^\sigma_\xi\delta^\rho_\alpha)], \tag{A27}$$

to finish the contractions in $\hat{T}^{\mu\alpha\beta}C_{\mu\alpha\beta}$. Since $\hat{T}^{\mu\alpha\beta}C_{\mu\alpha\beta}$ is at least $\mathcal{O}(Q)$, it is enough to expand the distribution functions to $\mathcal{O}(Q)$ (A18). After taking the momentum integral according to Appendix A 1, one will obtain the $C^{(9)}$ term in (28), with $C^{(9)}$ in (30).

c. Temperature gradients and acceleration

Both of the tensors $C^{\text{Tgra}}_{\mu\lambda}$ and $C^{\text{acc}}_{\mu\lambda}$ defined in (27) can be expanded in a general structure, namely

$$T_{\mu\lambda}(P) = T_1g_{\mu\lambda} + T_2u_\mu u_\lambda + T_3\hat{P}_{\perp\mu}\hat{P}_{\perp\lambda} + T_4u_\mu\hat{P}_{\perp\lambda} + T_5\hat{P}_{\perp\mu}u_\lambda + T_6\epsilon_{\mu\lambda\alpha\beta}u^\alpha\hat{P}^\beta_\perp, \tag{A28}$$

where one can find only projector $\epsilon_{\mu\lambda\alpha\beta}u^\alpha\hat{P}^\beta_\perp$ have nonvanishing coefficient under the momentum integral. Using the relation $\epsilon^{\mu\lambda\rho\sigma}u_\rho\hat{P}_{\perp\sigma}\epsilon_{\mu\lambda\alpha\beta}u^\alpha\hat{P}^\beta_\perp = 2$, (26) can be casted into

$$C^{\text{Tgra+acc}}_{A\mu} = C_{\text{Tgra}}\epsilon_{\mu\lambda\alpha\beta}u^\alpha\hat{P}^\beta_\perp\partial^\lambda\ln T + C_{\text{acc}}\epsilon_{\mu\lambda\alpha\beta}u^\alpha\hat{P}^\beta_\perp Du^\lambda, \tag{A29}$$

with

$$C_{\text{Tgra}} = -\frac{4e^4}{(2\pi)^5}\int dq_0d^3qd^3k'\frac{1}{2p_0'2k_0'2k_0}\delta(p_0-k_0-q_0)\delta(p_0'-k_0'+q_0)\frac{1}{2}\epsilon^{\mu\lambda\rho\sigma}u_\rho\hat{P}_{\perp\sigma}C^{\text{Tgra}}_{\mu\lambda}(-\beta)\bar{f}_K\bar{f}_{K'}f_{P'}f_P,$$
$$C_{\text{acc}} = -\frac{4e^4}{(2\pi)^5}\int dq_0d^3qd^3k'\frac{1}{2p_0'2k_0'2k_0}\delta(p_0-k_0-q_0)\delta(p_0'-k_0'+q_0)\frac{1}{2}\epsilon^{\mu\lambda\rho\sigma}u_\rho\hat{P}_{\perp\sigma}C^{\text{acc}}_{\mu\lambda}(-\beta)\bar{f}_K\bar{f}_{K'}f_{P'}f_P. \tag{A30}$$

Using (A27) to complete the contraction, and carrying out the momentum integral according to Appendix A 1, one will obtain the $C^{(10)}$ and $C^{(11)}$ term in (28), with $C^{(10)}$ and $C^{(11)}$ in (30).

APPENDIX B: GAUGE ISSUE

One can explicitly check that the collision term is gauge independent. In this section, we show for example that terms related to vorticity is gauge independent. Photon propagator in temporal axial gauge, Coulomb gauge and covariant gauge are given by

$$\begin{aligned}
\text{temporal axial gauge: } G_{\mu\nu} &= \frac{-1}{Q^2}P^T_{\mu\nu} + \frac{-1}{Q^2}\left(\frac{Q^2}{q^2}u_\mu u_\nu - \frac{Q^2}{q_0 q^2}u_{(\mu}Q_{\nu)} + \frac{Q^2}{q_0^2 q^2}Q_\mu Q_\nu\right), \\
\text{Coulomb gauge: } G_{\mu\nu} &= \frac{-1}{Q^2}P^T_{\mu\nu} + \frac{-1}{Q^2}\frac{Q^2}{q^2}u_\mu u_\nu, \\
\text{covariant gauge: } G_{\mu\nu} &= \frac{-1}{Q^2}P^T_{\mu\nu} + \frac{-1}{Q^2}\left(\frac{Q^2}{q^2}u_\mu u_\nu - \frac{q_0}{q^2}u_{(\mu}Q_{\nu)} + \frac{q^2}{q_0^2 Q^2}Q_\mu Q_\nu\right).
\end{aligned} \tag{B1}$$

The above covariant gauge corresponds to the Landau gauge $\xi = 0$

$$G_{\mu\nu} = \frac{1}{Q^2}\left(g_{\mu\nu} - (1-\xi)\frac{Q_\mu Q_\nu}{Q^2}\right), \tag{B2}$$

while in the calculation we have adopted Feynman gauge $\xi = 1$. The point is to work out the one-loop corrected photon propagator $G^{(0,1)}_{\mu\nu}$ in various gauges and to check whether the different terms among various gauges are vanishing under momentum integral. The expression of $G^{(0,1)}_{\mu\nu}$ in Landau gauge is given by (11). Feynman gauge and Landau gauge differs only in tensor structure of $Q_\mu Q_\nu$. While the three gauges in (B1) differs by $Q_\mu Q_\nu$, $u_{(\mu}Q_{\nu)}$ and crossing terms with $P^T_{\mu\nu}$ when multiplying two photon propagators.

To check the gauge invariance of zeroth order photon propagator $G^{(0)<}_{\mu\nu}(Q) = D^{22}_{\mu\beta}(Q)D^{11}_{\alpha\nu}(Q)\Pi^{(0)<\alpha\beta}(Q)$, one can find that contracting $\Pi^{(0)<\alpha\beta}$ with $Q_\alpha Q_\beta Q_\mu Q_\nu$ and $g_{\mu\alpha}Q_\beta Q_\nu + g_{\nu\beta}Q_\mu Q_\alpha$ will both leads to vanishing results under the δ-function. Thus Feynman gauge and Landau gauge give the same $G^{<(0)}_{\mu\nu}$. Temporal axial gauge and Coulomb gauge can be shown to give also the same $G^{<(0)}_{\mu\nu}$.

We then check the vorticity terms in first order photon propagator $G^{(1)<}_{\mu\nu}(Q) = D^{22}_{\mu\beta}(Q)D^{11}_{\alpha\nu}(Q)\Pi^{(1)<\alpha\beta}(Q)$. The vorticity related terms in $\Pi^{(1)<\alpha\beta}(Q)$ is

$$\Pi^{(1)<\alpha\beta}_\omega(Q) = -2i\epsilon^{\alpha\rho\beta\sigma}\int_{P',K'} K'_\sigma(P'\cdot\omega u_\rho - P'\cdot u\omega_\rho)\bar{f}_{K'}f'_{P'} - P'_\rho(K'\cdot\omega u_\sigma - K'\cdot u\omega_\sigma)f_{P'}f'_{K'}. \tag{B3}$$

As only self-energy components $\Sigma_{A\mu}$ and $\Sigma_{T\mu\nu}$ contains the first order photon propagator. The collision term involving $G^{(1)<}_{\mu\nu}(Q)$ in (4) can be extracted out using the expression of the components of self-energy (13), giving

$$\begin{aligned}
&\int_{K,Q} i((m^2 - P\cdot K)\epsilon_{\mu\nu\rho\sigma}Q^\nu - P_\mu\epsilon_{\lambda\nu\rho\sigma}Q^\nu P^\lambda)(G^{(1)<\mu\nu}(Q)f_P\bar{f}_K - G^{(1)>\mu\nu}(Q)\bar{f}_P f_K) \\
&= \int_{K,Q} i((m^2 - P\cdot K)\epsilon_{\mu\nu\rho\sigma}Q^\nu - P_\mu\epsilon_{\lambda\nu\rho\sigma}Q^\nu P^\lambda)G^{\mu\alpha}_R G^{\nu\beta}_A(\Pi^{(1)<}_{\alpha\beta}(Q)f_P\bar{f}_K - \Pi^{(1)>}_{\alpha\beta}(Q)\bar{f}_P f_K).
\end{aligned} \tag{B4}$$

Although the expression of $G^{(1)<}_{\mu\nu}(Q)$ depends on gauge choice, one can explicitly show that the collision term involving $G^{(1)<}_{\mu\nu}(Q)$ (B4) is not gauge depending. This can be proved by contracting

$$\Pi^{(1)<\alpha\beta}_\omega f_P\bar{f}_K - \Pi^{(1)>\alpha\beta}_\omega\bar{f}_P f_K \propto \frac{1}{2T}\epsilon^{\alpha\beta\rho\sigma}(Q\cdot\omega K'_\rho u_\sigma - K'\cdot\omega Q_\rho u_\sigma + K'\cdot u Q_\rho\omega_\sigma - Q\cdot u K'_\rho\omega_\sigma)\bar{f}_{K'}\bar{f}_K f_P f_{P'} \tag{B5}$$

with $Q_\alpha Q_\beta Q_\mu Q_\nu$, $g_{\mu\alpha} Q_\beta Q_\nu + g_{\nu\beta} Q_\mu Q_\alpha$, $u_{(\alpha} Q_{\mu)} u_{(\beta} Q_{\nu)}$ and crossing terms, respectively, then further projecting to $((m^2 - P \cdot K)\epsilon_{\mu\nu\rho\sigma} Q^\nu - P_\mu \epsilon_{\lambda\nu\rho\sigma} Q^\nu P^\lambda)$. One will find that it will either be directly vanishing by symmetry, or be proportional to $\vec{q} \times \vec{k}' \cdot \vec{\omega}$, which is vanishing under momentum integral. In this way, the vorticity related term is also gauge independent. Other terms related to first order gradient are also proved to be gauge independent in a similar way.

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