Radiative decays of the neutral $Z_c(3900)$ and $Z_c(4020)$

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We study the radiative decays $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi'_{cJ})$ ($J = 0, 1, 2$), with the assumption that $Z_c(3900)$ and $Z_c(4020)$ couple strongly to $D\bar{D}^* + \text{c.c.}$ and $D^*\bar{D}^*$ channels, respectively. By considering the contributions of intermediate charmed mesons triangle loops within an effective Lagrangian approach, it is shown that the calculated partial widths of $Z_c(3900) \to \gamma\chi_{cJ}$ are about a few hundred keV, while the obtained partial widths $Z_c(4020) \to \gamma\chi_{cJ}$ are about tens of keV. The predicted partial widths of $Z_c(3900) \to \gamma\chi'_{c0,1}$ are less than 1 keV, which is mainly due to the very small phase space. For $Z_c(4020) \to \gamma\chi'_{c0,2}$, the calculated partial widths are usually smaller than 1 keV. For the $Z_c(4020) \to \gamma\chi'_{c1}$ process, the obtained partial widths can reach up to the order of 10 keV. Furthermore, the dependence of these ratios between different decay modes on the masses of $Z_c(3900)$ or $Z_c(4020)$ are also investigated, which may be a good quantity for the experiments. It is hoped that the calculations here could be tested by future experiments.

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I. INTRODUCTION

The discovery of $X(3872)$ in 2003 opened the gate to the abundance of the XYZ structures in the heavy quarkonium region [1]. Many of them cannot be accommodated in the conventional quark model as $Q\bar{Q}$ and thus turn out to be excellent candidates for exotic states. A large amount of experimental and theoretical studies are devoted to those XYZ states [2–9]. Among these states, the charged charmoniumlike states $Z_c(3900)$ [10,11] and $Z_c(4020)$ [12] have attracted special attention due to their four-quark nature. In 2013, the BESIII Collaboration first observed a new charged state $Z_c(3900)$ in the $\pi^\pm J/\psi$ invariant mass spectra of the $e^+e^- \to \pi^+\pi^- J/\psi$ reaction [10], and it was confirmed by the Belle Collaboration in the same process [11]. Later, the charged $Z_c(3900)$ was also observed in the invariant mass spectrum of $D\bar{D}^*$ in the open charm process $e^+e^- \to \pi^\pm(D\bar{D}^*)^\mp$ [13]. In addition, the first evidence of the neutral $Z_c(3900)$ decaying into $J/\psi\pi^0$, was reported in Ref. [14] using the CLEO-c data. The BESIII Collaboration reported the observation of neutral $Z_c(3900)$ in the $e^+e^- \to \pi^0\pi^0 J/\psi$ [15] and $e^+e^- \to \pi^0(D\bar{D}^*)^0$ [16], respectively. Through the partial wave analysis of process $e^+e^- \to \pi^+\pi^- J/\psi$, the quantum numbers of $Z_c(3900)$ are determined as $I^G(J^{PC}) = 1^+(1^{+-})$ [17,18].

The BESIII Collaboration found charged $Z_c(4020)$ states in the invariant mass spectrum of $\pi^\pm h_c$ in the process of $e^+e^- \to \pi^+\pi^- h_c$ [12]. The charged $Z_c(4020)$ states were confirmed in the $D^*\bar{D}^*$ invariant mass spectrum of the $e^+e^- \to \pi^\pm(D^*\bar{D}^*)^\mp$ reaction [19]. Its neutral partner was reported in the $\pi^0 h_c$ invariant mass spectrum of the $e^+e^- \to \pi^0\pi^0 h_c$ reaction [20] and in the $(D^*\bar{D}^*)^0$ invariant mass spectrum of the $e^+e^- \to (D^*\bar{D}^*)^0\pi^0$ reaction [21], respectively.

After the observation of the $Z_c(3900)$ and $Z_c(4020)$ states, many theoretical discussions have been carried out to explore their inner structures [22–42]. Since the measured masses of $Z_c(3900)$ and $Z_c(4020)$ lie slightly above the $D\bar{D}^*$ and $D^*\bar{D}^*$ mass thresholds, it may indicate that these two states are good candidates of the hadronic molecule [22–27]. Besides the molecule explanation, these two states are also identified as tetraquark states [28–33], or kinematical threshold effects [34–36]. Recently, the production and decay behaviors of $Z_c(3900)$ and $Z_c(4020)$

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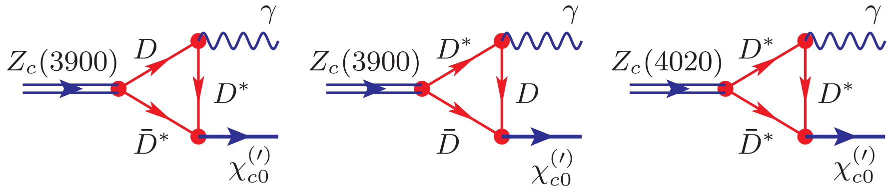

FIG. 1. The hadron-level diagrams of the radiative decay processes $Z_c(3900)/Z_c(4020) \to \gamma\chi_{c0}^{(\prime)}$. The charge conjugated diagrams are not shown but included in the calculations.

states were extensively investigated in $D\bar{D}^*$ and $D^*\bar{D}^*$ hadronic molecule scenarios in Refs. [37–42], where the theoretical calculations are consistent with the corresponding experimental measurements.

It is known that the intermediate meson loop transition is regarded as an important nonperturbative transition mechanism which has a long history [43–45] and recently it has been widely used to study the production and decays of exotic states [46–57]. The radiative transitions between neutral $Z_c(3900)/Z_c(4020)$ and the charmonia states are particular modes compared to the charged $Z_c(3900)/Z_c(4020)$. The quark and antiquark in the different components of neutral $Z_c(3900)/Z_c(4020)$ can annihilate into a photon, and the rest charm and anticharm quarks form a charmonium in the final state. In Ref. [53], the radiative decays of $Z_c(3900) \to \gamma\chi_{c0,1}$ was studied in a hadronic molecule picture using an effective Lagrangian approach. In Ref. [54], the radiative transitions from the neutral exotic $Z_c(4020)$ resonance to $X(3872)$ were investigated.

Along this line, in this work, we estimate the radiative decays $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi'_{cJ})$ $(J = 0, 1, 2)$ with the nonrelativistic effective field theory (NREFT) under the assumption that $Z_c(3900)$ and $Z_c(4020)$ couple strongly to $D\bar{D}^* +$ c.c. and $D^*\bar{D}^*$ channels in S wave, respectively. Based on this picture, in the present work, the quantum numbers of the neutral $Z_c(4020)$ state are taken to be $I^G(J^{PC}) = 1^+(1^{+-})$, which are consistent with those previous theoretical results, for instance, in Refs. [23,24,53–55].

The rest of this paper is organized as follows. In Sec. II, we present the theoretical framework used in this work. In Sec. III, numerical results are presented, and a brief summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

A. Triangle diagrams

We study the radiative decays of $Z_c(3900)$ and $Z_c(4020)$ states within the effective Lagrangian approach. Based on the strong couplings of $Z_c(3900)$ and $Z_c(4020)$ to $D\bar{D}^*$ and $D^*\bar{D}^*$, respectively, their radiative decays can be proceeded by those triangle diagrams as shown in Fig. 1, where there are three charmed mesons in the triangle loop. To be specific, we denote the one connecting the initial $Z_c(3900)/Z_c(4020)$ and the photon as $M1$, the one connecting the $Z_c(3900)/Z_c(4020)$ and the final $\chi_{cJ}^{(\prime)}$ as $M2$, and the exchanged meson between γ and χ_{cJ} as $M3$. For example, in Fig. 1(a), $M1$, $M2$, and $M3$ are the D, $\bar{D}^*$, and D^*, respectively. In addition, all the loops contributing to each decay are listed in Table I.

B. Effective interaction Lagrangians

In order to calculate the triangle loops shown in Fig. 1, we first need the effective couplings of the $Z_c(3900)$ and $Z_c(4020)$ states to $D\bar{D}^*$ and $D^*\bar{D}^*$ channels, respectively. The effective Lagrangians describing the couplings of $Z_c(3900)/Z_c(4020)$ to $D\bar{D}^*$ and $D^*\bar{D}^*$ channels via S wave can be easily written as [58],

$$\mathcal{L}_{Z_c^{(\prime)}} = z'\epsilon^{ijk}\bar{V}_a^{\dagger i} Z_c^{\prime j} V_a^{\dagger k} + z[\bar{V}_a^{\dagger i} Z_c^i P_a^\dagger - \bar{P}_a^\dagger Z_c^i V_a^{\dagger i}] + \text{H.c.}, \quad (1)$$

where Z_c and Z'_c stand for $Z_c(3900)$ and $Z_c(4020)$, respectively. V_a and P_a are the vector and pseudoscalar charmed mesons, respectively, i.e., $P_a(V_a) = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$. In the two-component notation of Ref. [59], the charmed mesons are represented by $H_a = \vec{V}_a \cdot \vec{\sigma} + P_a$, with $\vec{\sigma}$ being the Pauli matrices, and a is the light flavor index. Besides, z and z' are the effective couplings. With the above effective Lagrangians, we can obtain

$$\Gamma_{Z_c(3900)\to D^0\bar{D}^{*0}} = \frac{|z|^2}{4\pi}\frac{|\vec{q}_D|}{M_{Z_c(3900)}} M_D M_{D^*}, \quad (2)$$

$$\Gamma_{Z_c(4020)\to D^{*0}\bar{D}^{*0}} = \frac{|z'|^2}{4\pi}\frac{|\vec{q}_{D^*}|}{M_{Z'_c(4020)}} M_{D^*}^2, \quad (3)$$

TABLE I. All the possible loops contributing to each transition. The mesons are listed as $[M1, M2, M3]$.

$Z_c(3900) \to \gamma\chi_{c0}^{(\prime)}$	$[D, \bar{D}^*, D^*]$, $[D^*, \bar{D}, D]$
$Z_c(3900) \to \gamma\chi_{c1}^{(\prime)}$	$[D^*, \bar{D}, D^*]$
$Z_c(3900) \to \gamma\chi_{c2}^{(\prime)}$	$[D, \bar{D}^*, D^*]$
$Z_c(4020) \to \gamma\chi_{c0}^{(\prime)}$	$[D^*, \bar{D}^*, D^*]$
$Z_c(4020) \to \gamma\chi_{c1}^{(\prime)}$	$[D^*, \bar{D}^*, D]$
$Z_c(4020) \to \gamma\chi_{c2}^{(\prime)}$	$[D^*, \bar{D}^*, D^*]$

with $\vec{q}_D$ and $\vec{q}_{D^*}$ being the three-momenta of the D^0 and D^{*0} mesons in the rest frame of $Z_c(3900)$ and $Z_c(4020)$, respectively. Here, we have assumed that the total widths of $Z_c(3900)$ and $Z_c(4020)$ are saturated by the decays $Z_c(3900) \to D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ and $Z_c(4020) \to D^{*0}\bar{D}^{*0}$. With these values for $\Gamma_{Z_c(3900)\to D^0\bar{D}^{*0}+\bar{D}^0 D^{*0}} = 35 \pm 19$ MeV [16] and $\Gamma_{Z_c(4020)\to D^{*0}\bar{D}^{*0}} = 23 \pm 6$ MeV [21], one can get the relevant coupling constants as follows,

$$|z| = (1.67 \pm 0.45)\ \mathrm{GeV}^{-1/2}, \tag{4}$$

$$|z'| = (0.43 \pm 0.06)\ \mathrm{GeV}^{-1/2}, \tag{5}$$

where the errors are obtained with the uncertainties of the partial width of $Z_c(3900) \to D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ and $Z_c(4020) \to D^{*0}\bar{D}^{*0}$ decays [16,21]. Note that to get these above coupling constants, we have used $M_{Z_c(3900)} = 3885.7$ MeV [16], $M_{Z_c(4020)} = 4025.5$ MeV [21], $M_D = 1864.83$ MeV, and $M_{D^*} = 2006.85$ MeV as quoted in the Particle Data Group (PDG) [60].

On the other hand, the leading order Lagrangian for the coupling of the P-wave charmonium fields to the charmed and anticharmed mesons can be constructed considering parity, charge conjugation, and spin symmetry [61], which can be written as

$$\mathcal{L}_\chi = i\frac{g_1}{2}\mathrm{Tr}[\chi^{\dagger i} H_a \sigma^i \bar{H}_a] + \mathrm{H.c.}, \tag{6}$$

where $\bar{H}_a = -\vec{\bar{V}}_a \cdot \vec{\sigma} + \bar{P}_a$ is the anticharmed mesons fields. g_1 is the coupling constant of the ground P-wave charmonia to the charmed and anticharmed mesons. The Lagrangians for the coupling of the radial excited charmonia to the charmed and anticharmed mesons have the same form as Eq. (6) with the coupling constants changed to those for the excited states g_1'. Then, the Lagrangian of χ_{cJ} reads

$$\mathcal{L}_\chi = i g_1 \chi_{c2}^{\dagger ij}(V_a^i \bar{V}_a^j + V_a^j \bar{V}_a^i) + \sqrt{2} g_1 \chi_{c1}^{\dagger i}(V_a^i \bar{P}_a + P_a \bar{V}_a^i) + \frac{i}{\sqrt{3}} g_1 \chi_{c0}^{\dagger}(\vec{V}_a \cdot \vec{\bar{V}}_a + 3 P_a \bar{P}_a), \tag{7}$$

where the trace and symmetry properties $\chi_{c2}^{ij}\delta^{ij} = 0$ and $\chi_{c2}^{ij}\epsilon^{ijk} = 0$ have been used in the derivations. In this work we take $g_1 = 4\ \mathrm{GeV}^{-1/2}$ which is from an estimation using the vector meson dominance in Ref. [62]. With $M_{\chi_{c0}'} = 3.869$ GeV and $\Gamma_{\chi_{c0}'\to D\bar{D}} = 22.3$ MeV calculated in the linear potential (LP) model [63,64], we get $g_1' \simeq 1.28\ \mathrm{GeV}^{-1/2}$.

Finally, we also need the effective Lagrangians for these vertexes of charmed mesons and photon, which are [48,59,65]

$$\mathcal{L}_\gamma = \frac{e\beta}{2}\mathrm{Tr}[H_a^\dagger H_b \vec{\sigma} \cdot \vec{B} Q_{ab}] + \frac{eQ'}{2m_Q}\mathrm{Tr}[H_a^\dagger \vec{\sigma} \cdot \vec{B} H_a], \tag{8}$$

where $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ is the light quark charge matrix, and Q' is the heavy quark electric charge (in units of e). β is an effective coupling constant and, in this work, we take $\beta = 3.0\ \mathrm{GeV}^{-1}$ which is determined in the nonrelativistic constituent quark model and is adopted in the study of radiative D^* decays [65]. In Eq. (8), the first term is the magnetic moment coupling of the light quarks, while the second one is the magnetic moment coupling of the heavy quark and hence is suppressed by $1/m_Q$.

Then follow the works of Ref. [58], where the decays of $Z_b(10610)/Z_b(10650) \to \gamma\chi_{bJ}(J = 1, 2, 3)$ were investigated within the same framework as here. The transition amplitudes for $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi_{cJ}')$ are similar to those given in Ref. [58], where one just needs to change the bottom quark to charm quark sector. Thus the decay amplitudes of these triangle diagrams shown in Fig. 1 can be easily obtained. We present the explicit transition amplitudes for $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi_{cJ}')$ in Appendix.

Finally, the partial decay widths of $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi_{cJ}')$ are given by

$$\Gamma(Z_c(3900) \to \gamma\chi_{cJ}^{(\prime)}) = \frac{E_\gamma |\mathcal{M}_{Z_c(3900)\to\gamma\chi_{cJ}^{(\prime)}}|^2}{24\pi M_{Z_c(3900)}^2}, \tag{9}$$

$$\Gamma(Z_c(4020) \to \gamma\chi_{cJ}^{(\prime)}) = \frac{E_\gamma' |\mathcal{M}_{Z_c(4020)\to\gamma\chi_{cJ}^{(\prime)}}|^2}{24\pi M_{Z_c(4020)}^2}, \tag{10}$$

where E_γ and E_γ' are the photon energies in the $Z_c(3900)$ and $Z_c(4020)$ rest frame, respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

The $X(3927)$ was observed in the $\gamma\gamma \to D\bar{D}$ process by Belle [66] and *BABAR* [67] Collaborations, and has been a good candidate for the χ_{c2}' state [68]. In this work, we take $M_{\chi_{c2}'} = 3.927$ GeV as measured by the experiments [60]. In the observed spectrum of the charmonia, the only candidate of the χ_{c1}' with well-established quantum numbers is $X(3872)$, which was discovered by the Belle Collaboration [1]. However, since the proximity to the $D\bar{D}^*$ threshold, the interpretation of the $X(3872)$ as a molecular state or virtual state is very intriguing. The measured mass of $X(3872)$ is a much lighter mass than potential quark model predictions [63,68,69]. Thus we will not directly identify the $X(3872)$ as χ_{c1}', and the mass of χ_{c1}' will also be allowed to vary. To be specific, a range from 3.83 to 3.93 GeV for the mass of χ_{c1}' and $X(3872)$ will be chosen that covers the predicted values from quark models [63,68,69].

TABLE II. The final charmonium masses (in unit of GeV) are taken from the PDG [60], then calculated with screened potential (SP) in Ref. [68], linear potential (LP) in Ref. [69], and the results with SP and LP in Ref. [63].

Name	J^{PC}	Exp. [60]	[69]	[68]	LP [63]	SP [63]
χ_{c0}	0^{++}	3.415	3.424	3.433	3.415	3.415
χ_{c1}	1^{++}	3.511	3.505	3.510	3.516	3.521
χ_{c2}	2^{++}	3.556	3.556	3.554	3.552	3.553
χ'_{c0}	0^{++}	3.862?	3.852	3.842	3.869	3.848
χ'_{c1}	1^{++}	...	3.925	3.901	3.937	3.914
χ'_{c2}	2^{++}	3.927	3.972	3.937	3.967	3.937

The $X(3860)$ observed in the process $e^+e^- \to J/\psi D\bar{D}$ by the Belle Collaboration [70] serves as a good candidate for the χ'_{c0} state. The measured mass and width fit the expectation of the χ'_{c0} state predicted in the potential models [63]. Thus, we will take the mass of χ'_{c0} in a range from 3800 to 3900 MeV which covers the predicted values of the mass of χ'_{c0} from these quark models [63,68,69] and the experimental measurements [70]. In Table II, we list the obtained charmonium masses in previous works. One can see that these values from different models are consistent with each other.

In Table III, we listed the calculated partial widths of $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}$, which are obtained with $M_{Z_c(3900)} = 3885.7$ MeV, $M_{Z_c(4020)} = 4025.5$ MeV, $M_{\chi_{c0}} = 3414.7$ MeV, $M_{\chi_{c1}} = 3510.7$ MeV, and $M_{\chi_{c2}} = 3556.2$ MeV. It is found that these partial decay widths of $Z_c(3900) \to \gamma\chi_{cJ}$ are about a few hundred keV, while the obtained partial decay widths of $Z_c(4020) \to \gamma\chi_{cJ}$ are about tens of keV.

Next we study the mass effects of $\chi_{cJ}^{(\prime)}$ on these partial decay widths. In Fig. 2, we show the predicted partial widths of $Z_c(3900) \to \gamma\chi'_{c0}$ (solid line) and $Z_c(4020) \to \gamma\chi'_{c0}$ (dashed line) as a function of the mass of χ'_{c0}. The results show that the partial widths of $Z_c(3900)/Z_c(4020) \to \gamma\chi'_{c0}$ are less than 1 keV. For the decay of $Z_c(3900) \to \gamma\chi'_{c0}$ in Fig. 1, there are two subdiagrams: $[D\bar{D}^*]D^*$ and $[D^*\bar{D}]D$. The threshold of $D\bar{D}$ is approached at the lower end. Consequently, the curve for the width shows an increasing tendency at the lower end. While for $Z_c(4020) \to \gamma\chi'_{c0}$ in Fig. 1, only $[D^*\bar{D}^*]D^*$ subdiagram contributes. Because of the threshold of $D^*\bar{D}^*$ being far away the range of $M_{\chi'_{c0}}$ that we have chosen, the results show a monotonic behavior. Besides,

TABLE III. The decay widths of $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}$ in the unit of keV.

Decay mode	$\gamma\chi_{c0}$	$\gamma\chi_{c1}$	$\gamma\chi_{c2}$
$Z_c(3900)$	499.5	608.6	593.4
$Z_c(4020)$	33.4	99.4	108.9

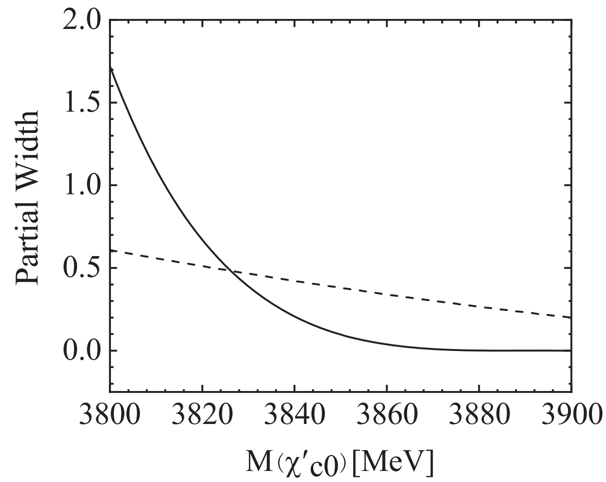

FIG. 2. Dependence of the decay widths of $Z_c(3900) \to \gamma\chi'_{c0}$ (solid line) and $Z_c(4020) \to \gamma\chi'_{c0}$ (dashed line) on the mass of χ'_{c0}.

with $M_{\chi'_{c0}} = 3.862$ GeV obtained in Ref. [70], the predicted partial widths are

$$\begin{aligned} \Gamma(Z_c(3900) \to \gamma\chi'_{c0}) &= 0.03 \text{ keV}, \\ \Gamma(Z_c(4020) \to \gamma\chi'_{c0}) &= 0.33 \text{ keV}. \end{aligned} \tag{11}$$

In Fig. 3, we show the numerical results for the partial widths of $Z_c(3900) \to \gamma\chi'_{c1}$ (solid line) and $Z_c(4020) \to \gamma\chi'_{c1}$ (dashed line) as a function of the mass of χ'_{c1}. The predicted partial widths of $Z_c(3900) \to \gamma\chi'_{c1}$ are only a few keV, while the partial widths of $Z_c(4020) \to \gamma\chi'_{c1}$ can reach up to 12 keV. For the $Z_c(3900) \to \gamma\chi'_{c1}$ process, the $[D^*\bar{D}]D^*$ subdiagram contributes and has very small phase space, while for the $Z_c(4020) \to \gamma\chi'_{c1}$ process, the

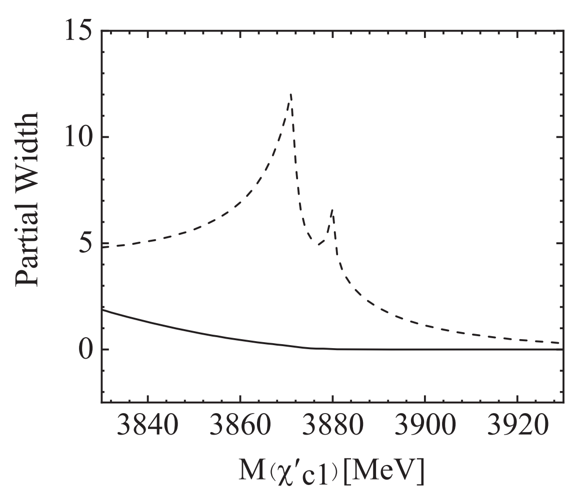

FIG. 3. Dependence of the decay widths of $Z_c(3900) \to \gamma\chi'_{c1}$ (solid line) and $Z_c(4020) \to \gamma\chi'_{c1}$ (dashed line) on the mass of the χ'_{c1}.

$[D^*\bar{D}^*]D$ subdiagram contributes. From Fig. 3 one can see that there are two obvious cusps near the thresholds of $D^{*0}\bar{D}^0$ and $D^{*+}D^-$, respectively. These cusps appear due to the triangle singularities near the thresholds of neutral and charged $D\bar{D}^*$ mesons. And if we take $M_{\chi'_{c1}} = 3.872$ GeV as the mass of $X(3872)$, the predicted partial widths are

$$\Gamma(Z_c(3900) \to \gamma\chi'_{c1}) = 0.11 \text{ keV},$$
$$\Gamma(Z_c(4020) \to \gamma\chi'_{c1}) = 8.76 \text{ keV}. \tag{12}$$

In Fig. 4, we present the dependence of the decay widths of $Z_c(4020) \to \gamma\chi'_{c2}$ on the mass of χ'_{c2}. The predicted partial widths of $Z_c(4020) \to \gamma\chi'_{c2}$ are less than 1 keV. The $[D^*\bar{D}^*]D^*$ subdiagram contributes to this process. From Fig. 4, it is shown that the partial widths monotonically decrease with the mass of χ'_{c2}. On the other hand, if we take $M_{\chi'_{c2}} = 3.927$ GeV as the mass of the $X(3927)$ state, the predicted partial width is

$$\Gamma(Z_c(4020) \to \gamma\chi'_{c2}) = 0.62 \text{ keV}. \tag{13}$$

From Eqs. (9) and (10) we know that these partial decay widths are proportional to the product of the coupling constants $|z^{(\prime)}g_1^{(\prime)}|^2$, which will be cancelled in the ratio between different partial decay widths. Therefore, the ratios among these partial decay widths are interesting, and we define

$$R_1 = \frac{\Gamma(Z_c(3900) \to \gamma\chi'_{c1})}{\Gamma(Z_c(3900) \to \gamma\chi'_{c0})}, \tag{14}$$

$$r_1 = \frac{\Gamma(Z_c(4020) \to \gamma\chi'_{c1})}{\Gamma(Z_c(4020) \to \gamma\chi'_{c0})}, \tag{15}$$

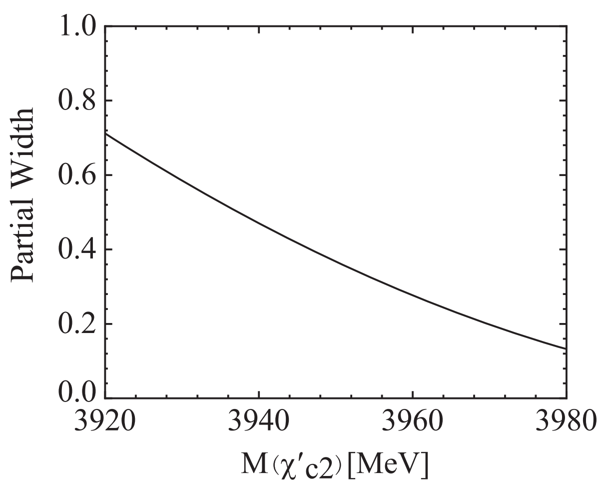

FIG. 4. Dependence of the decay widths of $Z_c(4020) \to \gamma\chi'_{c2}$ on the mass of χ'_{c2}.

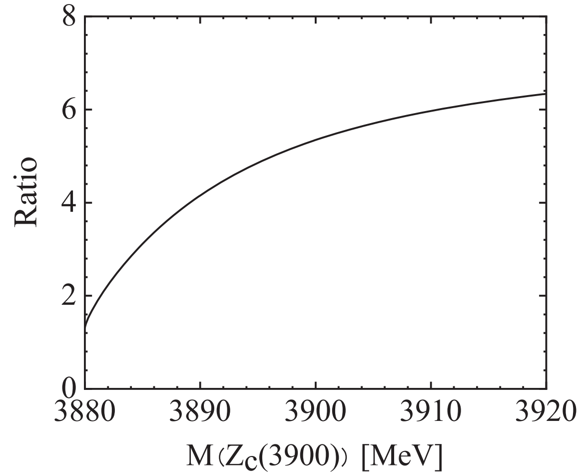

FIG. 5. Numerical results of R_1 as a function of the mass of $Z_c(3900)$.

$$r_2 = \frac{\Gamma(Z_c(4020) \to \gamma\chi'_{c2})}{\Gamma(Z_c(4020) \to \gamma\chi'_{c0})}. \tag{16}$$

The numerical results of ratio R_1 in terms of the mass of $Z_c(3900)$ are shown in Fig. 5. It is seen that there is no cusp structure because the mass of $Z_c(3900)$ chosen here is above the mass threshold of $D\bar{D}^*$. In addition, the ratio is larger than one in the considered mass range of $Z_c(3900)$.

In Fig. 6, we present the theoretical results of ratios r_1 (solid curve) and r_2 (dashed curve) as a function of the mass of $Z_c(4020)$. Ratio r_1 can reach up to about one thousand, which shows that $Z_c(4020) \to \gamma\chi'_{c1}$ is dominant and could be more easily measured by experiments. For ratio r_1, there is a double-cusp structure, which corresponds to the thresholds of the neutral and charged $D^*\bar{D}^*$ mesons. On the other hand, ratio r_2 is much less dependent on the mass of $Z_c(4020)$ and its value is about one.

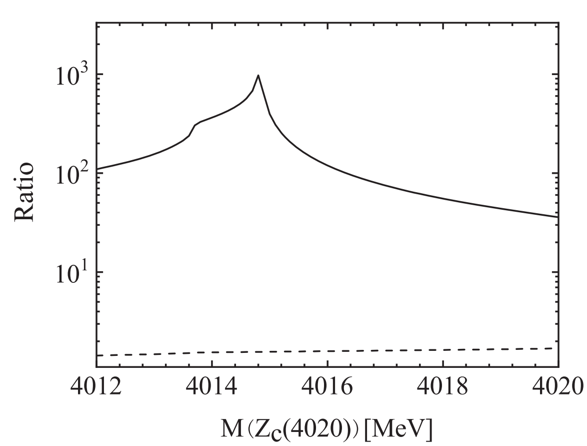

FIG. 6. Numerical results for r_1 (solid line) and r_2 (dashed line) as a function of the mass of $Z_c(4020)$.

IV. SUMMARY

In this work, we have investigated the radiative decays $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}(\gamma\chi'_{cJ})$ ($J = 0$, 1, 2), with the assumption that $Z_c(3900)$ and $Z_c(4020)$ couple strongly to $D\bar{D}^* +$ c.c. and $D^*\bar{D}^*$ channels, respectively. Including the contributions of intermediate charmed mesons triangle loops, these partial decay widths of $Z_c(3900) \to \gamma\chi_{cJ}$ and $Z_c(4020) \to \gamma\chi_{cJ}$ are calculated within an effective Lagrangian approach. With the masses quoted in the PDG [60], the obtained partial decay widths of $Z_c(3900) \to \gamma\chi_{cJ}$ are about a few hundred keV, while the obtained partial widths of $Z_c(4020) \to \gamma\chi_{cJ}$ are about tens of keV. In addition, for the $Z_c(3900) \to \gamma\chi'_{c0,1}$ and $Z_c(4020) \to \gamma\chi'_{c0,2}$ decays, the partial decay widths are less than 1 keV, which is mainly due to the very small phase space. For the $Z_c(4020) \to \gamma\chi'_{c1}$ process, the partial width can reach up to about 10 keV. Moreover, the dependence of these ratios between different decay modes on the masses of $Z_c(3900)$ or $Z_c(4020)$ are also investigated, which may be a good quantity for the experiments. The calculations here could be tested by future experiments.

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APPENDIX: THE TRANSITION AMPLITUDES

Here we give the amplitudes for the transitions $Z_c(3900)/Z_c(4020) \to \gamma\chi_{cJ}$. ϵ_1, ϵ_2, and ϵ_3 are the polarization vector of the initial state, final photon, and final charmonium state, respectively.

(i) $Z_c(3900) \to \gamma\chi_{c0}$

$$\mathcal{M}_{[D,\bar{D}^*,D^*]} = \frac{1}{\sqrt{6}} ezg_1\epsilon_{ijk}\epsilon_1^i\epsilon_2^j q^k\left(\beta Q + \frac{Q'}{m_c}\right) \times I(q, D, \bar{D}^*, D^*),$$

$$\mathcal{M}_{[D^*,\bar{D},D]} = -\frac{3}{\sqrt{6}} ezg_1\epsilon_{ijk}\epsilon_1^i\epsilon_2^j q^k\left(\beta Q + \frac{Q'}{m_c}\right) \times I(q, D^*, \bar{D}, D). \tag{A1}$$

(ii) $Z_c(3900) \to \gamma\chi_{c1}$

$$\mathcal{M}_{[D^*,\bar{D},D^*]} = -ezg_1[\epsilon_1\cdot q\epsilon_2\cdot\epsilon_3 - q\cdot\epsilon_3\epsilon_1\cdot\epsilon_2] \times\left(\beta Q - \frac{Q'}{m_c}\right) I(q, D^*, \bar{D}, D^*). \tag{A2}$$

(iii) $Z_c(3900) \to \gamma\chi_{c2}$

$$\mathcal{M}_{[D,\bar{D}^*,D^*]} = -\sqrt{2}ezg_1\epsilon_{ijk}\epsilon_3^{il}\epsilon_2^j q^k\epsilon_{1l}\left(\beta Q + \frac{Q'}{m_c}\right) \times I(q, D, \bar{D}^*, D^*). \tag{A3}$$

(iv) $Z_c(4020) \to \gamma\chi_{c0}$

$$\mathcal{M}_{[D^*,\bar{D}^*,D^*]} = -\frac{1}{\sqrt{6}} iez'g_1\epsilon_{ijk}\epsilon_1^i(q^k\epsilon_2^j - q^j\epsilon_2^k) \times\left(\beta Q - \frac{Q'}{m_c}\right) I(q, D^*, \bar{D}^*, D^*). \tag{A4}$$

(v) $Z_c(4020) \to \gamma\chi_{c1}$

$$\mathcal{M}_{[D^*,\bar{D}^*,D]} = iez'g_1[\epsilon_1\cdot\epsilon_2 q\cdot\epsilon_3 - \epsilon_2\cdot\epsilon_3 q\cdot\epsilon_1] \times\left(\beta Q + \frac{Q'}{m_c}\right) I(q, D^*, \bar{D}^*, D). \tag{A5}$$

(vi) $Z_c(4020) \to \gamma\chi_{c2}$

$$\mathcal{M}_{[D^*,\bar{D}^*,D^*]} = \sqrt{2}iez'g_1\epsilon_{ijk}\epsilon_1^i\epsilon_3^{jl}[q_l\epsilon_2^k - q^k\epsilon_{2l}] \times\left(\beta Q - \frac{Q'}{m_c}\right) I(q, D^*, \bar{D}^*, D^*). \tag{A6}$$

In the above amplitudes, the basic three-point loop function $I(q)$ is as follows [71]:

$$I(q) = i\int\frac{d^d l}{(2\pi)^d}\frac{1}{(l^2 - m_1^2 + i\epsilon)[(P-l)^2 - m_2^2 + i\epsilon]}\frac{1}{[(l-q)^2 - m_3^2] + i\epsilon} = \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3}\frac{1}{\sqrt{a}}\left[\tan^{-1}\left(\frac{c'-c}{2\sqrt{ac}}\right) + \tan^{-1}\left(\frac{2a + c' - c}{2\sqrt{a(c'-a)}}\right)\right], \tag{A7}$$

where the $\mu_{ij} = m_i m_j/(m_i + m_j)$ are the reduced masses, $b_{12} = m_1 + m_2 - M$, $b_{23} = m_2 + m_3 + q^0 - M$, and the M represents the mass of the initial particle. $a = (\mu_{23}/m_3)^2\vec{q}^2$, $c = 2\mu_{12}b_{12}$, $c' = 2\mu_{23}b_{23} + \mu_{23}\vec{q}^2/m_3$. m_1, m_2, and m_3 represent the masses of up, down, and right charmed mesons in the triangle loop, respectively.

It is worth mentioning that considering the nonrelativistic normalization of the charmonium and charmed meson fields, a factor $\sqrt{M_i M_f} m_1 m_2 m_3$ should be multiplied in each amplitude. Therefore, $I(q, M1, M2, M3) = \sqrt{M_i M_f} m_1 m_2 m_3 I(q)$, where M_i and M_f represent the masses of initial and final particles, respectively.

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