

Hidden variables of gravity and geometry and the cosmological constant problem

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We extend General Relativity by promoting Planck scale and the cosmological constant into integration constants, interpreted as fluxes of 4-forms hiding in the theory. When we include the charges of the 4-forms, these “constants” can vary discretely from region to region. We explain how the cosmological constant problem can be solved in this new framework. When the cosmological constant picks up contributions from two different 4-forms, with an irrational ratio of charges, the spectrum of its values is a very fine discretuum. When the charges are mutually irrational, $\frac{2\kappa_{\text{eff}}^2\kappa^2|Q_i|}{3T_i^2} < 1$, the discharge processes populating our discretuum will dynamically relax Λ , ceasing as Λ approaches zero. Thus, the theory exponentially favors a huge hierarchy $\Lambda/M_{\text{Pl}}^4 \ll 1$ instead of $\Lambda/M_{\text{Pl}}^4 \simeq 1$.

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In this paper, we discuss the salient features of our recent generalization [1] of standard General Relativity (GR) [2,3] where Planck scale, the cosmological term, and the matter sector couplings are discretely varying quantities throughout spacetime. We note that Hilbert’s formulation of GR [2] can be extended using an arbitrary 4-form $\mathcal{F} = d\mathcal{A}$ as the covariant integration measure in the action, which remains locally a constant multiple of the standard measure $\sqrt{g}d^4x$ when we add another 4-form $\mathcal{G} = d\mathcal{B}$, coupled to \mathcal{F} via the interaction $\mathcal{F} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} \mathcal{G}_{\mu\nu\lambda\sigma}$. This ensures that the ratio $\mathcal{F}/d^4x\sqrt{g}$ is constant, being the local value of Planck scale. The flux of \mathcal{G} adds to the cosmological constant. Our Pancosmic GR generalizes¹ the so-called “unimodular” formulation of GR [13–21]. Finally, we couple matter to geometry using the conformal 4-form/matter coupling, which is locally indistinguishable from GR, and is form invariant in the Quantum Field Theory (QFT) loop expansion [22,23]. The theory is safe from ghosts in this particular case [1]. This provides at least the first step in the demonstration that our mechanism may have a unitary UV completion into some more fundamental approach to quantizing gravity.

When we include membranes charged under 4-forms \mathcal{F} and \mathcal{G} their quantum-mechanical nucleation lead to local

changes of the fluxes of \mathcal{F} and \mathcal{G} , which jump up or down in the regions inside the membranes. As a result, in the interior of these membrane-bound bubbles the effective strength of gravity and the value of cosmological constant, and also the values of couplings and scales of local matter theory, jump relative to the outside.

A sequence of such nested expanding bubbles will scan both κ^2 and Λ over a wide range of parameters, realizing a toy model of the multiverse of eternal inflation [24]. This toy multiverse can solve the cosmological constant problem [25–27]. To this end we include another 4-form which is degenerate with the cosmological constant, with its corresponding system of charges. When the ratio of charges of our two 4-forms which add up in the total cosmological constant is an irrational number, the spectrum of values of Λ is a very fine discretuum [28]. When $\frac{2\kappa_{\text{eff}}^2\kappa^2|Q_i|}{3T_i^2} < 1$, which is the robust parameter ratio choice, the bounce action which controls the rates of the transitions that change Λ has a pole at $\Lambda = 0$, driving Λ to 0^+ dynamically, and stopping there.

In this case, the distribution of the terminal values is effectively described by the semiclassical Euclidean partition function [29–32], which has an essential singularity² at $\Lambda/\kappa_{\text{eff}}^4 \rightarrow 0$. The theory exponentially favors vacua with $\frac{\Lambda_{\text{total}}}{M_{\text{Pl}}^4} \rightarrow 0 \ll 1$. The Newton constant can be fixed separately, by observation, or by using a “weak” anthropic principle. Cosmological constant, however, is driven to zero without any need for anthropics. The theory avoids the empty universe problem of [32], and can admit inflation.

²This requires a careful evaluation of the Euclidean action [33,34].

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¹Some alternatives to minimal measure in the action were noted in [4–12].

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The cosmological constant problem reduces to “*Why now?*,” whose answers might involve late-time physics.

We note that our mechanism also avoids naturally the venerated Weinberg no-go theorem [27] for the adjustment of the cosmological constant, by exploiting loopholes in the assumption of the theorem. The adjustment occurs by quantum Brownian drift, instead of smooth field variation. Further, the evolution involves a special point in phase space, $\Lambda = 0^+$, which is the quantum attractor where the bubble nucleation stops. These two features alone invalidate the theorem of [27].

A simplified example of the action which generalizes Einstein-Hilbert action [2,3] is [1]

$$S = \int \mathcal{F} \left(R - \frac{1}{4!} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} \mathcal{G}_{\mu\nu\lambda\sigma} \right) - \int d^4x \sqrt{g} \mathcal{L}_{\text{QFT}} + S_{\text{membranes}}. \quad (1)$$

Here, $\mathcal{F} = d\mathcal{A}$, $\mathcal{G} = d\mathcal{B}$, and \mathcal{M} is a UV scale controlling the maximal flux the theory can admit. Careful variation yields field equations, which without membrane sources are

$$\begin{aligned} \frac{e^{\rho\zeta\gamma\delta}}{4!\sqrt{g}} \mathcal{F}_{\rho\zeta\gamma\delta} \left(\frac{\epsilon^{\alpha\beta\lambda\sigma} \mathcal{G}_{\alpha\beta\lambda\sigma}}{4!\sqrt{g}} \delta^\mu{}_\nu - 2R^\mu{}_\nu \right) &= T^\mu{}_\nu, \\ R - \frac{\epsilon^{\mu\nu\lambda\sigma}}{4!\sqrt{g}} \mathcal{G}_{\mu\nu\lambda\sigma} &= 2\lambda, \\ -\frac{\epsilon^{\mu\nu\lambda\sigma}}{4!\sqrt{g}} \mathcal{F}_{\mu\nu\lambda\sigma} &= \frac{\kappa^2}{2}. \end{aligned} \quad (2)$$

The last two equations are integrated variations with respect to \mathcal{A} and \mathcal{B} , and include the integration constants κ^2 and λ . A direct substitution of these two into the first gives

$$\kappa^2 (R^\mu{}_\nu - \frac{1}{2} R \delta^\mu{}_\nu) = -\kappa^2 \lambda \delta^\mu{}_\nu + T^\mu{}_\nu, \quad (3)$$

which are the local equations of GR. Yet, both Planck scale and the cosmological constant are integration constants. Our equations (2), (3) describe an infinity of GRs parametrized by κ^2 , λ , which do not mix with each other until the membranes are turned on.

The membrane action is

$$S_{\text{membranes}} = -\mathcal{T}_A \int d^3\xi \sqrt{\gamma}_A - \mathcal{Q}_A \int \mathcal{A} - \mathcal{T}_B \int d^3\xi \sqrt{\gamma}_B - \mathcal{Q}_B \int \mathcal{B}, \quad (4)$$

where

$$\int \mathcal{A} = \frac{1}{6} \int d^3\xi \mathcal{A}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma}, \quad (5)$$

and likewise for \mathcal{B} . Here, \mathcal{T}_i , \mathcal{Q}_i are the membrane tension and charge, respectively, while ξ^α are the intrinsic membrane coordinates and embedding maps are $x^\mu = x^\mu(\xi^\alpha)$. We take $\mathcal{T}_i > 0$ to exclude negative local energy.

Classically, membranes are sources fixed by initial conditions. However, quantum mechanically they can nucleate in background fields [35,36]. This changes the distribution of sources and the evolution of bubble interiors. As a result any background will evolve by growing subspaces with different κ^2 and λ .

The analysis simplifies when we transition to the magnetic dual variables of the 4-forms \mathcal{F} and \mathcal{G} [1]. We use Lagrange multipliers and rewrite the 4-form sector of (4) in the first-order formalism. Each pair of variables \mathcal{F} , \mathcal{A} and \mathcal{G} , \mathcal{B} are treated as independent dynamical variables and $\mathcal{F} = d\mathcal{A}$ and $\mathcal{G} = d\mathcal{B}$ follow as constraints enforced by Lagrange multipliers, \mathcal{P}_A , \mathcal{P}_B . The general first-order path integral is

$$Z = \int \dots [\mathcal{D}\mathcal{A}] [\mathcal{D}\mathcal{B}] [\mathcal{D}\mathcal{F}] [\mathcal{D}\mathcal{G}] [\mathcal{D}\mathcal{P}_A] [\mathcal{D}\mathcal{P}_B] \times e^{iS(\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{G}, \dots) + i \int \mathcal{P}_A (\mathcal{F} - d\mathcal{A}) + i \int \mathcal{P}_B (\mathcal{G} - d\mathcal{B})} \dots \quad (6)$$

Different dual pictures ensue from different order of integration, as in, e.g., the formulations of flux monodromy models of inflation [37–39]. So, to dualize we integrate out \mathcal{F} and \mathcal{G} , and redefine the Lagrange multipliers according to $\mathcal{P}_A = 2\lambda$, $\mathcal{P}_B = \frac{\kappa^2}{2}$. In terms of the dual variables our action, with the membrane terms from (4), is [1]

$$\begin{aligned} S &= \int d^4x \left\{ \sqrt{g} \left(\frac{\kappa^2}{2} R - \kappa^2 \lambda - \mathcal{L}_{\text{QFT}} \right) \right. \\ &\quad \left. - \frac{\lambda}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma} - \frac{\kappa^2}{12} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{B}_{\nu\lambda\sigma} \right\} \\ &\quad + S_{\text{boundary}} - \mathcal{T}_A \int d^3\xi \sqrt{\gamma}_A - \mathcal{Q}_A \int \mathcal{A} \\ &\quad - \mathcal{T}_B \int d^3\xi \sqrt{\gamma}_B - \mathcal{Q}_B \int \mathcal{B}. \end{aligned} \quad (7)$$

S_{boundary} is a generalization of the Israel-Gibbons-Hawking boundary action [40–42],

$$\begin{aligned} S_{\text{boundary}} &= \int d^3\xi \left(\left[\frac{\lambda}{3} \epsilon^{\alpha\beta\gamma} \mathcal{A}_{\alpha\beta\gamma} \right] + \left[\frac{\kappa^2}{12} \epsilon^{\alpha\beta\gamma} \mathcal{B}_{\alpha\beta\gamma} \right] \right) \\ &\quad - \int d^3\xi \sqrt{\gamma} [\kappa^2 K]. \end{aligned} \quad (8)$$

[...] is the jump across a membrane; λ , κ^2 are inside of [...] since they jump if a \mathcal{Q}_i is emitted.

To include the matter sector, we replace the matter Lagrangian of our Eq. (1) by $\sqrt{g} \mathcal{L}_{\text{QFT}}(g^{\mu\nu}) \rightarrow \sqrt{\hat{g}} \mathcal{L}_{\text{QFT}}(\hat{g}^{\mu\nu})$, where $\hat{g}_{\mu\nu} = g_{\mu\nu} \sqrt{\frac{\kappa}{\mathcal{M}}}$ and, as noted, \mathcal{M} is

a UV scale controlling the perturbative expansion of the full effective action in the powers of \mathcal{F} . This ensures the theory retains a positive-definite gravitational coupling which keeps ghosts at bay [1]. The matter loop corrections preserve this form of the action, as long as the regulator depends on κ/\mathcal{M} in the same way as the matter Lagrangian [22,23]. We can also generalize the gravitational sector, substituting $\frac{\kappa^2}{2}R \rightarrow \frac{M_{\text{Pl}}^2 + \kappa^2}{2}R$:

$$S = \int \left\{ \sqrt{g} \left(\frac{M_{\text{Pl}}^2 + \kappa^2}{2} R - \kappa^2 \lambda - \frac{\kappa^2}{\mathcal{M}^2} \mathcal{L}_{\text{QFT}} \left(\frac{\mathcal{M}}{\kappa} g^{\mu\nu} \right) \right) - \frac{\lambda}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma} - \frac{\kappa^2}{12} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{B}_{\nu\lambda\sigma} \right\} + S_{\text{boundary}} - \mathcal{T}_A \int d^3 \xi \sqrt{\gamma_A} - \mathcal{Q}_A \int \mathcal{A} - \mathcal{T}_B \int d^3 \xi \sqrt{\gamma_B} - \mathcal{Q}_B \int \mathcal{B}. \quad (9)$$

The variation of (9) yields [1]

$$(M_{\text{Pl}}^2 + \kappa^2) G^\mu{}_\nu = -\kappa^2 \lambda \delta^\mu{}_\nu + T^\mu{}_\nu + \dots, \\ \hat{\mathcal{F}}_{\mu\nu\lambda\sigma} = \frac{\kappa^2}{2} \sqrt{g} \epsilon_{\mu\nu\lambda\sigma}, \\ \hat{\mathcal{G}}_{\mu\nu\lambda\sigma} = \frac{2\kappa^2 \lambda - \kappa^2 R - T/4}{4\kappa^2} \sqrt{g} \epsilon_{\mu\nu\lambda\sigma}, \\ 2n^\mu \partial_\mu \lambda = \mathcal{Q}_A \delta(r - r_0), \\ \frac{1}{2} n^\mu \partial_\mu \kappa^2 = \mathcal{Q}_B \delta(r - r_0). \quad (10)$$

The Israel-Gibbons-Hawking action changes slightly by $\kappa^2 \rightarrow \kappa_{\text{eff}}^2 = M_{\text{Pl}}^2 + \kappa^2$.

To Wick rotate the action we use $t = -ix_E^0$, and turn the crank [1]. Defining the Euclidean action by $iS = -S_E$ and restricting to locally maximally symmetric backgrounds, $\langle \mathcal{L}_{\text{QFT}}^E \rangle = \Lambda_{\text{QFT}}$, with Λ_{QFT} the matter sector vacuum energy to an arbitrary loop order,

$$S_E = \int d^4 x_E \left\{ \sqrt{g} \left(-\frac{\kappa_{\text{eff}}^2}{2} R_E + \kappa^2 \lambda + \Lambda_{\text{QFT}} \right) - \frac{\lambda}{3} \epsilon_E^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma}^E - \frac{\kappa^2}{12} \epsilon_E^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{B}_{\nu\lambda\sigma}^E \right\} + S_{\text{boundary}} + \mathcal{T}_A \int d^3 \xi_E \sqrt{\gamma_A} - \frac{\mathcal{Q}_A}{6} \int d^3 \xi_E \mathcal{A}_{\mu\nu\lambda}^E \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon_E^{\alpha\beta\gamma} + \mathcal{T}_B \int d^3 \xi_E \sqrt{\gamma_B} - \frac{\mathcal{Q}_B}{6} \int d^3 \xi_E \mathcal{B}_{\mu\nu\lambda}^E \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon_E^{\alpha\beta\gamma}. \quad (11)$$

With our couplings, $\Lambda_{\text{QFT}} = \kappa^2 \frac{M_{\text{UV}}^4}{\mathcal{M}^2} + \dots = \kappa^2 \mathcal{H}_{\text{QFT}}^2$, where M_{UV}^4 is the QFT UV cutoff and ellipsis denote subleading terms [22,23]. Thus, $\Lambda = \kappa^2 (\lambda + \mathcal{H}_{\text{QFT}}^2) = \kappa^2 \lambda_{\text{eff}}$.

The membrane-induced transitions link geometries with $\kappa_{\text{out/in}}^2 \Lambda_{\text{out/in}}$ [*out/in* denote parent and offspring geometries (exterior and interior of the membranes, respectively)]. The configurations with local $O(4)$ symmetry dominate since they have minimal Euclidean action [43–45]. Both *in* and *out* can be described with the metrics $ds_E^2 = dr^2 + a^2(r) d\Omega_3$, where $d\Omega_3$ is the line element on a unit S^3 . Here, a solves $(a')^2 - \frac{1}{a^2} = -\Lambda/3\kappa_{\text{eff}}^2$. The prime is the r derivative [1]. The variations of (11) give also the membrane-induced contributions. We find [1]

$$\lambda_{\text{out}} - \lambda_{\text{in}} = \frac{\mathcal{Q}_A}{2}, \quad \kappa_{\text{out}}^2 - \kappa_{\text{in}}^2 = 2\mathcal{Q}_B, \\ \kappa_{\text{eff out}}^2 \frac{a'_{\text{out}}}{a} - \kappa_{\text{eff in}}^2 \frac{a'_{\text{in}}}{a} = -\frac{\mathcal{T}_A + \mathcal{T}_B}{2}. \quad (12)$$

We wrote the discontinuities as if the emission of A and B membranes were collocated simply to save space. We ignore

the discontinuity in \mathcal{B} since the p -form terms in the action cancel out exactly against the charges and do not affect the bounce action [1].

In semiclassical approximation, the membrane nucleation rates are $\Gamma \sim e^{-S_{\text{bounce}}}$ [43–45], where $S(\text{bounce}) = S(\text{instanton}) - S(\text{parent})$. Instanton actions are the Euclidean actions computed on a solution with a number of membranes, starting with one and counting off. Their classification was initiated in [35,36] for the theories with 4-form fluxes screening the cosmological constant [34–36,46,47]. We find important differences in our case because the cosmological constant depends on membrane charges linearly, instead of quadratically [1].

We focus here on the most interesting case of $dS \rightarrow dS$ decay when $q = \frac{2\kappa_{\text{eff}}^2 \kappa^2 |\mathcal{Q}_A|}{3\mathcal{T}_A^2} < 1$. In this case only the $(-+)$ instantons of Fig. 1 are kinematically allowed for A -membrane transitions [1], and the geometries can be glued together to form an instanton if³

³Corresponding to $\zeta_{\text{out}} = -1$, $\zeta_{\text{in}} = +1$ in the notation of [1,35,36].

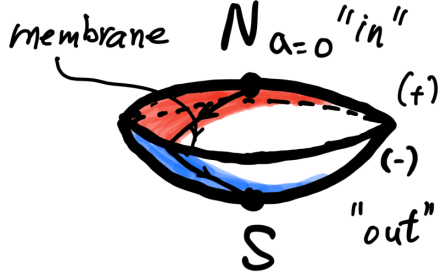


FIG. 1. A $q < 1$ instanton comprises two sections of S^4 .

$$\begin{aligned} \sqrt{1 - \frac{\Lambda_{\text{out}} a^2}{3\kappa_{\text{eff}}^2}} &= \frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2} (1 - q)a, \\ \sqrt{1 - \frac{\Lambda_{\text{in}} a^2}{3\kappa_{\text{eff}}^2}} &= \frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2} (1 + q)a. \end{aligned} \quad (13)$$

Crucially, due to (bi)linear dependence on charges—instead of quadratic—other $dS \rightarrow dS$ instantons are *forbidden* when $q < 1$ [1]. This has dramatic consequences: as a result, the resulting theory has a quantum attractor at $\Lambda = 0^+$.

We now uncover this behavior. First, it turns out to be useful to solve Eqs. (13) for a^2 , which is the radius of the membrane connecting parent and offspring. We find

$$\begin{aligned} \frac{1}{a^2} &= \frac{\Lambda_{\text{out}}}{3\kappa_{\text{eff}}^2} + \left(\frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2}\right)^2 (1 - q)^2 \\ &= \frac{\Lambda_{\text{in}}}{3\kappa_{\text{eff}}^2} + \left(\frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2}\right)^2 (1 + q)^2. \end{aligned} \quad (14)$$

Hence, there are two regimes of bubble formation. When a^2 is comparable to de Sitter radii, Eq. (14) shows that $\sim(1 - \frac{\Lambda_{\text{out}} a^2}{3\kappa_{\text{eff}}^2})^{1/2} \ll 1$ and the bounce action is approximated by the difference of one half of the parent and offspring horizon areas divided by $4G_N$,

$$\begin{aligned} S_{\text{bounce}} &\simeq -\frac{12\pi^2 \kappa_{\text{eff}}^4 \Delta\Lambda}{\Lambda_{\text{out}} \Lambda_{\text{in}}}, \\ \Delta\Lambda &= \Lambda_{\text{out}} - \Lambda_{\text{in}} = \frac{1}{2} \kappa^2 \mathcal{Q}_A. \end{aligned} \quad (15)$$

Since $q < 1$, as long as $\Lambda_{\text{out}} \gg 3\kappa_{\text{eff}}^2 (\frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2})^2$, in this regime the discharge of the cosmological constant is fast because $S_{\text{bounce}} < 0$. The cosmological constant is repelled down from the Planckian scales. The reverse processes increasing Λ ($\Delta\Lambda < 0$) also occur but their bounce action is the negative of (15) and they are rarer. The overall trend is the decrease of Λ .

This stage ends when $\Lambda < 3\kappa_{\text{eff}}^2 (\frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2})^2$, after which nucleations proceed via production of small bubbles, with the bounce action [1]

$$S_{\text{bounce}} \simeq \frac{24\pi^2 \kappa_{\text{eff}}^4}{\Lambda_{\text{out}}} \left(1 - \frac{8\kappa_{\text{eff}}^2 \Lambda_{\text{out}}}{3\mathcal{T}_A^2}\right), \quad (16)$$

and $S_{\text{bounce}} > 0$ because $\Lambda < 3\kappa_{\text{eff}}^2 (\frac{\mathcal{T}_A}{4\kappa_{\text{eff}}^2})^2$. Therefore, S_{bounce} diverges as $\Lambda_{\text{out}} \rightarrow 0^+$ —as we asserted above. So, the bubbling rate $\Gamma \sim e^{-S_{\text{bounce}}}$ has an essential singularity at $\Lambda_{\text{out}} \rightarrow 0^+$, where the rate goes to zero. Hence, when $q < 1$ small Λ is metastable, and the locally Minkowski space is absolutely stable to discharges.

Once $q < 1$, the only way it can be disturbed is if κ_{eff}^2 increases by the emission of \mathcal{Q}_B . Another possible problem is if emissions yield $\kappa_{\text{eff}}^2 < 0$, leading to a pandemic of spin-2 ghosts. Arranging the values of \mathcal{Q}_B and \mathcal{T}_B we can prevent both of these dangers. The dangerous processes are mediated by large bubbles, which are completely blocked when $16 \frac{\kappa_{\text{eff}}^4 |\mathcal{Q}_B|}{\mathcal{T}_B^2} \ll 1$. In this limit the B -bubble radius at nucleation is

$$\begin{aligned} \frac{1}{a^2} &= \left(\frac{\mathcal{T}_B}{4\kappa_{\text{eff}}^2 \text{out}}\right)^2 \left(1 + \mathcal{O}\left(\frac{\kappa_{\text{eff}}^4 \text{out} \mathcal{Q}_B}{\mathcal{T}_B^2}\right)\right) \\ &= \left(\frac{\mathcal{T}_B}{4\kappa_{\text{eff}}^2 \text{in}}\right)^2 \left(1 + \mathcal{O}\left(\frac{\kappa_{\text{eff}}^4 \text{in} \mathcal{Q}_B}{\mathcal{T}_B^2}\right)\right). \end{aligned} \quad (17)$$

Since only small bubbles can nucleate when $\frac{\Lambda}{3\kappa_{\text{eff}}^2} < (\frac{\mathcal{T}_B}{4\kappa_{\text{eff}}^2})^2$, κ_{eff}^2 stays constant. The condition $16 \frac{\kappa_{\text{eff}}^4 |\mathcal{Q}_B|}{\mathcal{T}_B^2} \ll 1$ also blocks the bubble size from below, ensuring that $a > 1/\kappa_{\text{eff}}$. This regime is disconnected both from Planckian physics and from $\kappa^2 < 0$, at least in the semiclassical approximation which we pursue [1]. Further, in this limit kinematics restricts $dS \rightarrow dS$ discharges to only be mediated by instantons of Fig. 1. Hence, \mathcal{Q}_B emissions again completely shut off at Minkowski. Finally, by arranging $\frac{\kappa_{\text{eff}}^4 \text{in} \mathcal{Q}_B}{\mathcal{T}_B^2} \ll \frac{\kappa_{\text{eff}}^4 \text{in} \mathcal{Q}_A}{\mathcal{T}_A^2} < 1$ we can make \mathcal{Q}_B discharges much slower than \mathcal{Q}_A discharges.

So, in our generalization of GR both the cosmological constant and Newton constant change by membrane emissions. The discharges are quantum mechanical and nonperturbative, and cease in the classical limit and flat space, $\Lambda/\kappa^4 \rightarrow 0$. This limit of the cosmological constant evolution is attained by quantum Brownian drift, and is the quantum attractor. This fits with ideas that an eternal, stable de Sitter space may not exist in a UV complete theory [48–54], and reminds of the wormhole approach to quantum gravity [55–63].

Let us now show how this discharges the cosmological constant. First we define the problem in our theory. The total cosmological constant is $\Lambda_{\text{total}} = \kappa^2 (\frac{\mathcal{M}_{\text{UV}}^4}{\mathcal{M}^2} + \frac{V}{\mathcal{M}^2} + \lambda)$, where we include the QFT vacuum energy $\sim \mathcal{M}_{\text{UV}}^4$, any nonvanishing QFT potential $\sim V$, and our variable λ . Since λ and κ^2 change discretely, by $\Delta\lambda = \mathcal{Q}_A/2$, $\Delta\kappa^2 = 2\mathcal{Q}_B$, we

have $\lambda = \lambda_0 + N \frac{Q_A}{2}$, $\kappa^2 = \kappa_0^2 + 2\mathcal{N}Q_B$, with N and \mathcal{N} arbitrary integers. The net cosmological constant term is $\Lambda_{\text{total}} = (\kappa_0^2 + 2\mathcal{N}Q_B) \left(\frac{\Lambda_0}{\mathcal{M}^2} + N \frac{Q_A}{2} \right)$, where $\Lambda_0 = \mathcal{M}_{\text{UV}}^4 + V + \mathcal{M}^2 \lambda_0$ and $\Delta \Lambda_{\text{total}} = (\kappa_0^2 + 2\mathcal{N}Q_B) Q_A / 2$. To get $\Lambda_{\text{total}} / \kappa_{\text{eff}}^4 < 10^{-120}$, we must either pick a tiny Q_A or fine-tune $(\kappa_0^2 + 2\mathcal{N}Q_B) \Lambda_0 / \mathcal{M}^2$. Variations of κ_{eff}^2 cannot help since we cannot suppress the curvature of the universe without reducing the gravitational force between two masses.

There is a simple solution to this obstacle, however. We add one more 4-form which contributes to λ , but couple it to a different membrane with parameters $T_{\hat{A}}, Q_{\hat{A}}$:

$$S = S - \int d^4x \sqrt{g} \left(\kappa^2 \hat{\lambda} + \frac{\hat{\lambda}}{3} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \hat{A}_{\nu\lambda\sigma} \right) - T_{\hat{A}} \int d^3\xi \sqrt{\gamma_{\hat{A}}} - Q_{\hat{A}} \int \hat{A}. \quad (18)$$

The membranes \hat{A} behave exactly as the A ones. We demand $\hat{q} < 1$ in addition to $q < 1$, and that $\frac{Q_{\hat{A}}}{Q_A} = \omega$ is an irrational number, as in the irrational axion proposal [28] (see also Ref. [9]). The cosmological constant ‘‘quantization’’ law now becomes

$$\Lambda_{\text{total}} = (\kappa_0^2 + 2\mathcal{N}Q_B) \left(\frac{\Lambda_0}{\mathcal{M}^2} + \frac{Q_A}{2} (N + \hat{N}\omega) \right). \quad (19)$$

Since ω is irrational, for any real number ρ there exist integers N, \hat{N} such that $N + \hat{N}\omega$ is arbitrarily close to ρ [28,64]. Thus, integers N, \hat{N} exist such that $N + \hat{N}\omega$ is arbitrarily close to $-\frac{2\Lambda_0}{Q_A \mathcal{M}^2}$. The set of Λ_{total} is dense, with values arbitrarily close to zero! Further, there is no axion ‘‘gauging’’ these dense discrete shifts and no emerging global symmetries [28,65]. For any initial value of Λ , there exist many sequences of discharging membranes, in any order, which will yield terminal values N, \hat{N} for which the cosmological constant is arbitrarily close to zero, and very long lived.

Due to the irrational ratio of charges, *all* previously separated superselection sectors now mix together, transitioning between each other by utilizing both A, \hat{A} charges. The slower nucleation processes when Λ is well below the cutoff also allow up jumps, which raise Λ , and so the superselection sectors will form a very fine discretuum. The states with $M_{\text{pl}}^4 \gg \Lambda > 0$ will be very long lived, decaying predominantly to $\Lambda \rightarrow 0^+$. The key ingredient here is the pole of the bounce action, Eq. (16), specific to the $(-+)$ of Fig. 1. Its presence means that $\Lambda \rightarrow 0^+$ is the dynamical attractor. This is captured by the behavior of the semi-classical Euclidean partition function.

Indeed, consider $Z = \int \dots DAD\hat{A}DBD\lambda D\hat{\lambda} D\kappa^2 Dg e^{-S_E}$, the Euclidean variant of the magnetic dual-partition function. In the saddle-point approximation,

$$Z = \sum_{\text{instantons}} \sum_{\lambda, \hat{\lambda}, \kappa^2} e^{-S_E(\text{instanton})}, \quad (20)$$

the idea was to sum over classical extrema of the action. In our case this begins with summing over the Euclidean instantons with any number of membranes included. The $O(4)$ invariant solutions should minimize the action, and so this is a reasonable leading-order approximation [43–45]. Thus, Z will invariably be dominated by our instantons.

This sum is challenging, but we can get a feel for the individual terms. Inverting the bounce action, $S(\text{instanton}) = S(\text{bounce}) + S(\text{parent})$. Without offspring, the instanton action equals the parent action, which is just the negative of the horizon area divided by $4G_N$, $S(\text{parent}) = -24\pi^2 \frac{\kappa_{\text{eff}}^4}{\Lambda_{\text{out}}}$. For n generations, summing over the family tree, $S(\text{instanton}, n) = \sum_n S(\text{offspring}, n) + S(\text{progenitor})$. By ‘‘offspring’’ we mean nested segments separated from the parent by membranes. The ‘‘progenitor’’ geometry is the primordial parent initiating the tree. Hence, by Eq. (16) $S(\text{instanton}) \lesssim -64\pi^2 \kappa_{\text{eff}}^6 / T_i^2$, and so an extended lineage may yield $S(\text{instanton}, n) \rightarrow -64\pi^2 \kappa_{\text{eff}}^6 \left(\frac{n_A}{T_A^2} + \frac{n_{\hat{A}}}{T_{\hat{A}}^2} \right) \rightarrow -24\pi^2 \frac{\kappa_{\text{eff}}^4}{\Lambda_{\text{terminal}}}$. The number of generations will be bounded by $-24\pi^2 \frac{\kappa_{\text{eff}}^4}{\Lambda_{\text{terminal}}}$, since nucleations can go on until $\Lambda_{\text{terminal}} \rightarrow 0^+$, at which point the rate vanishes.

Thus the sum (20), $Z \sim \sum e^{24\pi^2 \frac{\kappa_{\text{eff}}^4}{\Lambda} + \dots}$, will be heavily skewed toward $\Lambda \rightarrow 0^+$, reflecting that discharges cease in the Minkowski limit. Ergo, in our generalization of GR, Z heavily prefers $\frac{\Lambda}{\kappa_{\text{eff}}^4} \rightarrow 0$; de Sitter is unstable and quantum mechanics + GR dynamically relax Λ to zero. The instability stops as $\Lambda \rightarrow 0^+$, and the final (near-) Minkowski space is extremely long lived. To explain $G_N = \frac{1}{8\pi M_{\text{pl}}^2} \simeq 10^{-38} \text{ (GeV)}^{-2}$ we could either just decide to measure it, or invoke a mild version of the ‘‘Weak Anthropic Principle,’’ which can fix G_N by placing the Earth in the habitable zone around the Sun.

The ‘‘empty universe’’ problem [32] is averted since the relaxation of Λ involves large successive jumps, and the tiny terminal Λ comes from the irrational ratio of charges. So, the cosmological constant does not always dominate, but just sometimes [46]. The relaxation process is a random walk. Finally, we note that in addition to starting from a high scale, an empty universe could ‘‘restart’’ itself by a rare quantum jump which increases the cosmological constant since the up jumps are also possible,⁴ and then in subsequent evolution an inflationary stage is found [66].

⁴Note that this may have implications for the cosmological arrow of time, as it might yield a cosmic rebirth, rare as it is. As this issue is irrelevant for the present discussion, we ignore it here, intending to return to it at a later time.

In this sense, this channel for inflation may seem *a priori* rare, but since the system can continue exploring the phase space, even a “rare” event can be found eventually [67]. In any case, our universe would evolve towards Minkowski, $\Lambda \rightarrow 0$, so that the potential problems with more likely, smaller-scale fluctuations dubbed “Boltzmann Brains” may be avoided [67–70]. Hence, the avoidance of the “empty universe” problem opens the door for embedding “normal”

cosmology in our framework. We will return to the precise details of how to introduce inflation and postinflationary cosmology, as well as the questions about the UV completion of the mechanism in future work.

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