Investigations of charmless decays of $X(3872)$ via intermediate meson loops

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The charmless decay processes of $X(3872)$ provide us a good platform to study the nature and the decay mechanism of $X(3872)$. Based on a molecular nature of $X(3872)$ as a $\bar{D}D^*$ bound state, we have investigated the charmless decays $X(3872) \to VV$ and VP via intermediate $D^*\bar{D}$ + c.c. meson loops, where V and P stand for light vector and pseudoscalar mesons, respectively. We discuss three cases, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$) and neutral components dominant ($\theta = \pi/6$), where θ is a phase angle describing the proportion of neutral and charged constituents. The proportion of neutral and charged constituent has an influence on the decay widths of $X(3872) \to VV$ and VP. With the coupling constant of $X(3872)$ to the $\bar{D}D^*$ channel obtained under the molecule ansatz of $X(3872)$ resonance, the predicted decay widths of $X(3872) \to VV$ are about tens of keVs, while the decay width can reach a few hundreds of keVs for $X(3872) \to VP$. The dependence of these ratios between different decay modes of $X(3872) \to VV$ and $X(3872) \to VP$ to the mixing angle θ is also investigated. It is expected that the theoretical calculations here can be tested by future experiments.

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I. INTRODUCTION

In 2003, the $X(3872)$ state was first observed by the Belle collaboration in the $J/\psi\pi^+\pi^-$ invariant mass spectrum of the $B \to KX(3872) \to K\pi^+\pi^- J/\psi$ decay [1]. Then, it was confirmed in the $J/\psi\pi^+\pi^-$ channel from $p\bar{p}$ collisions by CDF and D0 collaborations [2,3], and e^+e^- collisions by the *BABAR* collaboration [4,5]. Its quantum numbers were determined to be $I^G(J^{PC}) = 0^+(1^{++})$ by the LHCb collaboration [6]. There are two salient features of $X(3872)$, one is that it has very narrow width ($\Gamma_X < 1.2$ MeV), the other one is that its mass is extremely close to the mass threshold of the $D^0\bar{D}^{*0}$ channel.

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The interpretation of the nature of $X(3872)$ is still an open question. Since its quantum numbers are $J^{PC} = 1^{++}$ and its mass is very close to the $D^0\bar{D}^{*0}$ threshold, one natural explanation is that it is a $D\bar{D}^*$ hadronic molecule as discussed in Refs. [7–34]. In general, a hadronic molecule can couple to other components which have the same quantum numbers. For instance, the possibility of a charmonium $c\bar{c}$ excited state admixture was investigated in Refs. [35,36]. It was also pointed out that the $D^\pm D^{*\mp}$ and $D_s^+ D_s^{*-}$ components are necessary to explain the branching ratio of $X(3872)$ to $J/\psi\rho$ and $J/\psi\omega$ [37–39]. On the other hand, the $X(3872)$ is also considered as a tetraquark state [40–43]. However, searching for the charged partners of $X(3872)$ shows negative results [44]. Besides, the $X(3872)$ was also viewed as a conventional charmonium state [45–47].

In Ref. [48], the isospin violating decay process of $X(3872) \to J/\psi\rho$ was estimated using final state interactions (FSI) by considering the intermediate $D\bar{D}^*$ meson loop, where it was found that the contribution from FSI is tiny. The radiative decays $X(3872) \to \gamma\psi/\psi'$ were investigated in Refs. [18,32,49], and the results support the molecular picture of $X(3872)$. While in Refs. [50–52], the pionic transition from $X(3872)$ to χ_{cJ} was studied.

In Ref. [50] it was concluded that these decay rates exhibit significantly different patterns depending on a pure charmonium or a multiquark structure of $X(3872)$.

All of these above theoretical studies of $X(3872)$ focus on its charmful decay modes. To better understand the nature of $X(3872)$, the study of its other decay modes is needed. For example, the charmless decays can also provide us a good platform to further study the nature of $X(3872)$. In this work, under the molecule ansatz of the $X(3872)$, which is a bound state of $\bar{D}D^*$, we will investigate the charmless decays of $X(3872) \to VV$ and VP (V and P stand for the vector meson and pseudoscalar meson) via intermediate charmed meson loops in an effective Lagrangian approach.

This article is organized as follows. In Sec. II, based on a molecular nature of $X(3872)$ as a $\bar{D}D^*$ bound state, we present the related decay amplitudes obtained with the effective Lagrangians constructed in the heavy quark limit and chiral symmetry. In Sec. III, we show our numerical results and discussions, and the last section is devoted to a short summary.

II. THEORETICAL FRAMEWORK

A. Coupling constant and decay diagrams

For a state slightly below an S-wave two-hadron threshold, the effective coupling constant of this state to the two-body channel, $g_{\rm eff}$, is related to the probability of finding the two-hadron component in the physical wave function of the bound state, c_i^2, and the binding energy $\epsilon = m_1 + m_2 - M$ [53–55],

$$g_{\rm eff}^2 = 16\pi c_i^2 (m_1 + m_2)^2 \sqrt{\frac{2\epsilon}{\mu}}, \tag{1}$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of m_1 and m_2.

We assume that the $X(3872)$ is a S-wave molecular state with quantum numbers $J^{PC} = 1^{++}$ given by the superposition of $D^0\bar{D}^{*0}$ and $D^\pm D^{*\mp}$ hadronic configurations as

$$|X(3872)\rangle = \frac{\cos\theta}{\sqrt{2}} |D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\rangle + \frac{\sin\theta}{\sqrt{2}} |D^{*+}D^- + D^- D^{*+}\rangle, \tag{2}$$

where θ is a phase angle describing the proportion of neutral and charged constituents. For example, $\theta = 0$ stands for $X(3872)$ as a pure $\bar{D}^{*0}D^0/\bar{D}^0D^{*0}$, while $\theta = \pi/4$ and $\theta = -\pi/4$ correspond to the isospin singlet and isospin triplet states, respectively. Then, one can parametrize the coupling of $X(3872)$ to the charmed mesons in terms of the following Lagrangian:

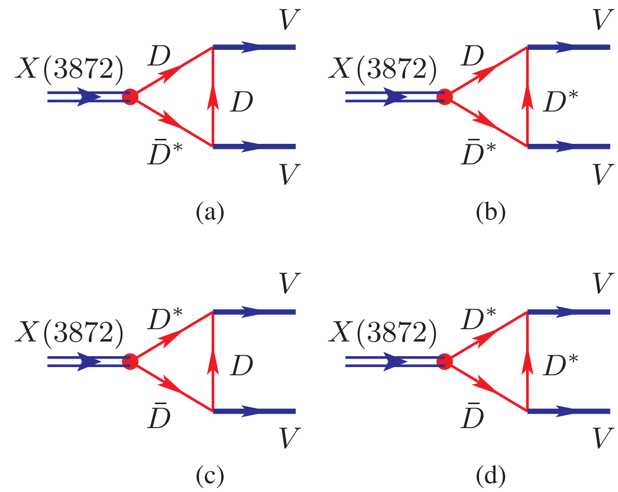

FIG. 1. Diagrams contributing to the charmless decay $X(3872) \to VV$ with $D\bar{D}^* + \text{c.c.}$ as intermediate states.

$$\mathcal{L}_{X(3872)} = \frac{g_n}{\sqrt{2}} X_\mu^\dagger (D^{*0\mu}\bar{D}^0 + D^0\bar{D}^{*0\mu}) + \frac{g_c}{\sqrt{2}} X_\mu^\dagger (D^{*+\mu}D^- + D^+ D^{*-\mu}), \tag{3}$$

where g_n and g_c are the coupling constants of $X(3872)$ with its neutral and charged components, respectively.

Using the masses of the $X(3872)$ and the charmed mesons as in Refs. [55,56], we obtain the mass difference between the $X(3872)$ and the $\bar{D}^{*0}D^0/\bar{D}^0D^{*0}$ (neutral) and $D^{*-}D^+/D^{*+}D^-$ threshold to be 0.16 and 8.21 MeV, respectively. Assuming that $X(3872)$ is a pure $D^0\bar{D}^{*0}$ or $D^\pm D^{*\mp}$ molecule, we obtain

$$|g_{\rm eff}^n| = 3.70 \text{ GeV}, \quad \text{with} \quad c^2_{D^0\bar{D}^{*0}} = 1, \tag{4}$$

$$|g_{\rm eff}^c| = 9.91 \text{ GeV}, \quad \text{with} \quad c^2_{D^\pm D^{*\mp}} = 1. \tag{5}$$

As a result, the coupling constants appearing in Eq. (3) are as follows[1]:

$$g_n = |g_{\rm eff}^n| \cos\theta, \qquad g_c = |g_{\rm eff}^c| \sin\theta. \tag{6}$$

With the above $\bar{D}D^*$ molecular picture for $X(3872)$, these $X(3872) \to VV$ and VP decays can proceed via $X(3872) \to \bar{D}D^* \to VV$ or VP through triangle loop diagrams, which are shown in Figs. 1 and 2, respectively. In this mechanism, $X(3872)$ goes into $\bar{D}D^*$ at a first step, then $\bar{D}$ and D^* are converged to VV or VP in the final state by exchanging a charmed meson. Note that, in Figs. 1 and 2, we have considered only the leading contributions as discussed in Refs. [57–59].

[1] These coupling constants are assumed to be real.

B. The interaction Lagrangians and decay amplitudes

The Lagrangians relevant to the light vector and pseudoscalar mesons can be constructed based on the heavy quark limit and chiral symmetry,

$$\begin{aligned}\mathcal{L} = & -i g_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^i \partial^\mu \mathcal{P}_{ij}\mathcal{D}_\mu^{*j\dagger} - \mathcal{D}_\mu^{*i}\partial^\mu \mathcal{P}_{ij}\mathcal{D}^{j\dagger}) + \frac{1}{2} g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\varepsilon_{\mu\nu\alpha\beta}\mathcal{D}_i^{*\mu}\partial^\nu\mathcal{P}^{ij}\overleftrightarrow{\partial}^{\alpha} \mathcal{D}_j^{*\beta\dagger} - i g_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger \overleftrightarrow{\partial}^{\partial} \mu \mathcal{D}^j(\mathcal{V}^\mu)^i_j \\ & -2 f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)^i_j(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^{\alpha}\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^{\alpha}\mathcal{D}^j) + i g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}^{\mu}\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j + 4i f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}^{*\dagger}_{i\mu}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)^i_j\mathcal{D}_\nu^{*j} + \text{H.c.},\end{aligned} \tag{7}$$

with the convention $\varepsilon_{0123} = 1$, where $\mathcal{P}$ and $\mathcal{V}_\mu$ are 3×3 matrices for the octet pseudoscalar and nonet vector mesons, respectively,

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \tag{8}$$

$$\mathcal{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{9}$$

In the heavy quark and chiral limits, the couplings of the charmed meson to the light vector mesons have the relationship [60,61]

$$g_{\mathcal{D}\mathcal{D}V} = g_{\mathcal{D}^*\mathcal{D}^*V} = \frac{\beta g_V}{\sqrt{2}}, \tag{10}$$

$$f_{\mathcal{D}^*\mathcal{D}V} = \frac{f_{\mathcal{D}^*\mathcal{D}^*V}}{m_{\mathcal{D}^*}} = \frac{\lambda g_V}{\sqrt{2}}, \tag{11}$$

$$g_{\mathcal{D}^*\mathcal{D}\mathcal{P}} = \frac{2g}{f_\pi}\sqrt{m_{\mathcal{D}} m_{\mathcal{D}^*}}, \tag{12}$$

$$g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}} = \frac{g_{\mathcal{D}^*\mathcal{D}\mathcal{P}}}{\sqrt{m_{\mathcal{D}} m_{\mathcal{D}^*}}}. \tag{13}$$

In this work, we take parameters $\beta = 0.9$, $\lambda = 0.56\ \text{GeV}^{-1}$, $g = 0.59$, and $g_V = m_\rho / f_\pi$ with $f_\pi = 132$ MeV, as used in previous works [60,62].

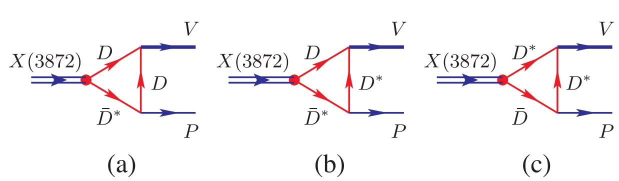

FIG. 2. Diagrams contributing to the charmless decay $X(3872) \to VP$ with $D\bar{D}^* + \text{c.c.}$ as intermediate states.

Then one can easily write the explicit transition amplitudes for $X(3872)(p_1) \to [D^{(*)}(q_1)\bar{D}^{(*)}(q_3)]D^{(*)}(q_2) \to V_1(p_2)V_2(p_3)$ shown in Fig. 1 as follows:

$$\begin{aligned}\mathcal{M}_a = & \int \frac{d^4 q_2}{(2\pi)^4}[g_{\text{eff}}\epsilon_{1\alpha}][g_{DDV}(q_1 - q_2)_\mu \epsilon_2^{*\mu}] \\ & \times [2 f_{D^*DV}\epsilon_{\kappa\lambda\rho\sigma} i p_3^\kappa \epsilon_3^{*\lambda}(q_2 + q_3)^\rho]\frac{1}{q_1^2 - m_1^2} \\ & \times \frac{1}{q_2^2 - m_2^2}\frac{(g^{\alpha\sigma} - q_3^\alpha q_3^\sigma / m_3^2)}{q_3^2 - m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{M}_b = & \int \frac{d^4 q_2}{(2\pi)^4}[g_{\text{eff}}\epsilon_{1\alpha}][2 f_{D^*DV}\epsilon_{\mu\nu\xi\phi} p_2^\mu \epsilon_2^{*\nu}(q_1 - q_2)^\xi] \\ & \times [g_{D^*D^*V}(q_3 + q_2)^\rho g_{\lambda\sigma}\epsilon_{3\rho}^* \\ & + 4 f_{D^*D^*V}(p_{3\lambda} g_\sigma^\rho - p_{3\sigma} g_\lambda^\rho)\epsilon_{3\rho}^*]\frac{i}{q_1^2 - m_1^2} \\ & \times \frac{(g^{\phi\sigma} - q_2^\phi q_2^\sigma / m_2^2)}{q_2^2 - m_2^2}\frac{(g^{\alpha\lambda} - q_3^\alpha q_3^\lambda / m_3^2)}{q_3^2 - m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{M}_c = & \int \frac{d^4 q_2}{(2\pi)^4}[g_{\text{eff}}\epsilon_{1\alpha}][-2 f_{D^*DV}\epsilon_{\mu\nu\xi\phi} p_2^\mu \epsilon_2^{*\nu}(q_1 - q_2)^\xi] \\ & \times [(g_{DDV}(q_3 + q_2)_\kappa \epsilon_3^{*\kappa}]\frac{(g^{\alpha\phi} - q_1^\alpha q_1^\phi / m_1^2)}{q_1^2 - m_1^2} \\ & \times \frac{i}{q_2^2 - m_2^2}\frac{1}{q_3^2 - m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{M}_d = & \int \frac{d^4 q_2}{(2\pi)^4}[g_{\text{eff}}\epsilon_{1\alpha}][g_{D^*D^*V}(q_1 - q_2)^\xi g_{\nu\phi}\epsilon_{2\xi}^* \\ & - 4 f_{D^*D^*V}(p_{2\nu} g_\phi^\xi - p_{2\phi} g_\nu^\xi)\epsilon_{2\xi}^*] \\ & \times [-2 f_{D^*DV}\epsilon_{\kappa\lambda\rho\sigma} p_3^\kappa \epsilon_3^{*\lambda}(q_2 + q_3)^\rho] \\ & \times \frac{(g^{\alpha\phi} - q_1^\alpha q_1^\phi / m_1^2)}{q_1^2 - m_1^2}\frac{(g^{\nu\sigma} - q_2^\nu q_2^\sigma / m_2^2)}{q_2^2 - m_2^2} \\ & \times \frac{i}{q_3^2 - m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{17}$$

where p_1 (ε_1), p_2 (ε_2) and p_3 (ε_3) are the four-momenta (polarization vector) of the initial state $X(3872)$, final state

V_1 and V_2, respectively. q_1, q_2 and q_3 are the four-momenta of the up, right and down charmed mesons in the triangle loop, respectively.

The explicit transition amplitudes for $X(3872)(p_1) \to [D^{(*)}(q_1)\bar{D}^{(*)}(q_3)]D^{(*)}(q_2) \to V(p_2)P(p_3)$ shown in Fig. 2 are as follows:

$$\begin{aligned}\mathcal{M}_a = &\int \frac{d^4 q_2}{(2\pi)^4}[g_{\rm eff}\epsilon_{1\alpha}][-g_{DDV}(q_1-q_2)_\mu \epsilon_2^{*\mu}]\\ &\times [g_{D^*DP}p_3^\kappa]\frac{1}{q_1^2-m_1^2}\frac{i}{q_2^2-m_2^2}\\ &\times \frac{(g_\kappa^\alpha - q_3^\alpha q_{3\kappa}/m_3^2)}{q_3^2-m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{M}_b = &\int \frac{d^4 q_2}{(2\pi)^4}[g_{\rm eff}\epsilon_{1\alpha}][2f_{D^*DV}\epsilon_{\mu\nu\xi\phi}p_2^\mu \epsilon_2^{*\nu}(q_1-q_2)^\xi]\\ &\times \left[\frac{1}{2}g_{D^*D^*P}\epsilon_{\kappa\lambda\rho\sigma}p_3^\lambda(q_3+q_2)^\rho\right]\frac{-i}{q_1^2-m_1^2}\\ &\times \frac{(g^{\phi\kappa}-q_2^\phi q_2^\kappa/m_2^2)}{q_2^2-m_2^2}\frac{(g^{\alpha\sigma}-q_3^\alpha q_3^\sigma/m_3^2)}{q_3^2-m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{M}_c = &\int \frac{d^4 q_2}{(2\pi)^4}[g_{\rm eff}\epsilon_{1\alpha}][g_{D^*D^*V}(q_1-q_2)^\xi g_{\nu\phi}\epsilon_{2\xi}^*\\ &-4f_{D^*D^*V}(p_{2\nu}g_\phi^\xi - p_{2\phi}g_\nu^\xi)\epsilon_{2\xi}^*][g_{D^*DP}p_3^\kappa]\\ &\times \frac{(g^{\alpha\phi}-q_1^\alpha q_1^\phi/m_1^2)}{q_1^2-m_1^2}\frac{(g_\kappa^\nu - q_2^\nu q_{2\kappa}/m_2^2)}{q_2^2-m_2^2}\\ &\times \frac{i}{q_3^2-m_3^2}\mathcal{F}(q^2),\end{aligned} \tag{20}$$

where $\mathcal{F}(q^2)$ is the form factor introduced to depict the off-shell effects of the exchanged mesons as well as the structure effects of the involved mesons. The form factor $\mathcal{F}(q^2)$ is parametrized as

$$\mathcal{F}(q^2) = \left(\frac{m^2-\Lambda^2}{q^2-\Lambda^2}\right)^n, \tag{21}$$

normalized to unity at $q^2 = m^2$ [61], where m and q are mass and momenta of the exchanged mesons. The cutoff Λ can be further reparametrized as $\Lambda = m_{D^{(*)}} + \alpha\Lambda_{\rm QCD}$ with $\Lambda_{\rm QCD} = 0.22$ GeV. The model parameter α is usually expected to be of the order of unity [61,63–66], but its concrete value cannot be estimated by the first principle. In practice, the value of α is usually determined by comparing theoretical estimates with the corresponding experimental measurements. However, no charmless decay mode of $X(3872)$ is known so far. For the rescattering processes studied in this work, it is found that the monopole form ($n=1$) or dipole form ($n=2$) for $\mathcal{F}(q^2)$ is utilized, the numerical results are very sensitive to the values of parameter α, and we have to use a very small value; otherwise, these partial decay widths will be very large, even more than the total width of $X(3872)$. In order to avoid too large dependence of the parameter α, we take $n=3$ in the numerical calculations.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will mainly discuss three cases where θ is 0, $\pi/6$ and $\pi/4$. When $\theta = 0$, it indicates that $X(3872)$ is a pure bound state with only a neutral component. When $\theta = \pi/4$, the proportions of the neutral and charged components are the same. There are both neutral and charged components at $\theta = \pi/6$, but the proportion of the neutral component is dominant.

In Fig. 3, we plot the α dependence of the partial decay widths of $X(3872) \to VV$ and $X(3872) \to VP$ with $\theta = 0$, respectively. In the range of $\alpha = 0.6$–1.2, the predicted partial decay widths of $X(3872) \to VV$ are about a few KeV, while the partial decay widths can reach a few tens of KeV for $X(3872) \to VP$. The $X(3872) \to K^{*0}\bar{K}^{*0}$ transition proceeds via $[D^+D^{*-}]D_s^{(*)}$ intermediate mesons, while the $X(3872) \to K^{*+}K^{*-}$ transition proceeds via $[D^0\bar{D}^{*0}]D_s^{(*)}$ intermediate mesons. So in the case of

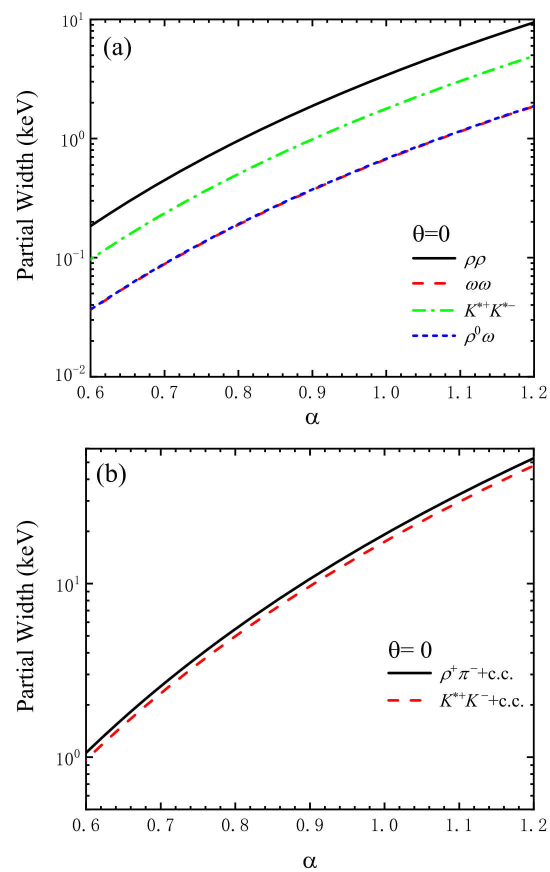

FIG. 3. The α dependence of decay widths (in unit of keV) of $X(3872) \to VV$ and $X(3872) \to VP$ with $\theta = 0$.

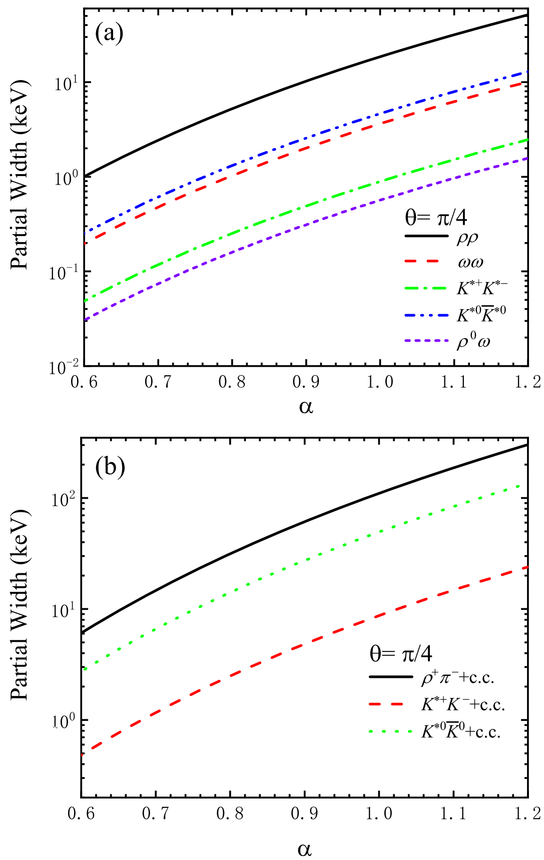

FIG. 4. The α dependence of decay widths (in unit of keV) of $X(3872) \to VV$ and $X(3872) \to VP$ with $\theta = \pi/4$.

$\theta = 0$, there is no neutral $K^{*0}\bar{K}^{*0}$ channel as shown in Fig. 3(a); the same reason for $X(3872) \to K^{*0}\bar{K}^0$ in Fig. 3(b). From Fig. 3(a), one can see that the partial decay width of $X(3872) \to \rho\rho$ is larger than those of $X(3872) \to K^{*+}K^{*-}$ and $\omega\omega$ decay modes. This is because both the charged $\rho^+\rho^-$ and neutron $\rho^0\rho^0$ channels contribute to the $\rho\rho$ channel. For the $X(3872) \to \rho^0\rho^0$ decay, its partial decay width is almost equal to the decay of $X(3872) \to \omega\omega$. In addition, for the $X(3872) \to K^{*+}K^{*-}$ decays, there are only contributions from the exchanging of charged charm mesons. In the case of $\theta = 0$, only neutral charmed meson loops contribute to the isospin-violating channel $X(3872) \to \rho^0\omega$. As a result, the obtained decay widths are almost the same as that of the channel $X(3872) \to \omega\omega$.

In Fig. 4, we plot the α dependence of the partial decay widths of $X(3872) \to VV$ and $X(3872) \to VP$ with $\theta = \pi/4$. In the range of $\alpha = 0.6$–1.2, the predicted partial decay widths of $X(3872) \to VV$ are about a few tens of KeV, while the partial decay widths can reach several hundred KeV for $X(3872) \to VP$. The behavior is similar to that of Fig. 3. Since the case of $\theta = \pi/4$ corresponds to equal neutral and charged components in $X(3872)$, so the channels $X(3872) \to K^{*+}K^{*-}$ and $X(3872) \to K^{*0}\bar{K}^{*0}$ have nonzero decay widths. The $X(3872) \to K^{*0}\bar{K}^{*0}$ transition proceeds via $[D^+D^{*-}]D_s^{(*)}$ intermediate mesons, while the $X(3872) \to K^{*+}K^{*-}$ transition proceeds via $[D^0\bar{D}^{*0}]D_s^{(*)}$ intermediate mesons. The mass of $X(3872)$ is much closer to the mass threshold of $D^0\bar{D}^{*0}$ than D^+D^{*-}, so the threshold effects of $X(3872) \to K^{*+}K^{*-}$ will be larger than that of $X(3872) \to K^{*0}\bar{K}^{*0}$. However, the couplings constant values obtained from Eq. (6) have the relation $g_n < g_c$. Thus with the same value of α, the obtained partial decay width of $X(3872) \to K^{*0}\bar{K}^{*0}$ is about several times larger than that of $X(3872) \to K^{*+}K^{*-}$. However, for the $X(3872) \to \rho\rho$ decay, there are contributions from exchanging both charged charm mesons and neutral charm mesons, and these two contributions give the instructive interference of the decay amplitudes. A similar situation occurs in $X(3872) \to VP$ as shown in Fig. 3(b). A similar situation occurs in $X(3872) \to VP$ as shown in Fig. 4(b). In the case of $\theta = \pi/4$, the charged and neutral charmed meson loops should cancel out exactly in the isospin symmetry limit for the isospin-violating channel $X(3872) \to \rho^0\omega$. In other words, the mass difference between the u and d quark will lead to $m_{D^{(*)\pm}} \neq m_{D^{(*)0}}$ due to the isospin symmetry breaking. As a result, the charged and neutral charmed meson loops cannot completely cancel out, and the residue

TABLE I. The branching ratios for $X(3872) \to VV$ and $X(3872) \to VP$ with different θ values. The α range is taken to be 0.6–1.2 here.

Final states	$\theta = 0$	$\theta = \pi/6$	$\theta = \pi/4$
$\rho\rho$	$(0.15–7.86) \times 10^{-3}$	$(0.06–3.20) \times 10^{-2}$	$(0.83–4.29) \times 10^{-2}$
$K^{*+}K^{*-}$	$(0.08–4.11) \times 10^{-3}$	$(0.06–3.08) \times 10^{-3}$	$(0.04–2.05) \times 10^{-3}$
$K^{*0}\bar{K}^{*0}$	$\cdots$	$(0.11–5.36) \times 10^{-3}$	$(0.02–1.07) \times 10^{-2}$
$\omega\omega$	$(0.03–1.55) \times 10^{-3}$	$(0.12–6.28) \times 10^{-3}$	$(0.16–8.41) \times 10^{-3}$
$\rho^0\omega$	$(0.03–1.56) \times 10^{-3}$	$(0.02–1.25) \times 10^{-4}$	$(0.03–1.31) \times 10^{-3}$
$\rho^\pm\pi^\mp$	$(0.09–4.40) \times 10^{-2}$	$(0.004–1.87) \times 10^{-1}$	$(0.05–2.53) \times 10^{-1}$
$K^{*+}K^- + \mathrm{c.c.}$	$(0.08–3.99) \times 10^{-2}$	$(0.06–2.99) \times 10^{-2}$	$(0.04–1.99) \times 10^{-2}$
$K^{*0}\bar{K}^0 + \mathrm{c.c.}$	$\cdots$	$(0.11–5.66) \times 10^{-2}$	$(0.02–1.13) \times 10^{-1}$

part will contribute to the isospin-violating amplitudes. The partial widths of the isospin-violating channel $X(3872) \to \omega\rho^0$ as shown in Fig. 4(a) are suppressed.

Using the center value of the total decay width of $X(3872)$ that was reported recently by the LHCb collaboration [67,68], we obtain the branching ratios for $X(3872) \to VV$ and VP in the cases of $\theta = 0$, $\pi/6$ and $\pi/4$, respectively. We take the range of α as 0.6–1.2, then the numerical results are shown in Table I. Our theoretical numerical results show that with the increase of θ, the partial decay widths of $K^{*+}K^{*-}$ and $K^{*+}K^- + \mathrm{c.c.}$ channels decrease because there are only neutral charmed meson loops in $X(3872) \to K^{*+}K^{*-}$ and $X(3872) \to K^{*+}K^-$. Also, the $X(3872)$ coupling constant to the neutral channel g_n is proportional to $\cos\theta$.

In Fig. 5, we present the partial decay widths of the $X(3872) \to VV$ in terms of the mass of $X(3872)$, where we have fixed the value of α as 1.0. The coupling constant of $X(3872)$ in Eq. (1) and the threshold effects can simultaneously influence the mass of $X(3872)$ dependence of the decay widths. Generally speaking, with increasing the mass

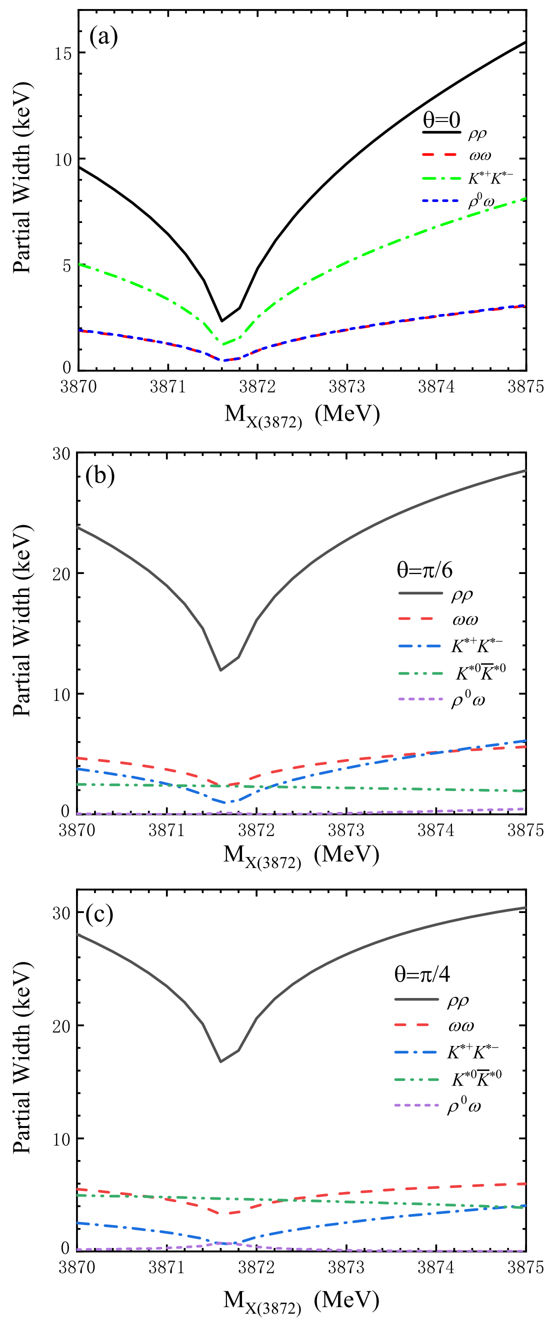

FIG. 5. The $M_{X(3872)}$ dependence of the decay widths (in unit of keV) of $X(3872) \to VV$ with $\alpha = 1.0$.

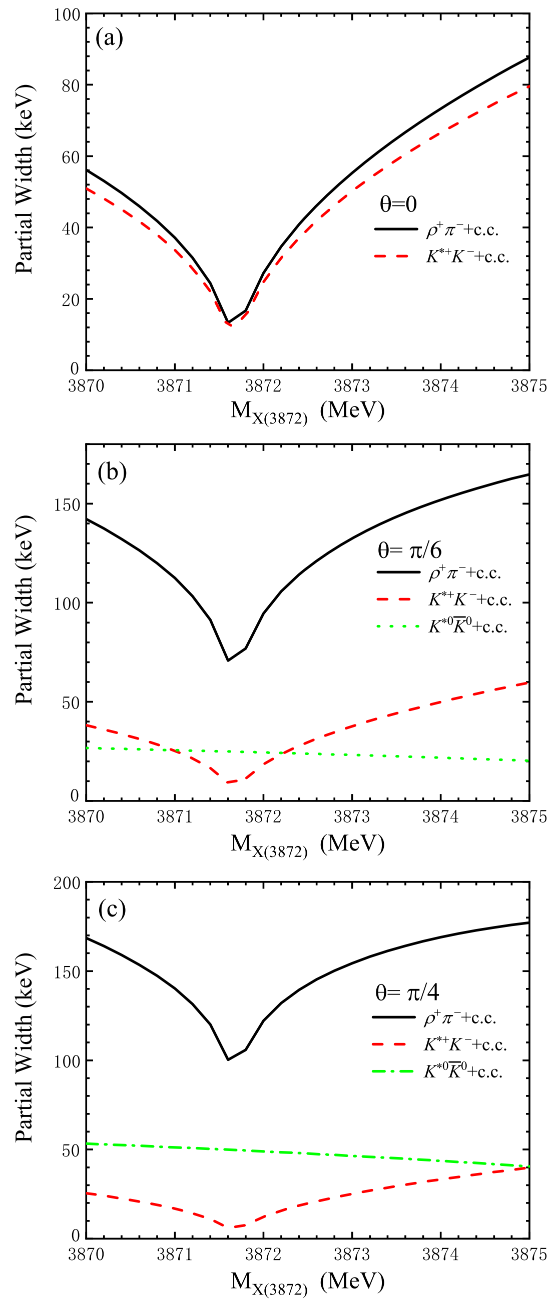

FIG. 6. The same as Fig. 5 but for $X(3872) \to VP$.

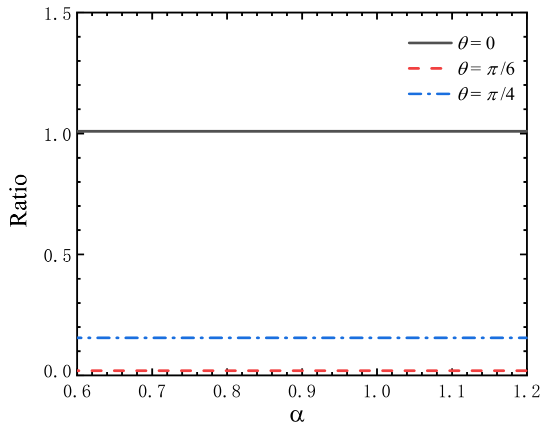

FIG. 7. The α dependence of the ratio R_1 defined in Eq. (22).

difference between $X(3872)$ and $D^{*0}\bar{D}^0$ mesons, i.e., increasing the binding energy, the coupling strength of $X(3872)$ increases, and the threshold effects decrease. Both the coupling strength of $X(3872)$ and the threshold effects vary quickly in the small binding energy region and slowly in the large binding energy region. As a result, the behavior of the partial widths is relatively sensitive at small binding energy, while it becomes smooth at large binding energy. The single-cusp structure locates at the thresholds of the $D^{*0}\bar{D}^0$ mesons for most of the decay channels except for the $K^{*0}\bar{K}^{*0}$ channel. This is because the $X(3872) \to K^{*0}\bar{K}^{*0}$ transition proceeds via $[D^+D^{*-}]D_s^{(*)}$ intermediate mesons. A similar behavior of partial widths occurs in $X(3872) \to VP$ as shown in Fig. 6.

It would be interesting to further clarify the uncertainties arising from the introduction of form factors by studying the α dependence of the ratios between different partial decay widths. For the decays $X(3872) \to VV$, we define the following ratios to the partial decay widths of $X(3872) \to \omega\omega$:

$$
\begin{aligned}
R_1 &= \frac{\Gamma(X(3872) \to \omega\rho^0)}{\Gamma(X(3872) \to \omega\omega)}, \\
R_2 &= \frac{\Gamma(X(3872) \to \rho\rho)}{\Gamma(X(3872) \to \omega\omega)}, \\
R_3 &= \frac{\Gamma(X(3872) \to K^{*+}K^{*-})}{\Gamma(X(3872) \to \omega\omega)}, \\
R_4 &= \frac{\Gamma(X(3872) \to K^{*0}\bar{K}^{*0})}{\Gamma(X(3872) \to \omega\omega)}.
\end{aligned} \tag{22}
$$

For the decays of $X(3872) \to VP$, the following ratios are defined:

$$
\begin{aligned}
r_1 &= \frac{\Gamma(X(3872) \to K^{*+}K^- + \text{c.c.})}{\Gamma(X(3872) \to \rho\pi)}, \\
r_2 &= \frac{\Gamma(X(3872) \to K^{*0}\bar{K}^0 + \text{c.c.})}{\Gamma(X(3872) \to \rho\pi)}.
\end{aligned} \tag{23}
$$

The ratios R_1 in terms of α are plotted in Fig. 7. The results of Fig. 7 show that the ratios are completely insensitive to this dependence. This stabilities of the ratios in terms of α indicate a reasonably controlled cutoff for each channel by the form factor to some extent. On the other hand, one can see that, in Fig. 7, there is extremely strong dependence of the ratio on the isospin mixing angle, θ, which is of more fundamental significance than the parameter α. This stability stimulates us to study the mixing angle θ dependence.

Next, we turn to the dependence of these ratios defined in Eqs. (22) and (23) to the mixing angle θ with a fixed α. In Fig. 8, we present the theoretical results of the ratio R_i ($i = 1, 2, 3, 4$) defined in Eq. (22) and r_i ($i = 1, 2$) defined in Eq. (23) as a function of the mixing angle θ with a fixed value $\alpha = 1.0$. It is interesting to note that the results of the ratio $R_2 = \frac{\Gamma(X(3872)\to\rho\rho)}{\Gamma(X(3872)\to\omega\omega)}$ are not dependent on the value of θ.

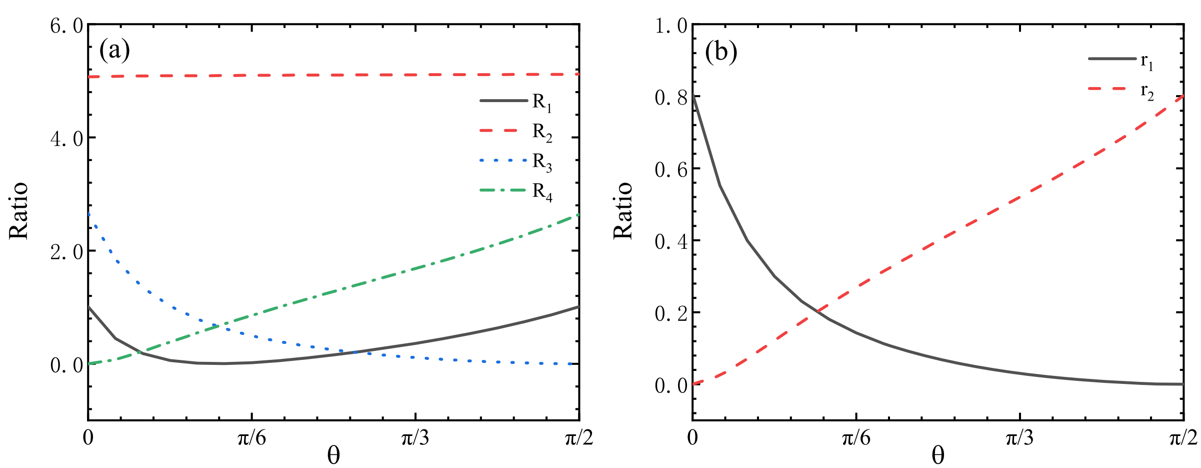

FIG. 8. (a) The ratio R_i ($i = 1, 2, 3, 4$) defined in Eq. (22) as a function of the mixing angle θ with $\alpha = 1.0$. (b) The ratio r_i ($i = 1, 2$) defined in Eq. (23) as a function of the mixing angle θ with $\alpha = 1.0$.

These ratios shown in Fig. 8 may be tested by the future experimental measurements and can be used to determine the value of the mixing angle.

IV. SUMMARY

Based on a molecular nature of $X(3872)$, we have investigated the charmless decays of $X(3872) \to VV$ and VP. For $X(3872)$, we considered three cases, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$) and neutral components dominant ($\theta = \pi/6$), where θ is a phase angle describing the proportion of neutral and charged constituents. We explore the rescattering mechanism within the effective Lagrangian based on the heavy quark symmetry and chiral symmetry. We can see that although the decay widths increase with the increase of α when we fix the phase angle θ, our theoretical results show that the cutoff parameter α dependence of the partial widths is not drastically sensitive, which indicates the dominant mechanism driven by the intermediate meson loops with a fairly good control of the ultraviolet contributions. When $X(3872)$ is a pure neutral bound state, the predicted partial decay widths of $X(3872) \to VV$ are about a few keV, while the partial decay widths can reach a few tens of keV for $X(3872) \to VP$. When there are both neutral and charged components in $X(3872)$, the predicted decay widths of $X(3872) \to VV$ are about tens of keV, while the decay widths can reach a few hundreds of keV for $X(3872) \to VP$.

Moreover, the dependence of these ratios between different charmless decay modes of $X(3872)$ to the charged and neutral mixing angle for the $X(3872)$ in the molecular picture is also investigated, which may be tested by future experiments and can be used to determine the value of the mixing angle.

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