Complete nonlinear action for a supersymmetric multiple D0-brane system

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We present a complete nonlinear action for the dynamical system of nearly coincident multiple D0-branes (mD0) which possesses, besides manifest spacetime (target superspace) supersymmetry, also the worldline supersymmetry, a counterpart of the local fermionic κ symmetry of single D0-brane (Dirichlet superparticle). The action contains an arbitrary nonvanishing function $\mathcal{M}(\mathcal{H})$ of the relative motion Hamiltonian $\mathcal{H}$. The ten-dimensional ($D = 10$) mD0 model with particular form of $\mathcal{M}(\mathcal{H})$ can be obtained by dimensional reduction from the action of the $D = 11$ multiple M-wave (mM0) system.

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I. INTRODUCTION

Dirichlet p-branes or Dp- branes [1] are the supersymmetric extended objects on which the fundamental $D = 10$ superstring can have its ends attached [2,3]. Their especially important role in string theory [4] was appreciated after the famous paper by Polchinski [5], where it was shown that they carry nontrivial charges with respect to Ramond-Ramond (RR) fields (see [6] for a comprehensive review).

The worldvolume action for a single super-Dp-brane is known [7–13] to be given by the sum of the supersymmetrized Dirac-Born-Infeld (DBI) term and a Wess-Zumino term describing the coupling to RR fields. Both terms contain the field strength of $d = (p + 1)$ dimensional worldvolume gauge field and in the weak field limit, after fixing the static gauge the first DBI term reduces to the action of the supersymmetric Abelian gauge field theory. Also the Wess-Zumino term in this gauge is expressed through the fields of the Abelian super-Yang-Mills multiplet.

The quest for an effective action for the multiple Dp-brane system, i.e., the system of N nearly coincident Dp-branes and strings ending on these Dp-branes, can be followed back to the seminal paper by Witten [14] where he argued that the gauge fixed description of its weak field limit is given by the non-Abelian U(N) super-Yang-Mills (SYM) action. Despite a number of very interesting results obtained during the past 26 years [15–29], the complete nonlinear supersymmetric action for the dynamical system of multiple Dp-branes (mDp) is not known presently even for the simplest case of $p = 0$ [30].

In this paper we present a nonlinear action which possesses several properties expected from the action of the mD0 system. Particularly, it is manifestly invariant under Poincaré symmetry, SU(N) gauge symmetry, and spacetime (type IIA target superspace) supersymmetry, and also possesses local worldline supersymmetry generalizing the κ symmetry of single D0-brane (massive type II $D = 10$ superparticle) action [31]. This latter fact is especially important because it guarantees that the ground state of this dynamical system is supersymmetric which is expected in the case of multiple D0-brane systems.

The rest of the paper is organized as follows. In Sec. II we present the complete supersymmetric and nonlinear candidate action for multiple D0-brane systems. The rigid spacetime supersymmetry and local worldsheet supersymmetry transformations leaving this action invariant are described in Sec. III. The technical details on the derivation of these results can be found in Appendix D which uses the approach and ingredients described in Appendixes A–C. Sec. IV contains our conclusions and discussion of the results.

II. SUPERSYMMETRIC NONLINEAR ACTION

The nonlinear action which we have found is written in terms of center of energy variables of an mD0 system, which are the same as in the case of a single D0-brane, and matrix variables describing the relative motion of mD0 constituents. The set of center of energy variables contains coordinate functions describing the embedding of the center of energy worldline in flat type IIA superspace, bosonic 10-vector, and two fermionic Majorana-Weyl spinors

$$Z^M(\tau) = (x^\mu(\tau), \theta^{1\alpha}(\tau), \theta^2_\alpha(\tau)), \tag{1}$$

$\mu = 0, \ldots, 9$, $\alpha = 1, \ldots, 16$, as well as the spinor moving frame variables which we will describe below. The relative motion variables are matrix fields from the 1d extended ($\mathcal{N} = 16$) SU(N) SYM multiplet, the set of which can be split on matter fields, $9 + 9$ bosonic and 16 fermionic Hermitean traceless $N \times N$ matrix fields

$$\mathbb{X}^i(\tau), \qquad \mathbb{P}^i(\tau), \qquad \mathbf{\Psi}_q(\tau), \tag{2}$$

$i = 1, \ldots, 9$, $q = 1, \ldots, 16$, and the bosonic anti-Hermitean traceless $N \times N$ matrix 1-form

$$\mathbb{A} = \mathrm{d}\tau \mathbb{A}_\tau(\tau) \tag{3}$$

containing the $su(N)$ valued worldline gauge field $\mathbb{A}_\tau(\tau)$. Besides SU(N) gauge transformations, the matrix fields are transformed by local SO(9) transformations according to their vector and spinor indices $i = 1, \ldots, 9$ and $q = 1, \ldots, 16$. These will also act on spinor frame variables and describe the gauge symmetry of the mD0 action.

The action has the form

$$\begin{aligned} S_{\mathrm{mD0}} = {} & m \int_{\mathcal{W}^1} E^0 - im \int_{\mathcal{W}^1} (\mathrm{d}\theta^1\theta^2 - \theta^1\mathrm{d}\theta^2) \\ & + \frac{1}{\mu^6} \int_{\mathcal{W}^1} \left(\mathrm{tr}(\mathbb{P}^i \mathrm{D}\mathbb{X}^i + 4i\mathbf{\Psi}_q \mathrm{D}\mathbf{\Psi}_q) + \frac{2}{\mathcal{M}} E^0 \mathcal{H} \right) \\ & - \frac{1}{\mu^6} \int_{\mathcal{W}^1} \frac{\mathrm{d}\mathcal{M}}{\mathcal{M}} \mathrm{tr}(\mathbb{P}^i \mathbb{X}^i) + \frac{1}{\mu^6} \int_{\mathcal{W}^1} \frac{1}{\sqrt{2\mathcal{M}}} (E^{1q} - E^2_q) \\ & \times \mathrm{tr}\left(-4i(\gamma^i \mathbf{\Psi})_q \mathbb{P}^i + \frac{1}{2} (\gamma^{ij} \mathbf{\Psi})_q [\mathbb{X}^i, \mathbb{X}^j] \right), \end{aligned} \tag{4}$$

where m and μ are constants of dimension of mass and

$$\mathcal{H} = \frac{1}{2} \mathrm{tr}(\mathbb{P}^i \mathbb{P}^i) - \frac{1}{64} \mathrm{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 - 2\mathrm{tr}(\mathbb{X}^i \mathbf{\Psi} \gamma^i \mathbf{\Psi}) \tag{5}$$

has the meaning of the relative motion Hamiltonian.

Actually the first line of (4) formally coincides with the action of a single D0-brane, i.e., a massive $D = 10$ type IIA superparticle in its moving frame formulation [28,37] (see below for the description of E^0 in it and Appendix B for some details). In this case m plays the role of the superparticle mass. In contrast, the constant μ characterizes the interaction of the center of energy and relative motion sector as well as the self-interaction of this latter. Notice that to simplify and to make more transparent the dependence of the action on this parameter we have chosen noncanonical dimensions for the matrix matter fields (2). In particular, with this choice of dimensions of matrix fields, the relative motion Hamiltonian $\mathcal{H}$ (5) is μ independent. However its dimension becomes (mass^6) so that $\mathcal{H}/\mu^6$ is dimensionless.

$\mathcal{M}$ in (4) is *an arbitrary* nonvanishing *function* of this dimensionless combination of the relative motion Hamiltonian and coupling constant,

$$\mathcal{M} = \mathcal{M}(\mathcal{H}/\mu^6). \tag{6}$$

A particular case of the action (4) with

$$\mathcal{M} = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{\mathcal{H}}{\mu^6}} \tag{7}$$

can be obtained by dimensional reduction of the 11D multiple M-wave (multiple M0-branes or mM0) system action from [38,39] similar to dimensional reduction of its $D = 4$ counterpart described in [29]. Another representative of the family (4) with $\mathcal{M} = m$ was studied in [28] where it was noticed that it cannot be obtained by dimensional reduction from 11D mM0 action.

Coming back to the first line of (4), in it E^0 is the projection of (the pull back of) 10D Volkov-Akulov 1-form

$$E^0 = \Pi^\mu u^0_\mu, \qquad \Pi^\mu = \mathrm{d}x^\mu - i\mathrm{d}\theta^1 \sigma^\mu \theta^1 - i\mathrm{d}\theta^2 \tilde{\sigma}^\mu \theta^2 \tag{8}$$

to one of the vector fields, $u^0_\mu(\tau)$, of moving frame attached to the worldline. That is described by a Lorentz group valued 10×10 matrix

$$(u^0_\mu, u^i_\mu) \in \mathrm{SO}(1,9) \tag{9}$$

composed of the moving frame vectors which obey

$$u^{\mu 0} u^0_\mu = 1, \qquad u^{\mu 0} u^i_\mu = 0, \qquad u^{\mu i} u^j_\mu = -\delta^{ij}. \tag{10}$$

The spinor moving frame described by Spin(1,9) valued matrix

$$v_\alpha{}^q \in \mathrm{Spin}(1,9) \tag{11}$$

provides a kind of square root of the above described moving frame in the sense of Cartan-Penrose-like relations (see Appendix A for more details)

$$u^0_\mu \sigma^\mu_{\alpha\beta} = v_\alpha{}^q v_\beta{}^q, \qquad u^i_\mu \sigma^\mu_{\alpha\beta} = v_\alpha{}^q \gamma^i_{qp} v_\beta{}^p, \tag{12}$$

$$v^q_\alpha \tilde{\sigma}^{\alpha\beta}_\mu v^p_\beta = u^0_\mu \delta_{qp} + u^i_\mu \gamma^i_{qp}. \tag{13}$$

In distinction to their $D = 4$ counterparts (described in [40] and, e.g., [29]) Eqs. (12) impose strong constraints on the spinor moving frame field $v_\alpha{}^q = v_\alpha{}^q(\tau)$ reducing the number of its components from the original $16 \times 16 = 256$ to $45 = \dim(\mathrm{SO}(1,9))$.

This spinor frame matrix field $v_\alpha{}^q(\tau)$ and its inverse $v^\alpha_q(\tau)$ are used to construct the fermionic forms E^{1q} and E^q_2 which enter the last term of the action (4),

$$E^{1q} = \mathrm{d}\theta^{1\alpha} v_\alpha{}^q, \qquad E^2_q = \mathrm{d}\theta^2_\alpha v_q{}^\alpha. \tag{14}$$

The covariant derivatives in the second line of (4)

$$\mathrm{D}\mathbb{X}^i := \mathrm{d}\tau \mathrm{D}_\tau \mathbb{X}^i := \mathrm{d}\mathbb{X}^i - \Omega^{ij}\mathbb{X}^j + [\mathbb{A}, \mathbb{X}^i], \tag{15}$$

$$\mathrm{D}\Psi_q := \mathrm{d}\tau \mathrm{D}_\tau \mathbf{\Psi}_q := \mathrm{d}\mathbf{\Psi}_q - \frac{1}{4}\Omega^{ij}\gamma^{ij}_{qp}\mathbf{\Psi}_p + [\mathbb{A}, \mathbf{\Psi}_q], \tag{16}$$

contain, beside the SU(N) gauge field (3), also the composite SO(9) connection (Cartan form)

$$\Omega^{ij} = u^{\mu i}\mathrm{d}u^j_\mu. \tag{17}$$

III. LOCAL WORLDLINE SUPERSYMMETRY

The action (4) is manifestly invariant under the rigid super-Poincaré supergroup transformations, including spacetime (target 10D IIA superspace) supersymmetry with constant fermionic parameters $\epsilon^{\alpha 1}$ and $\epsilon_\alpha{}^2$ acting nontrivially only on the center of energy variables,

$$\begin{aligned} &\delta_\epsilon \theta^{1\alpha} = \epsilon^{\alpha 1}, \qquad \delta_\epsilon \theta^2_\alpha = \epsilon_\alpha{}^2, \qquad \delta_\epsilon v^q_\alpha = 0, \\ &\delta_\epsilon x^\mu = i\theta^1\sigma^\mu\epsilon^1 + i\theta^2\tilde{\sigma}^\mu\epsilon^2. \end{aligned} \tag{18}$$

It is also invariant under the SU(N) gauge symmetry acting on the matrix matter fields by its adjoint representation, provided the $su(N)$ valued 1-form $\mathbb{A}$ transforms as SU(N) connection, as well as under the SO(9) symmetry acting by vector representation on index i of u^i_μ, $\mathbb{X}^i$, $\mathbb{P}^i$ and by its spinor representation on index q of $\mathbf{\Psi}_q$ and v^q_α.

Furthermore the action is invariant under local fermionic worldline supersymmetry parametrized by fermionic function $\kappa^q(\tau)$ carrying spinor index of SO(9). It acts on the center of energy variables exactly in the same manner as irreducible κ symmetry of a single D0-brane in its spinor moving frame formulation [28,37] [hence the notation $\kappa^q(\tau)$],

$$\begin{aligned} &\delta_\kappa \theta^{1\alpha} = \kappa^q v^\alpha_q/\sqrt{2}, \qquad \delta_\kappa \theta^2_\alpha = -\kappa^q v_\alpha{}^q/\sqrt{2}, \\ &\delta_\kappa x^\mu = i\delta_\kappa\theta^1\sigma^\mu\theta^1 + i\delta_\kappa\theta^2\tilde{\sigma}^\mu\theta^2, \\ &\delta_\kappa v^q_\alpha = 0 \Rightarrow \delta_\kappa u^0_\mu = 0 = \delta_\kappa u^i_\mu. \end{aligned} \tag{19}$$

The action of worldline SUSY on the matrix fields includes essentially nonlinear terms, some of which are proportional to the derivative of the function $\mathcal{M}$ with respect to its argument and, hence to additional power of $\frac{1}{\mu^6}$,

$$\delta\mathcal{M}(\mathcal{H}/\mu^6) = \frac{1}{\mu^6}\mathcal{M}'(\mathcal{H}/\mu^6)\delta\mathcal{H}, \qquad \mathcal{M}'(y) = \frac{\mathrm{d}}{\mathrm{d}y}\mathcal{M}(y). \tag{20}$$

The worldline supersymmetry transformations of the matrix matter fields are (see Appendix D for their derivation by method described in Appendix C)

$$\delta_\kappa \mathbb{X}^i = \frac{4i}{\sqrt{\mathcal{M}}}\kappa\gamma^i\mathbf{\Psi} + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\delta_\kappa\mathcal{H}\mathbb{X}^i - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa\mathcal{K}\mathbb{P}^i, \tag{21}$$

$$\begin{aligned} \delta_\kappa \mathbb{P}^i = &-\frac{1}{\sqrt{\mathcal{M}}}[\kappa\gamma^{ij}\mathbf{\Psi}, \mathbb{X}^j] - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\delta_\kappa\mathcal{H}\mathbb{P}^i \\ &+ \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa\mathcal{K}\left(\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - \gamma^i_{pq}\{\mathbf{\Psi}_p, \mathbf{\Psi}_q\}\right), \end{aligned} \tag{22}$$

$$\begin{aligned} \delta_\kappa \mathbf{\Psi}_q = &-\frac{1}{2\sqrt{\mathcal{M}}}(\kappa\gamma^i)_q\mathbb{P}^i - \frac{i}{16\sqrt{\mathcal{M}}}(\kappa\gamma^{ij})_q[\mathbb{X}^i, \mathbb{X}^j] \\ &- \frac{i}{4\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa\mathcal{K}[(\gamma^i\Psi)_q, \mathbb{X}^i]. \end{aligned} \tag{23}$$

Here

$$\delta_\kappa\mathcal{H} = \frac{1}{\sqrt{\mathcal{M}}}\frac{\mathrm{tr}(\kappa^q\mathbf{\Psi}_q([\mathbb{X}^i, \mathbb{P}^i] - 4i\{\mathbf{\Psi}_q, \mathbf{\Psi}_q\}))}{1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}} \tag{24}$$

with

$$\mathfrak{H} := \mathrm{tr}(\mathbb{P}^i\mathbb{P}^i) + \frac{1}{16}\mathrm{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 + 2\mathrm{tr}(\mathbb{X}^i\mathbf{\Psi}\gamma^i\mathbf{\Psi}) \tag{25}$$

is the worldline supersymmetry variation of the relative motion Hamiltonian (5) and

$$\Delta_\kappa\mathcal{K} = \frac{1}{2\sqrt{\mathcal{M}}}\frac{\mathrm{tr}(4i(\kappa\gamma^i\mathbf{\Psi})\mathbb{P}^i + \frac{5}{2}(\kappa\gamma^{ij}\mathbf{\Psi})[\mathbb{X}^i, \mathbb{X}^j])}{1 + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H}}. \tag{26}$$

This latter is related to the worldline supersymmetry variation of $\mathcal{K} = \mathrm{tr}(\mathbb{X}^i\mathbb{P}^i)$ by

$$\Delta_\kappa\mathcal{K} = \delta_\kappa(\mathrm{tr}(\mathbb{X}^i\mathbb{P}^i)) + \frac{1}{2\sqrt{\mathcal{M}}}i\kappa^q\nu_q, \tag{27}$$

where

$$i\nu_q := \mathrm{tr}\left(-4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i + \frac{1}{2}(\gamma^{ij}\mathbf{\Psi})_q[\mathbb{X}^i, \mathbb{X}^j]\right). \tag{28}$$

In terms of the above blocks the worldline supersymmetry variation of the SU(N) connection 1-form (gauge field) can be written as (see Appendix D for its derivation)

$$\delta_\kappa \mathbb{A} = -\frac{2}{\mathcal{M}\sqrt{\mathcal{M}}} E^0(\kappa^q \mathbf{\Psi}_q) \frac{(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H})}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})} + \frac{1}{\sqrt{2}\mathcal{M}}(E^{1q}-E^2_q)(\gamma^i\kappa)_q \mathbb{X}^i - (E^{1q}-E^2_q)\frac{1}{\mu^6}\frac{\mathcal{M}'}{\sqrt{2}\mathcal{M}^2}\frac{1}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})} \times \kappa^p \mathbf{\Psi}_{(q}\mathrm{tr}(4i(\gamma^i\mathbf{\Psi})_{p)}\mathbb{P}^i + \frac{5}{2}(\gamma^{ij}\mathbf{\Psi})_{p)}[\mathbb{X}^i,\mathbb{X}^j]). \quad (29)$$

IV. CONCLUSION AND DISCUSSION

Thus, we have found that the action (4) is invariant, besides the manifest spacetime (target superspace type IIA) supersymmetry (18), also under 16-parametric local worldline supersymmetry transformations (19), (21)–(23), and (29). Its counterpart in the case of single p-branes, local fermionic κ symmetry, is considered as an exclusive property of the supersymmetric extended objects of string/M theory. It guarantees that the ground state of the dynamical system preserves a part (one-half) of the spacetime supersymmetry.

The form of this worldline supersymmetry depends strongly on the choice of the function $\mathcal{M}(\mathcal{H}/\mu^6)$ in the action (4). This is restricted by the requirement of nonsingularity $\mathcal{M} \neq 0$ but otherwise is arbitrary [41].

The simplest model obtained by setting $\mathcal{M} = m = \text{const}$ was studied earlier in [28]. In this case $\mathcal{M}' = 0$ and worldline supersymmetry transformations of the matrix fields (21)–(23), (29) simplify drastically and provide the local supersymmetry generalization of the rigid $d = 1$ $\mathcal{N} = 16$ supersymmetry of 10D SU(N) SYM model reduced to $d = 1$. The local supersymmetry of the action is provided by coupling of this 1d SYM to the composed worldline supergravity on the worldline induced by the center of energy motion. This is described by 1d graviton 1-form (einbein) E^0 and 16 1d gravitini 1-forms $E^{1q} - E^2_q$ constructed from the center of energy variables according to (8) and (14).

Thus the nonlinearity of the previously proposed candidate action with $\mathcal{M} = m = \text{const}$ [28] does not go beyond that of the non-Abelian Yang-Mills theory. In contrast, the action (4) with a generic function $\mathcal{M}(\mathcal{H}/\mu^6)$, particularly the one with (7), which can be obtained by dimensional reduction from 11D mM0 action of [38], shows essential nonlinearity beyond the level of the SYM one, as it has been expected for the multiple D0-system. It is impressive that such a nonlinearity can be reached with preserving the local worldline supersymmetry characteristic for an mD0 system, and that this can be done for essentially arbitrary function $\mathcal{M}(\mathcal{H}/\mu^6)$. Also, the above mentioned connection with an 11D mM0 system, the details of which will be published in a forthcoming paper [42], is another important advantage of the functional (4) as a candidate mD0 action.

The problem of what choice of the function $\mathcal{M}(\mathcal{H}/\mu^6)$ leads to the true mD0-brane action requires additional study. A natural way to make this choice through using T duality (which was the main argument for construction of bosonic actions in [15]) requires as a first step to construct the candidate action for type IIB multiple D1-branes (mD1), the problem we are planning to address in the future. A more detailed study of the properties of the model (4) with arbitrary function $\mathcal{M}(\mathcal{H}/\mu^6)$, including the solution of its equations of motion and describing its BPS states, can be also useful to single out the true mD0-brane action or to clarify why so big set of models possesses the expected properties.

For a moment, an especially interesting in a string/M-theoretic perspective looks at the model (4) with function $\mathcal{M}(\mathcal{H}/\mu^6)$ given in (7) because, as we will show in the forthcoming paper [42], this can be obtained by dimensional reduction of the action for multiple M0-branes (multiple M wave or mM0) constructed in [38]. However, this argument implies the uniqueness of the action [38] as the one having the properties expected for an mM0 system. On the other hand, in light of the found multiplicity of the 10D actions with the properties expected for mD0 systems, it is tempting to search for possible essentially nonlinear generalizations of the 11D mM0 action of [38].

Also the generalization of the action (4) for the case of multiple Dp-brane systems with $1 < p \leq 9$ and for the case of curved target IIA supergravity superspace are intriguing and important problems.

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APPENDIX A: 10D SPINOR MOVING FRAME VARIABLES

The multiple D0-brane action, presented in the main text, is presently known only in its spinor moving frame formulation involving the auxiliary variables which we are going to describe in some detail.

The Spin(1,9)/Spin(9) spinor moving frame variables and their moving frame vector companions appropriate to the description of D0 brane and multiple D0 (mD0) systems are elements of, respectively, 16×16 and 10×10 matrices (11) and (9) (see [28,37])

$$v_\alpha{}^q \in \mathrm{Spin}(1,9) \quad \text{and} \quad (u^0_\mu, u^i_\mu) \in \mathrm{SO}(1,9). \quad (A1)$$

Here $i = 1,\ldots,9$ and $q = 1,\ldots,16$ are vector and spinor indices of SO(9) group while $\mu,\nu = 0,1,\ldots,9$ and $\alpha,\beta = 1,2,\ldots,16$ are 10-vector and 10D Majorana-Weyl spinor indices.

The condition that moving frame variables form the SO(1,9) valued matrix implies (10) and

$$u^0_\mu u^0_\nu - u^i_\mu u^i_\nu = \eta_{\mu\nu} = \mathrm{diag}(1,-1,\ldots,-1). \tag{A2}$$

The spinor moving frame variables obey the constraints

$$u^{(\nu)}_\mu \sigma^\mu_{\alpha\beta} = v^q_\alpha \sigma^{(\nu)}_{qp} v^p_\beta, \qquad u^{(\nu)}_\mu \tilde{\sigma}^{qp}_{(\nu)} = v^q_\alpha \tilde{\sigma}^{\alpha\beta}_\mu v^p_\beta, \tag{A3}$$

which express the SO(1,9) Lorentz invariance of the 10D generalization of the relativistic Pauli matrices $\sigma^\mu_{\alpha\beta} = \sigma^\mu_{\beta\alpha}$ and $\tilde{\sigma}^{\alpha\beta}_\mu = \tilde{\sigma}^{\beta\alpha}_\mu$,

$$\sigma^{(\mu}_{\alpha\gamma}\tilde{\sigma}^{\nu)\gamma\beta} := \frac{1}{2}\left(\sigma^\mu\tilde{\sigma}^\nu + \sigma^\nu\tilde{\sigma}^\mu\right)_\alpha{}^\beta = \eta^{\mu\nu}\delta_\alpha{}^\beta, \tag{A4}$$

and also makes the spinor frame matrix to describe double covering of the Lorentz group element represented by the moving frame matrix (see [40,43,44]). Roughly speaking, this statement can be formulated by saying that spinor frame variables (also called Lorentz harmonics [40,43,44]) are square roots of the moving frame variables (also called vector harmonics [45]).

Choosing the SO(9) invariant representation

$$\sigma^{(\mu)}_{qp} = (\delta_{qp}, \gamma^i_{qp}) = \tilde{\sigma}^{qp}_{(\mu)}, \tag{A5}$$

where $\gamma^i_{qp} = \gamma^i_{pq}$ are $d = 9$ gamma matrices,

$$\gamma^i_{qp} = \gamma^i_{pq}, \qquad \gamma^{(i}\gamma^{j)} = \delta^{ij}\mathbb{I}_{16\times 16}, \tag{A6}$$

we find that Eqs. (A3) acquire the form of (12) and

$$v^q_\alpha \tilde{\sigma}^{\alpha\beta}_\mu v^p_\beta = u^0_\mu \delta_{qp} + u^i_\mu \gamma^i_{qp}. \tag{A7}$$

Similarly, we find

$$u^0_\mu \tilde{\sigma}^{\mu\alpha\beta} = v_q{}^\alpha v_q{}^\beta, \qquad u^i_\mu \tilde{\sigma}^{\mu\alpha\beta} = -v_q{}^\alpha \gamma^i_{qp} v_p{}^\beta. \tag{A8}$$

Notice that

$$v_q{}^\beta = v^q_\alpha \tilde{\sigma}^{\mu\alpha\beta} u^0_\mu, \qquad \text{obeying} u^0_\mu \sigma^\mu_{\alpha\beta} v_q{}^\beta = v_\alpha{}^q, \tag{A9}$$

is the inverse spinor moving frame matrix $v_\alpha{}^q \in \mathrm{Spin}(1,9)$:

$$v_q{}^\alpha v_\alpha{}^p = \delta_q{}^p \Leftrightarrow v_\alpha{}^q v_q{}^\beta = \delta_\alpha{}^\beta. \tag{A10}$$

The derivatives of the moving frame and of the spinor moving frame variables are expressed in terms of Cartan forms

$$\Omega^i = u^0_\mu \mathrm{d}u^{\mu i}, \Omega^{ij} = u^i_\mu \mathrm{d}u^{\mu j}, \tag{A11}$$

by

$$\mathrm{D}u^0_\mu := \mathrm{d}u^0_\mu = u^i_\mu \Omega^i, \quad \mathrm{D}u^i_\mu := \mathrm{d}u^i_\mu + u^j_\mu \Omega^{ji} = u^0_\mu \Omega^i \tag{A12}$$

and

$$\mathrm{D}v_\alpha{}^q := \mathrm{d}v_\alpha{}^q + \frac{1}{4}\Omega^{ij} v_\alpha{}^p \gamma^{ij}_{pq} = \frac{1}{2}\gamma^i_{qp} v_\alpha{}^p \Omega^i \tag{A13}$$

$$\Rightarrow \mathrm{D}v_q{}^\alpha := \mathrm{d}v_q{}^\alpha - \frac{1}{4}\Omega^{ij}\gamma^{ij}_{qp} v^\alpha_p = -\frac{1}{2} v^\alpha_p \gamma^i_{pq}\Omega^i. \tag{A14}$$

Taking exterior derivatives of Eqs. (A12) (see Appendix C for definitions) we can find the Maurer-Cartan equations

$$\mathrm{D}\Omega^i = \mathrm{d}\Omega^i + \Omega^j \wedge \Omega^{ji} = 0,$$
$$\mathrm{d}\Omega^{ij} + \Omega^{ik} \wedge \Omega^{kj} = -\Omega^i \wedge \Omega^j. \tag{A15}$$

APPENDIX B: SINGLE D0-BRANE IN SPINOR MOVING FRAME FORMULATION AND ITS κ SYMMETRY

The action of the moving frame formulation of the 10D D0-brane in flat type IIA superspace, which also appears as a part of the multiple D0-brane action (4) describing the center of mass dynamics of this system, reads [37]

$$S_{\mathrm{D0}} = \int_{\mathcal{W}^1} \mathcal{L}_{\mathrm{D0}} = m\int_{\mathcal{W}^1} E^0 - im\int_{\mathcal{W}^1} (\mathrm{d}\theta^{1\alpha}\theta^2_\alpha - \theta^{1\alpha}\mathrm{d}\theta^2_\alpha). \tag{B1}$$

Here, $\mathrm{d} = \mathrm{d}\tau\partial/\partial\tau =: \mathrm{d}\tau\partial_\tau$, τ is proper time variable parametrizing the D0-brane worldline $\mathcal{W}^1$ defined as a line in target $D = 10$ type IIA superspace $\Sigma^{(10|32)}$ with 10 bosonic and $16 + 16 = 32$ fermionic coordinates

$$Z^M = (x^\mu, \theta^{1\alpha}, \theta^2_\alpha) \tag{B2}$$

by corresponding coordinate functions

$$Z^M(\tau) = (x^\mu(\tau), \theta^{1\alpha}(\tau), \theta^2_\alpha(\tau)), \tag{B3}$$

$$\mathcal{W}^1 \in \Sigma^{(10|32)}: \qquad Z^M = Z^M(\tau). \tag{B4}$$

The constant m entering both terms of (B1) is the mass of the D0-brane and E^0 is the contraction

$$E^0 = \Pi^\mu u^0_\mu \tag{B5}$$

of the pull back to the worldline of the 10D Volkov-Akulov 1-form

$$\Pi^\mu = \mathrm{d}x^\mu - i\mathrm{d}\theta^1\sigma^\mu\theta^1 - i\mathrm{d}\theta^2\tilde{\sigma}^\mu\theta^2 \tag{B6}$$

with the vector field $u^0_\mu = u^0_\mu(\tau)$. The pull back of a differential form on target superspace is obtained by

substituting the coordinate functions for the coordinates; so that Eq. (B5) actually includes

$$\Pi^\mu = \mathrm{d}\tau\Pi^\mu_\tau = \mathrm{d}x^\mu(\tau) - i\mathrm{d}\theta^1(\tau)\sigma^\mu\theta^1(\tau) - i\mathrm{d}\theta^2(\tau)\tilde{\sigma}^\mu\theta^2(\tau). \tag{B7}$$

Notice that, to simplify the notation, below and below, as well as in the main text, we use the same symbols for the differential forms on the target superspace and their pull backs to the worldline $\mathcal{W}^1$. The same applies to the superspace coordinates (B2) and the coordinate functions (B3). Particularly, in the second term of (B1) $\theta^{1\alpha}$ and θ^2_α denote $\theta^{1\alpha}(\tau)$ and $\theta^2_\alpha(\tau)$.

A very important property of the action (B1) is that, besides manifest $D = 10$ $\mathcal{N} = 2$ spacetime supersymmetry, it is also invariant under the following local fermionic κ-symmetry transformations

$$\begin{aligned} &\delta_\kappa\theta^{1\alpha} = \kappa^q v_q^\alpha, \qquad \delta_\kappa\theta^2_\alpha = -\kappa^q v_\alpha{}^q, \\ &\delta_\kappa x^\mu = i\delta_\kappa\theta^1\sigma^\mu\theta^1 + i\delta_\kappa\theta^2\tilde{\sigma}^\mu\theta^2, \\ &\delta_\kappa v_\alpha{}^q = 0 \Rightarrow \delta_\kappa u^i_\mu = \delta_\kappa u^0_\mu = 0, \end{aligned} \tag{B8}$$

where $\kappa^q = \kappa^q(\tau)$ with $q = 1, \ldots, 16$ are arbitrary fermionic functions.

To prove the κ-invariance of the single D0-brane action and also the invariance of multiple D0-brane action under its generalization, the worldline supersymmetry, we have used the formalism of generalized Lie derivatives based on formal exterior derivatives of differential forms which we are going to describe in Appendix C.

APPENDIX C: DIFFERENTIAL FORMS AND VARIATIONS

Let Ξ_q be differential q form in a superspace with coordinates Z^M,

$$\Xi_q = \frac{1}{q!}\mathrm{d}Z^{M_q} \wedge \ldots \wedge \mathrm{d}Z^{M_1}\Xi_{M_1\ldots M_q}(Z), \tag{C1}$$

where $\wedge$ is the exterior product of the differential forms. In the simplest case of basic 1-forms given by differentials of the superspace coordinates,

$$\mathrm{d}Z^M \wedge \mathrm{d}Z^N = -(-1)^{\epsilon(M)\epsilon(N)}\mathrm{d}Z^N \wedge \mathrm{d}Z^M, \tag{C2}$$

where $\epsilon(M) \equiv \epsilon(Z^M)$ is the so-called Grassmann parity of Z^M defined by

$$\epsilon(x^\mu) = 0, \qquad \epsilon(\theta^{1\alpha}) = 1, \qquad \epsilon(\theta^2_\alpha) = 1 \tag{C3}$$

in the case of $D = 10$ type IIA superspace with coordinates $Z^M = (x^\mu, \theta^{\alpha 1}, \theta^2_\alpha)$. For any bosonic p and q forms

$$\Xi_q \wedge \Upsilon_p = (-1)^{qp}\Upsilon_p \wedge \Xi_q, \tag{C4}$$

in particular,

$$\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = -\mathrm{d}x^\nu \wedge \mathrm{d}x^\mu. \tag{C5}$$

In the case of the forms which can be also fermionic

$$\Xi_q \wedge \Upsilon_p = (-1)^{qp+\epsilon(\Xi_q)\epsilon(\Upsilon_p)}\Upsilon_p \wedge \Xi_q. \tag{C6}$$

In particular, (C2) implies that all products of the supercoordinate differentials are antisymmetric but

$$\mathrm{d}\theta^{1\alpha} \wedge \mathrm{d}\theta^{\beta 1} = \mathrm{d}\theta^{\beta 1} \wedge \mathrm{d}\theta^{1\alpha}, \qquad \mathrm{d}\theta^2_\alpha \wedge \mathrm{d}\theta^2_\beta = \mathrm{d}\theta^2_\beta \wedge \mathrm{d}\theta^2_\alpha,$$
$$\mathrm{d}\theta^{1\alpha} \wedge \mathrm{d}\theta^2_\beta = \mathrm{d}\theta^2_\beta \wedge \mathrm{d}\theta^{1\alpha}.$$

The exterior derivative of the differential forms, which maps q forms into $(q+1)$ forms, is defined by

$$\begin{aligned} \mathrm{d}\Xi_q &= \frac{1}{q!}\mathrm{d}Z^{M_q} \wedge \ldots \wedge \mathrm{d}Z^{M_1} \wedge \mathrm{d}Z^{M_0}\partial_{M_0}\Xi_{M_1\ldots M_q}(Z) \\ &\equiv \frac{1}{(q+1)!}\mathrm{d}Z^{M_{q+1}} \wedge \ldots \wedge \mathrm{d}Z^{M_1} \\ &\quad \times (q+1)\partial_{[M_1}\Xi_{M_1\ldots M_{q+1}\}}(Z), \end{aligned}$$

where $\partial_N = \frac{\partial}{\partial Z^N}$ and $[\ldots\}$ denotes graded antisymmetrization over the enclosed indices, in particular,

$$\Xi_{[MN\}} = \frac{1}{2}(\Xi_{MN} - (-)^{\epsilon(M)\epsilon(N)}\Xi_{NM}). \tag{C7}$$

The exterior derivative operator d obeys the nilpotency condition and the (generalized) Leibniz rule

$$\mathrm{dd} = 0, \qquad \mathrm{d}(\Xi_q \wedge \Xi_p) = \Xi_q \wedge \mathrm{d}\Xi_p + (-1)^p\mathrm{d}\Xi_q \wedge \Xi_p. \tag{C8}$$

The variation of differential forms under generic transformations of coordinates can be calculated using the so-called Lie derivative formula,

$$\delta\Xi_q = i_\delta(\mathrm{d}\Xi_q) + \mathrm{d}(i_\delta\Xi_q), \tag{C9}$$

where i_δ is the contraction with variation symbol defined by

$$i_\delta\Xi_q = \frac{1}{(q-1)!}\mathrm{d}Z^{M_q} \wedge \ldots \wedge \mathrm{d}Z^{M_2}\delta Z^{M_1}\Xi_{M_1\ldots M_q}(Z). \tag{C10}$$

Notice that this implies

$$i_\delta \mathrm{d}Z^M = \delta Z^M. \tag{C11}$$

The contraction i_δ maps differential q forms into $(q-1)$ forms and obeys its own counterpart of the Leibnitz rule:

$$i_\delta(\Xi_q \wedge \Xi_p) = \Xi_q \wedge i_\delta \Xi_p + (-1)^p i_\delta \Xi_q \wedge \Xi_p. \tag{C12}$$

The variation of the Lagrangian D form $\mathcal{L}$ of a D-dimensional field theory can be calculated using the Lie derivative formula with formal exterior derivative [46]

$$\delta\mathcal{L} = i_\delta(\mathrm{d}\mathcal{L}) + \mathrm{d}(i_\delta\mathcal{L}). \tag{C13}$$

The total derivative term $\mathrm{d}(i_\delta\mathcal{L})$ is not essential when we derive the equations of motion and can be conventionally omitted if one does not study effects of boundary contributions.

In the models with manifest gauge symmetry it is more convenient to define the variations of differential forms given by covariant Lie derivative

$$\delta\Xi_q^{\mathcal{A}} = i_\delta(\mathrm{D}\Xi_q^{\mathcal{A}}) + \mathrm{D}(i_\delta\Xi_q^{\mathcal{A}}), \tag{C14}$$

where D is covariant derivative including the connection of the gauge symmetry group and $\mathcal{A}$ is an index (or multi-index including the index) of a representation of the gauge group carried by the differential q form. Clearly for the Lagrangian D form, which is invariant under the gauge symmetry, $\mathrm{D}\mathcal{L} = \mathrm{d}\mathcal{L}$ and the covariant Lie derivative prescription coincides with the standard Lie derivative one (C13).

As a warm-up exercise let us apply this method to vary the Lagrangian 1-form of the action (B1) of a single D0-brane in flat 10D type IIA superspace [37]:

$$\mathcal{L}_{\mathrm{D0}} = mE^0 - im(\mathrm{d}\theta^1\theta^2 - \theta^1\mathrm{d}\theta^2)$$

with constant m.

The formal exterior derivative of $E^0 = \Pi^\mu u_\mu^0$ in the first term of the Lagrangian form is given by

$$\mathrm{d}E^0 = E^i \wedge \Omega^i - i(E^{1q} \wedge E^{1q} + E_q^2 \wedge E_q^2), \tag{C15}$$

where

$$E^i = \Pi^\mu u_\mu^i, \qquad E^{1q} = \mathrm{d}\theta^{1\alpha} v_\alpha{}^q, \qquad E_q^2 = \mathrm{d}\theta_\alpha^2 v_q{}^\alpha. \tag{C16}$$

To find that we have used

$$\mathrm{d}\Pi^\mu = -i\mathrm{d}\theta^1\sigma^\mu \wedge \mathrm{d}\theta^1 - i\mathrm{d}\theta^2\tilde{\sigma}^\mu \wedge \mathrm{d}\theta^2 \tag{C17}$$

as well as Eqs. (12) and (A12).

The derivative of the second, Wess-Zumino term of the D0-brane action is

$$-2im\mathrm{d}\theta^{1\alpha} \wedge \mathrm{d}\theta_\alpha^2 = -2imE^{1q} \wedge E_q^2. \tag{C18}$$

Now after an elementary algebra we find that the formal exterior derivative of the Lagrangian form of single D0-brane can be written as

$$\mathrm{d}\mathcal{L}_{\mathrm{D0}} = mE^i \wedge \Omega^i - im(E^{1q} + E_q^2) \wedge (E^{1q} + E_q^2), \tag{C19}$$

where Ω^i is the covariant Cartan form defined in (A11).

Then, using the Lie derivative formula (C13), we find

$$\begin{aligned}\delta\mathcal{L}_{\mathrm{D0}} = {} & m(E^i i_\delta\Omega^i - i_\delta E^i\Omega^i) \\ & - 2im(E^{1q} + E_q^2)(i_\delta E^{1q} + i_\delta E_q^2),\end{aligned} \tag{C20}$$

where $i_\delta\Omega^i$ defines essential variation of the spinor frame variable by $\delta v_\alpha{}^q = i_\delta \mathrm{D}v_\alpha{}^q = \frac{1}{2}\gamma^i_{qp} v_\alpha{}^p i_\delta\Omega^i$. This equation can be obtained from the i_δ contraction of (A13) by setting $i_\delta\Omega^{ij} = 0$.

To conclude, let us note that in this formalism the local fermionic κ-symmetry transformations δ_κ (B8) leaving invariant the D0-brane action (B1) can be described by ($i_\kappa \mathrm{d} := \delta_\kappa$)

$$\begin{aligned}& i_\kappa\Pi^\mu = \delta_\kappa x^\mu - i\delta_\kappa\theta^1\sigma^\mu\theta^1 - i\delta_\kappa\theta^2\tilde{\sigma}^\mu\theta^2 = 0 \;\Rightarrow\; i_\kappa E^0 = 0, \\ & i_\kappa E^i = 0, \qquad i_\kappa\Omega^i = 0, \qquad i_\kappa\Omega^{ij} = 0, \\ & i_\kappa E^{1q} = -i_\kappa E_q^2 = \kappa^q \;\Rightarrow\; i_\kappa(E^{1q} + E_q^2) = 0.\end{aligned} \tag{C21}$$

Indeed substituting the above i_κ for i_δ in (C20), we find $\delta_\kappa\mathcal{L}_{\mathrm{D0}} = 0$.

APPENDIX D: MULTIPLE D0-BRANE ACTION AND ITS WORLDLINE SUPERSYMMETRY

In this Appendix we present some details of the derivation of the worldline supersymmetry leaving invariant the candidate mD0 action (4).

1. Formal exterior derivative of the Lagrangian form of the mD0 action

The first stage is to calculate the formal exterior derivative of the Lagrangian form of the action (4), this is to say of 1-form

$$\begin{aligned}\mathcal{L}_{\mathrm{mD0}} = {} & mE^0 - im(\mathrm{d}\theta^1\theta^2 - \theta^1\mathrm{d}\theta^2) \\ & + \frac{1}{\mu^6}\Big[\mathrm{tr}(\mathbb{P}^i\mathrm{D}\mathbb{X}^i + 4i\mathbf{\Psi}_q\mathrm{D}\mathbf{\Psi}_q) + \frac{2}{\mathcal{M}}E^0\mathcal{H} \\ & - \frac{\mathrm{d}\mathcal{M}}{\mathcal{M}}\mathrm{tr}(\mathbb{P}^i\mathbb{X}^i) + \frac{1}{\sqrt{2\mathcal{M}}}(E^{q1} - E_q^2)\mathrm{tr}(-4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i \\ & + \frac{1}{2}(\gamma^{ij}\mathbf{\Psi})_q[\mathbb{X}^i, \mathbb{X}^j])\Big],\end{aligned} \tag{D1}$$

where $\mathcal{H}$ is given in Eq. (5). The covariant derivatives D of the bosonic and fermionic Hermitian traceless $N \times N$ matrix fields are defined in (15) and (16) with the use of 1d gauge field 1-form $\mathbb{A} = d\tau \mathbb{A}_\tau$ and Cartan forms (A11), so that, when calculating the exterior derivative of (D1), we have to use the Ricci identities

$$DD\mathbb{X}^i = \Omega^i \wedge \Omega^j \mathbb{X}^j + [\mathbb{F}, \mathbb{X}^i], \qquad DD\mathbf{\Psi}_q = \frac{1}{4}\Omega^i \wedge \Omega^j (\gamma^{ij}\mathbf{\Psi})_q + [\mathbb{F}, \mathbf{\Psi}_q]. \tag{D2}$$

Here $\mathbb{F} = d\mathbb{A} - \mathbb{A} \wedge \mathbb{A}$ is the formal 2-form field strength of the 1d gauge field $\mathbb{A}$ (which is calculated without using $\mathbb{A} = d\tau \mathbb{A}_\tau$, with the aim to apply it in the Lie derivative formula for variation of the Lagrangian 1-form). Equations (D2) are obtained using the Maurer-Cartan equations (A15).

After some algebra, the exterior derivative of the multiple D0-branes Lagrangian form (D1) can be found to be

$$\begin{aligned}
\mu^6 d\mathcal{L}_{\text{mD0}} = {} & \mu^6 m E^i \wedge \Omega^i - i\mu^6 m (E^{1q} + E^2_q) \wedge (E^{1q} + E^2_q) + \Omega^i \wedge \Omega^j \text{tr}(\mathbb{P}^i \mathbb{X}^j + i\mathbf{\Psi}\gamma^{ij}\mathbf{\Psi}) \\
& + \text{tr}(\mathbb{F}([\mathbb{X}^i, \mathbb{P}^i] - 4i\{\mathbf{\Psi}_q, \mathbf{\Psi}_q\})) - \text{tr}(D\mathbb{P}^i \wedge D\mathbb{X}^i) - 4i\text{tr}(D\mathbf{\Psi}_q \wedge D\mathbf{\Psi}_q) \\
& + \frac{2}{\mathcal{M}} E^i \wedge \Omega^i \mathcal{H} - i\frac{2}{\mathcal{M}} (E^{1q} \wedge E^{1q} + E^2_q \wedge E^2_q)\mathcal{H} - \frac{1}{2\sqrt{2\mathcal{M}}} (E^{1q} + E^2_q)\gamma^i_{qp} i\nu_p \wedge \Omega^i \\
& + \frac{2}{\mathcal{M}} \left(1 - \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H}\right) E^0 \wedge d\mathcal{H} + \frac{1}{\sqrt{2\mathcal{M}}} (E^{1q} - E^2_q) \wedge iD\nu_q + \frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}} d\mathcal{K} \wedge d\mathcal{H} \\
& + \frac{1}{\mu^6}\frac{1}{2\sqrt{2\mathcal{M}}}\frac{\mathcal{M}'}{\mathcal{M}} (E^{1q} - E^2_q) i\nu_q \wedge d\mathcal{H},
\end{aligned} \tag{D3}$$

where $\mathcal{K} \coloneqq \text{tr}(\mathbb{X}^i \mathbb{P}^i)$, ν_q is defined in (28) and $\mathcal{H}$ is the relative motion Hamiltonian (5). The derivatives of these "blocks," which also enter (D3), read

$$d\mathcal{H} = \text{tr}\left(\mathbb{P}^i D\mathbb{P}^i + \frac{1}{16} D\mathbb{X}^i [[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - D\mathbb{X}^i \gamma^i_{pq} \{\mathbf{\Psi}_p, \mathbf{\Psi}_q\} - 2D\mathbf{\Psi}_q [(\gamma^i \mathbf{\Psi})_q, \mathbb{X}^i]\right), \tag{D4}$$

$$d\mathcal{K} = \text{tr}(D\mathbb{X}^i \mathbb{P}^i + \mathbb{X}^i D\mathbb{P}^i), \tag{D5}$$

$$iD\nu_q = \text{tr}\left(-4i(\gamma^i \mathbf{\Psi})_q D\mathbb{P}^i - 4i(\gamma^i D\mathbf{\Psi})_q \mathbb{P}^i - D\mathbb{X}^i [(\gamma^{ij}\mathbf{\Psi})_q, \mathbb{X}^j] + \frac{1}{2}(\gamma^{ij} D\mathbf{\Psi})_q [\mathbb{X}^i, \mathbb{X}^j]\right). \tag{D6}$$

2. Worldline supersymmetry (κ-symmetry) transformations of the center of energy variables

The previous experience with lower-dimensional counterparts of the mD0 system [29] suggests to assume that the worldline supersymmetry acts on the center of energy variables of the mD0 system (i.e., on the superspace coordinate functions and spinor frame variables) as the κ symmetry of the single D0-brane action (see Sec. B) acts on their single-brane counterparts. Namely, we set [47]

$$\begin{aligned}
& i_\kappa \Pi^\mu = 0 \Rightarrow i_\kappa E^0 = 0, \qquad i_\kappa E^i = 0, \\
& i_\kappa \Omega^i = 0, \qquad i_\kappa \Omega^{ij} = 0 \Rightarrow \delta_\kappa u^0_\mu = 0, \qquad \delta_\kappa u^i_\mu = 0, \delta_\kappa v_\alpha{}^q = 0,
\end{aligned} \tag{D7}$$

and

$$i_\kappa E^{1q} = -i_\kappa E^2_q = \frac{\kappa^q}{\sqrt{2}} \Rightarrow \delta_\kappa \theta^{1\alpha} = \frac{\kappa^q}{\sqrt{2}} v^\alpha_q, \qquad \delta_\kappa \theta^2_\alpha = -\frac{\kappa^q}{\sqrt{2}} v_\alpha{}^q. \tag{D8}$$

These expressions are equivalent to (19), but they are more convenient to use in our method of calculation of the variation of Lagrangian form.

Then, using the Lie derivative formula (C13) with (D7), (D8) and furthermore identifying in it

$$i_\kappa D = \delta_\kappa, \qquad i_\kappa \mathbb{F} = \delta_\kappa \mathbb{A}, \qquad i_\kappa \mathbb{A} = 0, \tag{D9}$$

we find that, modulo total derivative, the variation δ_κ of the Lagrangian form $\mathcal{L}_{\rm mD0}$ reduces to

$$\begin{aligned}\mu^6\delta_\kappa\mathcal{L}_{\rm mD0} &= {\rm tr}(\delta_\kappa\mathbb{A}([\mathbb{X}^i,\mathbb{P}^i]-4i\{\mathbf{\Psi}_q,\mathbf{\Psi}_q\}))+{\rm tr}(\delta_\kappa\mathbb{P}^i{\rm D}\mathbb{X}^i-{\rm D}\mathbb{P}^i\delta_\kappa\mathbb{X}^i-8i{\rm D}\mathbf{\Psi}_q\delta_\kappa\mathbf{\Psi}_q)\\ &-i\frac{2\sqrt{2}}{\mathcal{M}}(E^{1q}-E^2_q)\kappa^q\mathcal{H}+\frac{2}{\mathcal{M}}\left(1-\frac{\mathcal{H}}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\right)E^0\delta_\kappa\mathcal{H}+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}{\rm d}\mathcal{K}\delta_\kappa\mathcal{H}\\ &+\frac{1}{\mu^6}\frac{1}{2\sqrt{2\mathcal{M}}}\frac{\mathcal{M}'}{\mathcal{M}}(E^{1q}-E^2_q)i\nu_q\delta_\kappa\mathcal{H}-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\delta_\kappa\mathcal{K}{\rm d}\mathcal{H}\\ &-\frac{1}{\mu^6}\frac{1}{2\sqrt{\mathcal{M}}}\frac{\mathcal{M}'}{\mathcal{M}}\kappa^q i\nu_q{\rm d}\mathcal{H}-\frac{1}{\sqrt{\mathcal{M}}}\kappa^q i{\rm D}\nu_q+\frac{1}{\sqrt{2\mathcal{M}}}(E^{1q}-E^2_q)i\delta_\kappa\nu_q.\end{aligned}\tag{D10}$$

The worldline supersymmetry transformation rules of the matrix fields can be found by requiring this variation to vanish. As this calculation is a bit subtle, we present below some details.

3. Worldline supersymmetry transformations of the matrix matter fields

To find the supersymmetry transformation leaving invariant $S_{\rm mD0}=\int\mathcal{L}_{\rm mD0}$, i.e., obeying $\delta_\kappa\mathcal{L}_{\rm mD0}=0$ (modulo total derivative), we have to set equal to zero the coefficients for all the independent 1-forms in (D10). Requiring to vanish the terms proportional to ${\rm D}\mathbb{P}^i$, ${\rm D}\mathbb{X}^i$, and ${\rm D}\mathbf{\Psi}_q$, we find the set of *equations* for the worldline supersymmetry transformations of the matrix "matter" fields of the form of relations (21), (22) and (23).

We stress that at this stage these are equations because their right-hand sides contain $\Delta_\kappa\mathcal{K}$ from (27) and $\delta_\kappa\mathcal{H}$ which in their turn are expressed in terms of $\delta_\kappa\mathbb{X}^i\mathbb{X}^i$, $\delta_\kappa\mathbb{P}^i$, and $\delta_\kappa\mathbf{\Psi}_q$.

To solve these equations it is convenient to calculate formally the variations of composite quantities $\delta_\kappa\mathcal{H}$ and $\Delta_\kappa\mathcal{K}$ with (21)–(23). In this way we find the following equations:

$$\Delta_\kappa\mathcal{K}=\frac{1}{2\sqrt{\mathcal{M}}}{\rm tr}\left(4i(\kappa\gamma^i\mathbf{\Psi})\mathbb{P}^i+\frac{5}{2}(\kappa\gamma^{ij}\mathbf{\Psi})[\mathbb{X}^i,\mathbb{X}^j]\right)-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa\mathcal{K}\mathfrak{H},\tag{D11}$$

$$\delta_\kappa\mathcal{H}=\frac{1}{2\sqrt{\mathcal{M}}}{\rm tr}(\kappa^q\mathbf{\Psi}_q([\mathbb{X}^i,\mathbb{P}^i]-4i\{\mathbf{\Psi}_p,\mathbf{\Psi}_p\}))-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\delta_\kappa\mathcal{H}\mathfrak{H},\tag{D12}$$

where $\mathfrak{H}$ is given in Eq. (25) [48]. These equations are solved by (26) and (24).

Thus, worldline supersymmetry transformations of the matrix matter fields are given by (21)–(23) with (26) and (24).

4. Worldline supersymmetry transformations of the worldvolume gauge field

Taking into account the above results for supersymmetry transformations of the matrix matter fields, we find that the remaining variation of the Lagrangian form (D10) can be written as

$$\begin{aligned}\mu^6\delta_\kappa\mathcal{L}_{\rm mD0} &= {\rm tr}(\delta_\kappa\mathbb{A}([\mathbb{X}^i,\mathbb{P}^i]-4i\{\mathbf{\Psi}_q,\mathbf{\Psi}_q\}))+\frac{2}{\mathcal{M}}\left(1-\frac{\mathcal{H}}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\right)E^0\delta_\kappa\mathcal{H}\\ &+\frac{1}{\sqrt{2\mathcal{M}}}(E^{1q}-E^2_q)\left(i\delta_\kappa\nu_q-\frac{4i}{\sqrt{\mathcal{M}}}\kappa^q\mathcal{H}+\frac{1}{\mu^6}\frac{1}{2}\frac{\mathcal{M}'}{\mathcal{M}}i\nu_q\delta_\kappa\mathcal{H}\right).\end{aligned}\tag{D13}$$

To proceed further, we calculate $i\delta_\kappa\nu_q$, which reads

$$\begin{aligned}i\delta_\kappa\nu_q &= -\frac{1}{\sqrt{\mathcal{M}}}(\kappa\gamma^i)_q{\rm tr}(\mathbb{X}^i([\mathbb{X}^j,\mathbb{P}^j]-4i\{\mathbf{\Psi}_r,\mathbf{\Psi}_r\}))+\frac{4i}{\sqrt{\mathcal{M}}}\kappa^q\mathcal{H}\\ &+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}{\rm tr}(4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i+(\gamma^{ij}\Psi)_q[\mathbb{X}^i,\mathbb{X}^j])\delta_\kappa\mathcal{H}-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\Delta_\kappa\mathcal{K}{\rm tr}(\mathbf{\Psi}_q([\mathbb{X}^i,\mathbb{P}^i]-4i\{\mathbf{\Psi}_q,\mathbf{\Psi}_q\}))\end{aligned}\tag{D14}$$

and substitute it to (D13) arriving at

$$\mu^6\delta_\kappa\mathcal{L}_{\rm mD0} = {\rm tr}\Bigg(([\mathbb{X}^i,\mathbb{P}^i] - 4i\{\mathbf{\Psi}_r,\mathbf{\Psi}_r\})\Bigg[\delta_\kappa\mathbb{A} + \frac{2}{\mathcal{M}\sqrt{\mathcal{M}}}E^0(\kappa^q\mathbf{\Psi}_q)\frac{(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H})}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})}$$
$$-\frac{1}{\sqrt{2}\mathcal{M}}(E^{1q}-E^2_q)(\gamma^i\kappa)_q\mathbb{X}^i + +(E^{1q}-E^2_q)\frac{1}{\mu^6}\frac{\mathcal{M}'}{2\mathcal{M}\sqrt{2\mathcal{M}}}$$
$$\times\left(-2\Delta_\kappa\mathcal{K}\mathbf{\Psi}_q + \frac{\kappa^p\mathbf{\Psi}_p}{\sqrt{\mathcal{M}}}\frac{{\rm tr}(4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i + \frac{5}{2}(\gamma^{ij}\mathbf{\Psi})_q[\mathbb{X}^i,\mathbb{X}^j])}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})}\right)\Bigg]\Bigg). \tag{D15}$$

The above expression vanishes if the SU(N) gauge field transforms under worldline supersymmetry as

$$\delta_\kappa\mathbb{A} = -\frac{2}{\mathcal{M}\sqrt{\mathcal{M}}}E^0(\kappa^q\mathbf{\Psi}_q)\frac{(1-\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathcal{H})}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})} + \frac{1}{\sqrt{2}\mathcal{M}}(E^{1q}-E^2_q)(\gamma^i\kappa)_q\mathbb{X}^i$$
$$+(E^{1q}-E^2_q)\frac{1}{\mu^6}\frac{\mathcal{M}'}{2\mathcal{M}\sqrt{2\mathcal{M}}}\left(2\Delta_\kappa\mathcal{K}\mathbf{\Psi}_q - \frac{\kappa^p\mathbf{\Psi}_p}{\sqrt{\mathcal{M}}}\frac{{\rm tr}(4i(\gamma^i\mathbf{\Psi})_q\mathbb{P}^i + \frac{5}{2}(\gamma^{ij}\mathbf{\Psi})_q[\mathbb{X}^i,\mathbb{X}^j])}{(1+\frac{1}{\mu^6}\frac{\mathcal{M}'}{\mathcal{M}}\mathfrak{H})}\right).$$

Substituting (26) in it, we arrive after some algebra at Eq. (29).

[1] Of these D1-branes are Dirichlet strings or D-strings, D2-branes are Dirichlet membranes, and D0-branes are so-called Dirichlet particles, massive supersymmetric particles. The set of higher p-branes contains the maximal D9-brane which is spacetime filling as the string theory is 10 dimensional. In the original language of the string model [4] the string ending on the D9-brane is called the superstring with free ends.

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[30] Two comments should be made in this respect. First this statement refers to the mDp action in its form similar to the action of a single Dp-brane and which has the weak field limit described in [14], so it does not apply to a very interesting construction on "-1 quantization level" proposed and elaborated in [25–27] (see [28] for more discussion). Also notice an action proposed in [28] which will appear as a particular (simplest) case of nonlinear actions presented in this work.

[31] The κ symmetry was discovered for massive superparticles in [32,33] and for a massless one in [34]. The identification of κ symmetry with worldline supersymmetry was established in [35]; see [36] for a review.

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[46] Here "formal" means that we do not use the formula $\mathrm{d} = \mathrm{d}\tau\partial_\tau$ in our case or its D-dimensional generalization in the case of D-dimensional field theory; if we did, this would clearly imply vanishing of any 2-form in our case or any $(D+1)$ form in D-dimensional space. In other words, the procedure implies formal extension of all the differential forms from the worldline to target superspace or, better to say, to some its extension, some supergroup manifold which also includes the coordinates corresponding to spinor moving frame variables (called Lorentz harmonic superspace in [40]; see also [45]). The differentials of the latter are expressed in terms of Cartan forms (A11).

[47] The rescaling of fermionic function $\kappa^q \mapsto \frac{\kappa^q}{\sqrt{2}}$ is performed to simplify the worldline supersymmetry transformation rules of the matrix fields.

[48] To obtain (D11) one has to use the identity $\gamma^i_{s(r}\gamma^i_{pq)} \equiv \delta_{s(r}\delta_{pq)}$ and also notice that $\mathrm{tr}(\mathbf{\Psi}_r\{\mathbf{\Psi}_p, \mathbf{\Psi}_q\}) = \mathrm{tr}(\mathbf{\Psi}_{(r}\{\mathbf{\Psi}_p, \mathbf{\Psi}_{q)}\})$ is completely symmetric with respect to (rpq) indices while $\mathrm{tr}([(\gamma^i\mathbf{\Psi})_q, \mathbb{X}^i][(\gamma^j\mathbf{\Psi})_q, \mathbb{X}^j]) = 0$ vanishes.