Toward a model of quarks and leptons

Y. H. Ahn,[1] Sin Kyu Kang,[2] and Hyun Min Lee[3]

[1]*Institute for Convergence Fundamental Study, Seoul National University of Science and Technology, Seoul 139-743, Korea*

[2]*School of Natural Science, Seoul National University of Science and Technology, Seoul 139-743, Korea*

[3]*Department of Physics, Chung-Ang University, Seoul 06974, Korea*

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We propose an extra-dimension framework on the orbifold S^1/Z_2 to understand the origin of the fermion mass and mixing hierarchies. Introducing the flavor symmetry G_F (=non-Abelian × Abelian) as well as the extra gauged $U(1)$ symmetries through the bulk, we regard the $SU(2)$-singlet and -doublet fermions in the Standard Model (SM) to be localized at the separate 3-branes and let the extra $SU(2)$-singlet flavored fermions in the bulk couple to the SM fermions at the 3-branes. The extra $U(1)$ symmetries satisfy the $U(1)$ gravitational anomaly free condition, playing a crucial role in achieving the desirable fermion mass and mixing hierarchies and making the flavored axion naturally light. The singlet scalar fields, the flavon fields, are responsible for the spontaneous breaking of G_F on the two 3-branes, while the $SU(2)$-singlet flavored fermions are integrated out to give rise to the effective Yukawa couplings for the SM fermions endowed with the information of G_F breaking in the two sectors. The flavored axion from the Peccei-Quinn symmetry is also proposed for solving the strong CP problem and being a dark matter candidate in our model.

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I. INTRODUCTION

The observed hierarchies in the masses and mixings of quarks and leptons are one of the most puzzling problems in particle physics. A plausible explanation is to introduce a new gauge symmetry which is spontaneously broken at some ultraviolet (UV) scale and leaves behind its global subgroup. In this case, in the low-energy effective theory, the symmetry structure is composed of the Standard Model (SM) gauge symmetry $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ and a remnant global symmetry which plays an essential role in making desirable flavor structure of quarks and leptons [1]. It is known that flavor-dependent $U(1)$ gauge symmetries and/or non-Abelian discrete symmetries can arise from the isometry of the compactified extra dimensions in string theory. In addition, the compactification of the extra dimensions can be accompanied by certain 3-branes [four-dimensional (4D) surfaces embedded in higher-dimensional spaces] [2].

Inspired by some string compactifications, in this paper, we propose a mechanism for generating the fermion mass and mixing hierarchies in the extra-dimension framework with the flavor symmetry. The extra gauged $U(1)$ symmetries are also introduced under the $U(1)$ gravitation anomaly free condition, and they are crucial to achieve the desirable fermion mass and mixing hierarchies in this scenario. These are the features in this work which are distinguishable from other approaches to tackle the flavor problem in the extra-dimension framework [3–16]. A simple toy model with vectorlike leptons for seesaw leptons has been recently proposed in light of the muon $g-2$ anomaly [17], but without involving a flavor symmetry.

For our purpose, we construct a higher-dimensional theory compactified on the orbifold S^1/Z_2 with a global symmetry group for flavors $G_F = U(1) \times$ non-Abelian finite group, which might be originated from certain string compactifications. A set of SM gauge singlet scalar fields $\mathcal{F}$ charged under G_F, the so-called flavon fields, are located at two 3-branes in the extra dimension. In the 4D effective Lagrangian, the flavor fields act on dimension-four (-three) operators well sewed by $G_F \times G_{\rm SM}$ at different orders to generate the effective interactions for the SM and the right-handed neutrinos as follows:

$$\mathcal{L}_{4D} = \tilde{c}_1 \mathcal{O}_3 \mathcal{F} \tilde{\mathcal{X}} + \mathcal{O}_4 \sum_{n=0}^{\rm finite} c_n \left(\frac{\mathcal{F}}{M_5}\right)^n \mathcal{X}_n, \qquad (1)$$

where $\mathcal{O}_{3(4)}$ are dimension-three (-four) operators, $\tilde{c}_1 (c_n)$ are the complex coefficients of order unity, and $\tilde{\mathcal{X}}(\mathcal{X}_n)$ are dimensionless parameters induced due to the nonlocal effects by the exchange of bulk messenger fields. $\mathcal{F}$ acquires vacuum expectation values (VEVs) $\langle\mathcal{F}\rangle$ from some dynamics, thereby breaking G_F. The information of G_F breakdown is transmitted to the two 3-branes via the mediation of bulk messenger fields, and then the observed SM fermion mixing and mass can naturally be generated from the 4D effective theory. The vacuum structure of the flavons plays a crucial role in achieving the SM fermion mass and mixing hierarchies. We remark that Eq. (1) is constructed in a supersymmetric framework mainly because the holomorphy of the superpotential is needed to generate the desirable Yukawa textures of the charged fermions which lead to the right hierarchies of the charged-fermion masses and mixing for the given flavor symmetry we consider in this work. We take into account a minimal supersymmetric model with flavon superfields where the hierarchy problem concerned with the Higgs boson is alleviated.

As a bonus in our scenario, the pseudo-Goldstone modes coming from the flavon fields are localized on the 3-branes, becoming candidates for flavored axions A_i (and a QCD axion) [18] with decay constants determined by $\langle\mathcal{F}\rangle$. However, it is well known that nonperturbative quantum gravitational anomaly effects [19] could spoil the axion solution to the strong CP problem. In order to keep the axion solution in our scenario, we need to suppress the explicit breaking effects of the axionic shift symmetry by gravity and consistently couple gravity to matter. To this, we impose the $U(1)$-mixed gravitational anomaly free conditions for the extra gauged $U(1)$ symmetries, in turn, obtaining the constraints on the $U(1)$ charges of quarks and leptons. Moreover, assuming that the low-energy effective neutrino mass is generated by the usual seesaw mechanism [20], the seesaw scale can be congruent to a flavored-axion decay constant where the right-handed neutrinos acquire Majorana masses when G_F is broken.

II. FLAVOR PHYSICS EMBEDDED INTO 5D THEORY

We consider a 5D theory for flavors compactified on the orbifold S^1/Z_2 where the extra dimension on the circle S^1 is identified y with $-y$ [21]. The orbifold fixed points at $y=0$ and $y=L$, the boundaries of the 5D spacetime, are the locations of two 3-branes. We assume that all the ordinary matter fields are localized at either brane, and they are charged under the flavor symmetry G_F. We specify $G_F = SL_2(F_3)\times U(1)_X$ where $SL_2(F_3)$ is the symmetry group of the double tetrahedron [22].

The metric solution to the 5D Einstein equations respecting the 4D Poincaré invariance in the x^μ direction is given by

$$ds^2 = e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{2}$$

where the extra dimension is compactified on an interval $y\in[0,L]$, the warp factor is given by $\sigma(y)=ky$ with $k=\sqrt{-\frac{\Lambda}{6M_5^3}}>0$ and Λ being the bulk cosmological constant, and the 4D Minkowski flat metric is $\eta_{\mu\nu}=\mathrm{diag}(+,-,-,-)$. Note that, in Eq. (2), we can always take $\sigma(0)=0$ by rescaling the x^μ coordinates. Then, the 4D reduced Planck mass $M_P\simeq 2.43\times 10^{18}$ GeV can be extracted in terms of the 5D Planck mass M_5 as

$$M_P^2 = M_5^3\int_{-L}^{L} dy\, e^{2\sigma(y)} = \frac{M_5^3}{k}(e^{2kL}-1), \tag{3}$$

where M_5 is assumed to be higher than the electroweak scale, but M_5 and k are lower than M_P. Then, the scale of flavor dynamics would be given by the UV cutoff M_5.

While the warped background in our scenario does not play a role in addressing the hierarchy problem studied in Ref. [21], the supersymmetry (SUSY) can alleviate the problem. As shown later, warped geometry makes charged-fermion masses dependent on the warping factor $\sigma(L)$. Although it is possible to embed a flavor symmetry G_F into flat extra-dimensional framework, we construct our model in the framework of the warped geometry so that the mass of bulk messenger fields M_i should be positive and inverted ordering ($M_1>M_2>M_3$), which leads naturally to fermion mass hierarchies as shown in Sec. VI. Note that it is not guaranteed to satisfy the positiveness of M_i as well as the $U(1)$-mixed gravitational anomaly free conditions for the $U(1)$ quantum numbers of the fields assigned in this model unless they are embedded in warp geometry. The compactification length L is associated with the VEV of a massless 4D scalar field. However, this modulus field has zero potential, and consequently, L is not determined by the dynamics of the model. As discussed in [23,24], the value of L can be stabilized with the help of a scalar potential generated by a newly introduced bulk scalar with interaction terms that are localized to the two 3-branes. However, the study of such a potential to guarantee the stability is beyond the scope of this paper. As will be shown later, in our scenario, L can be predicted in terms of a nonlocal model parameter by flavor physics.

Now, let us consider a concrete model for low-energy effective theory by introducing bulk fermions propagating in a 5D space with the metric Eq. (2). They are singlets under $SU(2)$ with hypercharges Y_f and masses M_{f_i}, and interact with the normal matter fields confined at the $y=0$ or $y=L$ brane. Bulk fermions with common hypercharges can be distinguished by quantum numbers of a flavor symmetry G_F that acts on flavor fields in a theory. Hence, we refer to the bulk fermion as a flavored bulk fermion. To keep electric charge conservation, we introduce two kinds of $SU(2)$-singlet flavored bulk fermions Ψ_{f_i} and their mirror partners F_{f_i} charged under $G_{\rm SM}\times G_F$, and they are distinguishable only by the opposite hypercharge of each other. Here, $f_i=u_i$ (up-type quark), d_i (down-type quark), and l (charged lepton). While Ψ_{f_i} interact with the normal matter fields confined at the $y=0$ or L brane, F_{f_i} does not do so due to $U(1)_Y$ symmetry. The exchange of the flavored bulk fermions between the two 3-branes can provide nonlocal interactions between right- and left-handed SM fermions, as can be seen in Eqs. (28) and (29). This is the origin of the fermion Yukawa couplings in the 4D effective Lagrangian.

As shown in Refs. [25,26], there is no anomaly in the five-dimensional bulk, and the anomaly on the orbifold fixed points $y=0,L$ is entirely independent of the shape of the chiral modes. Hence, in the five-dimensional bulk, the associated gauge anomalies will be automatically canceled due to the mirror charges of flavored bulk fermions. Unlike Ref. [23], in this work, the compactification length L can be constrained thanks to the introduction of flavored bulk fermions. Under the flavor symmetry G_F, that is, $SL_2(F_3)\times U(1)_X$, we assign the lepton bulk fermion denoted as Ψ_l (its mirror F_l) to $(\mathbf{3},-13X_1/2)$; the u-type bulk fermions denoted as Ψ_{u_1},Ψ_{u_2}, and Ψ_{u_3} (its mirrors F_{u_i}) to $(\mathbf{1},-6X_1)$, $(\mathbf{1}',-6X_1)$, and $(\mathbf{1}'',0)$, respectively; d-type bulk fermions denoted as Ψ_{d_1},Ψ_{d_2}, and Ψ_{d_3} (its mirrors F_{d_i}) to $(\mathbf{1},-5X_1)$, $(\mathbf{1}',-5X_1)$, and $(\mathbf{1}'',3X_1)$, respectively. And under the SM gauge symmetry, that is, $G_{\rm SM}=[SU(3)_C\times SU(2)_L]_{U(1)_Y}$, we assign Ψ_l and F_l to $(1,1)_{-1}$ and $(1,1)_1$, respectively, Ψ_{u_i} and F_{u_i} to $(3,1)_{2/3}$ and $(3,1)_{-2/3}$, respectively, and Ψ_{d_i} and F_{d_i} to $(3,1)_{-1/3}$ and $(3,1)_{1/3}$, respectively. In Table I, for flavon fields $\mathcal{F}=\Phi_S,\Theta(\tilde\Theta),\Phi_T,\rho,\eta,\chi,\tilde\chi$ and flavored bulk fermions Ψ_f, we present the representations of $SL_2(F_3)$ and quantum charges under $U(1)_{X_1}\times U(1)_{X_2}\times U(1)_{X_T}$.

The brane-localized superpotential for the driving flavon fields having $U(1)_R$ charge $+2$ invariant under $G_{\rm SM}\times SL_2(F_3)\times U(1)_X$ is given at orders up to $(1/M_5^3)$ by

$$\begin{aligned}W_v=&\,\delta(y)\{\Phi_0^S(g_{s_1}\Phi_S\Phi_S+g_{s_2}\tilde\Theta\Phi_S)+\Theta_0(g_{\Theta_1}\Phi_S\Phi_S+g_{\Theta_2}\Theta\Theta+g_{\Theta_3}\Theta\tilde\Theta+g_{\Theta_4}\tilde\Theta\,\tilde\Theta)\}\\&+\delta(y-L)\left\{\Phi_0^T\left(\mu_T\Phi_T+g_T\Phi_T\Phi_T\frac{\varrho}{M_5}+g_{T\chi}\Phi_T\frac{\chi\tilde\chi}{M_5}+\sum_{i=1}^3 g_{T_i}\Phi_T(\Phi_T\Phi_T)\frac{\varrho^2}{M_5^3}\right)\right.\\&+\varrho_0\left(\mu_\varrho\varrho+g_{\varrho\chi}\chi\tilde\chi\frac{\varrho}{M_5}+g_{\varrho_1}(\chi\tilde\chi)^2\frac{\varrho}{M_5^3}+g_{\varrho_2}\Phi_T\Phi_T\frac{\varrho^3}{M_5^3}\right)+\eta_0\left(\mu_\eta\eta+g_\eta\eta\Phi_T\frac{\varrho}{M_5}+g_{\eta_1}\eta\Phi_T\frac{\chi\tilde\chi}{M_5^3}\varrho\right)\\&\left.+\chi_0(g_\chi\chi\tilde\chi-\mu_\chi^2+g_{\chi_1}\Phi_T\Phi_T\frac{\varrho^2}{M_5^2}+g_{\chi_2}\left(\eta\eta\right)_{\mathbf{3}}\Phi_T\frac{\varrho}{M_5^2}\right)\right\},\end{aligned}\tag{4}$$

where μ_i are dimensionful parameters, and g_i are dimensionless coupling constants. Note that there are no local higher-dimensional operators induced by the flavon fields Φ_S and $\Theta(\tilde\Theta)$ localized at the $y=0$ brane due to the charge assignment of $U(1)_X$. Nonlocal higher-dimensional brane interactions via the one-loop exchange of the flavored bulk fermions, such as Eq. (D1), are allowed but absorbed into the leading-order terms of Eq. (4) by the redefinition of coefficients. Because of the assignment of quantum numbers under $SL_2(F_3)\times U(1)_X\times U(1)_R$, the usual superpotential term $\mu_H H_u H_d$ is not allowed, while the following operators driven by Θ_0 and Φ_0^S are allowed due to the separation achieved by means of an extra dimension,

$$\delta(y)\left(g_{S_0}\Phi_0^S\frac{\Phi_S\Theta}{M_5^2}+g_{\Theta_0}\Theta_0\left(\frac{\Theta}{M_5}\right)^2\right)H_uH_d.\tag{5}$$

When Φ_0^S and Θ_0 get VEVs, the operators (5) generate the μ term $\mu_H=(g_{S_0}\langle\Phi_0^S\rangle 3v_Sv_\Theta+g_{\Theta_0}\langle\Theta_0\rangle v_\Theta^2)/(2M_5^2)$ of the order of $m_Sv_\Theta^2/M_5^2$ with m_S being soft SUSY breaking mass. In the SUSY limit, the driving fields develop zero vacuum structures along the flat directions, but they are corrected to be of order m_S when the SUSY breaking effect lifts up the flat directions.

The flavor symmetry G_F is spontaneously broken by the nontrivial VEVs of flavons. Note that the $U(1)$ charges of the fields are determined so as to satisfy the $U(1)$ gravitation anomaly free condition and the empirical hierarchies of fermion masses and mixings. For instance, in a supersymmetric model, the brane-localized superpotential for flavons with G_F invariance is given in Eq. (4). From the minimization conditions of the F-term scalar potentials, the VEVs of Φ_S and $\tilde\Theta$ localized at the $y=0$ brane are obtained

TABLE I. Representations and quantum charges of SM singlet flavon fields and bulk fermions under G_F.

Field	$SL_2(F_3)$	$U(1)_{X_1}$	$U(1)_{X_2}$	$U(1)_{X_T}$	Brane (y)
Φ_S: $\Theta(\tilde\Theta)$	**3**:**1**	1	0	0	0
Φ_T: ϱ	**3**:**1**	0	0	1, −1	L
η	**2**′	0	0	0	L
χ, $\tilde\chi$	**1**	0	1, −1	0	L
Ψ_l	**3**	−13/2	0	0	
Ψ_{u_1}:Ψ_{d_1}	**1**	−6: −5	0	0	
Ψ_{u_2}:Ψ_{d_2}	**1**′	−6: −5	0	0	
Ψ_{u_3}:Ψ_{d_3}	**1**″	0: 3	0	0	

$$\langle\Phi_S\rangle=\frac{v_S}{\sqrt{2}}(1,1,1),\qquad\langle\Theta\rangle=\frac{v_\Theta}{\sqrt{2}},\qquad\langle\tilde{\Theta}\rangle=0,\qquad(6)$$

and $\kappa=v_S/v_\Theta$ in supersymmetric limit. For Φ_T, ϱ, $\chi(\tilde{\chi})$, and η localized at the $y=L$ brane,

$$\langle\Phi_T\rangle=\frac{v_T}{\sqrt{2}}(1,0,0),\qquad\langle\varrho\rangle=\frac{v_\varrho}{\sqrt{2}},$$
$$\langle\chi\rangle=\langle\tilde{\chi}\rangle=\frac{v_\chi}{\sqrt{2}},\qquad\langle\eta\rangle=\frac{v_\eta}{\sqrt{2}}(1,0).\qquad(7)$$

Denoting $\Delta_{\mathcal{F}}\equiv v_{\mathcal{F}}/(\sqrt{2}M_5)$ and following the procedure in Ref. [27], we obtain

$$\Delta_\Theta=\Delta_S/\kappa=|\delta_1^{\rm G}/\delta_2^{\rm G}|\sqrt{2/(1+\kappa^2)}\Delta\chi.\qquad(8)$$

The complex scalar fields are decomposed as follows:

$$\Phi_{T_i}=\frac{v_T}{\sqrt{2}}e^{i\frac{A_T}{f_T}}\left(1+\frac{h_T}{v_T}\right),\qquad\varrho=\frac{v_\varrho}{\sqrt{2}}e^{-i\frac{A_T}{f_T}}\left(1+\frac{h_\varrho}{v_\varrho}\right),$$
$$\Phi_{Si}=\frac{e^{i\frac{\phi_S}{v_S}}}{\sqrt{2}}(v_S+h_S),\qquad\Theta=\frac{e^{i\frac{\phi_\theta}{v_\Theta}}}{\sqrt{2}}(v_\Theta+h_\Theta),$$
$$\chi=\frac{v_\chi}{\sqrt{2}}e^{i\frac{\phi_\chi}{v_g}}\left(1+\frac{h_\chi}{v_g}\right),\qquad\tilde{\chi}=\frac{v_{\tilde{\chi}}}{\sqrt{2}}e^{-i\frac{\phi_\chi}{v_g}}\left(1+\frac{h_{\tilde{\chi}}}{v_g}\right),\qquad(9)$$

where we have set $\Phi_{S1}=\Phi_{S2}=\Phi_{S3}\equiv\Phi_{Si}$, and radial modes $h_\chi=h_{\tilde{\chi}}$ in the supersymmetric limit, and $v_g=\sqrt{v_\chi^2+v_{\tilde{\chi}}^2}$ and $f_T=\sqrt{v_T^2+v_\varrho^2}$. In addition, pseudo-Goldstone modes A_1 and A_2 are expressed in terms of the angular fields ϕ_S, ϕ_θ, and ϕ_χ as $A_1=\frac{v_S\phi_S+v_\Theta\phi_\theta}{v_{\mathcal{F}}}$ and $A_2=\phi_\chi$ with $v_{\mathcal{F}}\equiv\sqrt{v_S^2+v_\Theta^2}=v_\Theta\sqrt{1+\kappa^2}$. Under the $U(1)_X$ transformations, the flavored axion A_i shifts into $A_i\to A_i+F_{a_i}\xi_i$, where $F_{a_i}=f_{a_i}/\delta_i^{\rm G}$ $(i=1,\ 2)$ with $\xi_1=\delta_2^{\rm G}\alpha$ and $\xi_2=-\delta_1^{\rm G}\alpha$ (α is a transformation parameter), whereas A_T shifts into $A_T\to A_T+f_T\xi_T$.

TABLE II. Representations of quark, lepton, and electroweak two Higgs $H_{u(d)}$ fields under $SL_2(F_3)\times U(1)_{X_i}$ $(i=1,2,T)$. All fields are left-handed particles/antiparticles. All of them have zero $U(1)_R$.

Field	$SL_2(F_3)$	$U(1)_{X_1}$	$U(1)_{X_2}$	$U(1)_{X_T}$	Brane
$Q_1,\ Q_2,\ Q_3$	$\mathbf{1}\ \mathbf{1}'\ \mathbf{1}''$	$-8,\ -6,\ 0$	$0,\ 0,\ 0$	$0,\ 0,\ 0$	$y=0$
$\mathcal{D}^c,\ b^c$	$\mathbf{2}'\ \mathbf{1}'$	$5,\ -3$	$-14,\ 18$	$0,\ 0$	$y=L$
$\mathcal{U}^c,\ t^c$	$\mathbf{2}'\ \mathbf{1}'$	$6,\ 0$	$-6,\ 11$	$0,\ 0$	$y=L$
L	$\mathbf{3}$	$-\frac{15}{2}$	0	0	$y=0$
$e^c,\ \mu^c,\ \tau^c$	$\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}'$	$\frac{13}{2},\ \frac{13}{2},\ \frac{13}{2}$	$41,\ 31,\ 27$	$-2,\ 1,\ 1$	$y=L$
N^c	$\mathbf{3}$	$-\frac{1}{2}$	0	0	$y=0$
$H_{u(d)}$	$\mathbf{1}$	0	0	0	$y=0$

All ordinary matter and Higgs fields charged under $G_{\rm SM}$ are localized at either brane. Thanks to the orbifold compactification, we set all elementary fermions from a chiral set. Then, all SM $SU(2)$ singlets such as right-handed quarks (q^c) and right-handed charged leptons (l^c) are localized at the $y=L$ brane, while $SU(2)$ doublets such as left-handed quarks (Q_i), left-handed leptons (L), and two electroweak Higgs $H_{u(d)}$ are localized at the $y=0$ brane. Under $[SL_2(F_3)\times U(1)_X]_{U(1)_Y}$, we assign the left-handed quark $SU(2)_L$ doublets denoted as Q_1, Q_2, and Q_3 to $(\mathbf{1},-8X_1)_{1/6}$, $(\mathbf{1}',-6X_1)_{1/6}$, and $(\mathbf{1}'',0)_{1/6}$, respectively, while the right-handed up-type quark $SU(2)$ singlets are assigned as $\mathcal{U}^c=\{u^c,c^c\}$ and t^c to $(\mathbf{2}',6X_1-6X_2)_{-2/3}$ and $(\mathbf{1}',11X_2)_{-2/3}$, respectively, and the right-handed down-type quarks $\mathcal{D}^c=\{d^c,s^c\}$ and b^c to $(\mathbf{2}',5X_1-14X_2)_{1/3}$ and $(\mathbf{1}',-3X_1+18X_2)_{1/3}$, respectively, which are summarized in Table II. The brane-localized Yukawa superpotentials for up- and down-type quark fields and lepton fields invariant under $G_{\rm SM}\times SL_2(F_3)\times U(1)_X$ are constructed as

$$\begin{aligned}W_q^u=\delta(y)\Bigg\{&\left[Y_{33}^u\Psi_{u3}^cQ_3+Y_{22}^u\Psi_{u2}^cQ_2+Y_{11}^u\Psi_{u1}^cQ_1+(Y_{21}^u\Psi_{u2}^cQ_1+Y_{31}^u\Psi_{u3}^cQ_1+Y_{32}^u\Psi_{u3}^cQ_2)\frac{(\Phi_S\Phi_S)}{M_5^2}\right]H_u\Bigg\}\\&+\delta(y-L)\Bigg\{\hat{Y}_{t3}t^c\Psi_{u3}+\left[Y_{u2}(\mathcal{U}^c\eta)+\tilde{Y}_{u2}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\right]\Psi_{u2}\\&+\bar{Y}_{u2}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{u2}+\left[Y_{u1}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)+\tilde{Y}_{u1}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\right]\Psi_{u1}\Bigg\},\end{aligned}\qquad(10)$$

$$\begin{aligned}W_q^d=\delta(y)\Bigg\{&[Y_{33}^d\Psi_{d3}^cQ_3+Y_{22}^d\Psi_{d2}^cQ_2+Y_{11}^d\Psi_{d1}^cQ_1]H_d+[(Y_{21}^d\Psi_{d2}^c+Y_{31}^d\Psi_{d3}^c)Q_1(\Phi_S\Phi_S)+Y_{32}^d\Psi_{d3}^cQ_2(\Phi_S\Phi_S)]\frac{H_d}{M_5^2}\Bigg\}\\&+\delta(y-L)\Bigg\{\hat{Y}_{b3}b^c\Psi_{d3}+Y_{d2}(\mathcal{D}^c\eta)\Psi_{d2}+\tilde{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\Psi_{d2}+\bar{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{d2}\\&+Y_{d1}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\Psi_{d1}+\tilde{Y}_{d1}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{d1}\Bigg\},\end{aligned}\qquad(11)$$

$$W_\ell = \delta(y)\left\{Y_L(\Psi^c_\ell L)H_d + (y_\nu(N^cL) + \tilde{y}_\nu(N^cL)\Phi_S)\frac{H_u}{M_5} + \frac{1}{2}y_R(N^cN^c)\Phi_S + \frac{1}{2}(y_\Theta\Theta + y_{\tilde{\Theta}}\tilde{\Theta})(N^cN^c)\right\}$$
$$+\,\delta(y-L)\left\{Y_e e^c\left(\Psi_\ell\frac{\Phi_T\Phi_T}{M_5}\right) + \tilde{Y}_e e^c\left(\Psi_\ell\frac{\Phi_T\Phi_T}{M_5}\right)\left(\frac{\Phi_T\varrho}{M_5^2}\right) + Y_\mu\mu^c(\Psi_\ell\Phi_T)\frac{\varrho^2}{M_5^2}\right.$$
$$\left.+\,\tilde{Y}_\mu\mu^c\left(\Psi_\ell\frac{\eta\eta}{M_5^2}\right)\varrho + Y_\tau\tau^c(\Psi_\ell\Phi_T)\frac{\varrho^2}{M_5^2} + \tilde{Y}_\tau\tau^c\left(\Psi_\ell\frac{\eta\eta}{M_5^2}\right)\varrho\right\}. \tag{12}$$

In the above superpontentials, $Y^{u(d)}_{ij}$, $\tilde{Y}_{u(d)i}$, $\bar{Y}_{u(d)i}$, and $Y_{u(d)i}$ for $i, j = 1, 2, 3$, as well as $Y_{L(l)}$ and $\tilde{Y}_l$ have mass dimension $-1/2$, whereas $\hat{Y}_{u(d)i}$ has mass dimension $+1/2$. Notice that they are composed of combinations of the flavon fields in such a way that the associated Yukawa interaction terms are neutral under the charge assignments of $U(1)_X \times SL_2(F_3)$ given in Table II. Then, after flavons and Higgs fields get VEVs, the corresponding Yukawa terms result in the masses of the charged fermions as presented in Eq. (C2) with Eq. (C1). The Yukawa couplings y_i $(\tilde{y}_i)$ have mass dimension zero.

Clearly, it shows that SM fermions localized at the two branes could form ordinary interactions between left- and right-handed fermions via the exchange of their flavored bulk fermions. Then, the action for quark and lepton fields localized on the branes reads

$$S_Y = \int d^4x dy\sqrt{g}\left[\int d^2\vartheta(W^u_q + W^d_q + W_\ell) + \mathrm{H.c.}\right], \tag{13}$$

where ϑ is a Grassmann variable having mass dimension $-1/2$. Note that after spontaneous $SL_2(F_3) \times U(1)_X$ breaking, all other higher-dimensional operators localized at the two branes can be absorbed by redefining the coupling constants, and thus they cannot change the patterns of quark and lepton mass matrices at leading operators shown in Eqs. (10)–(12).

All ordinary matter and Higgs fields charged under $G_{\rm SM}$ with $+1$ and 0 charges under $U(1)_R$, respectively, are localized at either brane. Then, all the SM fermion mixings and masses can be generated by nonlocal effects involving both branes and local breaking effects of G_F due to flavon fields. For the orbifold compactification, we set all elementary fermions from a chiral set, and their group representations and quantum numbers are summarized in Table II. From the $U(1)_{X_k} \times [SU(3)_C]^2$ anomaly coefficient defined by

$$\delta^{\rm G}_k\delta^{ab} = 2\sum_{\psi_f} X_{k\psi_f}\mathrm{Tr}(T^aT^b), \tag{14}$$

in the QCD instanton backgrounds where the T^a are the generators of the representation of $SU(3)_C$ to which Dirac fermion ψ_f belongs with X-charge, we get $\delta^{\rm G}_1 = -9$ and $\delta^{\rm G}_2 = -11$ with the domain-wall number $N_{\rm DW} = 1$. In this model, $U(1)_{X_1} \times U(1)_{X_2} \equiv U(1)_{\tilde{X}}$ is a pure axial symmetry $U(1)_{\rm PQ}$. Under the $U(1)_{\rm PQ}$ transformation, the QCD axion field A shifts into $A \to A + \frac{f_A}{N_C}\alpha$ with $\frac{f_A}{N_C} = \frac{f_{a_i}}{\sqrt{2}\delta^{\rm G}_i} \equiv F_A$, where F_A is the QCD axion decay constant, and $f_A = \sqrt{2}\delta^{\rm G}_2 f_{a_1} = \sqrt{2}\delta^{\rm G}_1 f_{a_2}$ with $f_{a_1} = |X_1|v_{\mathcal{F}} = \sqrt{2}|X_1|v_\chi$ and $f_{a_2} = |X_2|v_g = |X_2|v_\Theta\sqrt{1+\kappa^2}$. Below the $U(1)_{\rm PQ}$ symmetry breaking scale, the effective interaction of the QCD axion is expressed via the chiral $U(1)_{\tilde{X}}$ rotation $\psi_f \to e^{i\gamma_5\tilde{X}_{\psi_f}\alpha/2}\psi_f$ with $\tilde{X}_{\psi_f} = \delta^{\rm G}_2 X_{1\psi_f} + \delta^{\rm G}_1 X_{2\psi_f}$ as

$$\mathcal{L}_{\rm eff} \supset \left(\vartheta_{\rm eff} + \frac{A_1}{f_{a_1}}\delta^{\rm G}_1 + \frac{A_2}{f_{a_2}}\delta^{\rm G}_2\right)\frac{g_s^2}{32\pi^2}G^{\mu\nu a}\tilde{G}^a_{\mu\nu}$$
$$\equiv \left(\vartheta_{\rm eff} + \frac{A}{F_A}\right)\frac{g_s^2}{32\pi^2}G^{\mu\nu a}\tilde{G}^a_{\mu\nu}, \tag{15}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor with a being an $SU(3)$-adjoint index and $\tilde{G}^a_{\mu\nu}$ is its dual. The $U(1)_{X_i}$ is broken down to its discrete subgroup Z_{N_i} in the backgrounds of the QCD instanton, and the quantities N_i (nonzero integers) associated with the axionic domain wall are given by $N_1 = 2|\delta^{\rm G}_1| = 18$ and $N_2 = |\delta^{\rm G}_2| = 11$ such that no axionic domain-wall problem occurs at the QCD phase transition. A color anomaly coefficient N_C of $U(1)_{\tilde{X}} \times [SU(3)_C]^2$ and an electromagnetic one E_A of $U(1)_{\tilde{X}} \times [U(1)_{\rm EM}]^2$ are defined by $N_C \equiv 2\mathrm{Tr}[\tilde{X}_{\psi_f}T^aT^a] = 2\delta^{\rm G}_1\delta^{\rm G}_2$ and $E_A = 2\sum_{\psi_f}\tilde{X}_{\psi_f}(Q^{\rm em}_{\psi_f})^2$ with $Q^{\rm em}_{\psi_f}$ being the $U(1)_{\rm EM}$ charge of field ψ_f, respectively. Then, their ratio becomes $E_A/N_C = -761/99$. On the other hand, an anomalous $U(1)_{X_T}$ is embedded only in the lepton sector, as shown in Table II, with an electromagnetic anomaly coefficient $E_T \equiv 2\sum_\ell X_\ell(Q^{\rm em}_\ell)^2$ of the $U(1)_{X_T} \times [U(1)_{\rm EM}]^2$. As shown in Appendix A, an explicit breaking term can generate a mass term making the axionlike particle (ALP) a pseudo-Goldstone boson A_T.

III. 4D EFFECTIVE THEORY FOR FLAVOR PHYSICS

Consider the 5D action for flavored bulk fermions $\Psi_{fi}(x,y)$ with large bulk fermion masses M_{f_i} ($f = u, d, \ell$ and $i = 1, 2, 3$)

$$S_\Psi = \int d^4x dy \sqrt{g} \bar{\Psi}_{fi} \left\{ \frac{i}{2} e^M_A \Gamma^A \overleftrightarrow{D}_M + M_{f_i}(y) \right\} \Psi_{fi}, \quad (16)$$

where the 5D metric g_{MN} is decomposed into vierbeins e^A_M: $g_{MN} = \eta_{AB} e^A_M e^B_N$, $\Gamma^A = (\gamma^\mu, i\gamma_5)$, and $\Gamma_A = (\gamma_\mu, -i\gamma_5)$ satisfy the Dirac-Clifford algebra $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$, where η^{AB} is the 5D flat metric $= \mathrm{diag}(\eta_{\mu\nu}, -1)$. In terms of $\Psi_{fi}(x,y)$, the Kaluza-Klein (KK) wave functions for the left- and right-handed bulk fermions forming a complete orthogonal set

$$\Psi_{fiL(R)}(x,y) = \frac{e^{-\frac{3}{2}\sigma(y)}}{\sqrt{L}} \sum_n \psi^n_{f_i L(R)}(x) f^n_{iL(R)}(y) \quad (17)$$

are chosen to obey the 4D equation of motion

$$S = \sum_n \int d^4x \bar{\psi}^n_i (i\gamma_\mu D^\mu - m^n_f) \psi^n_{f_i}, \quad (18)$$

where m^n_f is the 4D mass of the nth KK mode, with the normalization condition $\frac{1}{L}\int_0^L dy f^m_{iL(R)} f^n_{iL(R)} = \delta_{mn}$. We choose a gauge $A_5 = 0$ such that the KK modes are independent of the gauge fields. We vary the total action including Eqs. (13) and (16) and obtain the following equation of motion (EOM) and boundary condition by requiring $\delta S = 0$ for any $\delta\bar{\Psi}_{f_i}$: The EOM is

$$ie^{-\sigma}\gamma^\mu D_\mu \Psi_{fi} - \gamma_5 \partial_y \Psi_{fi} - \frac{1}{2}(\partial_y \sigma)\gamma_5 \Psi_{fi} + M_{f_i}\Psi_{fi} = 0, \quad (19)$$

and the boundary condition is given by

$$\delta S^{\mathrm{surface}}_\Psi + \delta S_Y = 0, \quad (20)$$

where $\delta S^{\mathrm{surface}}_\Psi = \frac{1}{2}\int d^4x e^{4\sigma} \delta\bar{\Psi}_{fi}\gamma_5 \Psi_{fi}|^{y=L}_{y=0}$ coming from Eq. (16). Notice that the action for the quarks and leptons in Eq. (13) is localized at the branes. Plugging Eq. (17) into Eq. (19), we obtain

$$\begin{aligned} &ie^{-\sigma}\gamma^\mu D_\mu \psi^n_{f_i}(x) f^n_i(y) \\ &= \gamma_5 \partial_y f^n_i(y) \psi^n_{f_i}(x) \\ &\quad + \frac{1}{2}\sigma' \gamma_5 \psi^n_{f_i}(x) f^n_i(y) - M_{f_i} \psi^n_{f_i}(x) f^n_i(y), \end{aligned} \quad (21)$$

where $\sigma' = \partial_y \sigma$. Using Eq. (18), Eq. (21) can be decomposed in terms of left- and right-handed spinors $f^n_{iL,R}$,

$$\begin{aligned} \left(\partial_y + \frac{1}{2}\sigma' - M_{f_i}\right) f^n_{iR}(y) &= e^{-\sigma} m^n_f f^n_{iL}(y), \\ \left(\partial_y + \frac{1}{2}\sigma' + M_{f_i}\right) f^n_{iL}(y) &= -e^{-\sigma} m^n_f f^n_{iR}(y). \end{aligned} \quad (22)$$

The nonzero KK modes can be obtained by solving the first-order coupled equations of motion Eq. (22) for the Dirac component profiles $f^n_{iL(R)}$, giving the 4D KK mass spectrum for $n \geq 1$, $m^n_f \sim ke^{-kL}$ if $kL \gg 1$; see Ref. [28]. At low energies, i.e., when $E \ll 1/L$, only the zero mode is important, while at higher energies $E \gtrsim 1/L$, all the KK modes become essential. Since we are interested in the energy scale much lower than $1/L$, the 4D covariant derivative term in Eq. (21) can be negligible. Then, by setting $M_{f_i} =$ constant, we approximately get

$$\begin{aligned} f^0_{iR}(y) &\approx f^0_{iR}(L) e^{\frac{1}{2}[\sigma(L)-\sigma(y)] - M_{f_i}(L-y)}, \\ f^0_{iL}(y) &\approx f^0_{iL}(0) e^{-\frac{1}{2}\sigma(y) - M_{f_i} y}. \end{aligned} \quad (23)$$

Choosing

$$\delta f^n_{iL}(0) = \delta f^n_{iR}(L) = 0 \quad (24)$$

as boundary conditions, all the left-handed and right-handed KK modes of Ψ_{fi} would vanish, respectively, at the $y = 0$ brane and at the $y = L$ brane, without mixing mass terms between bulk and brane fermions at the branes. In this case, only the right-handed (left-handed) modes could couple to the SM fields on the $y = 0$ ($y = L$) brane, respectively. However, when there are mixing mass terms between bulk and brane fermions like in our case, such boundary conditions are generalized by the variation of the action in Eq. (20). By adjusting Eq. (24) subject to the boundary condition in Eq. (20) in the presence of the mixing mass terms, we obtain, for example, for u-type quarks

$$\begin{aligned} (\delta S)_{\mathrm{b.c}} = \int d^4x \sqrt{g} \Bigg[&\delta\Psi^c_{u1}(x,0)\left\{\frac{1}{2}\Psi_{u1}(x,0) + Y^u_{11} Q_1 H_u\right\} + \delta\Psi^c_{u2}(x,0)\left\{\frac{1}{2}\Psi_{u2}(x,0) + Y^u_{22} Q_2 H_u + Y^u_{21} Q_1 \left(\frac{\Phi_S \Phi_S}{M_5^2}\right) H_u\right\} \\ &+ \delta\Psi^c_{u3}(x,0)\left\{\frac{1}{2}\Psi_{u3}(x,0) + Y^u_{33} Q_3 H_u + \cdots\right\} + \delta\Psi_{u1}(x,L)\left\{\frac{1}{2}\Psi^c_{u1}(x,L) + Y_{u1}\left(\mathcal{U}^c \eta \frac{\Phi_T Q}{M_5^2}\right) + \cdots\right\} \\ &+ \delta\Psi_{u2}(x,L)\left\{\frac{1}{2}\Psi^c_{u2}(x,L) + Y_{u2}(\mathcal{U}^c \eta) + \cdots\right\} + \delta\Psi_{u3}(x,L)\left\{\frac{1}{2}\Psi^c_{u3}(x,L) + \hat{Y}_{t3} t^c\right\} + \mathrm{H.c.}\Bigg], \end{aligned} \quad (25)$$

resulting in the boundary conditions for the flavored bulk fermions shown in Eq. (B1), and similarly for d-type quarks and leptons in Eqs. (B2) and (B3). As a consequence, in the presence of the mixing mass terms on the branes, the zero modes of bulk fermions become massive. Therefore, using the above nontrivial relations between bulk and brane fermions, we can integrate out the bulk fermions to get the effective interactions for the brane fermions in 4D. In the process of integrating out the bulk fermions, it is sufficient to focus on the zero modes of the bulk fermions in the following discussion because the massive KK modes are taken to be much heavier than the zero modes.

From Eq. (13) with Eqs. (10)–(12), after integration along y we obtain the 4D action given in Eq. (C3). Applying Eq. (23) to Eq. (17), we get

$$\Psi_{fiR}(x,0) \approx \Psi_{fiR}(x,L)e^{2\sigma(L)-M_{f_i}L},$$
$$\Psi_{fiL}(x,L) \approx \Psi_{fiL}(x,0)e^{-2\sigma(L)-M_{f_i}L}. \tag{26}$$

In order to derive 4D effective Lagrangians for the fermion Yukawa interactions, we integrate out the bulk fermions in Eq. (C3) by using Eqs. (B1)–(B3) and (26). Those equations show that the bulk fermions are given in terms of the SM fermions, Higgs fields, and flavon fields. Performing the rescaling of dimensional parameters such as

$$\mathcal{F} \to e^{\sigma(y)}\mathcal{F}, \qquad M_5 \to e^{\sigma(y)}M_5, \qquad \psi \to e^{\frac{3}{2}\sigma(y)}\psi,$$
$$\hat{Y}_i \to e^{\frac{1}{2}\sigma(y)}\hat{Y}_i, \qquad Y_i^f \to e^{-\frac{1}{2}\sigma(y)}Y_i^f \tag{27}$$

(also applied by the replacement of $Y_i^f \to \tilde{Y}_i^f$, $\bar{Y}_i^f$, and Y_i), we finally get 4D Yukawa interactions given as

$$\begin{aligned}
-\mathcal{L}^q_{4D} = {}& \left[y^u_{11}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}} + \tilde{y}^u_{11}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^3}\right)_{\mathbf{1}}\right]Q_1H_u + \left[y^u_{22}\left(\mathcal{U}^c\frac{\eta}{M_5}\right)_{\mathbf{1}''} + \tilde{y}^u_{22}\left(\mathcal{U}^c\frac{\eta\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}''} + \bar{y}^u_{22}\left(\mathcal{U}^c\frac{\eta\eta\eta}{M_5^3}\right)_{\mathbf{1}''}\right]Q_2H_u \\
& + y^u_{21}(\mathcal{U}^c\eta)_{\mathbf{1}''}Q_1(\Phi_S\Phi_S)_{\mathbf{1}'}\frac{H_u}{M_5^3} + \tilde{y}^u_{21}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}''}(\Phi_S\Phi_S)_{\mathbf{1}'}Q_1\frac{H_u}{M_5^2} + \bar{y}^u_{21}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^3}\right)_{\mathbf{1}''}(\Phi_S\Phi_S)_{\mathbf{1}'}Q_1\frac{H_u}{M_5^2} \\
& + \mathbf{y}^t_{33}t^cQ_3H_u + \mathbf{y}^t_{32}t^cQ_2(\Phi_S\Phi_S)_{\mathbf{1}'}\frac{H_u}{M_5^2} + \mathbf{y}^t_{31}t^cQ_1(\Phi_S\Phi_S)_{\mathbf{1}''}\frac{H_u}{M_5^2} \\
& + \mathbf{y}^b_{33}b^cQ_3H + \left[y^d_{22}\left(\mathcal{D}^c\frac{\eta}{M_5}\right)_{\mathbf{1}''} + \tilde{y}^d_{22}\left(\mathcal{D}^c\frac{\eta\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}''} + \bar{y}^d_{22}\left(\mathcal{D}^c\frac{\eta\eta\eta}{M_5^3}\right)_{\mathbf{1}''}\right]Q_2H_d \\
& + \left[y^d_{11}\left(\mathcal{D}^c\frac{\eta\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}} + \tilde{y}^d_{11}\left(\mathcal{D}^c\frac{\eta\eta\eta}{M_5^3}\right)_{\mathbf{1}}\right]Q_1H_d + \mathbf{y}^b_{32}b^c\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)_{\mathbf{1}'}Q_2H_d + \mathbf{y}^b_{31}b^c\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)_{\mathbf{1}''}Q_1H_d \\
& + \left[y^d_{21}\left(\mathcal{D}^c\frac{\eta}{M_5}\right)_{\mathbf{1}''} + \tilde{y}^d_{21}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^3}\right)_{\mathbf{1}''} + \bar{y}^d_{21}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^3}\right)_{\mathbf{1}''}\right]\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)_{\mathbf{1}'}Q_1H_d + \text{H.c.}
\end{aligned} \tag{28}$$

$$\begin{aligned}
-\mathcal{L}^l_{4D} = {}& \left[y_e\left(L\frac{\Phi_T\Phi_T}{M_5^2}\right)_{\mathbf{1}} + \tilde{y}_e\left(L\frac{\Phi_T\Phi_T}{M_5^2}\right)_{\mathbf{3}}\frac{\Phi_T\varrho}{M_5^2}\right]e^cH_d + \left[y_\mu(L\Phi_T)_{\mathbf{1}'}\frac{\varrho^2}{M_5^3} + \tilde{y}_\mu\left(L\frac{\eta\eta}{M_5^3}\right)_{\mathbf{1}'}\varrho\right]\mu^cH_d \\
& + \left[y_\tau(L\Phi_T)_{\mathbf{1}''}\frac{\varrho^2}{M_5^3} + \tilde{y}_\tau\left(L\frac{\eta\eta}{M_5^3}\right)_{\mathbf{1}''}\varrho\right]\tau^cH_d + y_\nu(N^cL)_{\mathbf{1}}\frac{\Theta}{M_5}H_u + \tilde{y}_\nu(N^cL)_{\mathbf{3}}\frac{\Phi_S}{M_5}H_u \\
& + \frac{1}{2}(y_\Theta\Theta + y_{\tilde{\Theta}}\tilde{\Theta})(N^cN^c)_{\mathbf{1}} + \frac{1}{2}y_R\Phi_S(N^cN^c)_{\mathbf{3}} + \text{H.c.}
\end{aligned} \tag{29}$$

with well-defined Yukawa-coupling functions

$$\mathbf{y}^\alpha_{ij} = 4Y^f_{ij}\hat{Y}_{\alpha k}e^{-M_{f_k}L}\cosh 2\sigma(L),$$
$$y^\alpha_{ij}/M_5 = 4Y^f_{ij}Y_{\alpha k}e^{-M_{f_k}L}\cosh 2\sigma(L),$$
$$y_\ell/M_5 = 4Y_\ell Y_Le^{-M_\ell L}\cosh 2\sigma(L), \tag{30}$$

where $\alpha = (d,s,b,u,c,t)$, $f = (u,d)$, $\ell = (e,\mu,\tau)$, and in Eq. (27) $\mathcal{F} = \{\Phi_S, \Theta(\tilde{\Theta}), H_{u(d)}\}$ (at the $y=0$ brane), $\{\Phi_T, \varrho, \eta, \chi, \tilde{\chi}\}$ (at $y=L$), and ψ stands for all fermion fields localized at both branes. Note that the 4D effective Lagrangians are derived from Eqs. (28) and (29) after the flavons acquire VEVs $\langle\mathcal{F}\rangle$ at both branes. Therefore, we expect that the mediation of bulk messenger fermions via Eqs. (26) and (B1)–(B3) and vacuum configuration of the flavon fields play a crucial role in adjusting the quark and lepton masses and mixings to their observed values.

IV. HIERARCHIES OF FERMION MASSES AND MIXINGS

A. Charged fermions

After electroweak symmetry breaking, we get the VEVs for two Higgs doublets as $\langle H_{u,d}\rangle = v_{u,d}$, where $v_u = v\sin\beta/\sqrt{2}$ and $v_d = v\cos\beta/\sqrt{2}$ with $v = 246$ GeV. Given the specific vacuum alignment given by Eqs. (6) and (7), we obtain the up(down)-type quark mass matrices and diagonal form of the charged-lepton mass matrix, in which each entry is proportional to $e^{-M_{f_i}L}\cosh 2\sigma(L)$; see Eq. (C1).

Under the hierarchy $M_{f_1}L > M_{f_2}L > M_{f_3}L > 0$, the quark mixings θ^q_{ij} in the standard form [29] can be obtained from the diagonalization of the quark mass matrices given in Eq. (C1);

$$\theta^q_{ij} \simeq 3\kappa^2 f(\hat{\mathbf{y}},\hat{y})\left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\left(\frac{2}{1+\kappa^2}\right)^{\frac{1}{2}}\Delta_\chi\right|^{[\mathcal{Q}_{X_1}(Q_j)-\mathcal{Q}_{X_1}(Q_i)]},$$
$$\delta^q_{CP} = \arg(f(\hat{\mathbf{y}},\hat{y})), \tag{31}$$

where $i \neq j = 1, 2, 3$, and $\mathcal{Q}_{X_i}(q)$ represents the $U(1)_{X_i}$ quantum number $\mathcal{Q}$ of field q, and $f_i(\hat{\mathbf{y}},\hat{y})$ are functions of the associated hat Yukawa couplings being of order unity [cf. Eqs. (1), (30), and (C1)]. Note that in the limit of $\kappa \to 0$, there is no quark mixing. Clearly, it shows the quark mixings only depend on the local effects of G_F. For the charged lepton, the mass ratios are given by

$$\frac{m_e}{m_\mu} \simeq \hat{y}_e\Delta_T^2\Delta_\chi^{10}/(\hat{y}_\mu\Delta_T\Delta_\varrho + \hat{\hat{y}}_\mu\Delta_\eta^2)\Delta_\varrho, \tag{32}$$

$$\frac{m_\mu}{m_\tau} \simeq (\hat{y}_\mu\Delta_T\Delta_\varrho + \hat{\hat{y}}_\mu\Delta_\eta^2)\Delta_\chi^4/(\hat{y}_\tau\Delta_T\Delta_\varrho + \hat{\hat{y}}_\tau\Delta_\eta^2). \tag{33}$$

In the limit $\Delta_T \sim \Delta_\varrho \sim \Delta_\eta$, and for order unity $\hat{y}_{\alpha=e,\mu,\tau}$ and $\hat{\hat{y}}_\alpha$, those ratios become $\frac{m_e}{m_\mu} \simeq \Delta_\chi^{10}\Delta_\varrho$ and $\frac{m_\mu}{m_\tau} \simeq \Delta_\chi^4$.

B. Neutrinos

In this model, the low-energy effective neutrino masses are generated by the usual seesaw mechanism [20] with the inclusion of right-handed $SU(2)$-singlet Majorana neutrinos N^c. The seesaw scale M should be larger than the electroweak scale but smaller than M_5. The fundamental gravity scale M_5 can be derived in terms of F_A or M:

$$M_5 = \frac{|\delta^{\rm G}_2|}{\sqrt{2}}\frac{F_A}{\Delta_\chi} = \frac{M}{\Delta_\Theta} \quad \text{with} \quad M \equiv |\delta^{\rm G}_1|\frac{F_A}{\sqrt{1+\kappa^2}}. \tag{34}$$

Contrary to the quark and charged-lepton sectors, the neutrino Yukawa couplings do not depend on the nonlocal effects of the extra dimension. After seesawing $\mathcal{M}_\nu \simeq -m_D^T M_R^{-1} m_D$ where M_R and m_D are, respectively, right-handed Majorana and Dirac neutrino mass matrices [see Eqs. (E1) and (E2)], in a basis where charged-lepton masses are real and diagonal, we obtain the effective light neutrino mass matrix

$$\mathcal{M}_\nu = m_0\frac{e^{i\pi}}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_0 e^{i\pi}}{1-e^{2i\phi}\tilde{\kappa}^2}\begin{pmatrix}a_\nu & -\frac{1}{2}a_\nu & -\frac{1}{2}a_\nu\\ -\frac{1}{2}a_\nu & a_\nu-1 & 1-\frac{1}{2}a_\nu\\ -\frac{1}{2}a_\nu & 1-\frac{1}{2}a_\nu & a_\nu-1\end{pmatrix}$$
$$+ m_0 e^{i\pi}\frac{\kappa^2}{1-e^{2i\phi}\tilde{\kappa}^2}\begin{pmatrix}-b_\nu - c_\nu\tilde{\kappa} & \frac{1}{2}(b_\nu+c_\nu\tilde{\kappa})+d_\nu & \frac{1}{2}(b_\nu+c_\nu\tilde{\kappa})-d_\nu\\ \frac{1}{2}(b_\nu+c_\nu\tilde{\kappa})+d_\nu & \frac{1}{2}(b_\nu-c_\nu\tilde{\kappa})-d_\nu & \frac{1}{2}c_\nu\tilde{\kappa}-b_\nu\\ \frac{1}{2}(b_\nu+c_\nu\tilde{\kappa})-d_\nu & \frac{1}{2}c_\nu\tilde{\kappa}-b_\nu & \frac{1}{2}(b_\nu-c_\nu\tilde{\kappa})+d_\nu\end{pmatrix}, \tag{35}$$

where $m_0 = |\hat{y}_\nu|^2\frac{v_u^2}{M}\left|\frac{X_2\delta^{\rm G}_1}{X_1\delta^{\rm G}_2}\Delta_\chi\right|^{16}(\frac{2}{1+\kappa^2})^8$, and each component is given by a_ν, b_ν, c_ν, and d_ν that are functions of κ, $\tilde{\kappa}$, ϕ, and other phases.

As a result, the neutrino masses m_{ν_i} $(i = 1, 2, 3)$ are obtained by the transformation $U^T_{\rm PMNS}\mathcal{M}_\nu U_{\rm PMNS} = {\rm diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, where $U_{\rm PMNS}$ is the mixing matrix of three mixing angles θ_{12} (solar), θ_{13} (reactor), θ_{23} (atmospheric), and three CP-odd phases (one δ_{CP} for the Dirac neutrinos and two $\varphi_{1,2}$ for the Majorana neutrinos) [29]. In the limit $\kappa \to 0$ (leading to $\tilde{\kappa} \to 0$ and $a_\nu \to \frac{2}{3}$), the light neutrino masses generated via Eq. (35) become degenerate with no neutrino mixings [and also no quark mixings; see Eq. (31)]. Hence, it is reasonable to take a nonzero κ to generate the observed neutrino mixing angles. Then, the neutrino mass eigenvalues of Eq. (35) can be expanded in terms of κ: For normal mass ordering (NO), $m^2_{\nu_3} = m^2_{\nu+} > m^2_{\nu_2} \equiv m^2_0 > m^2_{\nu_1} = m^2_{\nu-}$ with $m^2_{\nu\pm} = m^2_0(Q_\nu \pm \sqrt{J_\nu})/P_\nu$, and for inverted mass ordering (IO), $m^2_{\nu_2} \equiv m^2_0 > m^2_{\nu_1} = m^2_{\nu+} > m^2_{\nu_3} = m^2_{\nu-}$, where Q_ν, J_ν, and P_ν are functions of the parameters in Eq. (35). The limit $\tilde{y}_\nu \to 0$ in Eq. (29) is equivalent to $a_\nu \to \frac{2}{3}(1-e^{i\phi}\tilde{\kappa})$ and $\{b_\nu, c_\nu, d_\nu\} \to 0$, and it gives rise to the so-called tribimaximal mixing of neutrinos. Then, the deviation from the tribimaximal mixing [30] can be presented in terms of κ:

$$
\begin{aligned}
&|\theta_{23} - \pi/4| \propto |d_\nu|\kappa^2, \\
&\theta_{13} \propto |d_\nu|\kappa^2, \\
&\theta_{12} - \frac{1}{2}\tan^{-1}(2\sqrt{2}) \simeq \frac{2\sqrt{2}|3|a_\nu|(\kappa^2 - \tilde{\kappa}^2)S_\nu + \mathcal{O}(\kappa^3, \tilde{\kappa}^4)|}{R_\nu + 3|a_\nu|(\tilde{\kappa}^2 - \frac{\kappa^2}{2})S_\nu + \mathcal{O}(\kappa^3, \tilde{\kappa}^4)},
\end{aligned} \tag{36}
$$

where $R_\nu = \frac{1}{3} - \frac{3}{4}|a_\nu|^2 - \frac{2}{3}\tilde{\kappa}^2\cos^2\phi$, and S_ν is a function of $|b_\nu|, |c_\nu|, \tilde{\kappa}$, and relevant phases [especially, $S_\nu \to 0$ and $\mathcal{O}(\kappa^3, \tilde{\kappa}^4) \to 0$ for $\{b_\nu, c_\nu\} \to 0$].

V. FLAVORED AXION

The flavored axion A_1 produces the flavor-changing neutral Yukawa interaction

$$
-\mathcal{L}^{asd} \sim i\frac{A_1}{f_{a1}}\lambda(m_d - m_s)\bar{d}s + \text{H.c.}, \tag{37}
$$

where λ is the Cabibbo angle [29], which gives the strongest bound on the QCD axion decay constant F_A. From the present experimental upper bound $\text{Br}(K^+ \to \pi^+ A_i) < (3-6)\times 10^{-11}$ at 90% confidence level [31], we obtain $F_A \gtrsim (0.7-1.5)\times 10^{10}$ GeV.

From the Lagrangian terms for the axion and electron given by

$$
\left(\frac{\partial_\mu A_1}{2f_{a_1}} - 41\frac{\partial_\mu A_2}{2f_{a_2}} + \frac{\partial_\mu A_T}{v_t}\right)\bar{e}\gamma^\mu\gamma_5 e, \tag{38}
$$

we can identify the effective axion couplings as $g_{1ee} = m_e/(\sqrt{2}|\delta_1^G|F_A)$ and $g_{2ee} = 41m_e/(\sqrt{2}|\delta_2^G|F_A)$. Thus, the flavored axions' $A_{1,2,T}$ interactions with leptons can be searched for in stellar evolutions in astroparticle physics [32]. Some stellar cooling hints can be interpreted as axion-electron couplings $7.2\times 10^{-14} \lesssim |g_{iee}| \lesssim 2.2\times 10^{-13}$ [33] determining the favored values of F_A via the A_i couplings to electrons. Otherwise, the lower limit on F_A is set to about 10^8 GeV by SN1987A, while the upper bound on F_A is about 10^{12} GeV from the dark matter abundance. Combining the stellar cooling hints (from astrophysics) with the constraint from $K^+ \to \pi^+ + A_1$ process (from particle physics), we obtain the consistent axion decay constant as

$$
(0.7\text{–}1.5)\times 10^{10} \lesssim F_A[\text{GeV}] \lesssim 1.9\times 10^{10}. \tag{39}
$$

VI. NUMERICAL RESULTS

In this scenario, there are 16 degrees of freedom [eight local parameters $\{\Delta_\chi, \Delta_\Theta, \Delta_S, \kappa, \Delta_\eta, \Delta_T, \Delta_\varrho, \tan\beta\}$ plus eight nonlocal parameters $\{\sigma(L), M_{u(d)_{1,2,3}}, M_\ell\}$]. Among them, 14 parameters are left undetermined after imposing the two constraints given in Eq. (8). We take $\hat{y}(\hat{\mathbf{y}}) = \mathcal{O}(1)$, and $M_{f_1} > M_{f_2} > M_{f_3}$ with $f = u, d$ for our numerical analysis. For a fixed nonlocal parameter, the remaining 13 parameters can be determined from the observables in the charged-fermion sectors (nine charged-fermion masses, three mixing angles, and one CP phase in the quark sector). We perform a numerical simulation using the linear algebra tools [34] by adopting the empirical results of the 13 observables in the charged-fermion sectors as inputs. The numerical results of the model parameters for $M_{d_3}L = 28.375$ are given as $\sigma(L) = 18.25$, $\tan\beta = 3.1$, $\kappa = 0.4$, $\Delta_\chi = 0.52$, $\Delta_\eta = 0.51$, $\Delta_T = 0.35$, $\Delta_\varrho = 0.34$, $M_{u_1}L = 43.875$, $M_{u_2}L = 40.975$, $M_{u_3}L = 33.675$, $M_{d_1}L = 35.925$, $M_{d_2}L = 35.295$, and $M_\ell L = 18.375$. From our numerical analysis, we found that $\sigma(L)$ is constrained for a given set of Δ_k such that the positiveness and inverted ordering of M_i are not kept below some value of $\sigma(L)$ while keeping the required fermion mixing and mass hierarchies. In addition, they are not kept in the case where the same flavor symmetry introduced in this model is embedded in the framework of the flat extra dimension.

Plugging the above results and Eq. (39) into Eq. (34), we can derive the scale of M_5 as $(1-2)\times 10^{11}$ GeV$\lesssim M_5 \lesssim 2.8\times 10^{11}$ GeV, leading to the QCD axion mass $m_a = 5.44^{+2.33}_{-2.58}\times 10^{-4}$ eV. Then, we obtain the axion photon coupling $|g_{a\gamma\gamma}| = 1.13^{+0.48}_{-0.53}\times 10^{-12}$ GeV^{-1} for $m_u/m_d = 0.47$, as shown as the red line in Fig. 1. With the help of Eq. (3), the compactification length $L[m]$ is given by

$$
L[m] \simeq 5.82\times 10^{-13}\times\frac{e^{-\sigma(L)}\sigma(L)}{\sinh\sigma(L)}\left(\frac{10^{11}\text{ GeV}}{M_5}\right)^3. \tag{40}
$$

For the benchmark given above, the compactification length of the extra dimension is estimated as $L \sim 10^{-28}m$. On the other hand, in the lepton sector, we can predict δ_{CP} precisely in terms of θ_{23}. Namely, θ_{23} would favor $\sim 51°$ for $\delta_{CP} \sim 180°$ and $\theta_{23} \sim 45°$ for $\delta_{CP} \sim 90°$ and 270°, as shown in Fig. 2. We note that the pattern of the light neutrino mass spectrum NO ($m_{\nu_3} > m_{\nu_2} > m_{\nu_1}$) or IO ($m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$) can be

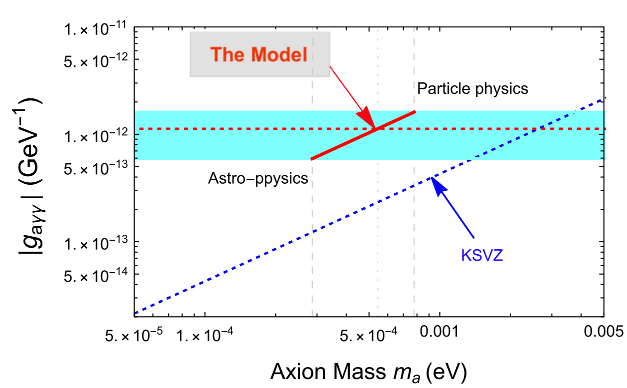

FIG. 1. Plot of $|g_{a\gamma\gamma}|$ versus m_a for Kim-Shifman-Vainshtein-Zakharov [35] (blue dotted line) and the model ("localized" red line) in terms of $E_A/N_C = 0$ and $-761/99$, respectively.

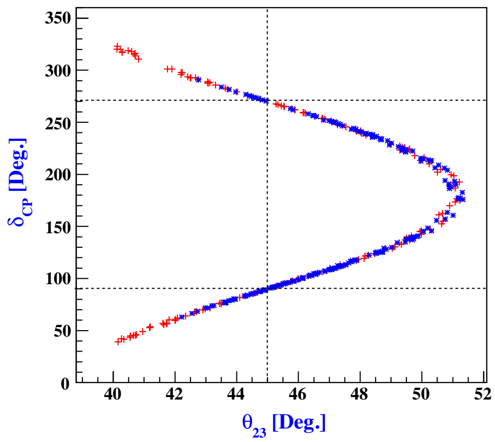

FIG. 2. Plot for predictions of δ_{CP} as a function of θ_{23}, for NO (red crosses) and IO (blue asters). For NO, $\theta_{23}[°]$ allows [40.1, 51.7], while for IO [42.2, 51.7].

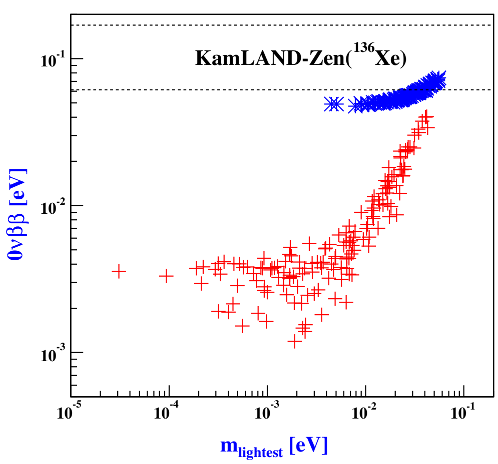

FIG. 3. Plot for the $0\nu\beta\beta$-decay rate as a function of the lightest neutrino mass m_{lightest} for NO ($m_{\text{lightest}} = m_1$ red crosses) and IO ($m_{\text{lightest}} = m_3$ blue asterisks), where the most stringent limit (90% confidence level) is given by KamLAND-Zen [36].

distinguished by the measurement of $0\nu\beta\beta$-decay rate, as can be seen in Fig. 3.

VII. CONCLUSION

We proposed an extra-dimension scenario to understand the origin of the fermion mass and mixing hierarchies by introducing the localized flavon fields and imposing the flavor symmetry $G_F(= \text{non-Abelian} \times \text{Abelian})$ through the bulk. We fixed the charges of the extra gauged $U(1)$ symmetries by the $U(1)$ gravitation anomaly free condition and found that they play a crucial role in achieving the desirable fermion mass and mixing hierarchies and protecting the Peccei-Quinn (PQ) symmetry from quantum gravity corrections.

We showed how the masses of charged fermions and quark mixing link to an extra dimension. For neutrino sectors, their masses and mixing do not depend on an extra dimension. On the other hand, although the mass of the flavored axion is not directly related to an extra dimension, there is a link among the seesaw scale, QCD axion decay constant, and extra dimension in our scenario.

When the bulk flavor symmetry G_F is broken due to the flavor fields localized at the 3-branes, we showed that the $SU(2)$-singlet flavored fermions in the bulk are integrated out to provide the effective Yukawa couplings for quarks and leptons, inherited with the G_F breaking in the two sectors. We also showed that there is a viable parameter space for the flavored axion in our model, which is consistent with the solution to the strong CP problem, the dark matter relic abundance with a misalignment mechanism, as well as the bounds from rare meson decays.

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APPENDIX A: A POSSIBLE SOURCE OF $U(1)_{X_T}$-EXPLICIT BREAKING

Spontaneous breakdown of the global $U(1)_{X_T}$ symmetry when $v_Q, v_T \neq 0$ are much higher than the electroweak scale leads to a Goldstone mode, such that the global $U(1)_{X_T}$ symmetry is explicitly broken and a possible source of $U(1)_{X_T}$-explicit breaking comes from soft breaking terms. In order for the massless ALP to become massive, we introduce relevant soft breaking terms induced by Φ_T and ϱ,

$$S_{\text{soft}} \supset -\int d^4x \bigg\{ \mu_1 g_T \Phi_0^T (\Phi_T \Phi_T)_{\mathbf{3}} + \mu_2 g_{\varrho\chi} \varrho_0 \chi \tilde{\chi} \frac{\varrho}{M_5} + \mu_3 g_{\varrho T} \varrho_0 (\Phi_T \Phi_T)_{\mathbf{1}} \left(\frac{\varrho}{M_5}\right)^3 + \text{H.c.} \bigg\}, \tag{A1}$$

where $\mu_i > 0$ have mass dimensions and $g_T, g_{\varrho\chi}, g_{\varrho T} < 0$ are dimensionless. Then, we have a simple cosine structure potential

$$V(A_T) \simeq 2m_S^2 \left(\mu_T \frac{v_T}{\sqrt{2}} + \mu_\varrho \frac{v_\varrho}{\sqrt{2}}\right) \left\{1 - \cos\left(\frac{X_T A_T}{f_T}\right)\right\}, \tag{A2}$$

where $\mu_1 \simeq \mu_2 \simeq \mu_3 \simeq m_S$ (soft supersymmetry breaking mass) was assumed and we set $h_{T,\varrho} = 0$. The potential Eq. (A2) is minimized at $A_T = 0$. A mass for the ALP is obtained,

$$m_{A_T}^2 = 2\left(\frac{m_S X_T}{f_T}\right)^2 \left(\mu_T \frac{v_T}{\sqrt{2}} + \mu_\varrho \frac{v_\varrho}{\sqrt{2}}\right), \tag{A3}$$

where μ_T and μ_ϱ are given such that $m_{A_T} \lesssim m_S$ is expected if $f_T \sim v_T$ and/or $\sim v_\varrho$. If μ_T and μ_σ become much smaller than v_T and v_ϱ, respectively, i.e., $|g_T|, |g_{\varrho\chi}|, |g_{\varrho T}| \ll 1$, a mass for the ALP could become $m_{A_T} \ll m_S$.

APPENDIX B: BOUNDARY CONDITIONS FOR THE FLAVORED BULK FERMIONS

From Eq. (25), the boundary conditions for the flavored bulk fermions are given for u-type quarks by

$$\begin{aligned}
\Psi_{u3}(x,0) &= -2\left\{Y^u_{33}Q_3 + [Y^u_{32}Q_2 + Y^u_{31}Q_1]\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)\right\}H_u, \\
\Psi_{u2}(x,0) &= -2\left\{Y^u_{22}Q_2 + Y^u_{21}Q_1\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)\right\}H_u, \\
\Psi_{u1}(x,0) &= -2Y^u_{11}Q_1H_u, \qquad \Psi^c_{u3}(x,L) = -2\hat{Y}_{t3}t^c, \\
\Psi^c_{u2}(x,L) &= -2\left\{Y_{u2}(\mathcal{U}^c\eta) + \tilde{Y}_{u2}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right) + \bar{Y}_{u2}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\right\}, \\
\Psi^c_{u1}(x,L) &= -2\left\{Y_{u1}\left(\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right) + \tilde{Y}_{u1}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\right\}.
\end{aligned} \tag{B1}$$

Similarly, for d-type quarks and leptons, we also get similar relations for the bulk fermions as follows:

$$\begin{aligned}
\Psi_{d3}(x,0) &= -2\left\{Y^d_{33}Q_3 + [Y^d_{32}Q_2 + Y^d_{31}Q_1]\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)\right\}H_d, \\
\Psi_{d2}(x,0) &= -2\left\{Y^d_{22}Q_2 + Y^d_{21}Q_1\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)\right\}H_d, \\
\Psi_{d1}(x,0) &= -2\left\{Y^d_{11}Q_1 + Y^d_{12}Q_2\left(\frac{\Phi_S\Phi_S}{M_5^2}\right)\right\}H_d, \\
\Psi^c_{d3}(x,L) &= -2\hat{Y}_{b3}b^c, \\
\Psi^c_{d2}(x,L) &= -2\left\{Y_{d2}(\mathcal{D}^c\eta) + \tilde{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right) + \bar{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)\right\}, \\
\Psi^c_{d1}(x,L) &= -2\left\{Y_{d1}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right) + \tilde{Y}_{d1}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)_{\mathbf{1}}\right\},
\end{aligned} \tag{B2}$$

$$\begin{aligned}
\Psi^c_\ell(x,L) &= -2\left\{e^c\left(Y_e\Phi_T\frac{\Phi_T}{M_5} + \tilde{Y}_e\frac{\Phi_T\Phi_T}{M_5^3}\Phi_T\varrho\right) + \mu^c\left(Y_\mu\Phi_T\frac{\varrho^2}{M_5^2} + \tilde{Y}_\mu\frac{\eta\eta}{M_5^2}\varrho\right) + \tau^c\left(Y_\tau\Phi_T\frac{\varrho^2}{M_5^2} + \tilde{Y}_\tau\frac{\eta\eta}{M_5^2}\varrho\right)\right\}, \\
\Psi_\ell(x,0) &= -2Y_LLH_d.
\end{aligned} \tag{B3}$$

APPENDIX C: FERMION MASS MATRICES AND QUARK MIXING MATRICES

We present the details of quark and lepton mass matrices obtained in 4D effective theory. From Eqs. (28) and (29), the up(down)-type quark $\mathcal{M}_{u(d)}$ and charged-lepton $\mathcal{M}_\ell$ mass matrices are expressed as

$$\mathcal{M}_d = \begin{pmatrix} m^d_{11} e^{i\left(3\frac{A_1}{v_{\mathcal{F}}}+14\frac{A_2}{v_g}\right)} e^{-M_{d_1}L} & 0 & 0 \\ m^d_{21} e^{i\left(3\frac{A_1}{v_{\mathcal{F}}}+14\frac{A_2}{v_g}\right)} e^{-M_{d_2}L} & m^d_{22} e^{i\left(\frac{A_1}{v_{\mathcal{F}}}+14\frac{A_2}{v_g}\right)} e^{-M_{d_2}L} & 0 \\ m^d_{31} e^{i\left(11\frac{A_1}{v_{\mathcal{F}}}-18\frac{A_2}{v_g}\right)} e^{-M_{d_3}L} & m^d_{32} e^{i\left(9\frac{A_1}{v_{\mathcal{F}}}-18\frac{A_2}{v_g}\right)} & m^d_{33} e^{i\left(3\frac{A_1}{v_{\mathcal{F}}}-18\frac{A_2}{v_g}\right)} e^{-M_{d_3}L} \end{pmatrix} \cosh 2\sigma(L),$$

$$\mathcal{M}_u = \begin{pmatrix} m^u_{11} e^{i\left(2\frac{A_1}{v_{\mathcal{F}}}+6\frac{A_2}{v_g}\right)} e^{-M_{u_1}L} & 0 & 0 \\ m^u_{21} e^{6i\frac{A_2}{v_g}} e^{-M_{u_2}L} & m^u_{22} e^{6i\frac{A_2}{v_g}} e^{-M_{u_2}L} & 0 \\ m^u_{31} e^{i\left(8\frac{A_1}{v_{\mathcal{F}}}-11\frac{A_2}{v_g}\right)} e^{-M_{u_3}L} & m^u_{32} e^{i\left(6\frac{A_1}{v_{\mathcal{F}}}-11\frac{A_2}{v_g}\right)} & m^u_{33} e^{-11i\frac{A_2}{v_g}} e^{-M_{u_3}L} \end{pmatrix} \cosh 2\sigma(L),$$

$$\mathcal{M}_\ell = \begin{pmatrix} m^e_{11} e^{i\left(\frac{A_1}{v_{\mathcal{F}}}-41\frac{A_2}{v_g}+2\frac{A_T}{v_t}\right)} & 0 & 0 \\ 0 & m^\mu_{22} e^{i\left(\frac{A_1}{v_{\mathcal{F}}}-31\frac{A_2}{v_g}-\frac{A_T}{v_t}\right)} & 0 \\ 0 & 0 & m^\tau_{33} e^{i\left(\frac{A_1}{v_{\mathcal{F}}}-27\frac{A_2}{v_g}-\frac{A_T}{v_t}\right)} \end{pmatrix} e^{-M_\ell L} \cosh 2\sigma(L), \tag{C1}$$

where $A_{1(2,T)}$ is massless mode $v_t = \sqrt{v_T^2 + v_\varrho^2}$, and

$$m^d_{11} \simeq (i\hat{y}^d_{11}\Delta_T\Delta_\varrho - \hat{\tilde{y}}^d_{11}\Delta^2_\eta)\Delta_\eta \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^3 \left(\frac{2}{1+\kappa^2}\right)^{\frac{3}{2}} \Delta^{17}_\chi v_d,$$

$$m^d_{21} \simeq -3\kappa^2\hat{y}^d_{21}\Delta_\eta \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^3 \left(\frac{2}{1+\kappa^2}\right)^{\frac{3}{2}} \Delta^{17}_\chi v_d,$$

$$m^d_{22} \simeq -\hat{y}^d_{22}\Delta_\eta \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right| \left(\frac{2}{1+\kappa^2}\right)^{\frac{1}{2}} \Delta^{15}_\chi v_d,$$

$$m^d_{31} \simeq 3\kappa^2\hat{\mathbf{y}}^b_{31} \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^{11} \left(\frac{2}{1+\kappa^2}\right)^{\frac{11}{2}} \Delta^{29}_\chi v_d,$$

$$m^d_{32} \simeq 3\kappa^2\hat{\mathbf{y}}^b_{32} \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^9 \left(\frac{2}{1+\kappa^2}\right)^{\frac{9}{2}} \Delta^{27}_\chi v_d,$$

$$m^d_{33} \simeq \hat{\mathbf{y}}^b_{33} \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^3 \left(\frac{2}{1+\kappa^2}\right)^{\frac{3}{2}} \Delta^{21}_\chi v_d,$$

$$m^u_{11} \simeq (i\hat{y}^u_{11}\Delta_T\Delta_\varrho - \hat{\tilde{y}}^u_{11}\Delta^2_\eta)\Delta_\eta \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^2 \frac{2}{1+\kappa^2} \Delta^8_\chi v_u,$$

$$m^u_{21} \simeq -3\kappa^2\hat{y}^u_{21}\Delta_\eta \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^2 \frac{2}{1+\kappa^2} \Delta^8_\chi v_u,$$

$$m^u_{22} \simeq -\hat{y}^u_{22}\Delta_\eta\Delta^6_\chi v_u,$$

$$m^u_{31} \simeq 3\kappa^2\hat{\mathbf{y}}^t_{31} \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^8 \left(\frac{2}{1+\kappa^2}\right)^4 \Delta^{19}_\chi v_u,$$

$$m^u_{32} \simeq 3\kappa^2\hat{\mathbf{y}}^t_{32} \left|\frac{\delta^{\rm G}_1}{\delta^{\rm G}_2}\right|^6 \left(\frac{2}{1+\kappa^2}\right)^3 \Delta^{17}_\chi v_u,$$

$$m^u_{33} \simeq \hat{\mathbf{y}}^t_{33}\Delta^{11}_\chi v_u,$$

$$
\begin{aligned}
m_{11}^{e} &\simeq \hat{y}_e \Delta_T^2 \Delta_\chi^{42} \left|\frac{\delta_1^{\mathrm{G}}}{\delta_2^{\mathrm{G}}}\right| \left(\frac{2}{1+\kappa^2}\right)^{\frac{1}{2}} v_d,\\
m_{22}^{\mu} &\simeq (\hat{y}_\mu \Delta_T \Delta_\varrho + \hat{\tilde{y}}_\mu \Delta_\eta^2)\Delta_\chi^{32}\Delta_\varrho \left|\frac{\delta_1^{\mathrm{G}}}{\delta_2^{\mathrm{G}}}\right| \left(\frac{2}{1+\kappa^2}\right)^{\frac{1}{2}} v_d,\\
m_{33}^{\tau} &\simeq (\hat{y}_\tau \Delta_T \Delta_\varrho + \hat{\tilde{y}}_\tau \Delta_\eta^2)\Delta_\varrho \Delta_\chi^{28} \left|\frac{\delta_1^{\mathrm{G}}}{\delta_2^{\mathrm{G}}}\right| \left(\frac{2}{1+\kappa^2}\right)^{\frac{1}{2}} v_d,
\end{aligned} \tag{C2}
$$

recalling that the hat Yukawa couplings are of order unity such as $\frac{1}{\sqrt{10}} \lesssim \hat{y}, \hat{\mathbf{y}} \lesssim \sqrt{10}$.

The 4D action in terms of the flavored bulk fermions at the boundaries reads

$$
\begin{aligned}
S_Y = \int d^4x\sqrt{g}\Bigg[\Bigg\{ & Y_{33}^u \Psi_{u3}^c(x,0)Q_3 + Y_{22}^u \Psi_{u2}^c(x,0)Q_2 + Y_{11}^u \Psi_{u1}^c(x,0)Q_1 + (Y_{21}^u \Psi_{u2}^c(x,0)Q_1 + Y_{31}^u \Psi_{u3}^c(x,0)Q_1\\
& + Y_{32}^u \Psi_{u3}^c(x,0)Q_2)\frac{(\Phi_S\Phi_S)}{M_5^2}\Bigg\}H_u + \hat{Y}_{t3} t^c \Psi_{u3}(x,L) + \left(Y_{u2}\mathcal{U}^c\eta + \tilde{Y}_{u2}\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\Psi_{u2}(x,L)\\
& + \bar{Y}_{u2}\left(\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{u2}(x,L) + \left(Y_{u1}\mathcal{U}^c\eta\frac{\Phi_T\varrho}{M_5^2} + \tilde{Y}_{u1}\mathcal{U}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{u1}(x,L) + \Bigg\{Y_{33}^d\Psi_{d3}^c(x,0)Q_3 + Y_{22}^d\Psi_{d2}^c(x,0)Q_2\\
& + Y_{11}^d\Psi_{d1}^c(x,0)Q_1 + ((Y_{21}^d\Psi_{d2}^c(x,0) + Y_{31}^d\Psi_{d3}^c(x,0))Q_1 + Y_{32}^d\Psi_{d3}^c(x,0)Q_2)\frac{\Phi_S\Phi_S}{M_5^2}\Bigg\}H_d + \hat{Y}_{b3}b^c\Psi_{d3}(x,L)\\
& + Y_{d2}(\mathcal{D}^c\eta)\Psi_{d2}(x,L) + \tilde{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\Psi_{d2}(x,L) + \bar{Y}_{d2}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{d2}(x,L) + Y_{d1}\left(\mathcal{D}^c\eta\frac{\Phi_T\varrho}{M_5^2}\right)\Psi_{d1}(x,L)\\
& + \tilde{Y}_{d1}\left(\mathcal{D}^c\eta\frac{\eta\eta}{M_5^2}\right)\Psi_{d1}(x,L) + Y_L\Psi_\ell^c(x,0)LH_d + (y_\nu(N^cL) + \tilde{y}_\nu(N^cL)\Phi_S)\frac{H_u}{M_5} + \frac{1}{2}y_R(N^cN^c)\Phi_S\\
& + \frac{1}{2}(y_\Theta\Theta + y_{\tilde{\Theta}}\tilde{\Theta})(N^cN^c) + Y_e e^c\Psi_\ell(x,L)\frac{\Phi_T\Phi_T}{M_5} + \tilde{Y}_e e^c\left(\Psi_\ell(x,L)\frac{\Phi_T\Phi_T}{M_5}\frac{\Phi_T\varrho}{M_5^2}\right) + Y_\mu\mu^c\Psi_\ell(x,L)\Phi_T\frac{\varrho^2}{M_5^2}\\
& + \tilde{Y}_\mu\mu^c\Psi_\ell(x,L)\frac{\eta\eta}{M_5^2}\varrho + Y_\tau\tau^c\Psi_\ell(x,L)\Phi_T\frac{\varrho^2}{M_5^2} + \tilde{Y}_\tau\tau^c\Psi_\ell(x,L)\frac{\eta\eta}{M_5^2}\varrho + \mathrm{H.c.}\Bigg].
\end{aligned} \tag{C3}
$$

Here, using Eq. (17), we get $f_{iR}^0(0) \approx f_{iR}^0(L)e^{\frac{1}{2}\sigma(L)-M_{f_i}L}$ and $f_{iL}^0(L) \approx f_{iL}^0(0)e^{-\frac{1}{2}\sigma(L)-M_{f_i}L}$ at the boundaries, which in turn lead to $\Psi_{u3}^c(x,0) \approx \Psi_{u3}^c(x,L)e^{2\sigma(L)-M_{u_3}L}$, $\Psi_{u3}(x,L) \approx \Psi_{u3}(x,0)e^{-2\sigma(L)-M_{u_3}L}$, etc. Plugging the above formulas for the Ψ fields at the boundaries as well as Eqs. (B1)–(B3) into Eq. (C3), we obtain the 4D Yukawa interactions given by Eqs. (28) and (29) after rescaling the dimensionful parameters Eq. (27).

From the quark mass matrix of Eq. (C1), its left-handed mixing matrix V_L^f with $f = u, d$ reads at leading order [37] with

$$
V_L^f = \begin{pmatrix}
\left(1-\frac{1}{2}C_f^2\tilde{\lambda}^2\right)e^{-i(\phi_2^f+\phi_3^f)} & -C_f\tilde{\lambda}e^{i(\phi_2^f+\phi_3^f)} & -B_f\tilde{\lambda}^3e^{-i\phi_2^f}\\
C_f\tilde{\lambda}e^{i(\phi_3^f-\phi_1^f)} & \left(1-C_f\frac{1}{2}\tilde{\lambda}^2\right)e^{i(\phi_3^f-\phi_1^f)} & -A_f\tilde{\lambda}^2e^{i(\phi_2^f-\phi_1^f)}\\
\tilde{\lambda}^3(A_fC_fe^{i(\phi_1^f+\phi_3^f)} + B_fe^{i(\phi_1^f+\phi_2^f-\phi_3^f)}) & A_f\tilde{\lambda}^2e^{i(\phi_1^f+\phi_3^f)} & e^{i(\phi_1^f+\phi_2^f)}
\end{pmatrix} + \mathcal{O}(\tilde{\lambda}^4), \tag{C4}
$$

where $C_f\tilde{\lambda}$, $B_f\tilde{\lambda}^3$, and $A_f\tilde{\lambda}^2$ correspond to the mixing angles θ_{12}^f, θ_{13}^f, and θ_{23}^f, respectively, in the standard parametrization of the mixing matrix. Then, setting $\tilde{\lambda} = \lambda(C_d - C_u)$, $\phi_3^u = \phi_3^d$, $\phi_2^u = \phi_2^d$, $\frac{A_d-A_u}{(C_d-C_u)^2} = A$, and $\frac{B_u-B_d}{(C_d-C_u)^3} = A\sqrt{\rho^2+\eta^2}$ and using $\frac{C_u}{(C_d-C_u)^2} \sim \frac{A_u}{(C_d-C_u)^2} \sim \tilde{\lambda}$ with the Wolfenstein parameters $\lambda, A, \rho, \eta, \delta_{CP}^q$ [29], one can obtain the quark mixing matrix $V_{\mathrm{CKM}} = V_L^u V_L^{d\dagger}$ by redefining quark fields $c \to e^{-i(\phi_2^d+2\phi_3^d-\phi_1^u)}c$, $s \to e^{i(\phi_1^d-\phi_2^d-2\phi_3^d)}s$, $t \to e^{-i(\phi_1^u+\phi_2^d+2\phi_3^d)}t$, and $b \to e^{-i(\phi_1^d+\phi_2^d+2\phi_3^d)}b$, and the CP-violating phase is given by $\delta_{CP}^q = \phi_2^d - 2\phi_3^d = \tan^{-1}(\eta/\rho)$. Subsequently, the down- and up-type quark masses are obtained for $|M_{f_1}L| > |M_{f_2}L| > |M_{f_3}L|$ in an analytical approximation as

$$m_b \simeq |\hat{\mathbf{y}}^b_{33}| \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right|^3 \left(\frac{2}{1+\kappa^2} \right)^{\frac{3}{2}} \Delta_\chi^{21} e^{-M_{d_3} L} \cosh 2\sigma(L) v_d,$$
$$m_s \simeq |\hat{y}^d_{22}| \Delta_\eta \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right| \left(\frac{2}{1+\kappa^2} \right)^{\frac{1}{2}} \Delta_\chi^{15} e^{-M_{d_2} L} \cosh 2\sigma(L) v_d,$$
$$m_d \simeq 2|\hat{y}^d_{11}| \Delta_T \Delta_\varrho \Delta_\eta \sin\phi_d \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right|^3 \left(\frac{2}{1+\kappa^2} \right)^{\frac{3}{2}} \Delta_\chi^{17} e^{-M_{d_1} L} \cosh 2\sigma(L) v_d, \tag{C5}$$

$$m_t \simeq |\hat{\mathbf{y}}^t_{33}| \Delta_\chi^{11} e^{-M_{u_3} L} \cosh 2\sigma(L) v_u,$$
$$m_c \simeq |\hat{y}^u_{22}| \Delta_\eta \Delta_\chi^6 e^{-M_{u_2} L} \cosh 2\sigma(L) v_u,$$
$$m_u \simeq 2|\hat{y}^u_{11}| \Delta_T \Delta_\varrho \Delta_\eta \sin\phi_u \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right|^2 \frac{2}{1+\kappa^2} \Delta_\chi^8 e^{-M_{u_1} L} \cosh 2\sigma(L) v_u, \tag{C6}$$

where in the last equations of Eqs. (C5) and (C6) $|\hat{y}^f_{11}| \Delta_T \Delta_\varrho = |\hat{\tilde{y}}^f_{11}| \Delta_\eta^2$ and $\tilde{\phi}_f = \phi_f + \pi/2$ with $f = u, d$ were used for simplicity. In addition, the ratio of electroweak Higgs field VEVs $\langle H_u \rangle / \langle H_d \rangle$ is approximately given in terms of the PDG [29] value by

$$\tan\beta \simeq \left(\frac{m_t}{m_b} \right)_{\rm PDG} \left| \frac{\hat{\mathbf{y}}^b_{33}}{\hat{\mathbf{y}}^t_{33}} \right| \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right|^3 \left(\frac{2}{1+\kappa^2} \right)^{\frac{3}{2}} \Delta_\chi^{10} e^{(M_{u_3} - M_{d_3}) L}.$$

APPENDIX D: HIGHER-DIMENSIONAL OPERATORS

Higher-dimensional operators arising from the one-loop exchange of the flavored bulk fermion pairs $\Psi_f^c \Psi_f$ and $F_f^c F_f$ read

$$W^{\rm h.o}_{\Psi,F} \supset \delta(y-L) \left\{ \tilde{\lambda}_\chi \frac{\chi\tilde{\chi}}{M_5^2} (\Psi_f^c \Psi_f)_{\mathbf{1}} + \tilde{\lambda}_\eta \frac{(\eta\eta)_{\mathbf{3}}}{M_5^2} (\Psi_f^c \Psi_f)_{\mathbf{3}} + \tilde{\lambda}_T \frac{(\Phi_T \Phi_T)_{\mathbf{1}} \varrho^2}{M_5^4} (\Psi_f^c \Psi_f)_{\mathbf{1}} + \cdots + (\Psi_f \leftrightarrow F_f) \right\}$$
$$+ \delta(y) \left\{ \text{leading-order interactions} \times \left(\frac{\Psi_f^c \Psi_f}{M_5^4} \right) + (\Psi_f \leftrightarrow F_f) \right\}, \tag{D1}$$

which can be absorbed into the leading-order operators by redefinition of the corresponding coefficients.

APPENDIX E: MAJORANA AND DIRAC NEUTRINO MASS TERMS

In the Lagrangian given as Eq. (29), the right-handed Majorana mass term reads with the help of VEV configurations Eq. (6) as

$$M_R = M \begin{pmatrix} 1 + \frac{2}{3}\tilde{\kappa} e^{i\phi} & -\frac{1}{3}\tilde{\kappa} e^{i\phi} & -\frac{1}{3}\tilde{\kappa} e^{i\phi} \\ -\frac{1}{3}\tilde{\kappa} e^{i\phi} & \frac{2}{3}\tilde{\kappa} e^{i\phi} & 1 - \frac{1}{3}\tilde{\kappa} e^{i\phi} \\ -\frac{1}{3}\tilde{\kappa} e^{i\phi} & 1 - \frac{1}{3}\tilde{\kappa} e^{i\phi} & \frac{2}{3}\tilde{\kappa} e^{i\phi} \end{pmatrix}, \tag{E1}$$

where Eq. (8) is used, and $\tilde{\kappa} \equiv \kappa |\hat{y}_R / \hat{y}_\Theta|$ and $\phi \equiv \arg(\hat{y}_R / \hat{y}_\Theta)$. Without loss of generality, setting $\hat{y}_\Theta = 1$, a common factor M in Eq. (E1) can be replaced by the QCD axion decay constant F_A as shown in Eq. (34). In addition, the Dirac neutrino mass term also reads with Eqs. (6) and (7) as

$$m_D = \hat{y}_\nu \Delta_\chi^8 \left| \frac{X_2 \delta_1^{\rm G}}{X_1 \delta_2^{\rm G}} \right|^8 \left(\frac{2}{1+\kappa^2} \right)^4 v_u$$
$$\times \begin{pmatrix} 1 + \frac{2}{3}\kappa y_\nu^s e^{i\phi_s} & -\kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} - \frac{1}{2} y_\nu^a e^{i\phi_a}\right) & -\kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} + \frac{1}{2} y_\nu^a e^{i\phi_a}\right) \\ -\kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} + \frac{1}{2} y_\nu^a e^{i\phi_a}\right) & \frac{2}{3}\kappa y_\nu^s e^{i\phi_s} & 1 - \kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} - \frac{1}{2} y_\nu^a e^{i\phi_a}\right) \\ -\kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} - \frac{1}{2} y_\nu^a e^{i\phi_a}\right) & 1 - \kappa\left(\frac{1}{3} y_\nu^s e^{i\phi_s} + \frac{1}{2} y_\nu^a e^{i\phi_a}\right) & \frac{2}{3}\kappa y_\nu^s e^{i\phi_s} \end{pmatrix}, \quad \text{(E2)}$$

where Eq. (8) is used, and $y_\nu^\alpha \equiv |\hat{\hat{y}}_{\nu\alpha}/\hat{y}_\nu|$ and $\phi_\alpha \equiv \arg(y_\nu^\alpha)$ with $\alpha = a, s$. y_ν^a, and y_ν^s correspond to the Yukawa coupling of the antisymmetric and symmetric operators $(N^c L)_{\mathbf{3a(s)}} \Phi_S H_u / M_5$ given in Eq. (12) which naturally cause the deviation from the tribimaximal mixing, such that it is responsible for the nonzero θ_{13} and a deviation of θ_{23} from the maximal mixing $\pi/4$.

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