

Renormalizable extension of the Abelian Higgs-Kibble model with a dimension-six operator

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A deformation of the Abelian Higgs Kibble model induced by a dimension-six derivative operator is studied. A novel differential equation is established fixing the dependence of the vertex functional on the coupling z of the dimension-six operator in terms of amplitudes at $z = 0$ (those of the power-counting renormalizable Higgs-Kibble model). The latter equation holds in a formalism where the physical mode is described by a gauge-invariant field. The functional identities of the theory in this formalism are studied. In particular, we show that the Slavnov-Taylor identities separately hold true at each order in the number of internal propagators of the gauge-invariant scalar. Despite being nonpower-counting renormalizable, the model at $z \neq 0$ depends on a finite number of physical parameters.

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I. INTRODUCTION

The experimental program at High-Luminosity and High-Energy LHC [1] will provide a unique opportunity to explore the Higgs sector of the electroweak theory, thus elucidating the electroweak spontaneous symmetry breaking (EWSSB) mechanism. To be sure, while the Standard Model's requirement of gauge invariance and power-counting renormalizability uniquely predicts the quartic Higgs potential as the source of EWSSB, many other alternatives exist in Beyond the Standard Model theories, where either new particles are introduced while preserving power-counting renormalizability (e.g., in the case of the two-Higgs doublet model or the minimal supersymmetric Standard Model) or additional power-counting violating interactions are switched on in the spirit of effective field theories [2–4].

In the latter case, the higher-dimensional operators introduced must fulfill the relevant symmetries of the specific theory considered, e.g., gauge symmetry, Lorentz covariance, and further possible discrete symmetries. The couplings of such operators are otherwise unconstrained additional physical parameters that must be fixed by suitable normalization conditions. Their number increases

order by order in the loop expansion, since more and more ultraviolet (UV) divergences arise in those effective models, as a consequence of the lack of power-counting renormalizability.

It sometimes happens that in a nonpower-counting renormalizable theory some of the coefficients of the higher-dimensional operators can be reduced to a smaller number of independent ones. This is what happens, for instance, in the reduction of couplings approach [5,6], since reduced couplings are functions of a primary one, satisfying a set of differential equations compatible with the renormalization group flow. Additionally, it is well known that the appropriate choice of field coordinates can greatly simplify the task of identifying the independent couplings of the model. For instance, one can obtain an equivalent theory with nonrenormalizable couplings from a power-counting renormalizable one by applying an invertible nonlinear field transformation. Provided that a suitable prescription is adopted when quantizing the former theory, physical observables in the two models do not differ, as stated by the so-called equivalence theorem [7–9]. In the case of spontaneously broken gauge theories, field-theoretic properties of the Higgs mechanism in different field representations, including those based on gauge-invariant variables, are discussed in Ref. [10].

As a preliminary study for the treatment of the full electroweak model, in this paper we consider a specific extension of the power-counting renormalizable Abelian Higgs-Kibble theory quantized in the simplest linear representation of the physical Higgs scalar via a gauge-invariant variable X_2 , reducing on shell to the gauge-invariant combination [11–20]

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$$X_2 \sim \frac{1}{v} \left(\phi^\dagger \phi - \frac{v^2}{2} \right), \quad (1.1)$$

where ϕ is the complex Higgs field

$$\phi = \frac{1}{\sqrt{2}} (\sigma + v + i\chi), \quad (1.2)$$

with v as the (gauge-invariant) minimum of the classical potential, σ as the field describing the physical scalar mode, and χ as the Goldstone field.

Within this framework, we find that there is a unique deformation of the power-counting renormalizable model that preserves at the quantum level the defining functional identities of the Abelian Higgs-Kibble model; more specifically, this deformation is induced by a modified kinetic term

$$-\frac{z}{2} X_2 \square X_2, \quad (1.3)$$

which corresponds, after the identification (1.1), to the dimension-six derivative operator

$$-\frac{z}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2} \right) \square \left(\phi^\dagger \phi - \frac{v^2}{2} \right). \quad (1.4)$$

Thus, z represents a parameter controlling the nonpower-counting renormalizable deformation induced by the dimension-six operator (1.4). At $z = 0$ one recovers the power-counting renormalizable Higgs-Kibble model, whereas at $z \neq 0$ power-counting renormalizability is lost; nevertheless, one can write a z -differential equation connecting the dependence of the one-particle irreducible (1-PI) amplitudes on the coefficient z and the mass M of the physical scalar. Then, under some reasonable assumptions on the boundary conditions, a unique solution exists for the 1-PI vertex functional (the generating functional of the 1-PI amplitudes) of the deformed theory, fulfilling the z -differential equation and all the remaining symmetries of the theory. Such vertex functional at $z \neq 0$ is constructed out of the Feynman amplitudes of the power-counting renormalizable theory at $z = 0$; most notably, each subsector of the theory at $z = 0$, labeled by the number ℓ of internal lines of the physical scalar X_2 propagating inside loops, can be lifted in a unique way to the corresponding subsector at $z \neq 0$.

The consistency of the solutions to the z -differential equation we will construct comes from the property that, at variance with the conventional representation of the scalar Higgs field ϕ , the ℓ subsectors of the theory are separately Slavnov-Taylor invariant. Hence, despite not being power-counting renormalizable, the theory at $z \neq 0$ can still be defined in terms of a finite number of physical parameters (those of the power-counting renormalizable theory and z). Thus, once extended to the electroweak gauge group, the distinct phenomenological implications of such a theory can be identified and, in principle, tested against the available experimental results.

The present paper is organized as follows. In Sec. II, we introduce the model and set our notations. Next, in Sec. III, we derive the differential equation controlling the dependence of the theory on the deformation parameter z . Renormalization of the z -differential equation and the ensuing constraints on the 1-PI amplitudes are derived in Sec. IV; the corresponding Slavnov-Taylor (ST) identity is then studied in Sec. V, together with its decomposition in a tower of relations among 1-PI Green's functions at fixed order in the number of internal X_2 lines. The compatibility of the z -differential equation with different renormalization schemes is studied in Sec. VI, and our conclusions and outlook are finally presented in Sec. VII.

II. EXTENSION OF THE ABELIAN HIGGS-KIBBLE MODEL

We will consider the Abelian Higgs-Kibble (HK) model [21,22] extended with the dimension-six operator in Eq. (1.4) as a useful playground in view of the treatment of the full $SU(2) \times U(1)$ electroweak theory with mass generation *à la* Higgs. In particular, we will use a gauge-invariant coordinate in order to describe the physical scalar model, in accord with the formalism of [13,23,24]; in fact, it is only within such an approach that one can obtain the differential equation constraining the dependence of the 1-PI amplitudes on z and, with it, all the results discussed in the following sections.

To this end, one introduces the field X_2 together with a Lagrange multiplier X_1 to obtain the vertex functional¹

$$\begin{aligned} \Gamma^{(0)} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) - \frac{M^2 - m^2}{2} X_2^2 - \frac{m^2}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 \right. \\ \left. + \frac{z}{2} \partial^\mu X_2 \partial_\mu X_2 - \bar{c} (\square + m^2) c + \frac{1}{v} (X_1 + X_2) (\square + m^2) \left(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \right) \right. \\ \left. + \frac{\xi b^2}{2} - b (\partial A + \xi e v \chi) + \bar{\omega} (\square \omega + \xi e^2 v (\sigma + v) \omega) + \bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \right) + \sigma^* (-e \omega \chi) + \chi^* e \omega (\sigma + v) \right]. \quad (2.1) \end{aligned}$$

¹That the extension of the scalar sector via the fields $X_{1,2}$ does not introduce additional physical degrees of freedom is shown in Appendix A.

In the above equation, D_μ is the covariant derivative

$$D_\mu = \partial_\mu - ieA_\mu. \quad (2.2)$$

A_μ is the Abelian gauge connection, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength, and ϕ is the complex Higgs field introduced in Eq. (1.2).

The model in Eq. (2.1) contains an enlarged scalar sector with respect to the usual ϕ formalism controlled by the fields $X_{1,2}$. X_1 enforces on shell the condition²

$$X_2 \sim \frac{1}{v} \left(\phi^\dagger \phi - \frac{v^2}{2} \right). \quad (2.3)$$

Consequently, X_2 can be thought of as a field coordinate parametrizing the scalar gauge-invariant combination $\phi^\dagger \phi - \frac{v^2}{2}$; in particular, at the linearized level, $X_2 \sim \sigma$.

By eliminating in Eq. (2.1) both X_1 and X_2 via their equations of motion, one recovers the usual vertex functional of the Abelian Higgs-Kibble model with the dimension-six derivative operator

$$\frac{z}{2} \partial^\mu X_2 \partial_\mu X_2 \sim \frac{1}{2v^2} \partial^\mu \left(\phi^\dagger \phi - \frac{v^2}{2} \right) \partial_\mu \left(\phi^\dagger \phi - \frac{v^2}{2} \right) \quad (2.4)$$

in the usual ϕ formalism. In particular, the two mass parameters m and M in the first line of Eq. (2.1) are chosen in such a way that by going on shell with the Lagrange multiplier X_1 one recovers the usual quartic Higgs potential $-\frac{M^2}{2v^2} (\phi^\dagger \phi - \frac{v^2}{2})^2$. The only physical parameter is thus M . In fact, it can be checked that the correlators of physical observables after going on shell with the fields $X_{1,2}$ do not depend on m [13]. Nevertheless, m must be introduced, as the power-counting renormalizable $m = 0$, $z = 0$ model would be unstable under radiative corrections.

The second line of Eq. (2.1) contains the deformation of the X_2 -kinetic term controlled by the parameter z . When such term is switched off ($z = 0$), we recover the power-counting renormalizable Higgs-Kibble model; on the other hand, at $z \neq 0$, the theory becomes nonrenormalizable and is defined by solving the z -differential equation (3.12) with appropriate boundary conditions as discussed in Sec. IV.

The main advantage of the X representation of the physical scalar mode is twofold. On the one hand, the full dependence on the additional parameter z is contained in

²Going on shell with X_1 yields a Klein-Gordon equation

$$(\square + m^2) \left(\phi^\dagger \phi - \frac{v^2}{2} - vX_2 \right) = 0,$$

so that the most general solution is $X_2 = \frac{1}{v} \left(\phi^\dagger \phi - \frac{v^2}{2} \right) + \eta$, η being a scalar field of mass m . However, in perturbation theory, the correlators of the mode η with any gauge-invariant operators vanish [14], so that one can safely set $\eta = 0$.

the quadratic part of the classical vertex functional, so that in the perturbative expansion using the mass eigenstate basis (leading to the diagonal propagators) the coupling z will enter in the X_2 propagator but not in the interaction vertices, contrary to what happens in the conventional ϕ representation of Eq. (1.4). On the other hand, since X_2 is gauge-invariant, the projection of the ST identity valid for the 1-PI amplitudes yields an additional set of relations that isolate separately invariant subsectors of the theory according to the number of internal X_2 lines (with the lowest order reproducing the Stückelberg theory, namely, no internal X_2 lines).

III. DIFFERENTIAL EQUATION FOR z

In the mass eigenstate basis, the dependence on the parameter z only arises via the X_2 propagator, see Eq. (2.1); indeed, an X_2 line circulating inside a general n -loop diagram will be characterized by a propagator $\Delta_{X_2 X_2}$ given by, see Eq. (B1),

$$\Delta_{X_2 X_2}(k^2, M^2) = \frac{i}{(1+z)k^2 - M^2}. \quad (3.1)$$

Introducing then the differential operator

$$\mathcal{D}_z^{M^2} = (1+z)\partial_z + M^2\partial_{M^2}, \quad (3.2)$$

one finds that $\Delta_{X_2 X_2}$ is an eigenvector of $\mathcal{D}_z^{M^2}$ with eigenvalue -1 ,

$$\mathcal{D}_z^{M^2} \Delta_{X_2 X_2}(k^2, M^2) = -\Delta_{X_2 X_2}(k^2, M^2). \quad (3.3)$$

Next, let us collectively denote with Φ the set of fields and external sources of the theory, and let us indicate with p_i (with $i = 1, \dots, r$) the external momenta, with $\Phi_i = \Phi(p_i)$ and $p_r = -\sum_{i=1}^{r-1} p_i$; in this way, an n -loop 1-PI Green's function $\Gamma_{\Phi_1 \dots \Phi_r}^{(n)}$ with r Φ_i insertions can be decomposed as the sum of all diagrams with (amputated) external legs $\Phi_1 \dots \Phi_r$ with $0, 1, 2, \dots, \ell$ internal X_2 propagators, i.e.,

$$\Gamma_{\Phi_1 \dots \Phi_r}^{(n)} = \sum_{\ell \geq 0} \Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)}. \quad (3.4)$$

Then, clearly,

$$\mathcal{D}_z^{M^2} \Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)} = -\ell \Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)}; \Rightarrow \mathcal{D}_z^{M^2} \Gamma_{\Phi_1 \dots \Phi_r}^{(n)} = -\sum_{\ell \geq 0} \ell \Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)}. \quad (3.5)$$

The most general solution of this equation reads (indicating explicitly only the dependence on the parameters z and M^2)

$$\Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)}(z, M^2) = \frac{1}{(1+z)^\ell} \Gamma_{\Phi_1 \dots \Phi_r}^{(n; \ell)}(0, M^2/1+z). \quad (3.6)$$

As before, each functional $\Gamma_{[k]}(t)$ can be expanded according to the double grading with respect to the loop number n and the ℓ sector; and the tower of Eqs. (3.15) yields a set of relations among the amplitudes of the nonrenormalizable theory at $z \neq 0$. At the lowest order, they depend on the amplitudes of the power-counting renormalizable theory at $z = 0$ through the term $\Gamma_{[0]}(1)$ on the rhs of Eq. (3.15a) and on those of the Stückelberg subsector Γ_0 . Explicit examples are provided in the next subsection.

A. Explicit checks

Let us carry out some explicit checks that the z -differential equation is indeed satisfied by the regularized one-loop 1-PI amplitudes. Constraints on the finite counterterms arising from the equation will be discussed later on, in Sec. IV.

1. Tadpoles

Consider the one-loop ($n = 1$) tadpoles of the field σ and the antifield \bar{c}^* . From the vertex functional (2.1), one sees that both tadpoles decompose at the one-loop order as the sum of a part with no internal X_2 lines and a one diagram with one ($\ell = 1$) internal X_2 line. Thus, Eq. (3.8) reads ($\Phi = \sigma, \bar{c}^*$)

$$\Gamma_{\Phi}^{(1)}(t) = \Gamma_{\Phi}^{(1)} = \Gamma_{\Phi}^{(1;0)} + \Gamma_{\Phi}^{(1;1)}, \quad (3.16)$$

which must be replaced everywhere in the entire tower of Eqs. (3.15), that is, one has

$$\Gamma_{[\dots]}(t) \rightarrow \Gamma_{[\dots]\Phi}^{(1)} = \Gamma_{[\dots]\Phi}^{(1;0)} + \Gamma_{[\dots]\Phi}^{(1;1)}. \quad (3.17)$$

Considering only the UV divergent parts (denoted with a bar), an explicit calculation gives (in the Feynman gauge $\xi = 1$)

$$\begin{aligned} \bar{\Gamma}_{\sigma}^{(1;0)} &= \frac{1}{16\pi^2} \frac{M_A^2}{v\epsilon} (m^2 + 6M_A^2); \\ \bar{\Gamma}_{\sigma}^{(1;1)} &= \frac{1}{16\pi^2} \frac{M^2}{(1+z)^3 v\epsilon} [m^2(1+z) + 2M^2], \end{aligned} \quad (3.18a)$$

$$\bar{\Gamma}_{\bar{c}^*}^{(1;0)} = -\frac{1}{16\pi^2} \frac{M_A^2}{\epsilon}, \quad \bar{\Gamma}_{\bar{c}^*}^{(1;1)} = -\frac{1}{16\pi^2} \frac{M^2}{(1+z)^2 \epsilon}. \quad (3.18b)$$

Expanding around $z = 0$ yields, up to $\mathcal{O}(z^2)$,

$$\bar{\Gamma}_{[0]\sigma}^{(1;1)} = \frac{1}{16\pi^2} \frac{M^2}{v\epsilon} (m^2 + 2M^2); \quad \bar{\Gamma}_{[0]\bar{c}^*}^{(1;1)} = -\frac{1}{16\pi^2} \frac{M^2}{\epsilon}, \quad (3.19a)$$

$$\bar{\Gamma}_{[1]\sigma}^{(1;1)} = -\frac{1}{8\pi^2} \frac{M^2}{v\epsilon} (m^2 + 3M^2); \quad \bar{\Gamma}_{[1]\bar{c}^*}^{(1;1)} = \frac{1}{8\pi^2} \frac{M^2}{\epsilon}, \quad (3.19b)$$

$$\bar{\Gamma}_{[2]\sigma}^{(1;1)} = \frac{3}{16\pi^2} \frac{M^2}{v\epsilon} (m^2 + 4M^2); \quad \bar{\Gamma}_{[2]\bar{c}^*}^{(1;1)} = -\frac{3}{8\pi^2} \frac{M^2}{\epsilon}. \quad (3.19c)$$

Then it is immediate to show

$$\bar{\Gamma}_{[1]\Phi}^{(1;1)} + M^2 \partial_{M^2} \bar{\Gamma}_{[0]\Phi}^{(1;1)} = -\bar{\Gamma}_{[0]\Phi}^{(1;1)}, \quad (3.20a)$$

$$2\bar{\Gamma}_{[2]\Phi}^{(1;1)} + \bar{\Gamma}_{[1]\Phi}^{(1;1)} + M^2 \partial_{M^2} \bar{\Gamma}_{[1]\Phi}^{(1;1)} = -\bar{\Gamma}_{[1]\Phi}^{(1;1)}. \quad (3.20b)$$

2. Two-point functions

More interesting is the case of the two-point functions, as in this case an explicit t dependence can arise from diagrams involving two internal X_2 lines,

$$\Gamma_{\Phi_1\Phi_2}^{(1)}(t) = \Gamma_{\Phi_1\Phi_2}^{(1;0)} + \Gamma_{\Phi_1\Phi_2}^{(1;1)} + t\Gamma_{\Phi_1\Phi_2}^{(1;2)}, \quad (3.21)$$

with the corresponding replacement in Eqs. (3.15),

$$\Gamma_{[\dots]}(t) \rightarrow \Gamma_{[\dots]\Phi_1\Phi_2}^{(1)}(t) = \Gamma_{[\dots]\Phi_1\Phi_2}^{(1;0)} + \Gamma_{[\dots]\Phi_1\Phi_2}^{(1;1)} + t\Gamma_{[\dots]\Phi_1\Phi_2}^{(1;2)}. \quad (3.22)$$

This is the case when $\Phi_1\Phi_2 = \bar{c}^*\bar{c}^*, \bar{c}^*\sigma$, and $\sigma\sigma$; instead, the two-point functions $\Phi_1\Phi_2 = \chi\chi, \chi A_\mu$, and $A_\mu A_\nu$ do not present such diagrams (and therefore the check of the z -differential equation proceeds as in the tadpole case).

Consider then the $\bar{c}^*\bar{c}^*$ function, for which an explicit calculation yields

$$\bar{\Gamma}_{\bar{c}^*\bar{c}^*}^{(1;0)} = \frac{1}{16\pi^2} \frac{1}{\epsilon}; \quad \bar{\Gamma}_{\bar{c}^*\bar{c}^*}^{(1;1)} = 0; \quad \bar{\Gamma}_{\bar{c}^*\bar{c}^*}^{(1;2)} = \frac{1}{16\pi^2} \frac{1}{(1+z)^2} \frac{1}{\epsilon}. \quad (3.23)$$

Then, noticing that there is no dependence on M^2 and that the $n = 1$ $\ell = 1$ term is convergent, Eq. (3.15a) reads

$$\int_0^1 dt t \bar{\Gamma}_{[1]\bar{c}^*\bar{c}^*}^{(1;2)} = -\bar{\Gamma}_{[0]\bar{c}^*\bar{c}^*}^{(1;2)}, \quad (3.24)$$

which is evidently satisfied. Equation (3.15b) reads instead

$$\int_0^1 dt t (2\bar{\Gamma}_{[2]\bar{c}^*\bar{c}^*}^{(1;2)} + \bar{\Gamma}_{[1]\bar{c}^*\bar{c}^*}^{(1;2)}) = -\bar{\Gamma}_{[1]\bar{c}^*\bar{c}^*}^{(1;2)}, \quad (3.25)$$

which is again easily verified. A similar procedure leads to the verification of all the remaining one-loop two-point functions.

IV. RENORMALIZATION

At $z = 0$ the dimension-six operator vanishes and the theory defined by the vertex functional (2.1) is power-counting renormalizable, as it coincides with the usual HK model. On the other hand, when $z \neq 0$ power-counting renormalizability is lost, and new UV divergences appear at each loop order (starting already at one loop) proportional to increasing powers of the external momenta. The theory can be still renormalized *à la* Weinberg [25]: the newly appearing divergent amplitudes must be regularized by subtracting appropriate counterterms; however, contrary to the renormalizable case, the corresponding *finite* parts of these amplitudes must also be fixed by appropriate renormalization conditions, reflecting the well-known fact that, being nonrenormalizable at $z \neq 0$, the vertex functional (2.1) gives rise to an effective field theory [2,26].

At $z \neq 0$, therefore, a general amplitude can be decomposed as follows:

$$\Gamma_{\Phi_1 \dots \Phi_r}^{(n)} = \sum_{\ell \geq 0} \left[\Gamma_{\Phi_1 \dots \Phi_r}^{(n;\ell)} - \sum_{k=1}^n \frac{1}{\epsilon^k} \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)} + F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} \right], \quad (4.1)$$

where $\Gamma_{\Phi_1 \dots \Phi_r}^{(n;\ell)}$ is the D -dimensional regularized n th order amplitude in the ℓ sector after the insertion of the counterterms up to order $n - 1$ in the loop expansion, $\bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)}$ are the residues of the poles in $1/\epsilon^k$ in the expansion of $\Gamma_{\Phi_1 \dots \Phi_r}^{(n;\ell)}$ around $D = 4$, and $F_{\Phi_1 \dots \Phi_r}^{(n;\ell)}$ are the finite counterterms that inherit the degree ℓ from the number of internal X_2 lines. Notice, in particular, that both $\bar{\Gamma}$ and F are Lorentz-covariant polynomials of degree δ_r (the UV degree of divergence of the corresponding amplitude) in the external momenta p_i , and that they obey the same differential equation (3.5) of the corresponding amplitudes, namely,

$$\mathcal{D}_z^{M^2} \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)} = -\ell \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)}; \quad \mathcal{D}_z^{M^2} F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} = -\ell F_{\Phi_1 \dots \Phi_r}^{(n;\ell)}, \quad (4.2)$$

and thus possess the structure (3.6) for their general solutions,

$$\bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)}(z, M^2) = \frac{1}{(1+z)^\ell} \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)}(0, M^2/1+z), \quad (4.3a)$$

$$F_{\Phi_1 \dots \Phi_r}^{(n;\ell)}(z, M^2) = \frac{1}{(1+z)^\ell} F_{\Phi_1 \dots \Phi_r}^{(n;\ell)}(0, M^2/1+z). \quad (4.3b)$$

Now, consider, for example, the case of the one-loop three-point σ 1-PI amplitude where one has contributions from diagrams with $\ell = 0-3$. Thus, we have

$$\Gamma_{\sigma_1 \sigma_2 \sigma_3}^{(1)} = \sum_{\ell=0}^3 \left[\Gamma_{\sigma_1 \sigma_2 \sigma_3}^{(1;\ell)} - \frac{1}{\epsilon} \bar{\Gamma}_{1;\sigma_1 \sigma_2 \sigma_3}^{(1;\ell)} + F_{\sigma_1 \sigma_2 \sigma_3}^{(1;\ell)} \right], \quad (4.4)$$

where the UV divergent and finite parts $\bar{\Gamma}$ and F are in this case polynomials of degree two in the independent momenta $p_{1,2}$ ($p_3 = -p_1 - p_2$), or

$$\bar{\Gamma}_{1;\sigma_1 \sigma_2 \sigma_3}^{(1;\ell)} = \gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{0(1;\ell)} + \gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{1(1;\ell)} (p_1^2 + p_2^2 + p_1 \cdot p_2), \quad (4.5a)$$

$$F_{\sigma_1 \sigma_2 \sigma_3}^{(1;\ell)} = f_{\sigma_1 \sigma_2 \sigma_3}^{0(1;\ell)} + f_{\sigma_1 \sigma_2 \sigma_3}^{1(1;\ell)} (p_1^2 + p_2^2 + p_1 \cdot p_2). \quad (4.5b)$$

An explicit calculation in the Feynman gauge then yields

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{0(1;0)} = -\frac{3}{16\pi^2 v^3} (m^4 - 2m^2 M_A^2 + 12M_A^4), \quad (4.6a)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{0(1;1)} = -\frac{3M^2}{4\pi^2 v^3 (1+z)^2} \left(m^2 + \frac{2M^2}{1+z} \right), \quad (4.6b)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{0(1;2)} = \frac{9}{16\pi^2 v^3 (1+z)^2} \left[m^4 + \frac{8m^2 M^2}{1+z} + \frac{12M^4}{(1+z)^2} \right], \quad (4.6c)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{0(1;3)} = -\frac{3}{4\pi^2 v^3 (1+z)^3} \left[m^4 + \frac{6m^2 M^2}{1+z} + \frac{8M^4}{(1+z)^2} \right], \quad (4.6d)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{1(1;0)} = 0, \quad (4.6e)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{1(1;1)} = -\frac{M^2}{2\pi^2 v^3 (1+z)^2}, \quad (4.6f)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{1(1;2)} = \frac{1}{8\pi^2 v^3 (1+z)^2} \left(m^2 + \frac{10M^2}{1+z} \right), \quad (4.6g)$$

$$\gamma_{1;\sigma_1 \sigma_2 \sigma_3}^{1(1;3)} = -\frac{1}{8\pi^2 v^3 (1+z)^3} \left(m^2 + \frac{6M^2}{1+z} \right). \quad (4.6h)$$

In particular, notice that summing the different layers in ℓ (like one would do in a “standard” approach) gives for the coefficient of the quadratic term in the independent momenta

$$\begin{aligned} \gamma_{1;\sigma_1\sigma_2\sigma_3}^{1(1)} &= \sum_{\ell=0}^3 \gamma_{1;\sigma_1\sigma_2\sigma_3}^{1(1;\ell)} \\ &= \frac{z}{8\pi^2 v^2 (1+z)^4} [2M^2(1-2z) + m^2(1+z)]. \end{aligned} \quad (4.7)$$

Consider then first the $z = 0$ case, where, without loss of generality, we can work in the minimal subtraction (MS) scheme (in fact, amplitudes at $z = 0$ in any other subtraction scheme can be reduced to those in the MS scheme by a suitable redefinition of fields and coupling constants). As expected, for a power-counting renormalizable model like the HK, Eq. (4.7) vanishes when $z = 0$; therefore, we obtain the condition on the finite parts

$$f_{1;\sigma_1\sigma_2\sigma_3}^{1(1)} \Big|_{z=0} = \sum_{\ell=0}^3 f_{1;\sigma_1\sigma_2\sigma_3}^{1(1;\ell)} \Big|_{z=0} = 0. \quad (4.8)$$

When $z \neq 0$, Eq. (4.7) is nonvanishing and, within an effective field-theory approach, we need to impose a renormalization condition for this new term without spoiling the ST identities. This can be achieved by fixing the coefficient $\lambda_6^{(1)}$ of the invariant that contains the term $\sigma^2 \square \sigma$, and namely [see Eq. (D4)],

$$\begin{aligned} \lambda_6^{(1)} \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right) (\phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi) \supset v \lambda_6^{(1)} \\ \times \int d^4x \sigma^2 \square \sigma; \quad f_{1;\sigma_1\sigma_2\sigma_3}^{1(1)} \equiv 2v \lambda_6^{(1)}. \end{aligned} \quad (4.9)$$

However, this fixes only the overall sum over ℓ ; its decomposition in terms of ℓ is, in general, not uniquely determined. If, on the other hand, each ℓ sector could be proven to be separately Slavnov-Taylor invariant, each of these subsectors would reproduce in the limit $z \rightarrow 0$ the corresponding subsectors of the amplitude in the power-counting renormalizable theory at $z = 0$. This implies that the condition (4.8) can be imposed at $z \neq 0$,

$$f_{1;\sigma_1\sigma_2\sigma_3}^{1(1)} = \sum_{\ell=0}^3 f_{1;\sigma_1\sigma_2\sigma_3}^{1(1;\ell)} = 0, \quad (4.10)$$

and at this point it is immediate to prove that the condition above implies, upon repeated application of the differential operator $\mathcal{D}_z^{M^2}$, that

$$f_{1;\sigma_1\sigma_2\sigma_3}^{1(1;\ell)} = 0 \quad \forall \ell. \quad (4.11)$$

This result is, in fact, generic and not limited to the three-point σ amplitude considered here for illustrative purposes; that is, within this formulation of the HK model at $z \neq 0$, finite parts are unambiguously set to zero by imposing the condition

$$\sum_{\ell \geq 0} F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} = 0; \Rightarrow F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} = 0 \quad \forall \ell. \quad (4.12)$$

In the next section we are going to prove that fixed ℓ sectors are indeed separately ST invariant. Before doing that, however, let us observe that the widely studied choice of normalization conditions for the Stückelberg theory (the $\ell = 0$ sector) in which one requires the matching of the 1-PI amplitudes of the Stückelberg model with those of the Higgs theory (seen as a UV completion of the former) [27–29] breaks the condition (4.12); in fact, it requires one to choose the finite parts of $F_{\Phi_1 \dots \Phi_r}^{(n;0)}$ in such a way that the Stückelberg and Higgs amplitudes at $z = 0$ coincide at some IR scale μ^2 in the Taylor expansion up to the superficial degree of UV divergence δ_r of the Stückelberg amplitude.

In order to enforce this matching condition, we need to fine-tune $F_{\Phi_1 \dots \Phi_r}^{(n;0)}$ in Eq. (4.1) according to (all amplitudes on the rhs are understood to be evaluated at $z = 0$)

$$F_{\Phi_1 \dots \Phi_r}^{(n;0)} \Big|_{p^2=\mu^2} = t^{\delta_r} \left(\Gamma_{\Phi_1 \dots \Phi_r}^{(n)} - \Gamma_{\Phi_1 \dots \Phi_r}^{(n;0)} + \sum_{k=1}^n \frac{1}{\epsilon^k} \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;0)} \right), \quad (4.13)$$

where t^{δ_r} denotes the Taylor expansion up to order δ_r around a symmetric point $p^2 = \mu^2$ in the momenta p_1, \dots, p_r . Then, using Eq. (4.1), we obtain

$$\begin{aligned} F_{\Phi_1 \dots \Phi_r}^{(n;0)} \Big|_{p^2=\mu^2} &= t^{\delta_r} \left[\sum_{\ell \geq 1} \left(\Gamma_{\Phi_1 \dots \Phi_r}^{(n;\ell)} - \sum_{k=1}^n \frac{1}{\epsilon^k} \bar{\Gamma}_{k;\Phi_1 \dots \Phi_r}^{(n;\ell)} \right) \right] \\ &+ \sum_{\ell \geq 1} F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} \Big|_{p^2=\mu^2}, \end{aligned} \quad (4.14)$$

where the last term in the above equation is a polynomial of degree δ_r in the momenta. In addition, the sum rule at $z = 0$,

$$\sum_{\ell \geq 0} F_{\Phi_1 \dots \Phi_r}^{(n;\ell)} \Big|_{z=0} = 0, \quad (4.15)$$

must also hold true, since, as mentioned above, we can assume, without loss of generality, that the power-counting renormalizable HK model at $z = 0$ is defined in the MS scheme. No unique choice of finite parts fulfilling simultaneously the two conditions (4.13) and (4.15) exists: at least two $F_{\Phi_1 \dots \Phi_r}^{(n;\ell)}$ must be different from zero with the remaining parts that can be arbitrarily fixed.

V. ℓ -SECTOR SLAVNOV-TAYLOR IDENTITIES

The 1-PI amplitudes involving at least one external X_2 leg are uniquely fixed by the X_2 equation of motion, Eq. (C4), so that we can concentrate on X_2 -independent amplitudes. At order n in the loop expansion, the 1-PI

amplitudes of the functional $\Gamma(t)$ of Eq. (3.11) at $X_2 = 0$ can be gathered into a vertex functional $\hat{\Gamma}(t)$ defined as

$$\begin{aligned}\hat{\Gamma}(t)^{(n)} &\equiv \Gamma_0^{(n)}|_{X_2=0} + t[\Gamma(t)^{(n)} - \Gamma_0^{(n)}]|_{X_2=0} \\ &= \Gamma_0^{(n)}|_{X_2=0} + \sum_{\ell \geq 1} t^\ell \Gamma^{(n;\ell)}|_{X_2=0}.\end{aligned}\quad (5.1)$$

$\hat{\Gamma}(t)^{(n)}$ can be recovered from the vertex functional of the complete theory $\Gamma(1)$ upon rescaling the internal X_2 propagators according to

$$\Delta_{X_2 X_2}(t) = \frac{it}{(1+z)p^2 - M^2} = \frac{i}{\frac{1+z}{t}p^2 - \frac{M^2}{t}}, \quad (5.2)$$

i.e., by rescaling the parameters z and M^2 according to

$$1+z \rightarrow \frac{1+z}{t}, \quad M^2 \rightarrow \frac{M^2}{t}. \quad (5.3)$$

Since both M^2 and z at tree level only appear in the quadratic part, the choice in the above equation entails that the parameter t only enters in the bilinear term and does not affect the interaction vertices. Moreover, since X_2 is gauge and Becchi-Rouet-Stora-Tyutin (BRST) invariant, this choice does not violate the ST identity. Therefore, at the regularized level, the ST identity holds true for the vertex functional $\hat{\Gamma}(t)$ in Eq. (5.1).

We can obtain a local scaling equation by applying the operator $\mathcal{D}_z^{M^2}$ to Eq. (5.1), since

$$\mathcal{D}_z^{M^2} \hat{\Gamma}(t) = \sum_{\ell \geq 1} t^\ell \mathcal{D}_z^{M^2} \Gamma^{(n;\ell)} = -\sum_{\ell \geq 1} \ell t^\ell \Gamma^{(n;\ell)} = -t \frac{\partial}{\partial t} \hat{\Gamma}(t), \quad (5.4)$$

so that

$$\left(\mathcal{D}_z^{M^2} + t \frac{\partial}{\partial t} \right) \hat{\Gamma}(t) = 0. \quad (5.5)$$

The most general solution of this equation has the form

$$\hat{\Gamma}(t) = \hat{\Gamma}\left(\frac{t}{M^2}, \frac{t}{1+z}\right). \quad (5.6)$$

Diagrams involving only tree-level interaction vertices and rescaled internal X_2 lines obviously fulfill Eq. (5.6). The fact that Eq. (5.6) holds true at the renormalized level implies that the rescaling in Eq. (5.3) survives quantization, namely, that also the counterterms are consistent with Eq. (5.6).

At $t = 0$, one recovers the Stückelberg sector of the theory; in addition, since the vertex functional $\hat{\Gamma}(t)$ admits a Taylor expansion in powers of t , Eq. (5.6) entails that each coefficient $\Gamma^{(n;\ell)}|_{X_2=0}$ of order t^ℓ satisfies the eigenvalue equation

$$\mathcal{D}_z^{M^2} \Gamma^{(n;\ell)}|_{X_2=0} = -\ell \Gamma^{(n;\ell)}|_{X_2=0}, \quad (5.7)$$

which is consistent with Eq. (3.5).

The subtraction of UV divergences by local counterterms with the boundary condition in Eq. (4.12) does not violate the ST identity, so that we can write at the renormalized level

$$\mathcal{S}(\hat{\Gamma}(t)) = 0. \quad (5.8)$$

In particular, notice that, since $\hat{\Gamma}(0) = \Gamma_0$ and $\hat{\Gamma}(1) = \Gamma(1)$, the functional $\hat{\Gamma}(t)$ interpolates between the Stückelberg theory and the fully deformed Abelian HK model at $z \neq 0$.

We can now expand the ST identity for $\hat{\Gamma}(t)$ in the number of loops and then order by order in t . This double expansion yields relations among 1-PI Green's functions that hold true separately. Let us then start at one-loop order and use the mass eigenstate basis, that is reflected in the shift (at $\xi \neq 0$) $b = b' + \frac{1}{\xi} \partial A + ev\chi$. We consider only amplitudes that are independent of $X_{1,2}$ since these amplitudes are recovered by the $X_{1,2}$ equations (C3) and (C4), so we can safely use σ in place of σ' . The ST identity becomes

$$\begin{aligned}\mathcal{S}_0(\hat{\Gamma}^{(1)}(t)) &= \int d^4x \left[\partial_\mu \omega \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta A_\mu} + \frac{\delta \Gamma^{(0)}}{\delta \sigma^*} \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta \sigma} \right. \\ &\quad + \frac{\delta \Gamma^{(0)}}{\delta \sigma} \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta \sigma^*} + \frac{\delta \Gamma^{(0)}}{\delta \chi^*} \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta \chi} \\ &\quad + \frac{\delta \Gamma^{(0)}}{\delta \chi} \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta \chi^*} \\ &\quad \left. + \left(b' + \frac{1}{\xi} \partial A + ev\chi \right) \frac{\delta \hat{\Gamma}^{(1)}(t)}{\delta \bar{\omega}} \right] = 0.\end{aligned}\quad (5.9)$$

At higher orders in the loop expansion, one must take into account the effects of the bilinear antifield-dependent terms in the ST identity (C5). Defining the bracket

$$(\hat{\Gamma}(t), \hat{\Gamma}(t)) \equiv \int d^4x \left[\frac{\delta \hat{\Gamma}(t)}{\delta \sigma^*} \frac{\delta \hat{\Gamma}(t)}{\delta \sigma} + \frac{\delta \hat{\Gamma}(t)}{\delta \chi^*} \frac{\delta \hat{\Gamma}(t)}{\delta \chi} \right], \quad (5.10)$$

at order $n > 1$ in the loop expansion the ST identity yields

$$\mathcal{S}_0(\hat{\Gamma}^{(n)}(t)) + \sum_{j=1}^{n-1} (\hat{\Gamma}^{(j)}(t), \hat{\Gamma}^{(n-j)}(t)) = 0. \quad (5.11)$$

A further expansion in powers of t yields a set of independent identities valid at order n , one for each ℓ ,

$$\mathcal{S}_0(\Gamma^{(n;\ell)}) + \sum_{j=1}^{n-1} \sum_{i=0}^{\ell} (\Gamma^{(j;i)}, \Gamma^{(n-j;\ell-i)}) = 0. \quad (5.12)$$

Such identities encode the conditions required to guarantee physical unitarity of the theory (i.e., the cancellation of the intermediate ghost states).

In Landau gauge major simplifications arise since the bilinear term vanishes, as a consequence of the fact that at order $n \geq 1$ amplitudes with at least one antifield external leg are zero, since there are no Feynman diagrams contributing to them. For the same reason

$$\mathcal{S}_0(\Gamma^{(n;\ell)}) = s(\Gamma^{(n;\ell)}), \quad (5.13)$$

implying that there are no radiative corrections to the classical BRST symmetry. The latter is a well-known property of the Landau gauge [30–32].

In particular, gauge invariance of the X_2 field entails that off-shell 1-PI subdiagrams with a given number of internal X_2 lines form separately gauge-invariant sectors. The lowest sector is the Stückelberg theory (no physical X_2 states). Then perturbation theory based on the classical action (2.1) will generate the higher order ℓ sectors (with one, two, etc. internal X_2 lines).

The interplay between the loop expansion and the grading in the number of internal X_2 lines in an arbitrary R_ξ gauge requires one to instead take into account the deformation of the classical BRST symmetry via the renormalization of antifield-dependent amplitudes and is encoded in the ST identity in Eq. (5.12). It should be stressed that the grading of the ST identity with respect to the number of internal X_2 lines does not depend on the z -differential equation and the specific quadratic deformation controlled by z , yet it holds for any X_2 potential provided that X_2 remains gauge invariant.

A. Two-point functions

As a specific example of the ST identity just derived, let us consider the two-point sector. By differentiating Eq. (5.9) with respect to ω and A_ν , one finds

$$-\partial_\mu \hat{\Gamma}_{A_\nu A_\mu}^{(1)}(t) + \Gamma_{\omega\chi^*}^{(0)} \hat{\Gamma}_{A_\nu\chi}^{(1)}(t) + \Gamma_{A_\nu\chi}^{(0)} \hat{\Gamma}_{\omega\chi^*}^{(1)}(t) - \frac{1}{\xi} \partial_\nu \hat{\Gamma}_{\omega\bar{\omega}}^{(1)}(t) = 0. \quad (5.14)$$

In a similar fashion by differentiating Eq. (5.9) with respect to ω , χ one gets

$$-\partial_\mu \hat{\Gamma}_{\chi A_\mu}^{(1)}(t) + \Gamma_{\omega\chi^*}^{(0)} \hat{\Gamma}_{\chi\chi}^{(1)}(t) + \Gamma_{\omega\sigma^*\chi}^{(0)} \hat{\Gamma}_\sigma^{(1)}(t) + \Gamma_{\chi\chi}^{(0)} \hat{\Gamma}_{\omega\chi^*}^{(1)}(t) + e v \hat{\Gamma}_{\omega\bar{\omega}}^{(1)}(t) = 0. \quad (5.15)$$

In Landau gauge instead no shift of the b field is required and the ST identities for the two-point functions are simpler (notice that the one-loop amplitudes with external antifield legs are zero in this gauge),

$$\begin{aligned} -\partial_\mu \hat{\Gamma}_{A_\nu A_\mu}^{(1)}(t)|_{\xi=0} + \Gamma_{\omega\chi^*}^{(0)} \hat{\Gamma}_{A_\nu\chi}^{(1)}(t)|_{\xi=0} &= 0, \\ -\partial_\mu \hat{\Gamma}_{\chi A_\mu}^{(1)}(t)|_{\xi=0} + \Gamma_{\omega\chi^*}^{(0)} \hat{\Gamma}_{\chi\chi}^{(1)}(t)|_{\xi=0} + \Gamma_{\omega\sigma^*\chi}^{(0)} \hat{\Gamma}_\sigma^{(1)}(t)|_{\xi=0} &= 0. \end{aligned} \quad (5.16)$$

We now project Eqs. (5.14)–(5.16) at zero and first order in t (there are no further contributions since the amplitudes involve at most one internal X_2 line). In the Feynman gauge, we then obtain ($\ell = 0, 1$)

$$-\partial_\mu \Gamma_{A_\nu A_\mu}^{(1;\ell)}|_{\xi=1} + \Gamma_{\omega\chi^*}^{(0)} \Gamma_{A_\nu\chi}^{(1;\ell)}|_{\xi=1} + \Gamma_{A_\nu\chi}^{(0)} \Gamma_{\omega\chi^*}^{(1;\ell)}|_{\xi=1} - \partial_\nu \Gamma_{\omega\bar{\omega}}^{(1;\ell)}|_{\xi=1} = 0, \quad (5.17a)$$

$$-\partial_\mu \Gamma_{\chi A_\mu}^{(1;\ell)}|_{\xi=1} + \Gamma_{\omega\chi^*}^{(0)} \Gamma_{\chi\chi}^{(1;\ell)}|_{\xi=1} + \Gamma_{\chi\chi}^{(0)} \Gamma_{\omega\chi^*}^{(1;\ell)}|_{\xi=1} + \Gamma_{\omega\sigma^*\chi}^{(0)} \Gamma_\sigma^{(1;\ell)}|_{\xi=1} + e v \Gamma_{\omega\bar{\omega}}^{(1;\ell)}|_{\xi=1} = 0, \quad (5.17b)$$

whereas in the Landau gauge, we get

$$-\partial_\mu \Gamma_{A_\nu A_\mu}^{(1;\ell)}|_{\xi=0} + \Gamma_{\omega\chi^*}^{(0)} \Gamma_{A_\nu\chi}^{(1;\ell)}|_{\xi=0} = 0, \quad (5.18a)$$

$$-\partial_\mu \Gamma_{\chi A_\mu}^{(1;\ell)}|_{\xi=0} + \Gamma_{\omega\chi^*}^{(0)} \Gamma_{\chi\chi}^{(1;\ell)}|_{\xi=0} + \Gamma_{\omega\sigma^*\chi}^{(0)} \Gamma_\sigma^{(1;\ell)}|_{\xi=0} = 0. \quad (5.18b)$$

At variance with the standard ϕ formalism, the identities above hold true separately for each sector with a given number of internal X_2 lines. They can be explicitly checked using the following results:

$$\begin{aligned} \bar{\Gamma}_{A_\mu A_\nu}^{(1;0)} &= -\frac{e^2 M_A^2}{8\pi^2} \frac{g^{\mu\nu}}{\epsilon} \delta_{\xi;1}; & \bar{\Gamma}_{A_\mu A_\nu}^{(1;1)} &= \frac{e^2}{24\pi^2(1+z)} [-(9M_A^2 + p^2)g^{\mu\nu} + p^\mu p^\nu] \frac{1}{\epsilon}, \\ \bar{\Gamma}_{\chi\chi}^{(1;0)} &= \frac{M_A^2}{16\pi^2 v^2} [6M_A^2 + \delta_{\xi;1}(m^2 - 2p^2)] \frac{1}{\epsilon}, \\ \bar{\Gamma}_{\chi\chi}^{(1;1)} &= \frac{1}{16\pi^2 v^2 (1+z)^3} \{M^2[(1+z)m^2 + 2M^2] - 2(3 - 2\delta_{\xi;1})M_A^2(1+z)^2 p^2\} \frac{1}{\epsilon}, \\ \bar{\Gamma}_{\chi A_\mu(p)}^{(1;0)} &= i \frac{e M_A^2}{8\pi^2 v} \frac{\delta_{\xi;1}}{\epsilon} p^\mu; & \bar{\Gamma}_{\chi A_\mu(p)}^{(1;1)} &= i \frac{e M_A^2 (3\delta_{\xi;0} + 2\delta_{\xi;1})}{8\pi^2 (1+z)v} \frac{1}{\epsilon} p^\mu, \\ \bar{\Gamma}_{\omega\bar{\omega}}^{(1;0)} &= 0; & \bar{\Gamma}_{\omega\bar{\omega}}^{(1;1)} &= -\frac{e^2 M_A^2}{8\pi^2 (1+z)} \frac{\delta_{\xi;1}}{\epsilon}; & \bar{\Gamma}_{\omega\chi^*}^{(1;0)} &= 0; & \bar{\Gamma}_{\omega\chi^*}^{(1;1)} &= -\frac{e^2 M_A}{8\pi^2 (1+z)} \frac{\delta_{\xi;1}}{\epsilon}, \end{aligned} \quad (5.19)$$

where $\delta_{\xi;1}$ ($\delta_{\xi;0}$) is one in the Feynman (Landau) gauge and zero otherwise.

VI. OTHER RENORMALIZATION SCHEMES

Separate invariance under the ST identities for each ℓ sector has some important consequences. For example, when chiral fermions are introduced, as in the Standard Model, the presence of the γ_5 matrix entails that the ST identities are broken by the intermediate regularization and that finite counterterms must be added to recover the validity of the ST identities themselves. In order to be consistent with the z -differential equation, such finite counterterms must also possess a grading in the number of internal X_2 lines; and the fact that for each ℓ sector a separate ST identity exists implies that the breaking of the ℓ th ST identity can be compensated by finite counterterms also belonging to the same ℓ sector.

At $z = 0$ the ST identities in the ℓ sector are fulfilled; namely,

$$\mathcal{S}_0(\Gamma^{(n;\ell)}|_{z=0}) + \sum_{j=1}^{n-1} \sum_{i=0}^{\ell} (\Gamma^{(j;i)}|_{z=0}, \Gamma^{(n-j;\ell-i)}|_{z=0}) = 0. \quad (6.1)$$

Then, since in both terms in Eq. (6.1) a common overall factor $1/(1+z)^\ell$ can be factorized and

$$\left[\mathcal{S}_0, \frac{\partial}{\partial M^2} \right] = \left[\mathcal{S}_0, \frac{\partial}{\partial z} \right] = 0, \quad (6.2)$$

a solution to the ST identities at $z \neq 0$ is provided by

$$\Gamma^{(n;\ell)}(z, M^2) = \frac{1}{(1+z)^\ell} \Gamma^{(n;\ell)}\left(0, \frac{M^2}{1+z}\right). \quad (6.3)$$

Similarly, all the other functional identities of the theory, e.g., the one reported in Eqs. (C1)–(C4), (C6), and (C7), can be expanded in the different ℓ sectors and each projected equation is satisfied by $\Gamma^{(n;\ell)}$ in Eq. (6.3) if $\Gamma^{(n;\ell)}|_{z=0}$ is a solution of the same equation at $z = 0$.

As Eq. (6.3) holds at the renormalized level, it can be taken as the definition of the 1-PI amplitudes of the theory at $z \neq 0$. Within such a formulation, one can then extend the construction presented to an arbitrary renormalization scheme of the power-counting renormalizable theory at $z = 0$, e.g., the on-mass shell renormalization scheme. In fact, finite renormalizations compatible with the symmetries of the theory are encoded into the relevant amplitudes contributing to $\Gamma^{(n;\ell)}(0, M^2)$ and are lifted at $z \neq 0$ by Eq. (6.3), as such normalization conditions inherit (as a consequence of the validity of the ST identities in each ℓ sector) a natural ℓ grading.

Consider as an example the on-mass shell normalization condition for the vector meson. Defining

$$\begin{aligned} \Gamma_{A^\mu A^\nu} &= g_{\mu\nu}(p^2 - M_A^2) + \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Sigma_T(p^2) \\ &+ \frac{p^\mu p^\nu}{p^2} \Sigma_L(p^2), \end{aligned} \quad (6.4)$$

one requires that the position of the pole of the physical components of the vector meson does not shift with respect to the one at tree level and that the residue of the propagator on the pole is one,

$$\text{Re} \Sigma_T(M_A^2) = 0; \quad \text{Re} \frac{\partial \Sigma_T(p^2)}{\partial p^2} \Big|_{p^2=M_A^2} = 0. \quad (6.5)$$

These conditions can be matched by finite renormalizations involving the invariants shown in Eq. (D4),

$$\begin{aligned} \lambda_4 \int d^4x (D^\mu \phi)^\dagger D_\mu \phi &\supset \frac{\lambda_4 v}{2} \int d^4x A_\mu^2; \\ \frac{\lambda_8}{2} \int d^4x F_{\mu\nu}^2 &\supset \lambda_8 \int d^4x A_\mu (\square g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu. \end{aligned} \quad (6.6)$$

Then, projecting into the different ℓ sectors, with $\ell = 0, 1$, which can be done due to the ℓ -sector ST invariance, one obtains

$$\begin{aligned} \text{Re} \Sigma_T^{(1;\ell)}(M_A^2) + v \lambda_4^{(1;\ell)} &= 0; \\ \text{Re} \frac{\partial \Sigma_T^{(1;\ell)}}{\partial p^2} \Big|_{p^2=M_A^2} - 2M_A^2 \lambda_8^{(1;\ell)} &= 0. \end{aligned} \quad (6.7)$$

As can be seen from the above equation, on-mass shell renormalization conditions respect the layers in ℓ of the ST identities and consequently the z -differential equation. Once the appropriate normalization conditions are enforced at order n in the loop expansion at $z = 0$, Eq. (6.3) fixes the 1-PI amplitudes of the theory at $z \neq 0$ in a unique way.

VII. CONCLUSIONS

When formulating the scalar sector of the Abelian HK model by means of a Lagrange multiplier X_1 and a gauge-invariant field coordinate X_2 , one obtains a new vertex functional (2.1) (with $z = 0$), which is on-shell equivalent to the original HK model, but satisfies a plethora of functional identities (see Appendix C) which constrain the behavior of otherwise seemingly unrelated amplitudes [13–15, 18].

At this stage, one might ask if other terms in the Lagrangian are allowed by the symmetries of the model formulated in the auxiliary fields formalism. It turns out that there is just one such term, namely, a kinetic term for X_2 ,

$$\frac{z}{2} \partial^\mu X_2 \partial_\mu X_2 \sim -\frac{z}{2} X_2 \square X_2. \quad (7.1)$$

This is akin to adding the following nonpower-counting renormalizable dimension-six derivative operator to the original model:

$$-\frac{z}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2} \right) \square \left(\phi^\dagger \phi - \frac{v^2}{2} \right). \quad (7.2)$$

However, in the $X_{1,2}$ formulation of the theory, both the dependence of the 1-PI amplitudes on the deformation parameter z as well as the subtraction of UV divergences are governed by a novel differential equation, which reads

$$[(1+z)\partial_z + M^2\partial_{M^2}]\Gamma_{\Phi_1\dots\Phi_r}^{(n)} = -\sum_{\ell \geq 0} \ell \Gamma_{\Phi_1\dots\Phi_r}^{(n;\ell)}, \quad (7.3)$$

where ℓ counts the number of internal X_2 lines. In addition, the model's ST identity separately holds for each sector of the vertex functional with a fixed number ℓ of internal X_2 lines. This is at variance with the standard formalism, in which the dependence on z affects both the bilinear and the interaction terms in the tree-level classical action (and hence no z -differential equation exists) and the ST identity cannot be filtered according to the number of internal physical Higgs propagators (as the physical Higgs field σ is not gauge invariant).

Then, the most general solution of the z -differential equation, namely,

$$\Gamma^{(n;\ell)}(z, M^2) = \frac{1}{(1+z)^\ell} \Gamma^{(n;\ell)}\left(0, \frac{M^2}{1+z}\right), \quad (7.4)$$

holds at the renormalized level and therefore provides a definition of the theory's 1-PI amplitudes at $z \neq 0$ that is valid in any renormalization scheme. Thus, even if power-counting renormalizability is lost, the model depends on the same number of physical parameters of its renormalizable realization at $z = 0$ plus the z parameter itself. To the best of our knowledge, this is a first.

Interestingly enough, the deformation (7.2), controlled by the parameter z and allowed by the symmetries of the

theory, only affects at the classical level the potential of the Higgs field, which becomes derivative dependent once one goes on shell with the auxiliary fields, while the interactions with the gauge field are unaffected by such deformation (again at the classical level). As such, it provides a candidate of a deformation of the usual quartic Higgs potential that might be relevant for the study of the electroweak spontaneous symmetry breaking. The extension to the $SU(2) \times U(1)$ gauge group and the phenomenological implications of the z deformation are currently under investigation. In particular, the magnitude of the one-loop oblique corrections together with the size of the different Higgs couplings at tree level (including the trilinear Higgs self-interaction that is expected to be studied in the forthcoming LHC run) can be used to fix the allowed region for the z parameter. Moreover, the novel z -differential equation allows one to control the radiative corrections of the theory. In the longer term, this paves the way to their systematic study and thus to the global electroweak fit based on the model at $z \neq 0$. We hope to report soon on our findings.

APPENDIX A: DEGREES OF FREEDOM IN THE X FORMALISM

The extension of the scalar sector via the fields $X_{1,2}$ does not introduce additional physical degrees of freedom. This can be seen at tree level by inspecting the propagators in the mass eigenstate basis, see Eq. (B1). We notice that $\Delta_{X_1 X_1}$ and $\Delta_{\sigma\sigma}$ differ by a sign and they, in fact, cancel out in the intermediate states; this is a consequence of the constraint $U(1)$ BRST symmetry

$$\delta X_1 = v c, \quad \delta c = 0, \quad \delta \bar{c} = \frac{1}{v} \left(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \right), \quad (A1)$$

all other fields and external sources being invariant under δ and c, \bar{c} being the constraint $U(1)$ ghost and antighost fields.

We notice that the whole dependence of the classical action (2.1) on m^2 is BRST exact, since

$$\begin{aligned} & \int d^4x \left\{ \frac{m^2}{2} X_2^2 - \frac{m^2}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 + \frac{1}{v} (X_1 + X_2) (\square + m^2) \left(\phi^\dagger \phi - \frac{v^2}{2} - v X_2 \right) - \bar{c} (\square + m^2) c \right\} \\ &= \int d^4x \delta \left[-\frac{m^2}{2v^2} \bar{c} \left(\phi^\dagger \phi - \frac{v^2}{2} + v X_2 \right) + \frac{1}{v} (X_1 + X_2) (\square + m^2) \bar{c} \right]. \end{aligned} \quad (A2)$$

Therefore, physical observables cannot depend on this parameter [33], in agreement with the explicit computations of [13].

In order to see that the physical states of the theory are unchanged with respect to the ϕ formalism, we make use of the BRST quantization approach [21,22,34]. We remark

that the BRST symmetry in Eq. (A1) holds true together with the usual BRST symmetry s associated with the Abelian gauge group

$$\begin{aligned} s A_\mu &= \partial_\mu \omega; & s \phi &= i e \omega \phi; & s \sigma &= -e \omega \chi; \\ s \chi &= e \omega (\sigma + v); & s \bar{\omega} &= b; & s b &= 0, \end{aligned} \quad (A3)$$

with all other fields and external sources being s invariant. ω and $\bar{\omega}$ are the ghost and antighost fields associated with the gauge symmetry, and b is the Nakanishi-Lautrup field.

Both BRST differentials s and \mathcal{J} are nilpotent and they anticommute, so that $s' = s + \mathcal{J}$ is nilpotent too. Then, the physical Hilbert space can be defined according to the BRST quantization prescription [21,22,34] as

$$\mathcal{H}_{\text{phys}} = \frac{\text{Ker } s'_0}{\text{Im } s'_0}, \quad (\text{A4})$$

where s'_0 is the linearized full BRST differential acting on the fields as follows:

$$\begin{aligned} s'_0 A_\mu &= \partial_\mu \omega; & s'_0 \chi &= ev\omega; & s'_0 \sigma &= 0; & s'_0 \bar{\omega} &= b; & s'_0 b &= 0, \\ s'_0 X_1 &= vc; & s'_0 c &= 0; & s'_0 \bar{c} &= \sigma - X_2; & s'_0 X_2 &= 0. \end{aligned} \quad (\text{A5})$$

From above equation one can easily see that the transverse polarizations of the massive gauge field belong to $\mathcal{H}_{\text{phys}}$ as well as the physical scalar σ . The latter can be equivalently parametrized at the linearized level by σ or X_2 , since their difference is s'_0 exact (being the image of \bar{c}), and thus X_2 and σ belong to the same equivalence class in $\mathcal{H}_{\text{phys}}$. χ is unphysical since it does not belong to the kernel of s'_0 . Moreover, the ghost ω is unphysical being (modulo a constant factor) the s'_0 image of χ ; thus $(\chi, ev\omega)$ form a so-called BRST doublet and, consequently, they do not contribute to the cohomology of s'_0 that determines $\mathcal{H}_{\text{phys}}$ (for a review, see, e.g., [35]). Similarly, the pairs $(\bar{\omega}, b)$, (X_1, c) , and $(\bar{c}, \sigma - X_2)$ form BRST doublets and thus drop out of $\mathcal{H}_{\text{phys}}$. We therefore conclude that the physical field content of the theory is still given by the transverse modes of the massive gauge fields and one physical massive Higgs scalar.

The last line of Eq. (2.1) contains the antifield-dependent terms, i.e., the terms coupling the antifields (external sources of the BRST transformation) and the nonlinear BRST variations of the fields that, being nonlinear, require an independent renormalization with respect to the fields themselves [33,36], controlled by the antifield Green's functions.

APPENDIX B: PROPAGATORS

The diagonalization of the quadratic part of the classical action in the sector spanned by σ , X_1 , and X_2 is achieved via the field redefinition $\sigma = \sigma' + X_1 + X_2$. σ' , X_1 , and X_2 are the mass eigenstates at tree level. Their propagators read

$$\begin{aligned} \Delta_{\sigma'\sigma'} &= \frac{i}{p^2 - m^2}; & \Delta_{X_1 X_1} &= -\frac{i}{p^2 - m^2}; \\ \Delta_{X_2 X_2} &= \frac{i}{(1+z)p^2 - M^2}. \end{aligned} \quad (\text{B1})$$

Diagonalization in the gauge sector is obtained by redefining the Nakanishi-Lautrup multiplier field

$$b' = b - \frac{1}{\xi} \partial A - ev\chi. \quad (\text{B2})$$

Then, the A_μ propagator is

$$\Delta_{\mu\nu} = -i \left(\frac{1}{p^2 - M_A^2} T_{\mu\nu} + \frac{1}{\frac{1}{\xi} p^2 - M_A^2} L_{\mu\nu} \right); \quad M_A = ev, \quad (\text{B3})$$

with

$$T_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}; \quad L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}, \quad (\text{B4})$$

whereas the Nakanishi-Lautrup, pseudo-Goldstone, and ghost propagators are

$$\Delta_{b'b'} = \frac{i}{\xi}; \quad \Delta_{\chi\chi} = \frac{i}{p^2 - \xi M_A^2}; \quad \Delta_{\bar{\omega}\omega} = \frac{i}{p^2 - \xi M_A^2}. \quad (\text{B5})$$

The Feynman gauge corresponds to $\xi = 1$, whereas the Landau gauge is $\xi = 0$. Finally, the ghost associated with the constraint BRST symmetry is free with a propagator

$$\Delta_{\bar{c}c} = \frac{-i}{p^2 - m^2}. \quad (\text{B6})$$

APPENDIX C: FUNCTIONAL IDENTITIES

In this appendix, we collect for the sake of reference the functional identities controlling the theory:

(i) The ST identity for the constraint BRST symmetry is

$$\begin{aligned} \mathcal{S}_c(\Gamma) &\equiv \int d^4x \left[vc \frac{\delta\Gamma}{\delta X_1} + \frac{\delta\Gamma}{\delta \bar{c}^*} \frac{\delta\Gamma}{\delta \bar{c}} \right] \\ &= \int d^4x \left[vc \frac{\delta\Gamma}{\delta X_1} - (\square + m^2)c \frac{\delta\Gamma}{\delta \bar{c}^*} \right] = 0, \end{aligned} \quad (\text{C1})$$

where in the latter equality we have used the fact that both the ghost c and the antighost \bar{c} are free,

$$\frac{\delta\Gamma}{\delta \bar{c}} = -(\square + m^2)c; \quad \frac{\delta\Gamma}{\delta c} = (\square + m^2)\bar{c}. \quad (\text{C2})$$

(ii) The X_1 equation of motion, which follows from Eq. (C1) by using the fact that the ghost c is free,

$$\frac{\delta\Gamma}{\delta X_1} = \frac{1}{v} (\square + m^2) \frac{\delta\Gamma}{\delta \bar{c}^*}. \quad (\text{C3})$$

(iii) The X_2 equation of motion,

$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v}(\square + m^2)\frac{\delta\Gamma}{\delta \bar{c}^*} - (\square + m^2)X_1 - ((1+z)\square + M^2)X_2 - v\bar{c}^*. \quad (\text{C4})$$

Notice that the z -term only affects the linear contribution on the right-hand side of the above equation and thus no new external source is needed to control its renormalization. Notice that further bilinear terms $\sim X_2 \square^n X_2, n \geq 2$ could be added to the classical action while still modifying only the linear part in X_2 of the above equation. However, such higher-derivative contributions induce in the spectrum modes with negative metrics and lead to mathematical inconsistencies [37]; thus, in this respect, the z deformation of the classical HK action studied here is unique.

(iv) The ST identity associated with the gauge group BRST symmetry

$$\mathcal{S}(\Gamma) = \int d^4x \left[\partial_\mu \omega \frac{\delta\Gamma}{\delta A_\mu} + \frac{\delta\Gamma}{\delta \sigma^*} \frac{\delta\Gamma}{\delta \sigma} + \frac{\delta\Gamma}{\delta \chi^*} \frac{\delta\Gamma}{\delta \chi} + b \frac{\delta\Gamma}{\delta \bar{\omega}} \right] = 0. \quad (\text{C5})$$

(v) The b equation,

$$\frac{\delta\Gamma}{\delta b} = \xi b - \partial A - \xi e v \chi. \quad (\text{C6})$$

(vi) The antighost equation,

$$\frac{\delta\Gamma}{\delta \bar{\omega}} = \square \omega + \xi e v \frac{\delta\Gamma}{\delta \chi^*}. \quad (\text{C7})$$

1. Descendant and ancestor amplitudes

Equations (C3) and (C4) fix the amplitudes involving X_1 and X_2 external legs in terms of $X_{1,2}$ -independent amplitudes. Hence, a hierarchy arises among 1-PI Green's functions: we call amplitudes with at least one X_1 or X_2 external legs descendant amplitudes, while the amplitudes without $X_{1,2}$ external legs are called ancestor amplitudes.

As an example, we derive the two-point $X_{1,2}$ amplitudes in terms of the 1-PI amplitudes with external \bar{c}^* legs. We start by differentiating Eq. (C3) at order $n \geq 1$ in the loop expansion with respect to $X_{1,2}$; we get

$$\Gamma_{X_1 X_1}^{(n)} = \frac{1}{v}(\square + m^2)\Gamma_{X_1 \bar{c}^*}^{(n)}; \quad \Gamma_{X_2 X_1}^{(n)} = \frac{1}{v}(\square + m^2)\Gamma_{X_2 \bar{c}^*}^{(n)}. \quad (\text{C8})$$

Next, we differentiate Eq. (C4) at order $n \geq 1$ obtaining

$$\Gamma_{X_2 X_2}^{(n)} = \frac{1}{v}(\square + m^2)\Gamma_{X_2 \bar{c}^*}^{(n)}. \quad (\text{C9})$$

Finally, differentiation of Eqs. (C3) and (C4) with respect to \bar{c}^* yields

$$\Gamma_{X_1 \bar{c}^*}^{(n)} = \Gamma_{X_2 \bar{c}^*}^{(n)} = \frac{1}{v}(\square + m^2)\Gamma_{\bar{c}^* \bar{c}^*}^{(n)}. \quad (\text{C10})$$

Substituting Eq. (C10) into Eq. (C8), we finally obtain

$$\Gamma_{X_1 X_1}^{(n)} = \Gamma_{X_1 X_2}^{(n)} = \Gamma_{X_2 X_2}^{(n)} = \frac{1}{v^2}(\square + m^2)^2 \Gamma_{\bar{c}^* \bar{c}^*}^{(n)}. \quad (\text{C11})$$

As anticipated, the two-point functions of the fields $X_{1,2}$ are completely fixed by the ancestor amplitude $\Gamma_{\bar{c}^* \bar{c}^*}^{(n)}$. We can therefore limit the analysis to the $X_{1,2}$ -independent 1-PI amplitudes, the latter being recovered algebraically by functional differentiation of Eqs. (C3) and (C4).

Next, because the constraint ghosts \bar{c} and c are free, at order $n \geq 1$ in the loop expansion the vertex functional Γ is c and \bar{c} independent. We can then take a derivative with respect to c of Eq. (C1) and then substitute Eq. (C2) to recover the X_1 equation of motion in Eq. (C3); thus we see that the constraint ST identity (C1) is equivalent to the X_1 equation.

Finally, the antighost equation (C7) at order $n \geq 1$,

$$\frac{\delta\Gamma^{(n)}}{\delta \bar{\omega}} = \xi e v \frac{\delta\Gamma^{(n)}}{\delta \chi^*}, \quad (\text{C12})$$

entails that the 1-PI vertex functional only depends on the antighost $\bar{\omega}$ only via the combination $\hat{\chi}^* = \chi^* + \xi e v \bar{\omega}$; and the b equation (C6) implies that at order $n \geq 1$ there is no dependence on the Nakanishi-Lautrup field

$$\frac{\delta\Gamma^{(n)}}{\delta b} = 0. \quad (\text{C13})$$

APPENDIX D: LIST OF INVARIANTS

We list here the set of invariants needed to control the UV divergences of operators up to dimension six at one-loop order. The list is taken from Ref. [15] upon setting to zero the source T_1 coupled to the derivative interaction $\sim T_1 (D^\mu \phi)^\dagger D_\mu \phi$ that is not present here. In order to match with the results of Ref. [15], one must also set the coupling constant g of the derivative interaction $\phi^\dagger \phi (D^\mu \phi)^\dagger D_\mu \phi$ equal to zero. We keep the numbering used in Ref. [15] and use a bar to denote the UV-divergent part of the coefficients.

In the cohomologically trivial sector, two invariants must be considered,

$$\bar{\rho}_0 \mathcal{S}_0 \int d^4x [\sigma^*(\sigma + v) + \chi^* \chi]; \quad \bar{\rho}_1 \mathcal{S}_0 \int d^4x (\sigma^* \sigma + \chi^* \chi). \quad (\text{D1})$$

The invariants involving the external source \bar{c}^* are

$$\bar{\vartheta}_1 \int d^4x \bar{c}^*; \quad \bar{\vartheta}_3 \int d^4x \frac{1}{2} (\bar{c}^*)^2; \quad \bar{\vartheta}_9 \int d^4x \frac{1}{3!} (\bar{c}^*)^3; \quad (D2)$$

and

$$\begin{aligned} \bar{\theta}_1 \int d^4x \bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2} \right); \quad \bar{\theta}_3 \int d^4x \bar{c}^* (D^\mu \phi)^\dagger D_\mu \phi; \\ \bar{\theta}_5 \int d^4x \bar{c}^* [(D^2 \phi)^\dagger \phi + \text{H.c.}]; \quad \bar{\theta}_7 \int d^4x \bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2; \\ \bar{\theta}_9 \int d^4x \bar{c}^* F_{\mu\nu}^2; \quad \bar{\theta}_{13} \int d^4x (\bar{c}^*)^2 \left(\phi^\dagger \phi - \frac{v^2}{2} \right). \end{aligned} \quad (D3)$$

The invariants only involving the fields of the theory are

$$\begin{aligned} \bar{\lambda}_1 \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right); \quad \bar{\lambda}_2 \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2; \\ \bar{\lambda}_3 \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^3; \quad \bar{\lambda}_4 \int d^4x (D^\mu \phi)^\dagger D_\mu \phi; \\ \bar{\lambda}_5 \int d^4x \phi^\dagger [(D^2)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu] \phi; \\ \bar{\lambda}_6 \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right) (\phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi); \\ \bar{\lambda}_7 \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right) (D^\mu \phi)^\dagger D_\mu \phi; \quad \bar{\lambda}_8 \int d^4x F_{\mu\nu}^2; \\ \bar{\lambda}_9 \int d^4x \partial^\mu F_{\mu\nu} \partial^\rho F_{\rho\nu}; \quad \bar{\lambda}_{10} \int d^4x \left(\phi^\dagger \phi - \frac{v^2}{2} \right) F_{\mu\nu}^2. \end{aligned} \quad (D4)$$

The coefficients are related to one-loop amplitudes according to the algebraic relations derived in [15]. Notice that they still hold in the presence of the deformation controlled by z since they arise from the projection of the linearized ST operator, which is unaffected by the gauge-invariant term $\sim z \partial^\mu X_2 \partial_\mu X_2$. We find in the cohomologically trivial sector

$$\bar{\rho}_1 = -\frac{1}{e v} \bar{\Gamma}_{\chi^* \omega}^{(1)} = \frac{1}{8\pi^2 v^2} \frac{M_A^2}{1+z} \frac{\delta_{\xi;1}}{\epsilon}; \quad (D5)$$

$$-m^2 v \bar{\rho}_0 + v \bar{\lambda}_1 = \bar{\Gamma}_\sigma^{(1)}; \quad (D6a)$$

$$\bar{\rho}_0 v^2 + \bar{\vartheta}_1 = \bar{\Gamma}_{\bar{c}^*}^{(1)}. \quad (D6b)$$

In the Feynman gauge one can safely set $\bar{\rho}_0 = 0$, while $\bar{\rho}_1 = 0$ in the Landau gauge [15], since there are no radiative corrections to the antifield-dependent amplitudes. Hence, we obtain ($\bar{\lambda}_1$ is gauge invariant)

$$\begin{aligned} \bar{\lambda}_1 &= \frac{1}{16\pi^2 v^2} \left[(m^2 + 6M_A^2) M_A^2 + m^2 \frac{M^2}{(1+z)^2} + \frac{2M^4}{(1+z)^3} \right] \frac{1}{\epsilon}; \\ \bar{\rho}_0 &= \frac{1}{m^2 v} (\bar{\Gamma}_\sigma^{(1)}|_{\xi=1} - \bar{\Gamma}_\sigma^{(1)}|_{\xi=0}) = \frac{M_A^2}{16\pi^2 v^2} \frac{\delta_{\xi;0}}{\epsilon}. \end{aligned} \quad (D7)$$

By using Eq. (D6b) we get

$$\bar{\vartheta}_1 = -\frac{1}{16\pi^2} \left[M_A^2 + \frac{M^2}{(1+z)^2} \right] \frac{1}{\epsilon}; \quad (D8)$$

$$\begin{aligned} \bar{\vartheta}_3 &= \bar{\Gamma}_{\bar{c}_1^* \bar{c}_2^*}^{(1)}|_{p_1^2=0} = \frac{1}{16\pi^2} \left[1 + \frac{1}{(1+z)^2} \right] \frac{1}{\epsilon}; \\ \bar{\vartheta}_5 &= \bar{\Gamma}_{\bar{c}_1^* \bar{c}_2^* \bar{c}_3^*}^{(1)}|_{p_2=p_3=0} = 0. \end{aligned} \quad (D9)$$

Next, writing

$$\bar{\Gamma}_{\bar{c}_3^* \chi_1 \chi_2}^{(1)} = \gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{0(1)} + \gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{1(1)} (p_1^2 + p_2^2) + \gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{2(1)} (p_1 \cdot p_2), \quad (D10)$$

we obtain

$$\bar{\theta}_1 = \gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{0(1)} - 2\bar{\rho}_0 - 2\bar{\rho}_1; \quad \bar{\theta}_3 = -\gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{2(1)}; \quad \bar{\theta}_5 = -\gamma_{1;\bar{c}_3^* \chi_1 \chi_2}^{1(1)}. \quad (D11)$$

An explicit calculation yields

$$\begin{aligned} \bar{\theta}_1 &= -\frac{1}{16\pi^2 v^2} \\ &\times \left[\underbrace{2M_A^2 + m^2}_{0\text{-sector}} - \underbrace{\frac{2M^2}{(1+z)^2}}_{1\text{-sector}} + \underbrace{\frac{m^2}{(1+z)^2} + \frac{4M^2}{(1+z)^3}}_{2\text{-sector}} \right] \frac{1}{\epsilon}, \end{aligned} \quad (D12)$$

where, as indicated, the first two terms in the square brackets arise from the Stückelberg subdiagrams (zero internal X_2 lines), the third from the subdiagrams with one internal X_2 lines, and the last two terms from subdiagrams with two internal X_2 lines. Notice that they are obtained from the corresponding expressions at $z=0$ by carrying out the replacement $M^2 \rightarrow \frac{M^2}{(1+z)}$ and multiplying each subdiagram by the appropriate prefactor $1/(1+z)^\ell$. Finally, observe that $\bar{\theta}_1$ does not depend on the gauge, as it should being the coefficient of a gauge-invariant operator. Similarly, we find

$$\bar{\theta}_3 = -\frac{1}{8\pi^2 v^2} \left[1 - \frac{1}{1+z} \right] \frac{1}{\epsilon}; \quad \bar{\theta}_5 = 0. \quad (D13)$$

Now we can fix $\bar{\theta}_7$ according to

$$2(\bar{\rho}_0 + \bar{\rho}_1) + \bar{\theta}_1 + 2v^2 \bar{\theta}_7 = \bar{\Gamma}_{\bar{c}_3^* \sigma_1 \sigma_2}^{(1)}|_{p_1=p_2=0}, \quad (D14)$$

obtaining

$$\bar{\theta}_7 = \frac{1}{4\pi^2 v^4} \left[\underbrace{-\frac{M^2}{(1+z)^2}}_{1\text{-sector}} + \underbrace{\frac{m^2}{(1+z)^2} + \frac{4M^2}{(1+z)^3}}_{2\text{-sector}} - \underbrace{\frac{m^2}{(1+z)^3} - \frac{3M^2}{(1+z)^4}}_{3\text{-sector}} \right] \frac{1}{\epsilon}, \quad (\text{D15})$$

where again we have collected factors of $1/(1+z)$ in such a way to identify their origin from the subdiagrams with a given number of internal X_2 legs. Notice that $\bar{\theta}_3$ and $\bar{\theta}_7$ vanish for $z \rightarrow 0$, in agreement with Ref. [15] when the coupling constant g of the operator $\sim \phi^\dagger \phi (D^\mu \phi)^\dagger D_\mu \phi$ is set to zero.

The amplitude

$$\bar{\Gamma}_{\bar{c}_3^* A_1^\mu A_2^\nu}^{(1)} = \frac{M_A^2}{8\pi^2 v^2} \frac{z}{1+z} \frac{g^{\mu\nu}}{\epsilon} \quad (\text{D16})$$

is momentum independent both in Landau and Feynman gauges, which implies $\bar{\theta}_9 = 0$; in addition,

$$\bar{\theta}_{13} = \frac{1}{2v} \bar{\Gamma}_{\sigma_3 \bar{c}_1^* \bar{c}_2^*}^{(1)} = \frac{1}{16\pi^2 v^2} \frac{1}{(1+z)^2} \left(-1 + \frac{1}{1+z} \right). \quad (\text{D17})$$

For $z \rightarrow 0$, one gets $\bar{\theta}_{13} = 0$, again in agreement with Ref. [15].

We now move to the sector of gauge-invariant operators depending on the fields only. The coefficients of the potential $\bar{\lambda}_2$ and $\bar{\lambda}_3$ are derived by solving the equations

$$2v^2 \bar{\lambda}_2 + \bar{\lambda}_1 - 2m^2 \bar{\rho}_1 - 5m^2 \bar{\rho}_0 = \bar{\Gamma}_{\sigma_1 \sigma_2}^{(1)}|_{p=0}, \quad (\text{D18a})$$

$$6v^3 \bar{\lambda}_3 + 6v \bar{\lambda}_2 - \frac{9m^2}{v} \bar{\rho}_1 - \frac{12m^2}{v} \bar{\rho}_0 = \bar{\Gamma}_{\sigma_1 \sigma_2 \sigma_3}^{(1)}|_{p_2=p_3=0}. \quad (\text{D18b})$$

It is convenient to express the results for each $\bar{\lambda}_{2,3}$ as the sum over the ℓ -sector contributions; writing

$$\bar{\lambda}_{2,3} = \sum_{\ell=0}^2 \bar{\lambda}_{2,3}^{(\ell)}, \quad (\text{D19})$$

we obtain

$$\begin{aligned} \bar{\lambda}_2^{(0)} &= \frac{1}{32\pi^2 v^4} \frac{m^4 + 12M_A^4}{\epsilon}, & \bar{\lambda}_2^{(1)} &= \frac{1}{8\pi^2 v^4} \left[\frac{m^2 M_A^2}{1+z} - \frac{m^2 M^2}{(1+z)^2} - \frac{2M^4}{(1+z)^3} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_2^{(2)} &= \frac{1}{32\pi^2 v^4} \left[\frac{m^4}{(1+z)^2} + \frac{8m^2 M^2}{(1+z)^3} + \frac{12M^4}{(1+z)^4} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_3^{(0)} &= -\frac{m^2 M_A^2}{16\pi^2 v^6} \frac{1}{\epsilon}, & \bar{\lambda}_3^{(1)} &= \frac{1}{16\pi^2 v^6} \left[\frac{m^2 M_A^2}{1+z} + \frac{4m^2 M^2}{(1+z)^2} + \frac{8M^4}{(1+z)^3} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_3^{(2)} &= -\frac{1}{8\pi^2 v^6} \left[\frac{m^4}{(1+z)^2} + \frac{8m^2 M^2}{(1+z)^3} + \frac{12M^4}{(1+z)^4} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_3^{(3)} &= \frac{1}{8\pi^2 v^6} \left[\frac{m^4}{(1+z)^3} + \frac{6m^2 M^2}{(1+z)^4} + \frac{8M^4}{(1+z)^5} \right] \frac{1}{\epsilon}. \end{aligned} \quad (\text{D20})$$

Notice that in the limit $z \rightarrow 0$ one recovers the results given in [15], in particular, λ_3 becomes zero; furthermore, the consistency conditions (the 1-PI amplitudes on the right-hand side are understood at zero external momenta)

$$\begin{aligned} 2v \bar{\lambda}_2 - \frac{3m^2}{v} \bar{\rho}_1 - \frac{4m^2}{v} \bar{\rho}_0 &= \bar{\Gamma}_{\sigma\chi\chi}^{(1)}, \\ 2\bar{\lambda}_2 - \frac{4m^2}{v^2} \bar{\rho}_1 - \frac{4m^2}{v^2} \bar{\rho}_0 &= \bar{\Gamma}_{\sigma\sigma\chi\chi}^{(1)} \end{aligned} \quad (\text{D21})$$

hold separately for each ℓ sector.

The coefficients $\bar{\lambda}_4$ and $\bar{\lambda}_5$ are obtained from the relations

$$2(\bar{\rho}_0 + \bar{\rho}_1) + \bar{\lambda}_4 = \frac{\partial \bar{\Gamma}_{\chi_1 \chi_2}^{(1)}}{\partial p^2} \Big|_{p^2=0}; \quad 3\bar{\lambda}_5 = \frac{\partial \bar{\Gamma}_{\chi_1 \chi_2}^{(1)}}{\partial (p^2)^2} \Big|_{p^2=0}. \quad (\text{D22})$$

The two-point amplitude $\Gamma_{\chi_1 \chi_2}^{(1)}$ has UV degree of divergence 2, hence $\bar{\lambda}_5 = 0$; moreover,

$$\bar{\lambda}_4 = -\frac{M_A^2}{8\pi^2 v^2} \left(1 + \frac{3}{1+z} \right) \frac{1}{\epsilon}. \quad (\text{D23})$$

In the limit $z \rightarrow 0$, we recover the results of Ref. [15] again by setting $g = 0$.

The coefficients $\bar{\lambda}_6$ and $\bar{\lambda}_7$ can be determined from the three-point function

$$\begin{aligned} \bar{\Gamma}_{\sigma_3 \chi_1 \chi_2}^{(1)} &= \gamma_{1; \sigma_3 \chi_1 \chi_2}^{0(1)} + \gamma_{1; \sigma_3 \chi_1 \chi_2}^{1(1)} (p_1^2 + p_2^2) \\ &+ \gamma_{1; \sigma_3 \chi_1 \chi_2}^{2(1)} p_1 \cdot p_2 + \mathcal{O}(p^4), \end{aligned} \quad (\text{D24})$$

where the momentum of the σ field has been eliminated in favor of $p_{1,2}$ by imposing momentum conservation. Then $\bar{\lambda}_6$ and $\bar{\lambda}_7$ can be determined according to

$$2v\bar{\lambda}_6 + \gamma_{1; \sigma_3 \chi_1 \chi_2}^{1(1)} = 0; \quad 2v\bar{\lambda}_6 + v\bar{\lambda}_7 + \gamma_{1; \sigma_3 \chi_1 \chi_2}^{2(1)} = 0. \quad (\text{D25})$$

Decomposing as before $\bar{\lambda}_{6,7}$ according to the grading in the internal X_2 lines,

$$\bar{\lambda}_{6,7} = \sum_{\ell=0}^2 \bar{\lambda}_{6,7}^{(\ell)}, \quad (\text{D26})$$

one gets

$$\begin{aligned} \bar{\lambda}_6^{(0)} &= 0; \quad \bar{\lambda}_6^{(1)} = \frac{1}{16\pi^2 v^4} \left[\frac{2M_A^2}{1+z} + \frac{M^2}{(1+z)^2} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_6^{(2)} &= -\frac{1}{16\pi^2 v^4} \left[\frac{2M_A^2}{(1+z)^2} + \frac{M^2}{(1+z)^3} \right], \\ \bar{\lambda}_7^{(0)} &= \frac{1}{8\pi^2 v^4} \frac{2M_A^2 - m^2}{\epsilon}; \quad \bar{\lambda}_7^{(1)} = \frac{1}{8\pi^2 v^4} \left[\frac{2M_A^2 + m^2}{1+z} + \frac{M^2}{(1+z)^2} \right] \frac{1}{\epsilon}, \\ \bar{\lambda}_7^{(2)} &= -\frac{1}{8\pi^2 v^4} \left[\frac{4M_A^2}{(1+z)^2} + \frac{M^2}{(1+z)^3} \right] \frac{1}{\epsilon}. \end{aligned} \quad (\text{D27})$$

Again we notice that in the limit $z \rightarrow 0$ $\bar{\lambda}_{6,7}$ are zero, in agreement with the results of Ref. [15].

There are no contributions of order p^4 in $\bar{\Gamma}_{A_1^\mu A_2^\mu}^{(1)}$, so $\bar{\lambda}_9 = 0$. In turn, $\bar{\lambda}_8$ can be fixed by the projection equation

$$[e^2 v^2 (2\bar{\rho}_0 + \bar{\lambda}_4) + (2\bar{\lambda}_8 + e^2 v^2 \bar{\lambda}_5) p_1^\mu] g^{\mu\nu} + 2(e^2 v^2 \bar{\lambda}_5 - \bar{\lambda}_8) p_1^\mu p_1^\nu = \bar{\Gamma}_{A_1^\mu A_2^\mu}^{(1)}, \quad (\text{D28})$$

yielding

$$\bar{\lambda}_8 = -\frac{M_A^2}{48\pi^2 v^2 (1+z)} \frac{1}{\epsilon}. \quad (\text{D29})$$

The limit $z \rightarrow 0$ reproduces the result of Ref. [15].

Finally, the coefficient $\bar{\lambda}_{10}$ is recovered from the amplitude $\bar{\Gamma}_{\sigma_3 A_1^\mu A_2^\nu}^{(1)}$,

$$\bar{\Gamma}_{\sigma_3 A_1^\mu A_2^\nu}^{(1)} = [\gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{0(1)} - 2\gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{1(1)} p_1 \cdot p_2 + \gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{2(1)} (p_1^2 + p_2^2)] g^{\mu\nu} + \gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{3(1)} p_1^\mu p_2^\nu + \gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{4(1)} p_1^\nu p_2^\mu, \quad (\text{D30})$$

as

$$\bar{\lambda}_{10} = \frac{\gamma_{1; \sigma_3 A_1^\mu A_2^\nu}^{1(1)}}{4v} = 0. \quad (\text{D31})$$

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