$N\bar{N}$ production in e^+e^- annihilation near the threshold revisited

A. I. Milstein[1,2,*] and S. G. Salnikov[1,2,†]

[1]*Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia*
[2]*Novosibirsk State University, 630090 Novosibirsk, Russia*

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Production of $p\bar{p}$ and $n\bar{n}$ pairs in e^+e^- annihilation near the threshold of the process is discussed with account for the new experimental data that appeared recently. Since a significant part of these new data was obtained at energies noticeably exceeding the threshold, we also take into account the form factor describing the amplitude of $N\bar{N}$ pair production at small distances. The effective optical potential, which describes a sharp dependence of the $N\bar{N}$ production cross sections near the threshold, consists of the central potential for S and D waves and the tensor potential. These potentials differ for states with isospin $I = 0$ and $I = 1$ of the $N\bar{N}$ pair. The optical potential describes well $N\bar{N}$ scattering phases, the cross sections of $p\bar{p}$ and $n\bar{n}$ production in e^+e^- annihilation near the threshold, and the electromagnetic form factors G_E and G_M for protons and neutrons, as well as the cross sections of the processes $e^+e^- \to 6\pi$ and $e^+e^- \to K^+K^-\pi^+\pi^-$.

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I. INTRODUCTION

A strong energy dependence of the cross sections of baryon-antibaryon and meson-antimeson pair production has been observed in many processes near the thresholds of the corresponding reactions. Some of these processes are $e^+e^- \to p\bar{p}$ [1–8], $e^+e^- \to n\bar{n}$ [9–11], $e^+e^- \to \Lambda_{(c)}\bar{\Lambda}_{(c)}$ [12–15], $e^+e^- \to B\bar{B}$ [16], and $e^+e^- \to \phi\Lambda\bar{\Lambda}$ [17]. This anomalous behavior can naturally be explained by small relative velocities of the produced particles. Therefore, they can interact strongly with each other for a sufficiently long time. As a result, the wave function of the produced pair changes significantly (the so-called final-state interaction). The idea of the final-state interaction as a source of anomalous energy dependence of the cross sections near the thresholds has been expressed in many papers [18–28]. However, the technical approaches used in these papers were different. It turned out that in almost all cases the anomalous behavior of the cross sections is successfully described by the final-state interaction.

Unfortunately, information on the potentials, which are responsible for the final-state interaction, is very limited. However, instead of trying to find these potentials from first principles, one can use some effective potentials, which are described by a small number of parameters. These parameters are found by comparison of the predictions with a large amount of experimental data. Such an approach has justified itself in all known cases.

One of the most complicated processes for investigation is $N\bar{N}$ pair production in e^+e^- annihilation near the threshold. To describe the process, it is necessary to take into account the central part of the potential for S and D waves and the tensor part of the potential. In addition, these potentials are different in the isoscalar and isovector channels. Another circumstance, that is necessary to take into account, is a large number of $N\bar{N}$ annihilation channels to mesons. As a result, instead of the usual real potentials, one has to use the so-called optical potentials containing the imaginary parts. Note that, in a narrow region near the thresholds of $p\bar{p}$ and $n\bar{n}$ production, the Coulomb interaction of p and $\bar{p}$ should also be taken into account as well as the proton and neutron mass difference.

The details of the approach that allows one to solve the specified problem are given in our paper [24]. However, in that paper the parameters of the potentials and the corresponding predictions for various characteristics of the processes were based on the old experimental data on the production of $p\bar{p}$ and $n\bar{n}$ pairs. Moreover, a significant part of the uncertainty in the parameters of the model was related to a poor accuracy of the experimental data on the cross section of $n\bar{n}$ pair production. Recently, new data have appeared on $n\bar{n}$ pair production in e^+e^- annihilation near the threshold [10,11]. These data differ significantly from the previous ones and have a fairly high accuracy compared to the previous experiments. Therefore, it became necessary to perform a new analysis of the numerous experimental data within our model.

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[*] A.I.Milstein@inp.nsk.su
[†] S.G.Salnikov@inp.nsk.su

The approach in Ref. [24] was based on the assumption that the amplitude of a hadronic system production at small distances weakly depends on the energy of the system near the threshold of the process. Therefore, in Ref. [24] this amplitude was considered as energy independent, and strong energy dependence of the cross section has appeared via the energy dependence of the wave function due to the final-state interaction. In order to use the new data obtained at energies significantly above the threshold (but in the nonrelativistic approximation), in the present paper we introduce the phenomenological dipole form factor which describes the amplitude of hadronic system production at small distances. More precisely, we consider the total kinetic energy of two produced particles less than 200 MeV, that provides the applicability of nonrelativistic approximation. There is also an interesting problem concerning the oscillations of the nucleon form factors in the timelike region (see Ref. [29] and references therein). However, these oscillations are significant in the energy region outside of that considered in our paper.

The aim of the present work is the analysis of $N\bar{N}$ real and virtual pair production in e^+e^- annihilation with the new experimental data taken into account. We show that our model, which contains a relatively small number of parameters, successfully describes the energy dependence of $N\bar{N}$ scattering phases (see Ref. [30] and references therein), the energy dependence of the cross sections of $p\bar{p}$ and $n\bar{n}$ pair production near the threshold [1–11], and the electromagnetic form factors G_E and G_M for protons and neutrons in the timelike region [1–5,8], as well as the anomalous behavior of the cross sections of the processes $e^+e^- \to 6\pi$ [6,31–33] and $e^+e^- \to K^+K^-\pi^+\pi^-$ [6,34,35].

II. DESCRIPTION OF THE MODEL

The wave function of the $N\bar{N}$ system produced in e^+e^- annihilation through one virtual photon has quantum numbers $J^{PC} = 1^{--}$ and contains four components, namely, $p\bar{p}$ pair in S and D waves and $n\bar{n}$ pair in S and D waves with the total spin $s = 1$. The $N\bar{N}$ pair with other quantum numbers can be produced via two-photon e^+e^- annihilation, but the corresponding cross section is strongly suppressed by the fine-structure constant and we do not consider this contribution in our paper. It is necessary to take into account $p\bar{p}$ and $n\bar{n}$ pairs together in the wave function due to the charge-exchange processes $p\bar{p} \leftrightarrow n\bar{n}$. Contributions of S and D waves must be taken into account together due to a tensor potential, which, for total angular momentum $J = 1$ and total spin $s = 1$, leads to mixing of states with orbital angular momenta $L = 0$ and $L = 2$. In the absence of the effects violating the isotopic invariance (the Coulomb $p\bar{p}$ interaction and the proton and neutron mass difference), the potential in the states with a certain isospin $I = 0$, 1 has the form

$$V^I = V_S^I(r)\delta_{L0} + V_D^I(r)\delta_{L2} + V_T^I(r)[6(\boldsymbol{s}\cdot\boldsymbol{n})^2 - 4], \tag{1}$$

where $\boldsymbol{s}$ is the spin operator of $N\bar{N}$ pair ($s = 1$), $\boldsymbol{n} = \boldsymbol{r}/r$, and $\boldsymbol{r} = \boldsymbol{r}_N - \boldsymbol{r}_{\bar{N}}$. The potentials $V_S^I(r)$, $V_D^I(r)$, and $V_T^I(r)$ correspond to interaction in the states with $L = 0$ and $L = 2$, as well as the tensor interaction. Note that the imaginary parts of the effective potentials in Eq. (1) account for the annihilation channels of the $N\bar{N}$ pair. Since the corresponding amplitudes strongly depend on quantum numbers of the $N\bar{N}$ pair, it is impossible to compare the effective optical potentials for different partial waves. This is why we consider in Eq. (1) only the terms relevant to $N\bar{N}$ production through one-photon e^+e^- annihilation.

With account for the effects violating the isotopic invariance, we have to solve not two independent systems for each isospin but one system of equations for the four-component wave function Ψ (see Ref. [24] for more details):

$$\begin{gathered}
[p_r^2 + \mu\mathcal{V} - \mathcal{K}^2]\Psi = 0, \qquad \Psi = (u^p, w^p, u^n, w^n)^T, \\
\mathcal{K}^2 = \begin{pmatrix} k_p^2\mathbb{I} & 0 \\ 0 & k_n^2\mathbb{I} \end{pmatrix}, \qquad \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
\mu = \frac{1}{2}(m_p + m_n), \qquad k_p^2 = \mu E, \qquad k_n^2 = \mu(E - 2\Delta), \qquad \Delta = m_n - m_p,
\end{gathered} \tag{2}$$

where the superscript index T denotes a transposition of a row, $p_r^2 = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $u^p(r), w^p(r)$ and $u^n(r), w^n(r)$ are the radial wave functions of the $p\bar{p}$ or $n\bar{n}$ pair with $L = 0$ and $L = 2$, respectively, m_p and m_n are the proton and neutron masses, respectively, E is the energy of a system counted from the $p\bar{p}$ threshold, and $\hbar = c = 1$. For the sake of simplicity of notation, in Ψ we have pointed out only the radial parts of the components of the wave function. Obviously, the angular parts of the corresponding terms are different and well known (see, e.g., Ref. [18]). However, an account for these angular parts is trivial and is performed in the potential $\mathcal{V}$ in Eq. (2). This potential $\mathcal{V}$ is the matrix 4×4 which accounts for the $p\bar{p}$ interaction and $n\bar{n}$ interaction as well as transitions $p\bar{p} \leftrightarrow n\bar{n}$. This matrix can be written in a block form as

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}^{pp} & \mathcal{V}^{pn} \\ \mathcal{V}^{pn} & \mathcal{V}^{nn} \end{pmatrix}, \tag{3}$$

where the matrix elements read

$$\mathcal{V}^{pp}=\frac{1}{2}(\mathcal{U}^1+\mathcal{U}^0)-\frac{\alpha}{r}\mathbb{I}+\mathcal{U}_{cf},\quad \mathcal{V}^{nn}=\frac{1}{2}(\mathcal{U}^1+\mathcal{U}^0)+\mathcal{U}_{cf},$$

$$\mathcal{V}^{pn}=\frac{1}{2}(\mathcal{U}^0-\mathcal{U}^1),$$

$$\mathcal{U}^I=\begin{pmatrix} V_S^I & -2\sqrt{2}V_T^I \\ -2\sqrt{2}V_T^I & V_D^I-2V_T^I \end{pmatrix},\quad \mathcal{U}_{cf}=\frac{6}{\mu r^2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4}$$

and α is the fine-structure constant, $\mathcal{U}_{cf}$ is the matrix of the centrifugal potential (its component is nonzero only for D-wave), and $\mathbb{I}$ is the unit matrix 2×2.

Equation (2) has four linearly independent regular at $r\to 0$ solutions Ψ_i ($i=1$–4) with asymptotics at $r\to\infty$ given in Ref. [24]. The proton and neutron electromagnetic form factors are expressed in terms of the components of these wave functions as follows:

$$\begin{aligned}
G_M^p &= \left\{g_p u_1^p(0)+g_n u_1^n(0)+\frac{1}{\sqrt{2}}[g_p u_2^p(0)+g_n u_2^n(0)]\right\}F_D(q),\\
G_E^p &= \frac{q}{2\mu}\{g_p u_1^p(0)+g_n u_1^n(0)-\sqrt{2}[g_p u_2^p(0)+g_n u_2^n(0)]\}F_D(q),\\
G_M^n &= \left\{g_p u_3^p(0)+g_n u_3^n(0)+\frac{1}{\sqrt{2}}[g_p u_4^p(0)+g_n u_4^n(0)]\right\}F_D(q),\\
G_E^n &= \frac{q}{2\mu}\{g_p u_3^p(0)+g_n u_3^n(0)-\sqrt{2}[g_p u_4^p(0)+g_n u_4^n(0)]\}F_D(q),\\
F_D(q) &= \frac{1}{(1-\frac{q^2}{q_0^2})^2},\qquad q=2\mu+E,\qquad q_0=840\ \mathrm{MeV}.
\end{aligned} \tag{5}$$

Here, $F_D(q)$ is the phenomenological dipole form factor that takes into account the energy dependence of the amplitude of the hadronic system production at small distances, $u_i^p(0)$ and $u_i^n(0)$ are the energy-dependent components of the wave function at $r=0$, and g_p and g_n are energy-independent fitting parameters.

The cross sections of $p\bar{p}$ and $n\bar{n}$ pair production, which we refer to as the elastic cross sections, have the form

$$\begin{aligned}
\sigma_{\mathrm{el}}^p &= \frac{4\pi k_p\alpha^2}{q^3}F_D^2(q)[|g_p u_1^p(0)+g_n u_1^n(0)|^2+|g_p u_2^p(0)+g_n u_2^n(0)|^2],\\
\sigma_{\mathrm{el}}^n &= \frac{4\pi k_n\alpha^2}{q^3}F_D^2(q)[|g_p u_3^p(0)+g_n u_3^n(0)|^2+|g_p u_4^p(0)+g_n u_4^n(0)|^2].
\end{aligned} \tag{6}$$

In the absence of the final-state interaction, we have $u_1^p(0)=u_3^n(0)=1$, and the remaining $u_i^p(0)$ and $u_i^n(0)$ vanish. The functions $u_3^p(0)$ and $u_1^n(0)$ differ from zero due to the charge-exchange process, while nonzero values of $u_2^p(0)$, $u_2^n(0)$, $u_4^p(0)$, and $u_4^n(0)$ are the consequence of the tensor forces. Note that $|G_E^p/G_M^p|$ and $|G_E^n/G_M^n|$ differ from unity solely due to the tensor forces. For $E=0$ these ratios are equal to unity, since at the threshold the contribution of the D wave vanishes.

In addition to the strong energy dependence of the cross sections σ_{el}^p and σ_{el}^n near the threshold, a strong energy dependence reveals also in the cross sections of meson production in e^+e^- annihilation near the $N\bar{N}$ pair production threshold [6,31–35]. Such a behavior is related to the production of a virtual $N\bar{N}$ pair below and above the threshold with the subsequent annihilation of this pair into mesons. Since the probability of virtual $N\bar{N}$ pair production strongly depends on energy, then the probability of meson production through the intermediate $N\bar{N}$ state also strongly depends on energy. Meanwhile, the probability of meson production through other mechanisms has weak energy dependence near the $N\bar{N}$ threshold. To find the cross section σ_{in}^I of meson production through the $N\bar{N}$ intermediate state (the inelastic cross section) with a certain isospin I, one can use the optical theorem. Because of this theorem, the cross sections $\sigma_{\mathrm{tot}}^I=\sigma_{\mathrm{el}}^I+\sigma_{\mathrm{in}}^I$ are expressed via the imaginary part of the Green's function $\mathcal{D}(r,r'|E)$ of the Schrödinger equation:

$$\begin{aligned}
\sigma_{\mathrm{tot}}^I &= \frac{2\pi\alpha^2}{q^3}F_D^2(q)\mathrm{Im}[(\mathcal{G}^I)^\dagger\mathcal{D}(0,0|E)\mathcal{G}^I],\\
(\mathcal{G}^0)^T &= \frac{g_p+g_n}{2}\cdot(1,0,1,0),\\
(\mathcal{G}^1)^T &= \frac{g_p-g_n}{2}\cdot(1,0,-1,0).
\end{aligned} \tag{7}$$

In fact, the optical theorem allows one to express the cross section of hadron production in one-photon annihilation via the imaginary part of the photon polarization operator. However, the contribution of nonrelativistic $N\bar{N}$ pairs to the photon polarization operator is proportional to the nonrelativistic Green's function of the $N\bar{N}$ system [36]. The cross sections σ_{el}^{I} have the form

$$\begin{aligned}\sigma_{\mathrm{el}}^{0} &= \frac{4\pi k_p\alpha^2}{q^3}F_D^2(q)\left|\frac{g_p+g_n}{2}\right|^2[|u_1^p(0)+u_1^n(0)|^2+|u_2^p(0)+u_2^n(0)|^2]\\ &\quad+\frac{4\pi k_n\alpha^2}{q^3}F_D^2(q)\left|\frac{g_p+g_n}{2}\right|^2[|u_3^p(0)+u_3^n(0)|^2+|u_4^p(0)+u_4^n(0)|^2],\\ \sigma_{\mathrm{el}}^{1} &= \frac{4\pi k_p\alpha^2}{q^3}F_D^2(q)\left|\frac{g_p-g_n}{2}\right|^2[|u_1^p(0)-u_1^n(0)|^2+|u_2^p(0)-u_2^n(0)|^2]\\ &\quad+\frac{4\pi k_n\alpha^2}{q^3}F_D^2(q)\left|\frac{g_p-g_n}{2}\right|^2[|u_3^p(0)-u_3^n(0)|^2+|u_4^p(0)-u_4^n(0)|^2].\end{aligned} \tag{8}$$

The Green's function satisfies the equation

$$[p_r^2+\mu\mathcal{V}-\mathcal{K}^2]\mathcal{D}(r,r'|E)=\frac{1}{rr'}\delta(r-r') \tag{9}$$

and is expressed in terms of regular and irregular solutions of the Schrödinger equation (2) (see Ref. [24] for details).

III. RESULTS AND DISCUSSION

The optical potentials $V(r)$ in Eq. (1) are expressed in terms of the potentials $\tilde{U}^0(r)$ and $\tilde{U}^1(r)$ associated with isoscalar and isovector exchange:

$$V(r)=\tilde{U}^0(r)+(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\tilde{U}^1(r), \tag{10}$$

where $\boldsymbol{\tau}_{1,2}$ are isospin Pauli matrices for the nucleon and antinucleon, respectively. Therefore, $V_{S,D,T}^{I}$ in Eq. (1) have the form

$$V_j^1(r)=\tilde{U}_j^0(r)+\tilde{U}_j^1(r),\qquad V_j^0(r)=\tilde{U}_j^0(r)-3\tilde{U}_j^1(r),\qquad j=S,D,T. \tag{11}$$

In our model, we use the simplest parametrization of the potentials $\tilde{U}^I(r)$:

$$\begin{aligned}\tilde{U}_j^0(r)&=(U_j^0-iW_j^0)\theta(a_j^0-r),\\ \tilde{U}_j^1(r)&=(U_j^1-iW_j^1)\theta(a_j^1-r)+U_j^\pi(r)\theta(r-a_j^1),\\ j&=S,D,T,\end{aligned} \tag{12}$$

where $\theta(x)$ is the Heaviside function, U_j^I, W_j^I, and a_j^I are free real parameters fixed by fitting the experimental data, and $U_j^\pi(r)$ are the terms in the pion-exchange potential. We do not present here the explicit form of the pion-exchange potential, since it is well known [see, e.g., Ref. [37] and Eq. (19) in our previous paper [23]].

To fit the parameters of our model, we use the following experimental data: $N\bar{N}$ scattering phases obtained by the Nijmegen group (see Ref. [30] and references therein), the cross sections of $p\bar{p}$ and $n\bar{n}$ production near the threshold [2–6,10,11], and modules of electromagnetic form factors $|G_E^p|$ and $|G_M^p|$ [4], as well as the ratios $|G_E^p/G_M^p|$ [2–5,8] and $|G_E^n/G_M^n|$ [11]. The resulting values of parameters are given in Table I. For these parameters we obtain $\chi^2/N_{df}=98/85$, where N_{df} is the number of degrees of freedom. The values a_j^I given in the table are of the order of 1 fm except of a_T^0. However, the corresponding tensor potential $U_T^0-iW_T^0$ is very weak, and the specific value of a_T^0 is not important for our predictions. We have checked this statement explicitly by reduction of the radius a_T^0 from 2.7 to 1 fm. Note that the cross sections of the processes discussed in our paper depend only on the relative phase of the constants g_p and g_n but not on its individual phases. This is why we set the phase of the constant g_p to be zero.

Figure 1 shows a comparison of our predictions for partial cross sections of $p\bar{p}$ scattering with the results of partial wave analysis [30] carried out by the Nijmegen group. Note that the accuracy of predictions in Ref. [30] is not well established, since these predictions differ slightly from the predictions of other groups (see, e.g., Ref. [38]). Therefore, there is not much sense to find a fit which is closer to the predictions of Ref. [30]. However, we have

TABLE I. The parameters of the model.

	$\tilde{U}_S^0$	$\tilde{U}_D^0$	$\tilde{U}_T^0$	$\tilde{U}_S^1$	$\tilde{U}_D^1$	$\tilde{U}_T^1$
U_i (MeV)	−196	80.8	−2.2	−36.3	401.6	15.2
W_i (MeV)	167.3	225.4	−2	−16.4	217.2	1.5
a_i (fm)	0.701	1.185	2.704	1.294	0.739	1.289
g_i	$g_p=14.1$			$g_n=3.6-1.1i$		

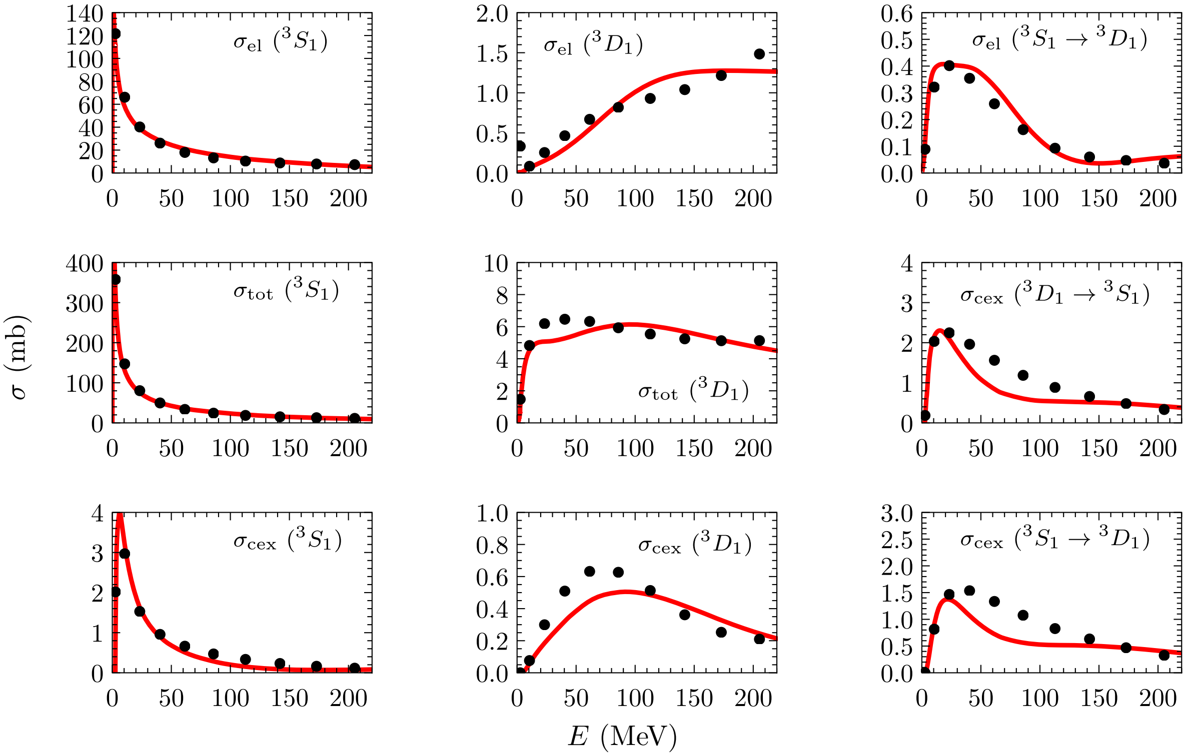

FIG. 1. The predictions for the cross sections of $p\bar{p}$ scattering compared with the Nijmegen data [30].

checked that our predictions for the cross sections of $N\bar{N}$ pair production in e^+e^- annihilation are stable with respect to slight modifications of the parameters of the model. Figure 2 shows the energy dependence of $p\bar{p}$ and $n\bar{n}$ pair production cross sections. Figure 3 shows $|G_E^p|$ and $|G_M^p|$, as well as the ratios $|G_E^p/G_M^p|$ and $|G_E^n/G_M^n|$. Good agreement of the predictions with the available experimental data is seen everywhere. Note that a kink in the cross section of $p\bar{p}$ production at an energy of about 3 MeV is related to the threshold of $n\bar{n}$ production ($2m_n - 2m_p = 2.6$ MeV).

As mentioned above, the optical theorem allows one to predict the contributions $\sigma_{\rm in}^I$ to the cross sections of meson production in e^+e^- annihilation associated with the $N\bar{N}$ pairs in an intermediate state. In Fig. 4, the cross sections $\sigma_{\rm tot}^I$, $\sigma_{\rm el}^I$, and $\sigma_{\rm in}^I$ are shown. It can be seen that in the channel with $I = 1$ there is a large dip in the cross section $\sigma_{\rm in}^1$ at the threshold of real $N\bar{N}$ pair production. At the same time, in the channel with $I = 0$ this dip is practically invisible.

A dip was found in the cross sections of the processes $e^+e^- \to 3(\pi^+\pi^-)$ [6,31,32], $e^+e^- \to 2(\pi^+\pi^-\pi^0)$ [31,33], and $e^+e^- \to K^+K^-\pi^+\pi^-$ [6,34,35]. Since in our approach we cannot predict the cross sections in each channel, for comparison of our predictions with experimental data we use the following procedure. We assume that strong energy dependence of the cross sections for the production of mesons in each channel near the $N\bar{N}$ threshold is related to a strong energy dependence of the amplitude of virtual $N\bar{N}$ pair production in an intermediate state. We also suppose that the amplitudes of virtual $N\bar{N}$ pair transitions to specific meson states weakly depend on energy near the threshold of $N\bar{N}$ production. Evidently, other contributions to meson production cross sections, which are not related to $N\bar{N}$ in an intermediate state, have also a weak energy dependence. Therefore, we approximate the cross section $\sigma_{\rm mesons}^I$ of meson production in a state with a certain isospin by the function

$$\sigma_{\rm mesons}^I = a \cdot \sigma_{\rm in}^I + b \cdot E^2 + c \cdot E + d, \qquad (13)$$

where a, b, c, and d are some fitting parameters, which depend on the specific final states.

The 6π final state has isospin $I = 1$ due to G-parity conservation. A comparison of our predictions for the 6π production cross section with the experimental data is shown in Fig. 5. For these processes, the fit shows that we can set $b = 0$, and the remaining parameters are $a = 0.14$, $c = 3.3 \times 10^{-3}$ nb/MeV, and $d = 0.84$ nb for $3(\pi^+\pi^-)$ production and $a = 0.4$, $c = 2 \times 10^{-3}$ nb/MeV, and $d = 3.8$ nb for the $2(\pi^+\pi^-\pi^0)$ case. It can be seen that there is good agreement between our predictions and experimental data. However, new experimental data for the cross section of $2(\pi^+\pi^-\pi^0)$ production would be useful, since the data obtained by *BABAR* and CMD-3 Collaborations differ noticeably from each other.

Consider now the process $e^+e^- \to K^+K^-\pi^+\pi^-$. Unlike the 6π state, the state $K^+K^-\pi^+\pi^-$ may be in both isospin

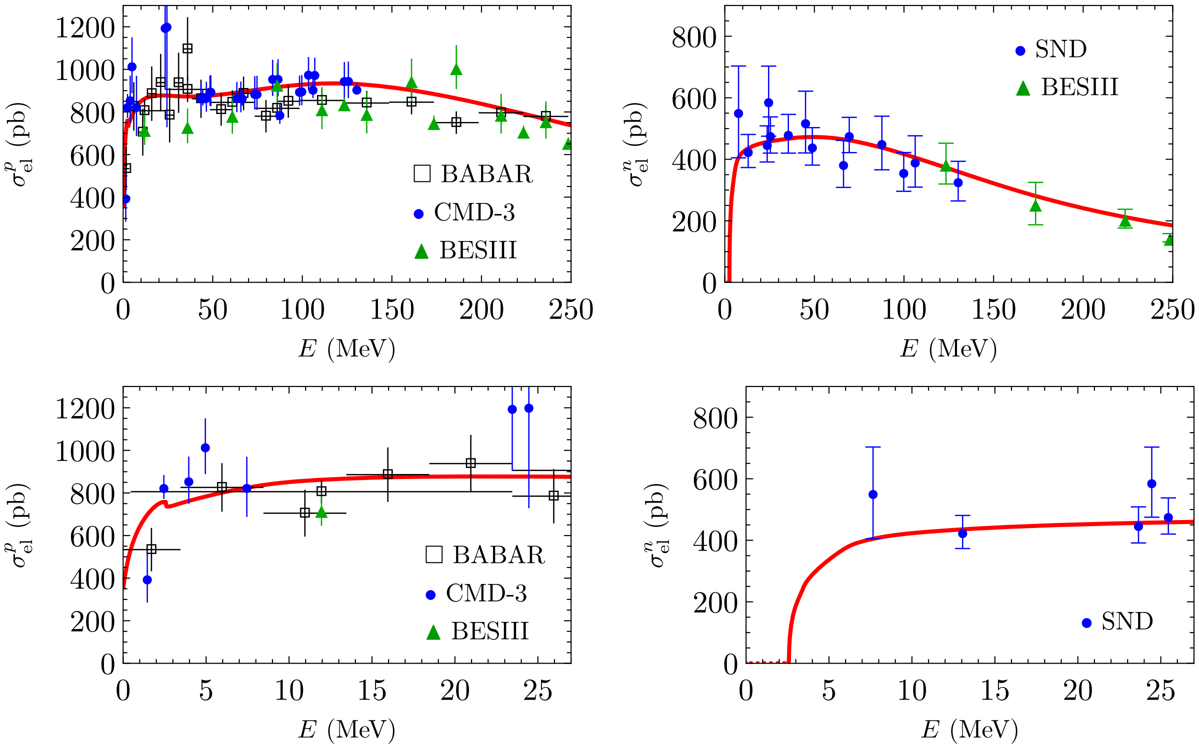

FIG. 2. The energy dependence of the cross sections of $p\bar{p}$ (left) and $n\bar{n}$ (right) pair production in e^+e^- annihilation. The near-threshold energy region is shown in more detail in the bottom row. The experimental data are taken from *BABAR* [2], CMD-3 [3,6], SND [11], and BESIII [4,5,10].

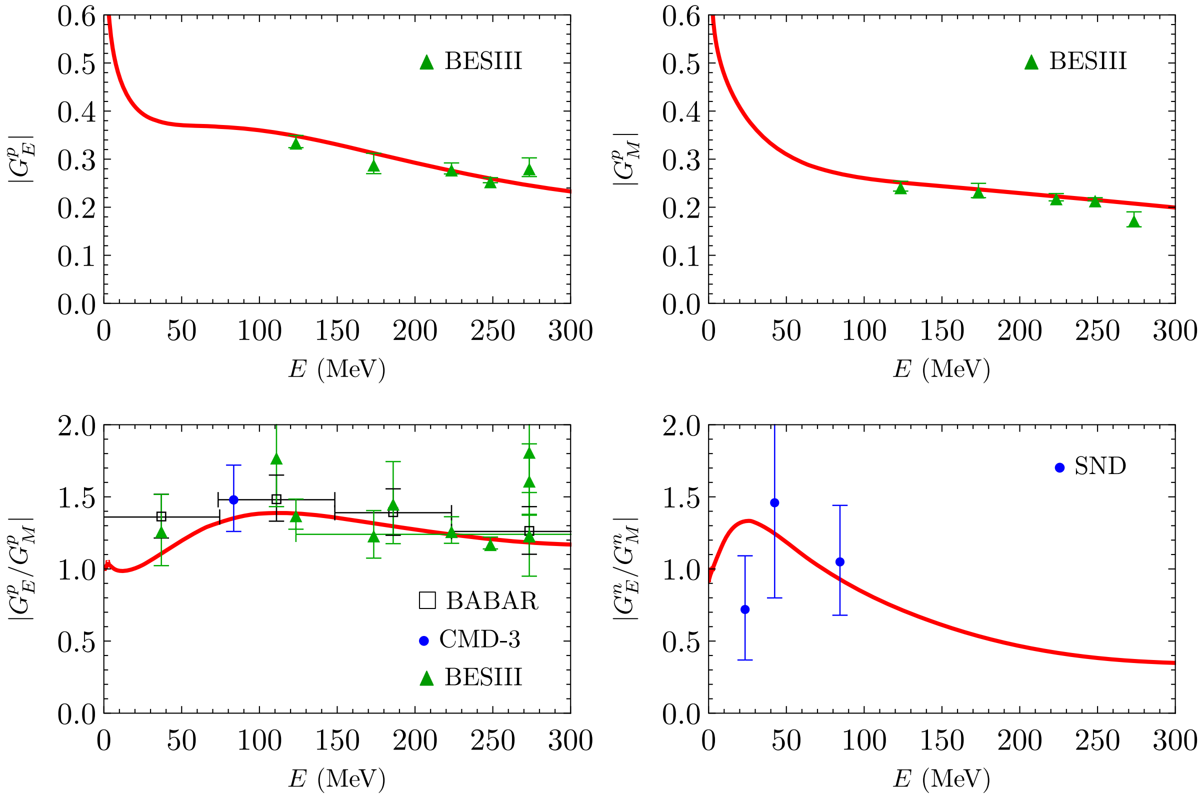

FIG. 3. The energy dependence of the form factors $|G^p_E|$ and $|G^p_M|$, as well as the ratios $|G^p_E/G^p_M|$ and $|G^n_E/G^n_M|$. The experimental data are taken from *BABAR* [2], CMD-3 [3], SND [11], and BESIII [4,5,8].

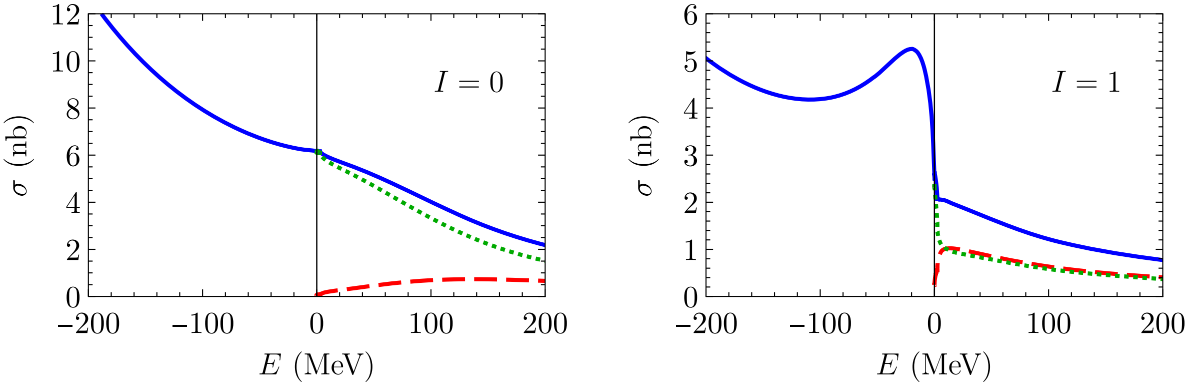

FIG. 4. The energy dependence of the cross sections $\sigma^I_{\rm tot}$ (solid line), $\sigma^I_{\rm el}$ (dashed line), and $\sigma^I_{\rm in}$ (dotted line) for isospins $I = 0$, 1.

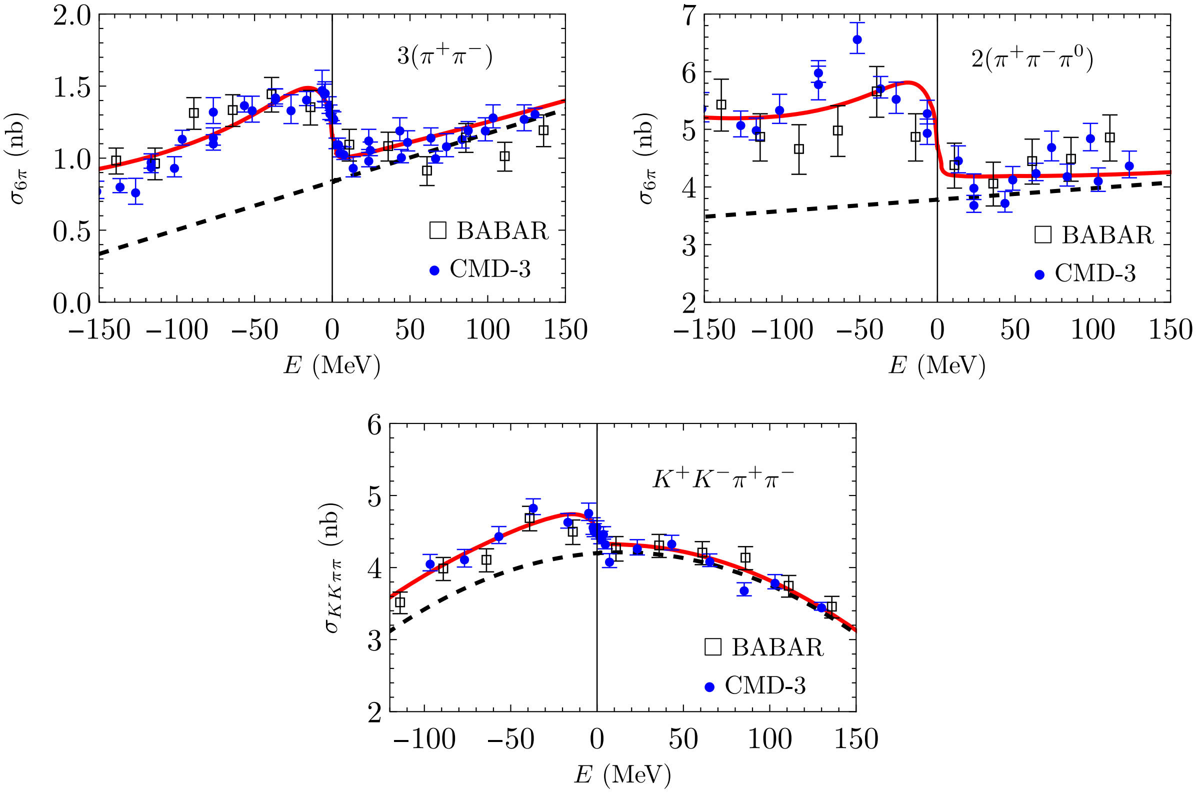

FIG. 5. The energy dependence of the cross sections for the processes $e^+e^- \to 3(\pi^+\pi^-)$, $e^+e^- \to 2(\pi^+\pi^-\pi^0)$, and $e^+e^- \to K^+K^-\pi^+\pi^-$. The experimental data are taken from Refs. [6,31–35], respectively.

states $I = 1$ and $I = 0$. Since our calculations show that the cross section $\sigma^0_{\rm in}$ has no sharp energy dependence near the $N\bar{N}$ threshold, then the contribution of state with $I = 0$ can be taken into account in the parameters b, c, and d. Thus, we can compare the cross section of the process $e^+e^- \to K^+K^-\pi^+\pi^-$ with formula (13) for $I = 1$. The fitting parameters for this process are $a = 0.11$, $b = -6.1 \times 10^{-5}$ nb/MeV^2, $c = 1.7 \times 10^{-3}$ nb/MeV, and $d = 4.2$ nb. Comparison of our predictions with experimental data is also shown in Fig. 5. Again, there is good agreement of our predictions and experimental results.

IV. CONCLUSION

Using new experimental data on the production of $p\bar{p}$ and $n\bar{n}$ pairs in e^+e^- annihilation, a simple model is suggested that successfully describes the cross sections of a few processes with production of real or virtual $N\bar{N}$ pairs. These processes are $e^+e^- \to p\bar{p}$, $e^+e^- \to n\bar{n}$, $e^+e^- \to 6\pi$, and $e^+e^- \to K^+K^-\pi^+\pi^-$ near the $N\bar{N}$ production threshold. Moreover, this model describes well the energy dependence of partial cross sections for nucleon-antinucleon scattering in states with $L = 0$, 2, $s = 1$, and $J = 1$, as well as the electromagnetic form factors of the proton and neutron in the timelike region. Since new experimental data were obtained at energies noticeably exceeding the $N\bar{N}$ production threshold, an effective dipole form factor was introduced. It accounts for the energy dependence of the amplitude of real or virtual $N\bar{N}$ pair production at small distances. Since the new data on $n\bar{n}$ production have noticeably better accuracy compared to the previous ones, our predictions became more accurate. The analysis of meson production in different channels shows that the strong energy dependence of the meson production cross sections near the $N\bar{N}$ threshold is related solely to a strong energy dependence of the amplitude of virtual $N\bar{N}$ pair production in an intermediate state.

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