New derivation of the twist-3 gluon fragmentation contribution to polarized hyperon production

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A novel method of formulating the twist-3 gluon fragmentation function contribution to hyperon polarization in the proton-proton collision is presented. The method employs a covariant gauge and takes full advantage of the Ward-Takahashi identities before performing the collinear expansion. It provides a robust way of constructing the general cross section formula and also a clear understanding for the absence of the ghostlike terms in the twist-3 cross section in the leading order with respect to the QCD coupling constant.

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I. INTRODUCTION

In our recent paper [1], we presented a formalism for calculating the twist-3 gluon fragmentation function (FF) contribution to the polarized hyperon production in the proton-proton collision,

$$p(p)+p(p')\to\Lambda^{\uparrow}(P_h,S_\perp)+X, \tag{1}$$

where p, p', and P_h are the momenta of the particles and $S_\perp$ is the transverse spin vector of final $\Lambda^\uparrow$. This contribution is diagrammatically shown in Fig. 1, and the corresponding twist-3 cross section can be calculated from the formula[1]

$$\begin{aligned}E_h\frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h}=&\frac{1}{16\pi^2S_E}\sum_{i,j=q,\bar{q},g}\int_0^1\frac{dx}{x}f_i(x)\int_0^1\frac{dx'}{x'}f_j(x')\\&\times\left[\Omega^\mu_\alpha\Omega^\nu_\beta\int_0^1 dz\mathrm{Tr}[\hat{\Gamma}^{\alpha\beta}(z)S_{\mu\nu}(P_h/z)]-i\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int_0^1 dz\mathrm{Tr}\left[\hat{\Gamma}^{\alpha\beta\gamma}_\partial(z)\frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{\mathrm{c.l.}}\right]\right.\\&\left.+\Re\left\{i\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int_0^1\frac{dz}{z}\int_z^\infty\frac{dz'}{z'}\left(\frac{1}{1/z-1/z'}\right)\mathrm{Tr}\left[\hat{\Gamma}^{\alpha\beta\gamma}_{Fabc}\left(\frac{1}{z'},\frac{1}{z}\right)S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z'},\frac{1}{z}\right)\right]\right\}\right],\end{aligned}\tag{2}$$

where $f_i(x)$ $(i=q,\bar{q},g)$ is the twist-2 unpolarized quark, antiquark, and gluon distributions in the unpolarized proton with the parton's momentum fraction x, $S_E=(p+p')^2$ is the center-of-mass energy squared, and Tr indicates the sum over all spinor or Lorentz indices depending on the channels. The correlation functions $\hat{\Gamma}^{\alpha\beta}(z)$, $\hat{\Gamma}^{\alpha\beta\gamma}_\partial(z)$, and $\hat{\Gamma}^{\alpha\beta\gamma}_{Fabc}(\frac{1}{z'},\frac{1}{z})$, respectively, define *intrinsic*, *kinematical*, and *dynamical* twist-3 gluon FFs. (For the precise definition, see Sec. II.) $S_{\mu\nu}(k)$ and $S^{abc}_{L\mu\nu\lambda}(\frac{1}{z'},\frac{1}{z})$ are the partonic hard parts, which are, at the beginning, convoluted with the Fourier transform of the hadronic matrix elements $\sim\langle 0|A^\nu_b(0)|hX\rangle\langle hX|A^\mu_a(\xi)|0\rangle$

[1]To get a gauge- and frame-independent twist-3 cross section, the $q\bar{q}g$-type FF contribution shown in Fig. 2 of Ref. [1] needs to be added to (2). Since the calculation of the contribution is straightforward, it is not considered in this paper. Readers should refer to Ref. [1] for that contribution.

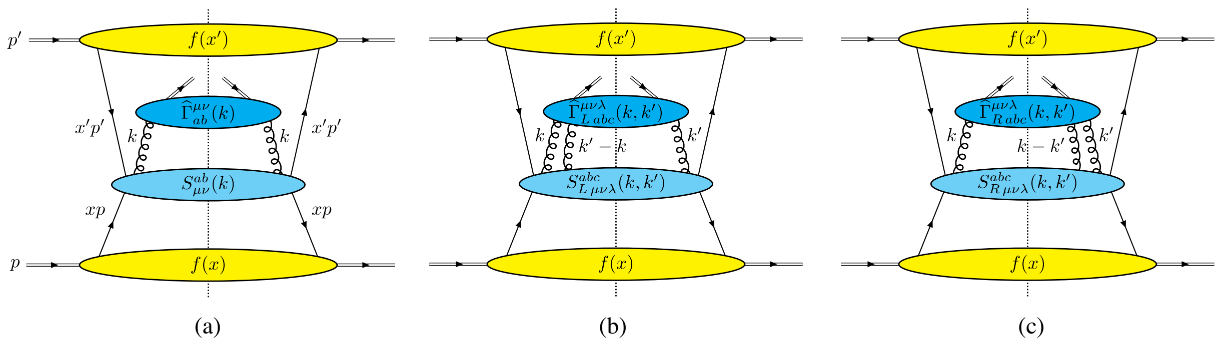

FIG. 1. Generic diagrams representing the twist-3 gluon FF contribution to $pp \to \Lambda^\uparrow X$ for the quark and antiquark distributions in the initial unpolarized protons. The gluon distribution functions in the initial protons also contribute. The diagrams (a), (b), and (c), respectively, correspond to $W_g^{(a)}$, $W_g^{(b)}$, and $W_g^{(c)}$ in (10).

and $\sim\langle 0|A_b^\nu(0)|hX\rangle\langle hX|A_a^\mu(\xi)gA_c^\lambda(\eta)|0\rangle$, respectively. The symbol Ω_α^μ is defined as $\Omega_\alpha^\mu = g_\alpha^\mu - P_h^\mu w_\alpha$ with another lightlike vector w satisfying $P_h \cdot w = 1$, and $|_{\mathrm{c.l.}}$ implies the collinear limit, $k \to P_h/z$. In Ref. [1], the formula (2) was applied to the process (1), and the cross section was calculated in the leading order (LO) with respect to the QCD coupling constant. This completed the LO twist-3 cross section for (1) together with the known results for the contribution from the twist-3 distribution function [2–4] and the twist-3 quark fragmentation function [5]. Since the formula (2) is a very general one, it can be easily adopted for other processes such as $e^+e^- \to \Lambda^\uparrow X$ [6], $ep \to e\Lambda^\uparrow X$ [7,8], etc.

To derive the general formula (2), we applied in Ref. [1] the collinear expansion to the hard parts $S_{\mu\nu}(k)$ and $S^{abc}_{L\mu\nu\lambda}(k,k')$. Using the Ward-Takahashi identities for the partonic hard parts, we could eventually rewrite the twist-3 cross sections in terms of the low derivatives of the hard parts and the gauge-invariant correlation functions of the gluon's field strengths as in (2). Actual calculation, however, is extremely complicated and lengthy and is not easy to see how the correlation functions of the gauge field A_a^μ is converted into those of the field strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$. Furthermore, vanishing of the ghostlike terms appearing in the Ward-Takahashi identities is essential to reach (2), which was not clearly shown in Ref. [1]. Therefore, an easier way of deriving (2) is very useful.

In this paper, we present a much more robust and concise way of deriving (2). In this method, we use Ward-Takahashi identities from the outset to convert gauge fields into a part of the field strengths, which results in substantial savings in the actual calculation. This procedure was once adopted for deriving the twist-3 three-gluon distribution contribution to the single spin asymmetry in $ep^\uparrow \to eDX$, where three-gluon distribution contributes as an only source for the asymmetry and appears as a "pole contribution"[9]. For the present case of the twist-3 gluon FF for (1), three types of FFs contribute as a "nonpole contribution," and hence the situation is much more complicated. Furthermore, our present method provides clear proof for the absence of the ghostlike terms which appear in the Ward-Takahashi identities. This is crucial to guarantee the gauge invariance of the twist-3 cross section.

The remainder of the paper is organized as follows. In Sec. II, a brief summary of the twist-3 gluon FFs which appear in (2) is given. In Sec. III and the Appendix, we present a novel derivation of (2) and prove the absence of the ghostlike terms in the LO twist-3 cross section. Section IV is devoted to a brief summary.

II. GLUON FRAGMENTATION FUNCTIONS

Here, we summarize the twist-3 gluon FFs in our notation which appear in (2) [1].[2] The twist-3 *intrinsic* gluon FFs are defined as

$$\hat{\Gamma}^{\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|([\infty w,0]F^{w\beta}(0))_a|h(P_h,S_h)X\rangle\langle h(P_h,S_h)X|(F^{w\alpha}(\lambda w)[\lambda w,\infty w])_a|0\rangle$$

$$= -g_\perp^{\alpha\beta}\hat{G}(z) - iM_h\epsilon^{P_h w\alpha\beta}(S_h\cdot w)\Delta\hat{G}(z) - iM_h\epsilon^{P_h w S_\perp [\alpha}w^{\beta]}\Delta\hat{G}_{3T}(z) + M_h\epsilon^{P_h w S_\perp \{\alpha}w^{\beta\}}\Delta\hat{G}_{3\bar{T}}(z), \tag{3}$$

where $N = 3$ is the number of colors for a quark, $|h(P_h,S_h)\rangle$ is the spin-1/2 hyperon state with the 4-momentum P_h ($P_h^2 = M_h^2$) and the spin vector S_h ($S_h^2 = -M_h^2$), and $[\lambda w, \infty w]$ is the gauge link in the adjoint representation connecting λw and ∞w. For the transversely polarized baryon, we use the spin vector $S_\perp$ normalized as $S_\perp^2 = -1$. In the twist-3 accuracy P_h

[2]See also Refs. [6,10,11] for earlier references and more details about the gluon FFs.

can be regarded as lightlike. For a baryon with large momentum, $P_h \simeq (|\vec{P}_h|, \vec{P}_h)$, another lightlike vector w is defined as $w = 1/(2|\vec{P}_h|^2)(|\vec{P}_h|, -\vec{P}_h)$, which satisfies $P_h \cdot w = 1$. $\hat{G}(z)$ and $\Delta\hat{G}(z)$ are twist 2, and $\Delta\hat{G}_{3T}(z)$ and $\Delta\hat{G}_{3\bar{T}}(z)$ are twist 3. We also note $\Delta\hat{G}_{3\bar{T}}(z)$ is naively T odd, contributing to hyperon polarization. Each function in (3) has a support on $0 < z < 1$.

The *kinematical* FFs are defined from the transverse derivative of the correlation functions of the field strengths,

$$\hat{\Gamma}^{\alpha\beta\gamma}_{\partial}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|([\infty w, 0]F^{w\beta}(0))_a|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|(F^{w\alpha}(\lambda w)[\lambda w, \infty w])_a|0\rangle \overleftarrow{\partial}^{\gamma}$$
$$= -i\frac{M_h}{2} g^{\alpha\beta}_{\perp}\epsilon^{P_h w S_\perp \gamma}\hat{G}^{(1)}_T(z) + \frac{M_h}{2}\epsilon^{P_h w \alpha\beta}S^{\gamma}_{\perp}\Delta\hat{G}^{(1)}_T(z) - i\frac{M_h}{8}(\epsilon^{P_h w S_\perp \{\alpha} g^{\beta\}\gamma}_{\perp} + \epsilon^{P_h w \gamma\{\alpha}S^{\beta\}}_{\perp})\Delta\hat{H}^{(1)}_T(z), \quad (4)$$

where each function is defined to be real and has a support on $0 < z < 1$.

The *dynamical* FFs are defined from the light-cone correlation functions of three field strengths,

$$\hat{\Gamma}^{\alpha\beta\gamma}_{Fabc}\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = \sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|F^{w\beta}_b(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F^{w\alpha}_a(\lambda w) g F^{w\gamma}_c(\mu w)|0\rangle$$
$$= \frac{if^{abc}}{N}\hat{\Gamma}^{\alpha\beta\gamma}_{FA}\left(\frac{1}{z_1}, \frac{1}{z_2}\right) + d^{abc}\frac{N}{N^2-4}\hat{\Gamma}^{\alpha\beta\gamma}_{FS}\left(\frac{1}{z_1}, \frac{1}{z_2}\right), \quad (5)$$

where the gauge link operators are suppressed for simplicity, and f^{abc} and d^{abc} are the antisymmetric and symmetric structure constants of color SU(N). The dynamical FFs can be defined as the decomposition of the two correlation functions in (5) as

$$\hat{\Gamma}^{\alpha\beta\gamma}_{FA}\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = \frac{-if_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|F^{w\beta}_b(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F^{w\alpha}_a(\lambda w) g F^{w\gamma}_c(\mu w)|0\rangle$$
$$= -M_h\left(\hat{N}_1\left(\frac{1}{z_1}, \frac{1}{z_2}\right) g^{\alpha\gamma}_{\perp}\epsilon^{P_h w S_\perp \beta} + \hat{N}_2\left(\frac{1}{z_1}, \frac{1}{z_2}\right) g^{\beta\gamma}_{\perp}\epsilon^{P_h w S_\perp \alpha} - \hat{N}_2\left(\frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2}\right) g^{\alpha\beta}_{\perp}\epsilon^{P_h w S_\perp \gamma}\right), \quad (6)$$

$$\hat{\Gamma}^{\alpha\beta\gamma}_{FS}\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = \frac{d_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|F^{w\beta}_b(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F^{w\alpha}_a(\lambda w) g F^{w\gamma}_c(\mu w)|0\rangle$$
$$= -M_h\left(\hat{O}_1\left(\frac{1}{z_1}, \frac{1}{z_2}\right) g^{\alpha\gamma}_{\perp}\epsilon^{P_h w S_\perp \beta} + \hat{O}_2\left(\frac{1}{z_1}, \frac{1}{z_2}\right) g^{\beta\gamma}_{\perp}\epsilon^{P_h w S_\perp \alpha} + \hat{O}_2\left(\frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2}\right) g^{\alpha\beta}_{\perp}\epsilon^{P_h w S_\perp \gamma}\right). \quad (7)$$

Correlation functions (6) and (7), respectively, define two independent set of the *complex* functions $\{\hat{N}_1, \hat{N}_2\}$ and $\{\hat{O}_1, \hat{O}_2\}$ due to the exchange symmetry of the field strengths. Functions $\hat{N}_1$ and $\hat{O}_1$ satisfy the relations

$$\hat{N}_1\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = -\hat{N}_1\left(\frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2}\right),$$
$$\hat{O}_1\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = \hat{O}_1\left(\frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2}\right). \quad (8)$$

The real parts of these four FFs are T even, and the imaginary parts are T odd, the latter being the sources of single spin asymmetries. $\hat{N}_{1,2}(\frac{1}{z_1}, \frac{1}{z_2})$ and $\hat{O}_{1,2}(\frac{1}{z_1}, \frac{1}{z_2})$ have a support on $\frac{1}{z_2} > 1$ and $\frac{1}{z_2} > \frac{1}{z_1} > 0$.

III. TWIST-3 GLUON FRAGMENTATION CONTRIBUTION TO $pp \to \Lambda^{\uparrow} X$

In this section, we present a robust way to derive the basic formula (2). The twist-3 gluon FF contribution to (1) can be written as

$$E_h\frac{d\sigma(p, p', P_h; S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E}\sum_{i,j=q,\bar{q},g}\int \frac{dx}{x} f_i(x) \times \int \frac{dx'}{x'} f_j(x') W_g(xp, x'p', P_h/z, S_\perp), \quad (9)$$

where W_g represents the partonic hard scattering followed by the fragmentation of a gluon into the final $\Lambda^{\uparrow}$.

Figure 1 -shows the generic structure of the LO diagrams for this contribution. Corresponding to Figs. 1(a)–1(c), W_g consists of three parts,

$$W_g(xp,x'p',P_h;S_\perp) \equiv W_g^{(a)} + W_g^{(b)} + W_g^{(c)}$$
$$= \int \frac{d^4k}{(2\pi)^4}[S^{ab}_{\mu\nu}(k)\hat{\Gamma}^{\mu\nu}_{ab}(k)] + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k') + S^{abc}_{R\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Rabc}(k,k')], \quad (10)$$

where $\hat{\Gamma}^{\mu\nu}_{ab}(k)$, $\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')$, and $\hat{\Gamma}^{\mu\nu\lambda}_{Rabc}(k,k')$ are the hadronic matrix elements of the gauge (gluon) fields with k and k' the 4-momenta of the gluons fragmenting into the final Λ, and $S^{ab}_{\mu\nu}(k)$, $S^{abc}_{L\mu\nu\lambda}(k,k')$, and $S^{abc}_{R\mu\nu\lambda}(k,k')$ are the corresponding partonic hard scattering parts with the color indices a, b, c and the Lorentz indices μ, ν, λ. In (10), the factor $1/2$ in front of $W_g^{(b)}$ and $W_g^{(c)}$ takes into account the exchange symmetry of the gluon fields in the fragmentation matrix elements. Hadronic matrix elements are defined as

$$\hat{\Gamma}^{\mu\nu}_{ab}(k) = \sum_X \int d^4\xi e^{-ik\xi}\langle 0|A^\nu_b(0)|hX\rangle\langle hX|A^\mu_a(\xi)|0\rangle, \quad (11)$$

$$\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k') = \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi}e^{-i(k'-k)\eta}\langle 0|A^\nu_b(0)|hX\rangle\langle hX|A^\mu_a(\xi)gA^\lambda_c(\eta)|0\rangle, \quad (12)$$

$$\hat{\Gamma}^{\mu\nu\lambda}_{Rabc}(k,k') = \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi}e^{-i(k'-k)\eta}\langle 0|A^\nu_b(0)gA^\lambda_c(\eta)|hX\rangle\langle hX|A^\mu_a(\xi)|0\rangle, \quad (13)$$

where the gauge coupling g associated with the attachment of the extra gluon line to the hard part is included in $\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')$ and $\hat{\Gamma}^{\mu\nu\lambda}_{Rabc}(k,k')$. Therefore, the hard parts $S^{ab}_{\mu\nu}(k)$, $S^{abc}_{L\mu\nu\lambda}(k,k')$, and $S^{abc}_{R\mu\nu\lambda}(k,k')$ are of $O(g^4)$ in the LO calculation. From Hermiticity, one has $\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')^\star = \hat{\Gamma}^{\nu\mu\lambda}_{Rbac}(k',k)$ and $S^{abc}_{L\mu\nu\lambda}(k,k')^\star = S^{bac}_{R\nu\mu\lambda}(k',k)$, which guarantees the reality of W_g. A standard procedure to extract the twist-3 effect is the collinear expansion of the hard parts with respect to k and k' around P_h. We followed the method in Ref. [1] to get (2).

Here, we present an alternative method which leads to (2) more easily. In this method, we fully use the Ward-Takahashi identities for the hard parts to convert some of the gluon field A^a_μ into a part of the field strength $F^a_{\mu\nu}$. Ward-Takahashi identities for the hard part read

$$k^\mu S^{ab}_{\mu\nu}(k) = k^\nu S^{ab}_{\mu\nu}(k) = 0, \quad (14)$$

$$(k'-k)^\lambda S^{abc}_{L\mu\nu\lambda}(k,k') = \frac{-if^{abc}}{N^2-1}S_{\mu\nu}(k') + G^{abc}_{\mu\nu}(k,k'), \quad (15)$$

$$k^\mu S^{abc}_{L\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1}S_{\lambda\nu}(k') + G^{cba}_{\lambda\nu}(k'-k,k'), \quad (16)$$

$$k'^\nu S^{abc}_{L\mu\nu\lambda}(k,k') = 0, \quad (17)$$

$$(k'-k)^\lambda S^{abc}_{R\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1}S_{\mu\nu}(k) - (G^{bac}_{\nu\mu}(k',k))^\star, \quad (18)$$

$$k'^\nu S^{abc}_{R\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1}S_{\mu\lambda}(k) + (G^{cab}_{\lambda\mu}(k-k',k))^\star, \quad (19)$$

$$k^\mu S^{abc}_{R\mu\nu\lambda}(k,k') = 0, \quad (20)$$

where $S_{\mu\nu}(k) \equiv S^{ab}_{\mu\nu}(k)\delta_{ab}$. The G terms are the ghostlike terms which appear due to the off-shell-ness and the nonphysical polarization of the gluon lines entering the fragmentation matrix elements. Actual forms of those ghostlike terms for $pp \to \Lambda^\uparrow X$ were given in Appendix A of Ref. [1] in the LO with respect to the QCD coupling. They are proportional to f^{abc} and satisfy the relation

$$k^\mu G^{abc}_{\mu\nu}(k,k') = k'^\nu G^{abc}_{\mu\nu}(k,k') = 0. \quad (21)$$

We will see that use of the relations (14)–(21) from the outset brings enormous saving in the actual calculation and clearer understanding on the absence of the ghostlike terms in the LO twist-3 cross section.[3]

We first consider $W_g^{(a)}$. The integration momentum k can be decomposed as

$$k^\mu = (k\cdot w)P^\mu_h + \Omega^\mu_\nu k^\nu, \quad (22)$$

[3]The absence of the ghost term contribution to the twist-3 cross sections was discussed for the 3-gluon distribution contribution to $\vec{p}p^\uparrow \to DX$ [12] and twist-3 quark FF contribution to $ep^\uparrow \to e\pi X$ [13].

where $\Omega^\mu_\nu \equiv g^\mu_\nu - P^\mu_h w_\nu$. Inserting (22) into (14), one gets

$$S^{ab}_{P_h\nu}(k) = \frac{-1}{k\cdot w}\Omega^\mu_\kappa k^\kappa S^{ab}_{\mu\nu}(k), \tag{23}$$

$$S^{ab}_{\mu P_h}(k) = \frac{-1}{k\cdot w}\Omega^\nu_\tau k^\tau S^{ab}_{\mu\nu}(k). \tag{24}$$

Then, we can write

$$\begin{aligned} S^{ab}_{\mu\nu}(k)\hat{\Gamma}^{\mu\nu}_{ab}(k) &= S^{ab}_{\mu\nu}(k) g^\mu_\alpha g^\nu_\beta \hat{\Gamma}^{\alpha\beta}_{ab}(k) = S^{ab}_{\mu\nu}(k)(P^\mu_h w_\alpha + \Omega^\mu_\alpha)(P^\nu_h w_\beta + \Omega^\nu_\beta)\hat{\Gamma}^{\alpha\beta}_{ab}(k) \\ &= \frac{1}{(k\cdot w)^2} S^{ab}_{\mu\nu}(k)\Omega^\mu_\kappa\Omega^\nu_\tau(-k^\kappa w_\alpha + k\cdot w g^\kappa_\alpha)(-k^\tau w_\beta + k\cdot w g^\tau_\beta)\hat{\Gamma}^{\alpha\beta}_{ab}(k), \end{aligned} \tag{25}$$

where we have used (23) and (24) in the last equality. Noting that one can write

$$(-k^\kappa w_\alpha + k\cdot w g^\kappa_\alpha)(-k^\tau w_\beta + k\cdot w g^\tau_\beta)\hat{\Gamma}^{\alpha\beta}_{ab}(k) = \sum_X \int d^4\xi e^{-ik\xi}\langle 0|F^{(0)\tau w}_b(0)|hX\rangle\langle hX|F^{(0)\kappa w}_a(\xi)|0\rangle \equiv \hat{\Gamma}^{\kappa\tau}_{Fab}(k), \tag{26}$$

where $F^{(0)\kappa\sigma}_a \equiv \partial^\kappa A^\sigma_a - \partial^\sigma A^\kappa_a$ is the $O(A)$ piece of the gluon's field strength, one obtains

$$\int \frac{d^4k}{(2\pi)^4}[S^{ab}_{\mu\nu}(k)\hat{\Gamma}^{\mu\nu}_{ab}(k)] = \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k\cdot w)^2}S^{ab}_{\mu\nu}(k)\Omega^\mu_\kappa\Omega^\nu_\tau\hat{\Gamma}^{\kappa\tau}_{Fab}(k)\right]. \tag{27}$$

To extract the twist-3 contribution, we apply the collinear expansion to the rhs of (27). Writing $k\cdot w = 1/z$, the contribution up to twist-3 from (27) can be obtained as

$$\begin{aligned} W^{(\mathrm{a})}_g &= \int \frac{d^4k}{(2\pi)^4}[S^{ab}_{\mu\nu}(k)\hat{\Gamma}^{\mu\nu}_{ab}(k)]^{\mathrm{twist-3}} \\ &= \int d\left(\frac{1}{z}\right) z^2\Omega^\mu_\kappa\Omega^\nu_\tau\left[S_{\mu\nu}(z)\hat{\Gamma}^{\kappa\tau}_F(z) + \Omega^\lambda_\rho \frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{k=P_h/z}\hat{\Gamma}^{\kappa\tau\rho}_{\partial F}(z)\right], \end{aligned} \tag{28}$$

where $S_{\mu\nu}(z) \equiv S^{ab}_{\mu\nu}(P_h/z)\delta^{ab}$ and

$$\hat{\Gamma}^{\kappa\tau}_F(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z}\langle 0|F^{(0)\tau w}_a(0)|hX\rangle\langle hX|F^{(0)\kappa w}_a(\lambda w)|0\rangle, \tag{29}$$

$$\hat{\Gamma}^{\kappa\tau\rho}_{\partial F}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z}\langle 0|F^{(0)\tau w}_a(0)|hX\rangle\langle hX|(-i)\partial^\rho F^{(0)\kappa w}_a(\lambda w)|0\rangle. \tag{30}$$

Equations (29) and (30) are, respectively, identified as the $O(g^0)$ parts of (3) and (4), and (28) represents the lowest-order contribution to the first and second terms in (2).

Next, we proceed to analyze $W^{(\mathrm{b})}_g$. Using (22) in (16), we have

$$(k\cdot w)S^{abc}_{LP_h\nu\lambda}(k,k') + \Omega^\mu_\rho k^\rho S^{abc}_{L\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1}S_{\lambda\nu}(k') + G^{cba}_{\lambda\nu}(k'-k,k'), \tag{31}$$

from which we obtain

$$S^{abc}_{LP_h\nu\lambda}(k,k') = \frac{1}{k\cdot w}\left(-\Omega^\mu_\rho k^\rho S^{abc}_{L\mu\nu\lambda}(k,k') + \frac{if^{abc}}{N^2-1}S_{\lambda\nu}(k') + G^{cba}_{\lambda\nu}(k'-k,k')\right). \tag{32}$$

Likewise, from (22) and (17), we have

$$S^{abc}_{L\mu P_h\lambda}(k,k') = \frac{-1}{k'\cdot w}\Omega^\nu_\tau k'^\tau S^{abc}_{L\mu\nu\lambda}(k,k'). \tag{33}$$

As in (25), Eq. (32) and (33) can be used to rewrite integrand of $W_g^{(\mathrm{b})}$ as

$$\begin{aligned}
S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k') &= S^{abc}_{L\mu\nu\lambda}(k,k')g^\mu_\kappa g^\nu_\tau g^\lambda_\sigma \hat{\Gamma}^{\kappa\tau\sigma}_{Labc}(k,k')\\
&= S^{abc}_{L\mu\nu\lambda}(k,k')(P^\mu_h w_\kappa + \Omega^\mu_\kappa)(P^\nu_h w_\tau + \Omega^\nu_\tau)(P^\lambda_h w_\sigma + \Omega^\lambda_\sigma)\hat{\Gamma}^{\kappa\tau\sigma}_{Labc}(k,k')\\
&= \frac{1}{k\cdot w}\frac{1}{k'\cdot w}S^{abc}_{L\mu\nu\lambda}(k,k')\Omega^\mu_\alpha\Omega^\nu_\beta(-k^\alpha w_\kappa + k\cdot w g^\alpha_\kappa)(-k'^\beta w_\tau + k'\cdot w g^\beta_\tau)\\
&\quad\times(P^\lambda_h w_\sigma + \Omega^\lambda_\sigma)\hat{\Gamma}^{\kappa\tau\sigma}_{Labc}(k,k') + \frac{1}{k\cdot w}\left(\frac{if^{abc}}{N^2-1}S_{\sigma\tau}(k') + G^{cba}_{\sigma\tau}(k'-k,k')\right)\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k').
\end{aligned} \tag{34}$$

Similarly to (26), we have

$$\begin{aligned}
&(-k^\alpha w_\kappa + k\cdot w g^\alpha_\kappa)(-k'^\beta w_\tau + k'\cdot w g^\beta_\tau)\hat{\Gamma}^{\kappa\tau\sigma}_{Labc}(k,k')\\
&\quad= \sum_X\int d^4\xi\int d^4\eta e^{-ik\xi}e^{-i(k'-k)\eta}\langle 0|F_b^{(0)\beta w}(0)|hX\rangle\langle hX|F_a^{(0)\alpha w}(\xi)gA^\sigma_c(\eta)|0\rangle\\
&\quad\equiv \hat{\Gamma}^{\alpha\beta\sigma}_{LFAabc}(k,k').
\end{aligned} \tag{35}$$

Using this equation, Eq. (34) can be rewritten as

$$\begin{aligned}
S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k') &= S^{abc}_{L\mu\nu\lambda}(k,k')\frac{1}{k\cdot w}\frac{1}{k'\cdot w}\Omega^\mu_\alpha\Omega^\nu_\beta(P^\lambda_h\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k') + \Omega^\lambda_\sigma\hat{\Gamma}^{\alpha\beta\sigma}_{LFAabc}(k,k'))\\
&\quad + \frac{1}{k\cdot w}\left(\frac{if^{abc}}{N^2-1}S_{\sigma\tau}(k') + G^{cba}_{\sigma\tau}(k'-k,k')\right)\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k').
\end{aligned} \tag{36}$$

Since (36) is integrated over k and k' in (10), one can change the integration variable as $k\to k'-k$ in the last term of (36) containing the ghostlike term. Furthermore, because of $\hat{\Gamma}^{w\tau\sigma}_{Labc}(k'-k,k') = \hat{\Gamma}^{\sigma\tau w}_{Lcba}(k,k')$, one can change this term as

$$\frac{1}{k\cdot w}G^{cba}_{\sigma\tau}(k'-k,k')\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k') \longrightarrow \frac{1}{k'\cdot w - k\cdot w}G^{cba}_{\sigma\tau}(k,k')\hat{\Gamma}^{\sigma\tau w}_{Lcba}(k,k') = \frac{1}{k'\cdot w - k\cdot w}G^{abc}_{\sigma\tau}(k,k')\hat{\Gamma}^{\sigma\tau w}_{Labc}(k,k'). \tag{37}$$

Using this form for the last term in (36) and applying the collinear expansion to the first term in (36) up to twist 3, one obtains

$$\begin{aligned}
[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')]^{\text{twist-3}} &= zz'\Omega^\mu_\alpha\Omega^\nu_\beta\left\{S^{abc}_{L\mu\nu P_h}\left(\frac{1}{z},\frac{1}{z'}\right) + \Omega^\lambda_\gamma k^\gamma\frac{\partial S^{abc}_{L\mu\nu P_h}(k,k')}{\partial k^\lambda}\bigg|_{\text{c.l.}} + \Omega^\lambda_\gamma k'^\gamma\frac{\partial S^{abc}_{L\mu\nu P_h}(k,k')}{\partial k'^\lambda}\bigg|_{\text{c.l.}}\right\}\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k')\\
&\quad + zz'\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\sigma S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right)\hat{\Gamma}^{\alpha\beta\sigma}_{LFAabc}(k,k')\\
&\quad + z\frac{if^{abc}}{N^2-1}S_{\sigma\tau}(k')\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k') + \frac{1}{1/z'-1/z}G^{abc}_{\sigma\tau}(k,k')\hat{\Gamma}^{\sigma\tau w}_{Labc}(k,k'),
\end{aligned} \tag{38}$$

where we have set $k\cdot w = \frac{1}{z}$ and $k'\cdot w = \frac{1}{z'}$. The first term of (38) can be further rewritten by the Ward-Takahashi identity (15). The collinear limit of (15) gives

$$S^{abc}_{L\mu\nu P_h}\left(\frac{1}{z},\frac{1}{z'}\right) = \frac{1}{1/z'-1/z}\left\{\frac{-if^{abc}}{N^2-1}S_{\mu\nu}(z') + G^{abc}_{\mu\nu}\left(\frac{1}{z},\frac{1}{z'}\right)\right\}. \tag{39}$$

Similarly from the collinear limit of the first derivatives of (15) with respect to k and k', one obtains

$$\left.\frac{\partial S^{abc}_{L\mu\nu P_h}(k,k')}{\partial k^\lambda}\right|_{\rm c.l.} = \frac{1}{1/z'-1/z}\left\{S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right) + \left.\frac{\partial G^{abc}_{\mu\nu}(k,k')}{\partial k^\lambda}\right|_{\rm c.l.}\right\}, \tag{40}$$

$$\left.\frac{\partial S^{abc}_{L\mu\nu P_h}(k,k')}{\partial k'^\lambda}\right|_{\rm c.l.} = \frac{1}{1/z'-1/z}\left\{-S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right) + \left.\frac{\partial G^{abc}_{\mu\nu}(k,k')}{\partial k'^\lambda}\right|_{\rm c.l.} - \frac{if^{abc}}{N^2-1}\left.\frac{\partial S_{\mu\nu}(k')}{\partial k'^\lambda}\right|_{\rm c.l.}\right\}. \tag{41}$$

In the last term in the rhs of (38), using the relation (21) and following a similar procedure as (34) and (35), one can rewrite

$$G^{abc}_{\mu\nu}(k,k')\hat{\Gamma}^{\mu\nu w}_{Labc}(k,k') = zz'G^{abc}_{\mu\nu}(k,k')\Omega^\mu_\alpha\Omega^\nu_\beta\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k'). \tag{42}$$

Using this form, one sees the collinear expansion of the last term in (38) yields the identical terms as the ghostlike terms in (39), (40), and (41). This way, one obtains the ghostlike terms (i.e., terms containing $G^{abc}_{\mu\nu}$) in (38) as

$$\begin{aligned}[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')]^{\rm ghost} = 2\Omega^\mu_\alpha\Omega^\nu_\beta\frac{zz'}{1/z'-1/z}\Bigg\{G^{abc}_{\mu\nu}\left(\frac{1}{z},\frac{1}{z'}\right)\\ +\Omega^\lambda_\gamma\left(k^\gamma\left.\frac{\partial G^{abc}_{\mu\nu}(k,k')}{\partial k^\lambda}\right|_{\rm c.l.} + k'^\gamma\left.\frac{\partial G^{abc}_{\mu\nu}(k,k')}{\partial k'^\lambda}\right|_{\rm c.l.}\right)\Bigg\}\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k').\end{aligned} \tag{43}$$

Using the actual forms of the ghostlike terms given in Appendix A of Ref. [1], we will show in the Appendix that (43) does not contribute to the LO twist-3 cross section. Hence, we will discard (43) below.

Remaining terms in (38) can be written as

$$\begin{aligned}[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')]^{\rm twist-3} &= \Omega^\mu_\alpha\Omega^\nu_\beta\frac{1}{1/z+i\epsilon}\frac{z'}{1/z'-1/z+i\epsilon}\left(\frac{-if^{abc}}{N^2-1}\right)S_{\mu\nu}(z')\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k')\\ &+\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\frac{1}{1/z+i\epsilon}\frac{z'}{1/z'-1/z+i\epsilon}S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right)\\ &\times\left\{(k-k')^\gamma\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k') + \left(\frac{1}{z'}-\frac{1}{z}\right)\hat{\Gamma}^{\alpha\beta\gamma}_{LFAabc}(k,k')\right\}\\ &+\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\frac{1}{1/z+i\epsilon}\frac{z'}{1/z'-1/z+i\epsilon}\left(\frac{-if^{abc}}{N^2-1}\right)k'^\gamma\left.\frac{\partial S_{\mu\nu}(k')}{\partial k'^\lambda}\right|_{\rm c.l.}\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k')\\ &+\Omega^\mu_\alpha\Omega^\nu_\beta\frac{z'^2}{1/z+i\epsilon}\left\{S_{\mu\nu}(z') + \Omega^\lambda_\gamma k'^\gamma\left.\frac{\partial S_{\mu\nu}(k')}{\partial k'^\lambda}\right|_{\rm c.l.}\right\}\\ &\times\frac{if^{abc}}{N^2-1}(-k'^\alpha w_\sigma + k'\cdot wg^\alpha_\sigma)(-k'^\beta w_\tau + k'\cdot wg^\beta_\tau)\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k'),\end{aligned} \tag{44}$$

where the third term in (38) was rewritten as the last term by using (14) and subsequent collinear expansion of $S_{\mu\nu}(k')$. We have introduced $i\epsilon$ in the denominators, which gives rise to the future pointing gauge links. Integration of (44) over k and k' proceeds as follows. We first write for the last term of (44)

$$\begin{aligned}&(-k'^\alpha w_\sigma + k'\cdot wg^\alpha_\sigma)(-k'^\beta w_\tau + k'\cdot wg^\beta_\tau)\hat{\Gamma}^{w\tau\sigma}_{Labc}(k,k')\\ &= \int d^4\xi\int d^4\eta e^{-ik\xi}e^{-i(k'-k)\eta}\sum_X\langle 0|F^{(0)w\beta}_b(0)|hX\rangle\\ &\quad\times\langle hX|g(\partial^w A^w_a(\xi))A^\alpha_c(\eta) - g(\partial^w A^\alpha_a(\xi))A^w_c(\eta) + gA^w_a(\xi)F^{(0)w\alpha}_c(\eta) + gF^{(0)w\alpha}_a(\xi)A^w_c(\eta)|0\rangle\\ &\equiv\hat{\Gamma}^{\alpha\beta}_{LF\partial abc}(k,k').\end{aligned} \tag{45}$$

Then, the contribution from the first term in {} of the last term of (44) reads

$$\begin{aligned}
&\int \frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\Omega^\mu_\alpha\Omega^\nu_\beta\frac{z'^2}{1/z+i\epsilon}\frac{if^{abc}}{N^2-1}S_{\mu\nu}(z')\hat{\Gamma}^{\alpha\beta}_{LF\partial abc}(k,k')\\
&=\int d\left(\frac{1}{z'}\right)\int d\left(\frac{1}{z}\right)\Omega^\mu_\alpha\Omega^\nu_\beta z'^2\frac{if^{abc}}{N^2-1}S_{\mu\nu}(z')\times i\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z'-1/z)}\\
&\quad\times\sum_X\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|gA^w_a(\lambda w)A^\alpha_c(\mu w)-gA^\alpha_a(\lambda w)A^w_c(\mu w)|0\rangle\\
&\quad+\int d\left(\frac{1}{z'}\right)\int d\left(\frac{1}{z}\right)\Omega^\mu_\alpha\Omega^\nu_\beta\frac{z'^2}{1/z+i\epsilon}\frac{if^{abc}}{N^2-1}S_{\mu\nu}(z')\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z'-1/z)}\\
&\quad\times\sum_X\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|gA^w_a(\lambda w)F_c^{(0)w\alpha}(\mu w)+gF_a^{(0)w\alpha}(\lambda w)A^w_c(\mu w)|0\rangle,
\end{aligned}\tag{46}$$

where, in the first term, we have performed integration by parts for λ integration, which kills the factor $\frac{1}{1/z+i\epsilon}$. Integration over $1/z$ of this equation can be done immediately. The second term in $\{\}$ of the last term of (44) can be integrated parallelly. Following this procedure, $W_g^{(\mathrm{b})}$ is obtained by the integral of (44) as

$$\begin{aligned}
W_g^{(\mathrm{b})}&=\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')]^{\text{twist-3}}\\
&=-\Omega^\mu_\alpha\Omega^\nu_\beta\int d\left(\frac{1}{z'}\right)f^{abc}z'^2S_{\mu\nu}(z')\times\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z'}\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|gA^w_a(\lambda w)A^\alpha_c(\lambda w)|0\rangle\\
&\quad+\Omega^\mu_\alpha\Omega^\nu_\beta\int d\left(\frac{1}{z'}\right)f^{abc}z'^2S_{\mu\nu}(z')\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z'}\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|gF_a^{(0)w\alpha}(\lambda w)\int_\infty^\lambda d\mu A^w_c(\mu w)|0\rangle\\
&\quad+\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int d\left(\frac{1}{z'}\right)z'^2if^{abc}\frac{\partial S_{\mu\nu}(k')}{\partial k'^\lambda}\bigg|_{\text{c.l.}}\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z'}\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|\partial^\gamma\{gA^w_a(\lambda w)A^\alpha_c(\lambda w)\}|0\rangle\\
&\quad-\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int d\left(\frac{1}{z'}\right)z'^2if^{abc}\frac{\partial S_{\mu\nu}(k')}{\partial k'^\lambda}\bigg|_{\text{c.l.}}\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z'}\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\\
&\quad\times\langle hX\bigg|\int_\infty^\lambda d\mu\{(\partial^\gamma F_a^{(0)w\alpha}(\lambda w))gA^w_c(\mu w)+F_a^{(0)w\alpha}(\lambda w)g\partial^\gamma A^w_c(\mu w)\}|0\rangle\\
&\quad-\frac{i}{2}\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int_1^\infty d\left(\frac{1}{z'}\right)\int_0^{1/z'}d\left(\frac{1}{z}\right)\frac{zz'}{1/z'-1/z}S^{abc}_{L\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right)\hat{\Gamma}^{\alpha\beta\gamma}_{LFabc}\left(\frac{1}{z},\frac{1}{z'}\right),
\end{aligned}\tag{47}$$

where

$$\hat{\Gamma}^{\alpha\beta\gamma}_{LFabc}\left(\frac{1}{z},\frac{1}{z'}\right)=\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z'-1/z)}\langle 0|F_b^{(0)w\beta}(0)|hX\rangle\langle hX|F_a^{(0)w\alpha}(\lambda w)gF_c^{(0)w\gamma}(\mu w)|0\rangle.\tag{48}$$

The first four terms in (47) come from the combination of the first, third, and the last terms in (44), while the last term in (47) is from the second term of (44). $\hat{\Gamma}^{\alpha\beta\gamma}_{LFabc}(\frac{1}{z},\frac{1}{z'})$ can be identified as the lowest-order part of the dynamical FF (5).

Calculation of $W_g^{(c)}$ can be performed in the same way. The result reads

$$\begin{aligned}
W_g^{(c)} =& \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}[S^{abc}_{R\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Rabc}(k,k')]^{\text{twist}-3}\\
=& \Omega^\mu_\alpha\Omega^\nu_\beta\int d\left(\frac{1}{z}\right)f^{abc}z^2S_{\mu\nu}(z)\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|A^w_b(0)gA^\beta_c(0)|hX\rangle\langle hX|F^{(0)w\alpha}_a(\lambda w)|0\rangle\\
&+\Omega^\mu_\alpha\Omega^\nu_\beta\sum_X\int d\left(\frac{1}{z}\right)f^{abc}z^2S_{\mu\nu}(z)\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\int_0^\infty d\mu A^w_c(\mu w)F^{(0)w\beta}_b(0)|hX\rangle\langle hX|gF^{(0)w\alpha}_a(\lambda w)|0\rangle\\
&-\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\sum\int d\left(\frac{1}{z}\right)z^2if^{abc}\frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{\text{c.l.}}\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|A^w_b(0)gA^\beta_c(0)|hX\rangle\langle hX|\partial^\gamma F^{(0)w\alpha}_a(\lambda w)|0\rangle\\
&-\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int d\left(\frac{1}{z}\right)z^2if^{abc}\frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{\text{c.l.}}\frac{1}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\int_0^\infty d\mu gA^w_c(\mu w)F^{(0)w\beta}_b(0)|hX\rangle\langle hX|\partial^\gamma F^{(0)w\alpha}_a(\lambda w)|0\rangle\\
&-\frac{i}{2}\Omega^\mu_\alpha\Omega^\nu_\beta\Omega^\lambda_\gamma\int_1^\infty d\left(\frac{1}{z}\right)\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{zz'}{1/z'-1/z}S^{abc}_{R\mu\nu\lambda}\left(\frac{1}{z},\frac{1}{z'}\right)\hat{\Gamma}^{\alpha\beta\gamma}_{RFabc}\left(\frac{1}{z},\frac{1}{z'}\right),
\end{aligned}\tag{49}$$

where

$$\hat{\Gamma}^{\alpha\beta\gamma}_{RFabc}\left(\frac{1}{z},\frac{1}{z'}\right)=\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z'-1/z)}\langle 0|F^{(0)w\beta}_b(0)gF^{(0)w\gamma}_c(\mu w)|hX\rangle\langle hX|F^{(0)w\alpha}_a(\lambda w)|0\rangle. \tag{50}$$

We can now compare the sum of $W_g^{(b)}$ (47) and $W_g^{(c)}$ (49) with (2). We first remember that the gluon's field strength is $F^{\mu\nu}_a=\partial^\mu A^\nu_a-\partial^\nu A^\mu_a+gf^{abc}A^\mu_bA^\nu_c=F^{(0)\mu\nu}_a+gf^{abc}A^\mu_bA^\nu_c$. The first terms of (47) and (49) are the $O(g)$ contribution to (2) from the $O(A^2)$ term of the field strength in the intrinsic FF (3). The second terms of (47) and (49) are the $O(g)$ contribution to (2) from the gauge links in the intrinsic FF. The third terms of (47) and (49) are the $O(g)$ contribution to (2) from the $O(A^2)$ term of the field strength in the kinematical FF (4). The fourth terms of (47) and (49) are the $O(g)$ contribution to (2) from the gauge links in the kinematical FF. To identify the fifth terms of (47) and (49), we note the following relations:

$$\hat{\Gamma}^{\alpha\beta\gamma}_{RFabc}\left(\frac{1}{z},\frac{1}{z'}\right)=\hat{\Gamma}^{\beta\alpha\gamma}_{LFbac}\left(\frac{1}{z'},\frac{1}{z}\right)^\star, \tag{51}$$

$$S^{abc}_{R\alpha\beta\gamma}\left(\frac{1}{z},\frac{1}{z'}\right)=S^{bac}_{L\beta\alpha\gamma}\left(\frac{1}{z'},\frac{1}{z}\right)^\star. \tag{52}$$

From these relations, the sum of the last terms in (47) and (49) is the $O(g)$ contribution to (2) from the dynamical FF (5). This way, the basic formula (2) has been proved in the leading order with respect to the QCD coupling constant.

The method used here for the twist-3 gluon FF contribution to $pp\to\Lambda^\uparrow X$ can also be applied to the twist-3 gluon distribution function contribution to the double-spin asymmetry in $\vec{p}p^\uparrow\to DX$, which occurs as a nonpole contribution [12]. Our method provides a clearer understanding for the absence of the ghostlike terms in the corresponding LO twist-3 cross section.

IV. SUMMARY

In this paper, we presented a new derivation of the basic formula (2) for the twist-3 gluon FF contribution to $pp\to\Lambda^\uparrow X$. Our method uses the Ward-Takahashi identities for the partonic hard parts from the outset before performing the collinear expansion. This method provides a robust shortcut to convert the correlation functions of the gauge (gluon) fields into the gauge-invariant correlation functions for the gluon's field strengths. Furthermore, it provides a clear understanding that the ghostlike terms appearing in the Ward-Takahashi identities do not contribute to the LO twist-3 cross section. Since this method is quite general, it will become a useful tool to extend the formula in the next-to-leading-order calculation.

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APPENDIX: ABSENCE OF THE GHOSTLIKE CONTRIBUTION AT LO TWIST 3

Here, we show that the ghostlike term (43) does not contribute to the twist-3 cross section. Actual forms of $G^{abc}_{\mu\nu}(k,k')$ are given in Eq. (A7) for the $q\bar{q}\to gg$ channel, Eq. (A9) for the $qg\to gq$ channel, and Eqs. (A10)–(A14) for the $gg\to gg$ channel in Ref. [1]. In all channels, they take the structure

$$G^{abc}_{\mu\nu}(k,k') = (k^2 g_{\mu\rho} - k_\mu k_\rho) f^{abc} \tilde{G}^\rho_\nu(k,k'), \qquad \text{(A1)}$$

where $\tilde{G}^\rho_\nu(k,k')$ is some function of k and k' (and xp and $x'p'$). Inserting this form into (43), one obtains

$$[S^{abc}_{L\mu\nu\lambda}(k,k')\hat{\Gamma}^{\mu\nu\lambda}_{Labc}(k,k')]^{\text{ghost}} = 2\Omega^\mu_\alpha\Omega^\nu_\beta \frac{f^{abc}z'}{1/z'-1/z}\left\{-k_\mu\tilde{G}_{P_h\nu}\left(\frac{1}{z},\frac{1}{z'}\right) + 2P_h\cdot k\tilde{G}_{\mu\nu}\left(\frac{1}{z},\frac{1}{z'}\right) - \Omega^\lambda_\gamma k^\gamma P_{h\mu}\tilde{G}_{\lambda\nu}\left(\frac{1}{z},\frac{1}{z'}\right)\right.$$
$$\left. - \frac{P_{h\mu}}{z}\left(\Omega^\lambda_\gamma k^\gamma \frac{\partial\tilde{G}_{P_h\nu}(k,k')}{\partial k^\lambda}\bigg|_{\text{c.l.}} + \Omega^\lambda_\gamma k'^\gamma \frac{\partial\tilde{G}_{P_h\nu}(k,k')}{\partial k'^\lambda}\bigg|_{\text{c.l.}}\right)\right\}\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k'). \qquad \text{(A2)}$$

From this form, one sees that all terms in $\{\cdots\}$ except for the first one contribute only at twist 4: $\Omega^\mu_\alpha P_{h\mu}$ extracts "$\alpha = -$" component from $\hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k')$, which is subleading, and $\Omega^\lambda_\gamma k^\gamma$ and $P_h\cdot k$, respectively, cause additional one- and two-power suppressions. For the first term which contributes at twist 3, we obtain from (35)

$$\Omega^\mu_\alpha k_\mu \hat{\Gamma}^{\alpha\beta w}_{LFAabc}(k,k') = \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi} e^{-i(k'-k)\eta} \langle 0|F^{(0)\beta w}_b(0)|hX\rangle\langle hX|(-i)\frac{\partial}{\partial\xi^\alpha}F^{(0)\alpha w}_a(\xi) g A^w_c(\eta)|0\rangle. \qquad \text{(A3)}$$

Here, we note that the QCD equation of motion $D_\alpha F^{\alpha w}_a + g\bar{\psi}t^a \not{\phi}\psi = 0$ implies $\partial_\alpha F^{(0)\alpha w}_a(\xi)$ is of $O(g)$, and hence the first term in $\{\cdots\}$ of (A2) becomes $O(g^6)$. Consistent treatment of this term requires the inclusion of all $O(g^6)$ diagrams, which is beyond the scope of this work. We thus conclude that it does not contribute to the LO cross section. This proves that (43) does not contribute to the LO twist-3 cross section.

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