Motion induced excitation and electromagnetic radiation from an atom facing a thin mirror

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(Received 6 July 2022; accepted 24 August 2022; published 8 September 2022)

We evaluate the probability of (de)excitation and photon emission from a neutral, moving, nonrelativistic atom, coupled to the quantum electromagnetic field and in the presence of a thin, perfectly conducting plane ("mirror"). These results extend, to a more realistic model, the ones we had presented for a scalar model, where the would-be electron was described by a scalar variable, coupled to an (also scalar) vacuum field. The latter was subjected to either Dirichlet or Neumann conditions on a plane. In our evaluation of the spontaneous emission rate produced when the accelerated atom is initially in an excited state, we pay attention to its comparison with the somewhat opposite situation, namely, an atom at rest facing a moving mirror.

DOI: 10.1103/PhysRevD.106.065005

I. INTRODUCTION

Several important effects are associated with the quantum vacuum fluctuations of the electromagnetic (EM) field, ranging from the microscopic realm (spontaneous emission by excited atoms, the Lamb shift, anomalous magnetic moments of elementary particles, and van der Waals interactions) to the Casimir interaction between neutral macroscopic bodies [1]. The Casimir-Polder force corresponds to a hybrid situation, since it involves an atom and a macroscopic medium. Different manifestations of the fluctuations of the EM field in analogous situations correspond to the effect of a change in boundary conditions on the probability of spontaneous emission from an atom. This has been studied for an atom in the presence of a perfectly conducting plane, or inside a cavity [2].

New effects appear when one introduces time dependence; for instance, when the atom or the macroscopic media are in motion: photon creation by accelerated mirrors (dynamical Casimir effect) and quantum friction for an atom and a surface (or between two surfaces) in relative motion at constant velocity [3]. In the present work, we are concerned with a dynamical situation, focusing on the changes in the decay probability (of an initially excited atom) as well as on the possibility of excitation of an atom which is initially in its ground state. Note that, for this kind of system, one can even have the production of photon pairs without any change in the atomic state, which is the microscopic analog of the dynamical Casimir effect [4].

In Ref. [5], the authors considered an atom at rest near an accelerating mirror and showed that uniformly accelerated motion of the mirror yields excitation of a static two-level atom with simultaneous emission of a real photon. They also found that the excitation probability oscillates as a function of the atomic position because of interference between contributions from the waves incident on and reflected from the mirror. In Ref. [6], an atom accelerating near a mirror is considered, and a radiative effect is reported. From an inertial point of view, the process arises from a collision of the negative vacuum energy of Rindler space with the mirror. There is a qualitative symmetry under interchange of accelerated and inertial subsystems, but it hinges on the accelerated detector's being initially in its own Rindler vacuum.

In a previous work [7], we presented a study on the excitation and decay probabilities for a moving atom in front of a planar mirror, in a simplified model: the "atom" was endowed with a scalar variable describing the electron, and it was coupled to a real quantum scalar field. Perfect conductor boundary conditions were replaced with Neumann and Dirichlet boundary conditions on the mirror's plane.

We paid particular attention to two different processes, both taking place in the lowest order in the coupling constant: transition of the atom from the ground state to the first excited state, with simultaneous emission of a

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photon, and spontaneous emission of an initially excited atom. We considered a small-amplitude motion of the atom and analyzed the spectral and directional dependence of the radiation on the motion.

In this paper, we generalize those results to the more realistic case of an atom coupled to the quantum electromagnetic field, taking into account the $\mathbf{v} \times \mathbf{B}$ interaction between the moving atom and the magnetic field, the so-called Röntgen term, which is a consequence of the vector character of the electromagnetic field and is crucial to maintain Lorentz covariance [8] to the relevant order in the velocity.

This paper is organized as follows. In Sec. II, we describe our model in terms of its classical action. In Sec. III, we compute the vacuum persistence amplitude from the imaginary part of the effective action. After recovering known results for an atom oscillating in free space, we also present the calculation for the case of the atom oscillating in front of a planar perfect mirror. In Sec. IV, we compute the transition probabilities for two processes: decay of an excited atom and excitation of an atom initially in its ground state. We compare the results for a moving atom in front of a static mirror with those for an oscillating mirror and a static atom [9,10], tracing the differences in terms of the Röntgen current. Section V contains the conclusions of our work.

II. MODEL AND ITS CLASSICAL ACTION

Our starting point to define the model shall be the action $\mathcal{S}_a$, for an atom coupled to the EM field, in the electric dipole approximation. This approximation, to be unambiguous, must be formulated on a comoving system. Indeed, let us assume that in the system where the atom is at rest there is an electric dipole moment $\mathbf{d}_0$ and a vanishing magnetic dipole: $\mathbf{m}_0 = 0$. Then, in the laboratory system, and to the first order in the velocity of the atom (in our conventions, the speed of light $c \equiv 1$), we shall have

$$\begin{aligned}\mathbf{d}(t) &= \mathbf{d}_0(t) + \mathbf{v}(t) \times \mathbf{m}_0(t) = \mathbf{d}_0(t) + \mathbf{v}(t) \times 0 = \mathbf{d}_0(t)\\ \mathbf{m}(t) &= \mathbf{m}_0(t) - \mathbf{v}(t) \times \mathbf{d}_0(t) = -\mathbf{v}(t) \times \mathbf{d}_0(t)\\ &= -\mathbf{v}(t) \times \mathbf{d}(t). \end{aligned} \tag{1}$$

Therefore, the action $\mathcal{S}_a$ (in the laboratory system) must also include a coupling, to the magnetic field, of the motion-induced magnetic dipole,

$$\begin{aligned}\mathcal{S}_a &= \int dt \bigg[\frac{m}{2}\dot{\mathbf{x}}^2(t) - V(\mathbf{x}(t)) + \mathbf{d}(t) \cdot \mathbf{E}(t, \mathbf{r}(t))\\ &\quad + \mathbf{m}(t) \cdot \mathbf{B}(t, \mathbf{r}(t))\bigg]\\ &= \int dt \bigg\{\frac{m}{2}\dot{\mathbf{x}}^2(t) - V(\mathbf{x}(t))\\ &\quad + e\mathbf{x}(t) \cdot [\mathbf{E}(t, \mathbf{r}(t)) + \dot{\mathbf{r}}(t) \times \mathbf{B}(t, \mathbf{r}(t))]\bigg\}, \end{aligned} \tag{2}$$

where $\mathbf{x}(t)$ denotes the position of the electron with respect to the (center of mass of the) atom, while $\mathbf{r}(t)$ does so for the atom with respect to the origin of the laboratory system. Regarding the potential V binding the electron, for the sake of simplicity, we shall assume here that it has a harmonic oscillator form: $V = \frac{m}{2}\Omega^2\mathbf{x}^2$.

On the other hand, the free EM field action $\mathcal{S}_{\rm em}(A)$ is given by

$$\mathcal{S}_{\rm em}(A) = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm g.f.}(A)\right], \tag{3}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which includes a gauge-fixing term $\mathcal{L}_{\rm g.f.}$. We want to consider the cases of an atom moving either in free space or in the presence of a perfect mirror. The second case shall be dealt with when integrating out the EM field fluctuations. Not unexpectedly, the outcome will turn out to be the sum of the free space result plus a "reflected" contribution (in the method of images sense).

III. EFFECTIVE ACTION AND ITS IMAGINARY PART

As a first step in the derivation of the effective action $\Gamma[\mathbf{r}(t)]$, which will only depend on the atom's trajectory, we first integrate out the electron's degrees of freedom, $\mathbf{x}(t)$, to obtain an intermediate effective action $\mathcal{S}_{\rm eff}(A;\mathbf{r})$.

Since we are assuming a harmonic oscillator form for V in (2), the functional integral over $\mathbf{x}$ becomes a Gaussian. The result of such an integral is (modulo an irrelevant constant) tantamount to replacing *in the action* the integrated variable in terms of its source, using the classical equation of motion for $\mathbf{x}$. The latter corresponds to a harmonic oscillator forced by a time-dependent force, which is the Lorentz force acting on $\mathbf{r}(t)$ (not on $\mathbf{x}$). Solving for $\mathbf{x}$ in terms of that force, and recalling that Feynman conditions are to be imposed on the time dependence of that solution, we find

$$\mathcal{S}_{\rm eff}(A;\mathbf{r}) = \mathcal{S}_{\rm em}(A) + \mathcal{S}_{\rm I}^{(a)}(A;\mathbf{r}), \tag{4}$$

with

$$\begin{aligned}\mathcal{S}_{\rm I}^{(a)}(A,\mathbf{r}) = \frac{ie^2}{2m}\int_{t,t'} &\Delta_\Omega(t-t')[\mathbf{E}(t,\mathbf{r}(t))\\ &+ \dot{\mathbf{r}}(t) \times \mathbf{B}(t,\mathbf{r}(t))] \cdot [\mathbf{E}(t',\mathbf{r}(t'))\\ &+ \dot{\mathbf{r}}(t') \times \mathbf{B}(t',\mathbf{r}(t'))], \end{aligned} \tag{5}$$

where we have used a shorthand notation for the integration over time, for example, $\int_{t,t'} \ldots \equiv \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \ldots$, and

$$\Delta_\Omega(t-t') = \int \frac{d\nu}{2\pi} e^{-i\nu(t-t')} \tilde{\Delta}_\Omega(\nu)$$
$$\tilde{\Delta}_\Omega(\nu) = \frac{i}{\nu^2 - \Omega^2 + i\epsilon}. \quad (6)$$

We can produce an explicit expression for $\Delta_\Omega(t-t')$, which will turn out to be quite useful:

$$\Delta_\Omega(t-t') = \frac{1}{2\Omega}[\theta(t-t')e^{-i\Omega(t-t')} + \theta(t'-t)e^{i\Omega(t-t')}]. \quad (7)$$

The final form of the effective action of the system, $\Gamma[\mathbf{r}(t)]$, is obtained by including the EM field fluctuations. Namely,

$$e^{i\Gamma[\mathbf{r}(t)]} = \frac{\int \mathcal{D}A e^{i\mathcal{S}_{\rm eff}(A;\mathbf{r})}}{\int \mathcal{D}A e^{i\mathcal{S}_{\rm eff}(A;\mathbf{r}_0)}}, \quad (8)$$

where $\mathbf{r}_0$ is the average position of the atom, which we will assume to be time independent. Then, up to first order in e^2, we obtain

$$\Gamma[\mathbf{r}(t)] = \frac{ie^2}{2m}\int_{t,t'} \Delta_\Omega(t-t')[\langle \mathbf{E}(t,\mathbf{r}(t))\cdot\mathbf{E}(t',\mathbf{r}(t'))\rangle$$
$$+ 2\langle \mathbf{E}(t,\mathbf{r}(t))\cdot\dot{\mathbf{r}}(t')\times\mathbf{B}(t',\mathbf{r}(t'))\rangle$$
$$+ \langle \dot{\mathbf{r}}(t)\times\mathbf{B}(t,\mathbf{r}(t))\cdot\dot{\mathbf{r}}(t')\times\mathbf{B}(t',\mathbf{r}(t'))\rangle$$
$$- \langle \mathbf{E}(t,\mathbf{r}_0)\cdot\mathbf{E}(t',\mathbf{r}_0)\rangle], \quad (9)$$

where the symbol $\langle \ldots \rangle$ denotes the functional averaging

$$\langle \ldots \rangle = \frac{\int \mathcal{D}A \ldots \exp\{i[\mathcal{S}_{\rm em}(A)]\}}{\int \mathcal{D}A \exp\{i[\mathcal{S}_{\rm em}(A)]\}}. \quad (10)$$

The presence of the mirror may be introduced in more than one way; in the previous definition of the functional averages, since the action is the one for free space, we have implicitly assumed that it is dealt with by a proper definition of the integration measure. Namely, the integral is over fields satisfying perfect boundary conditions on the mirror. Our choice of coordinates is such that the mirror occupies the $x_3 = 0$ plane (for the sake of simplicity, we shall make no distinction between lower and upper indices, from now on; thus, $x_3 \equiv x^3 \equiv z$). From (9), we see that we just need to perform functional averages for *pairs* of fields (each factor involves derivatives of the gauge field). Therefore, we shall only need the gauge field propagator with perfect conductor boundary conditions on the mirror.

Just before inserting the explicit expressions for the EM field correlators, it is convenient to perform an expansion in powers of the departures about the average position of the atom, $\mathbf{r}_0$. To that end, we set $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{y}(t)$, expand up to the second order in $\mathbf{y}(t)$, and discard terms that, by their very structure, cannot contribute to the imaginary part of the effective action.

Thus, with this in mind, we may present the expression for the effective action, expanded to the second order in $\mathbf{y}(t)$, as

$$\Gamma = \Gamma_{EE} + \Gamma_{EB} + \Gamma_{BB}, \quad (11)$$

where

$$\Gamma_{EE} = \frac{ie^2}{2m}\int_{t,t'} y_i(t)y_j(t')\Delta_\Omega(t-t') \times \left(\frac{\partial^2}{\partial r_i \partial r'_j}\langle \mathbf{E}(t,\mathbf{r})\cdot\mathbf{E}(t',\mathbf{r}')\rangle\right)\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0}, \quad (12)$$

$$\Gamma_{EB} = \frac{ie^2}{m}\int_{t,t'} y_i(t)\dot{y}_j(t')\Delta_\Omega(t-t')\varepsilon_{kjl} \times \left(\frac{\partial}{\partial r_i}\langle E_k(t,\mathbf{r})B_l(t',\mathbf{r}')\rangle\right)\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0}, \quad (13)$$

$$\Gamma_{BB} = \frac{ie^2}{2m}\int_{t,t'} \dot{y}_i(t)\dot{y}_j(t')\Delta_\Omega(t-t')\varepsilon_{kil}\varepsilon_{kjm} \times \langle B_l(t,\mathbf{r})B_m(t',\mathbf{r}')\rangle\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0}. \quad (14)$$

The previous formulas, valid in free space, hold true when a mirror is present, the difference between those two situations being the form of the EM field correlation functions. Let us first consider the free space case.

A. Free space

We evaluate each one of the three terms into which we have decomposed Γ in (11) in turn. They involve different correlation functions between components of the EM field in free space. Note that any time-local term appearing in those correlation functions [namely, a polynomial in $\delta(t-t')$ and its derivatives] will not contribute to the imaginary part, and we shall therefore discard them. We shall use a (0) to denote the free space version of an object, to distinguish it from the one when the perfectly conducting plane is present.

For $\Gamma^{(0)}_{EE}$, we have

$$\langle E_i(t,\mathbf{r}(t))E_j(t',\mathbf{r}(t'))\rangle^{(0)} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \times k^2\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Delta_k(t-t'), \quad (15)$$

where $\langle \ldots \rangle^{(0)}$ denotes correlation functions in free space. Here, Δ_k is defined as in (6), with $k \equiv |\mathbf{k}|$ playing the role of Ω: $\Delta_k(t-t') \equiv [\Delta_\Omega(t-t')]_{\Omega\to k}$. Therefore, from (15), we derive

$$\left(\frac{\partial^2}{\partial r_i \partial r'_j}\langle \mathbf{E}(t,\mathbf{r})\cdot\mathbf{E}(t',\mathbf{r}')\rangle^{(0)}\right)\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} = 2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\mathbf{k}^2 k_i k_j \Delta_k(t-t') = \frac{2}{3}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k^4\delta_{ij}\Delta_k(t-t'). \tag{16}$$

Inserting this into $\Gamma^{(0)}_{EE}$, we note that the resulting expression will contain the product $\Delta_\Omega \Delta_k$. This product may be simplified by using the property, valid for any pair Ω_1, Ω_2:

$$\Delta_{\Omega_1}(t-t')\Delta_{\Omega_2}(t-t') = \frac{\Omega_1+\Omega_2}{2\Omega_1\Omega_2}\Delta_{\Omega_1+\Omega_2}(t-t'). \tag{17}$$

In our case, this leads to

$$\Gamma^{(0)}_{EE} = \frac{ie^2}{6m\Omega}\int_{t,t'} y_i(t)y_i(t')\int\frac{d^3\mathbf{k}}{(2\pi)^3}(\Omega+k)k^3\Delta_{\Omega+k}(t-t'). \tag{18}$$

Writing then $\Delta_{\Omega+k}$ in terms of its Fourier transform, and Fourier transforming the departures, we obtain

$$\Gamma^{(0)}_{EE} = \frac{ie^2}{6m\Omega}\int\frac{d\nu}{2\pi}\tilde{y}^*_i(\nu)\tilde{y}_i(\nu)\int\frac{d^3\mathbf{k}}{(2\pi)^3}(\Omega+k)k^3\tilde{\Delta}_{\Omega+k}(\nu). \tag{19}$$

The imaginary part of Γ_{EE} is then straightforwardly obtained from the one of $\tilde{\Delta}_{\Omega+k}$:

$$\mathrm{Im}[\Gamma^{(0)}_{EE}] = \frac{\pi e^2}{6m\Omega}\int\frac{d\nu}{2\pi}\tilde{y}^*_i(\nu)\tilde{y}_i(\nu)\int\frac{d^3\mathbf{k}}{(2\pi)^3}(\Omega+k) \times k^3\delta[\nu^2-(\Omega+k)^2]. \tag{20}$$

Performing the integration over $\mathbf{k}$,

$$\mathrm{Im}[\Gamma^{(0)}_{EE}] = \int\frac{d\nu}{2\pi}m^{(0)}_{EE}(\nu)|\tilde{\mathbf{y}}(\nu)|^2 = \frac{e^2}{24\pi m\Omega}\int\frac{d\nu}{2\pi}\theta(|\nu|-\Omega)(|\nu|-\Omega)^5|\tilde{\mathbf{y}}(\nu)|^2. \tag{21}$$

For the computation of $\Gamma^{(0)}_{EB}$, we start from the correlation function:

$$\langle E_j(t,\mathbf{r})B_l(t',\mathbf{r}')\rangle^{(0)} = i\epsilon_{jlm}\int\frac{d^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \times k_m\partial_t\Delta_k(t-t'). \tag{22}$$

Upon insertion of this into the expression for $\Gamma^{(0)}_{EB}$, the product $\Delta_\Omega\partial_t\Delta_k$ arises. For this object, we use the property

$$\Delta_{\Omega_1}(t-t')\partial_t\Delta_{\Omega_2}(t-t') = \frac{1}{2\Omega_1}\partial_t\Delta_{\Omega_1+\Omega_2}(t-t') \tag{23}$$

to get

$$\Gamma^{(0)}_{EB} = \frac{ie^2}{3m\Omega}\int_{t,t'}y_i(t)\dot{y}_i(t')\int\frac{d^3\mathbf{k}}{(2\pi)^3}k^2\partial_t\Delta_{\Omega+k}(t-t'). \tag{24}$$

Integrating by parts and Fourier transforming,

$$\Gamma^{(0)}_{EB} = -\frac{ie^2}{3m\Omega}\int\frac{d\nu}{2\pi}\tilde{y}^*_i(\nu)\tilde{y}_i(\nu)\nu^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}k^2\tilde{\Delta}_{\Omega+k}(\nu), \tag{25}$$

whence the imaginary part then becomes

$$\mathrm{Im}[\Gamma^{(0)}_{EB}] = \int\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}(\nu)|^2 m^{(0)}_{EB}(\nu) = -\frac{e^2}{12\pi m\Omega}\int\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}(\nu)|^2\theta(|\nu|-\Omega)|\nu|(|\nu|-\Omega)^4. \tag{26}$$

Finally, to evaluate $\Gamma^{(0)}_{BB}$, we need the correlator of two magnetic fields. It is straightforward to see that

$$\varepsilon_{kil}\varepsilon_{kjm}\langle B_l(t,\mathbf{r})B_m(t',\mathbf{r}')\rangle^{(0)} = \frac{4}{3}\delta_{ij}\int\frac{d^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \times k^2\Delta_k(t-t'). \tag{27}$$

Therefore,

$$\Gamma^{(0)}_{BB} = \frac{2ie^2}{3m}\int_{t,t'}\dot{y}_i(t)\dot{y}_i(t')\Delta_\Omega(t-t')\int\frac{d^3\mathbf{k}}{(2\pi)^3}k^2\Delta_k(t-t'). \tag{28}$$

Proceeding in an analogous fashion as for the previous two terms, we find

$$\mathrm{Im}[\Gamma^{(0)}_{BB}] = \int\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}(\nu)|^2 m^{(0)}_{BB}(\nu) = \frac{e^2}{12\pi m\Omega}\int\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}(\nu)|^2\theta(|\nu|-\Omega)\nu^2(|\nu|-\Omega)^3. \tag{29}$$

Adding the three contributions to the imaginary part of the effective action, we get

$$\mathrm{Im}[\Gamma^{(0)}] = \mathrm{Im}[\Gamma^{(0)}_{EE}] + \mathrm{Im}[\Gamma^{(0)}_{EB}] + \mathrm{Im}[\Gamma^{(0)}_{BB}] = \frac{e^2}{24\pi m\Omega}\int_{-\infty}^{+\infty}\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}(\nu)|^2\theta(|\nu|-\Omega) \times(|\nu|-\Omega)^3(\nu^2+\Omega^2). \tag{30}$$

This coincides with the result obtained in Ref. [4], if one performs the angular integration of the probability distribution obtained there.

B. Perfect mirror

We now evaluate the different terms contributing to the imaginary part of the effective action when a perfect mirror is present. The difference is in the form of the EM field correlation functions. They may be obtained from the ones of the gauge field, which in turn can be constructed, for example, by using the method of images. To that end, it is convenient to introduce first a special notation to distinguish among spacetime coordinates. We shall use $x = (x_\parallel, x_3)$, with $x_\parallel$ denoting x_0, x_1, x_2, the coordinates for which there is translation invariance. When using indices, the ones from the beginning of the Greek alphabet, $\alpha, \beta, \ldots$, will be implicitly assumed to run over the values 0, 1, and 2. Besides, $a, b, \ldots$ will take the values 1 and 2 (these appear when dealing with spatial coordinates on the plane).

Then, the correlator in the presence of the mirror,

$$\langle A_\mu(x) A_\nu(y)\rangle \equiv D_{\mu\nu}(x_\parallel - y_\parallel; x_3, y_3), \tag{31}$$

may be written as

$$D_{\mu\nu}(x_\parallel - y_\parallel; x_3, y_3) = D^{(0)}_{\mu\nu}(x_\parallel - y_\parallel; x_3, y_3) + D^{(R)}_{\mu\nu}(x_\parallel - y_\parallel; x_3, y_3), \tag{32}$$

where $D^{(0)}_{\mu\nu}$ is the gauge-field propagator in free space and $D^{(R)}_{\mu\nu}$ is the reflected contribution:

$$D^{(R)}_{\mu\nu} = -g^\alpha_\mu g^\beta_\nu D^{(0)}_{\alpha\beta}(x_\parallel - y_\parallel; x_3, -y_3). \tag{33}$$

Because of the fact that the EM field correlators will be the sum of two terms, the first one identical to the free space one and the second a reflection (R) term, also the effective action and its imaginary part will share this property. Namely,

$$\Gamma = \Gamma^{(0)} + \Gamma^{(R)},$$
$$\Gamma^{(R)} = \Gamma^{(R)}_{EE} + \Gamma^{(R)}_{EB} + \Gamma^{(R)}_{BB}. \tag{34}$$

We now evaluate each one of the three reflection terms above, having in mind that they are to be added to the free space terms; namely, they have no meaning by themselves, and in particular, their imaginary parts could be negative.

$\Gamma^{(R)}_{EE}$ is obtained by using the reflection term instead of the free space correlator in the analogous formula we have already used for free space. Indeed,

$$\Gamma^{(R)}_{EE} = \frac{ie^2}{2m}\int_{t,t'} y_i(t) y_j(t') \Delta_\Omega(t-t') \times \left(\frac{\partial^2}{\partial r_i \partial r'_j}\langle \mathbf{E}(t,\mathbf{r})\cdot\mathbf{E}(t',\mathbf{r}')\rangle^{(R)}\right)\Bigg|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \tag{35}$$

Now, because of the presence of the mirror, three-dimensional rotation symmetry is lost. We will, as a consequence, have different contributions to $\Gamma^{(R)}_{EE}$ (and its imaginary part) depending on whether the motion is parallel or normal to the plane. It is rather straightforward to see that $\Gamma^{(R)}_{EE}$ becomes the sum of two independent contributions, one for each kind of motion,

$$\Gamma^{(R)}_{EE}[\mathbf{y}(t)] = \Gamma^{(R)}_{EE,\parallel}[\mathbf{y}_\parallel(t)] + \Gamma^{(R)}_{EE,\perp}[y_3(t)], \tag{36}$$

since

$$\begin{aligned}\Gamma^{(R)}_{EE}[\mathbf{y}(t)] = \frac{ie^2}{2m}\int_{t,t'}\Bigg\{&\frac{1}{2}\mathbf{y}_\parallel(t)\cdot\mathbf{y}_\parallel(t')\Delta_\Omega(t-t') \\ &\times\left(\frac{\partial^2}{\partial r_a \partial r'_a}\langle \mathbf{E}(t,\mathbf{r})\cdot\mathbf{E}(t',\mathbf{r}')\rangle^{(R)}\right)\Bigg|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \\ &+ y_3(t) y_3(t')\Delta_\Omega(t-t') \\ &\times\left(\frac{\partial^2}{\partial r_3 \partial r'_3}\langle \mathbf{E}(t,\mathbf{r})\cdot\mathbf{E}(t',\mathbf{r}')\rangle^{(R)}\right)\Bigg|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0}\Bigg\}.\end{aligned} \tag{37}$$

In particular, for parallel motion, we find

$$\Gamma^{(R)}_{EE,\parallel}[\mathbf{y}_\parallel(t)] = -\frac{ie^2}{4m}\int_{t,t'}\mathbf{y}_\parallel(t)\mathbf{y}_\parallel(t')\int\frac{d^3\mathbf{k}}{(2\pi)^3}\cos(2k_3 a) \times \mathbf{k}_\parallel{}^2(\mathbf{k}_\parallel{}^2 + 3k_3^2)\Delta_\Omega(t-t')\Delta_k(t-t'), \tag{38}$$

while for motion along the perpendicular, x_3 direction,

$$\Gamma^{(R)}_{EE,\perp}[y_3(t)] = \frac{ie^2}{2m}\int_{t,t'} y_3(t) y_3(t')\int\frac{d^3\mathbf{k}}{(2\pi)^3}\cos(2k_3 a)k_3^2 \times (\mathbf{k}_\parallel{}^2 + 3k_3^2)\Delta_\Omega(t-t')\Delta_k(t-t'). \tag{39}$$

By using a procedure entirely analogous to the one for the free space part, we find the respective imaginary parts. Note that both can, and will, depend on a, the distance of $\mathbf{r}_0$ to the mirror,

$$\mathrm{Im}[\Gamma^{(R)}_{EE,\parallel}] = \int\frac{d\nu}{2\pi}|\tilde{\mathbf{y}}_\parallel(\nu)|^2 m^\parallel_{EE}(\nu), \tag{40}$$

where

$$m^\parallel_{EE}(\nu) = -\frac{e^2}{32\pi m\Omega}\theta(|\nu|-\Omega)(|\nu|-\Omega)^5 f_1[(|\nu|-\Omega)a], \tag{41}$$

with

$$f_1(x) = 3\left(\frac{1}{x^4} - \frac{1}{2x^2}\right)\cos(2x) + \frac{1}{2}\left(-\frac{3}{x^5} + \frac{11}{2x^3}\right)\sin(2x). \tag{42}$$

For the perpendicular case, we have

$$\mathrm{Im}[\Gamma^{(R)}_{EE,\perp}] = \int \frac{d\nu}{2\pi}|\tilde{y}_3(\nu)|^2 m^{\perp}_{EE}(\nu), \tag{43}$$

where

$$m^{\perp}_{EE}(\nu) = \frac{e^2}{16\pi m\Omega}\int \frac{d\nu}{2\pi}\theta(|\nu|-\Omega)(|\nu|-\Omega)^5 f_2[(|\nu|-\Omega)a], \tag{44}$$

with

$$f_2(x) = \left(-\frac{3}{x^4} + \frac{5}{2x^2}\right)\cos(2x) + \frac{1}{4}\left(\frac{6}{x^5} - \frac{13}{x^3} + \frac{6}{x}\right)\sin(2x). \tag{45}$$

The term that involves the mixed correlator between the electric and magnetic fields may be written as follows:

$$\Gamma^{(R)}_{EB} = \frac{ie^2}{m}\int_{t,t'}\int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{2ik_3 a}\Delta_\Omega(t-t')\partial_t\Delta_k(t-t') \times y_j(t)\dot{y}_l(t')(k_j\delta_{la}k_a - 2k_j\delta_{l3}k_3). \tag{46}$$

The imaginary parts for parallel and normal motion become

$$\mathrm{Im}[\Gamma^{(R)}_{EB,\parallel}] = \int \frac{d\nu}{2\pi}|\tilde{\mathbf{y}}_\parallel(\nu)|^2 m^{\parallel}_{EB}(\nu), \tag{47}$$

where

$$m^{\parallel}_{EB}(\nu) = -\frac{e^2}{8\pi m\Omega}\theta(|\nu|-\Omega)|\nu|(|\nu|-\Omega)^4 f_3[(|\nu|-\Omega)a], \tag{48}$$

with

$$f_3(x) = -\frac{1}{4x^2}\cos(2x) + \frac{1}{8x^3}\sin(2x), \tag{49}$$

and

$$\mathrm{Im}[\Gamma^{(R)}_{EB,\perp}] = \int \frac{d\nu}{2\pi}|\tilde{y}_3(\nu)|^2 m^{\perp}_{EB}(\nu), \tag{50}$$

where

$$m^{\perp}_{EB}(\nu) = \frac{e^2}{8\pi m\Omega}\theta(|\nu|-\Omega)|\nu|(|\nu|-\Omega)^4 f_4[(|\nu|-\Omega)a], \tag{51}$$

with

$$f_4(x) = \frac{1}{x^2}\cos(2x) - \left(\frac{1}{2x^3} - \frac{1}{x}\right)\sin(2x). \tag{52}$$

Finally, for $\Gamma^{(R)}_{BB}$, we have

$$\Gamma^{(R)}_{BB} = -\frac{ie^2}{2m}\int_{t,t'}\int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{2ik_3 a}\Delta_\Omega(t-t')\Delta_k(t-t') \times \{\dot{y}_a(t)\dot{y}_a(t')(\mathbf{k}_\parallel{}^2 - k_3^2) - 2k_3^2\dot{y}_3(t)\dot{y}_3(t')\} \tag{53}$$

and the respective imaginary parts for parallel and normal motion,

$$\mathrm{Im}[\Gamma^{(R)}_{BB,\parallel}] = \int \frac{d\nu}{2\pi}|\tilde{\mathbf{y}}_\parallel(\nu)|^2 m^{\parallel}_{BB}(\nu), \tag{54}$$

where

$$m^{\parallel}_{BB}(\nu) = -\frac{e^2}{16\pi m\Omega}\theta(|\nu|-\Omega)\nu^2(|\nu|-\Omega)^3 f_5[(|\nu|-\Omega)a], \tag{55}$$

with

$$f_5(x) = -\frac{1}{x^2}\cos(2x) + \frac{1}{2}\left(\frac{1}{x^3} - \frac{1}{x}\right)\sin(2x), \tag{56}$$

and

$$\mathrm{Im}[\Gamma^{(R)}_{BB,\perp}] = \int \frac{d\nu}{2\pi}|\tilde{y}_3(\nu)|^2 m^{\perp}_{BB}(\nu), \tag{57}$$

where

$$m^{\perp}_{BB}(\nu) = \frac{e^2}{16\pi m\Omega}\theta(|\nu|-\Omega)\nu^2(|\nu|-\Omega)^3 f_6[(|\nu|-\Omega)a], \tag{58}$$

with

$$f_6(x) = \frac{1}{x^2}\cos(2x) + \left(-\frac{1}{2x^3} + \frac{1}{x}\right)\sin(2x). \tag{59}$$

In Fig. 1, we plot $m_1 = 1 + m^{\parallel}_{EE}/m^{(0)}_{EE}$ for the parallel motion and $m_2 = 1 + m^{\perp}_{EE}/m^{(0)}_{EE}$ for the normal one, both as functions of the dimensionless variable $x = a(|\nu| - \Omega)$. In both cases, the contribution from $m^{\parallel,\perp}_{EE}$ goes to zero as $a|\nu| \to \infty$. In the other limit, i.e., when $x \to 0$, we get $m^{\parallel}_{EE}/m^{(0)}_{EE} = -7/10$ and $m^{\perp}_{EE}/m^{(0)}_{EE} = 11/10$. This case appears as qualitatively similar to the Dirichlet contribution reported in Ref. [7], (see the comment in Ref. [11]). Figures 2 and 3 show behavior similar to the previous one. The rates goes to zero in the large limit and

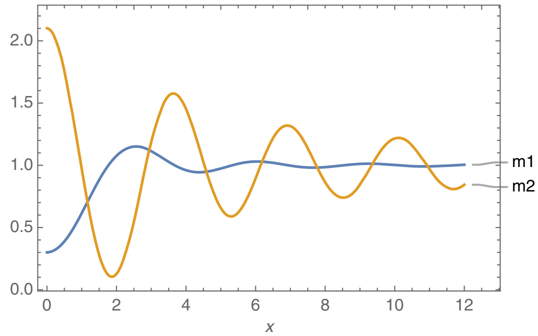


FIG. 1. We plot $m_1 = 1 + m_{EE}^{\parallel}/m_{EE}^{(0)}$ and $m_2 = 1 + m_{EE}^{\perp}/m_{EE}^{(0)}$ as functions of the dimensionless $x = a(|\nu| - \Omega)$.

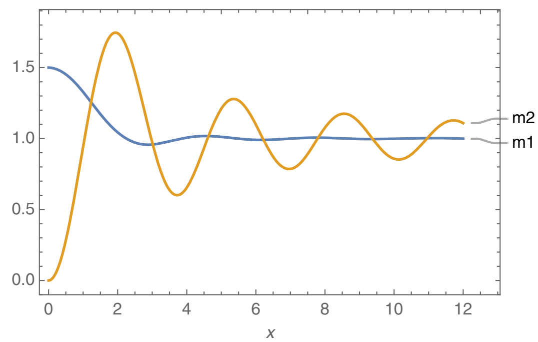


FIG. 2. $m_1 = 1 + m_{EB}^{\parallel}/m_{EB}^{(0)}$ and $m_2 = 1 + m_{EB}^{\perp}/m_{EB}^{(0)}$ as a function of the dimensionless $x = a(|\nu| - \Omega)$.

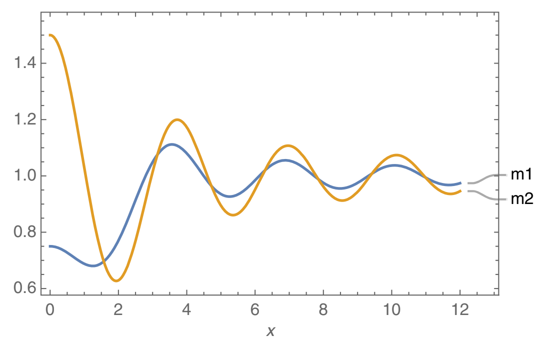


FIG. 3. Ratios $m_1 = 1 + m_{BB}^{\parallel}/m_{BB}^{(0)}$ and $m_2 = 1 + m_{BB}^{\perp}/m_{BB}^{(0)}$ as a function of the dimensionless $x = a(|\nu| - \Omega)$.

$m_{EB}^{\parallel}/m_{EB}^{(0)} = 1/2$, $m_{EB}^{\perp}/m_{EB}^{(0)} = -1$, and $m_{BB}^{\parallel}/m_{BB}^{(0)} = -1/4$ and $m_{BB}^{\perp}/m_{BB}^{(0)} = 1/2$ when $x \to 0$. These limits show different behavior with respect to the Dirichlet and Neumann cases reported in Ref. [7].

IV. TRANSITION AMPLITUDES

Let us now study the transition amplitudes and probabilities for the EM field model. The first-order transition matrix will now be given by

$$T_{fi} \equiv e \int dt \langle f|\mathbf{x}(t) \cdot [\mathbf{E}(t, \mathbf{r}(t)) + \dot{\mathbf{r}}(t) \times \mathbf{B}(t, \mathbf{r}(t))]|i\rangle, \tag{60}$$

where

$$|i\rangle = |i_a\rangle \otimes |i_{\mathrm{EM}}\rangle \quad |f\rangle = |f_a\rangle \otimes |f_{\mathrm{EM}}\rangle, \tag{61}$$

with the "a" and "EM" indices denoting the atom and electromagnetic field states.

For the electron's degrees of freedom, we have, in the interaction picture,

$$\mathbf{x}(t) = \frac{1}{\sqrt{2m\Omega}}(\mathbf{a}e^{-i\Omega t} + \mathbf{a}^\dagger e^{i\Omega t}). \tag{62}$$

Here, $\mathbf{a} = \sum_{l=1}^{3} a_l \hat{\mathbf{e}}_l$, where $\hat{\mathbf{e}}_l$ are three orthonormal vectors, since the Hamiltonian for the electron is essentially a three-dimensional harmonic oscillator. This implies that, when considering a transition from the vacuum to an excited state, that process will introduce a spatial direction, in other words, a polarization. On the other hand, for the gauge field in the Coulomb gauge, we use the expansion

$$\mathbf{A}(x) = \int d^2\mathbf{k}_\parallel \int_0^\infty dk_z \sum_\lambda [\alpha_\lambda(\mathbf{k})\mathbf{f}_{\mathbf{k}}^{(\lambda)}(x) + \mathrm{H.c.}], \tag{63}$$

where λ sums over the two independent modes for each $\mathbf{k}$, which are consistent with the perfect conductor condition at $z = 0$. In this gauge, that amounts to a vanishing, on that plane, of the components of the vector potential which are parallel to that surface.

Including a global factor to normalize the states, we may write those modes as follows (see, for example, Ref. [12]):

$$\begin{aligned}\mathbf{f}_{\mathbf{k}}^{(1)}(x) &= e^{-ikt}\sqrt{\frac{2}{(2\pi)^3 k}}(\hat{\mathbf{k}}_\parallel \times \hat{\mathbf{z}})\sin(k_z z)e^{i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel}\\ \mathbf{f}_{\mathbf{k}}^{(2)}(x) &= e^{-ikt}\sqrt{\frac{2}{(2\pi)^3 k}}k^{-1}[\hat{\mathbf{z}}|\mathbf{k}_\parallel|\cos(k_z z)\\ &\quad - i\hat{\mathbf{k}}_\parallel k_z \sin(k_z z)]e^{i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel}\end{aligned} \tag{64}$$

($\hat{\mathbf{k}}_\parallel$ and $\hat{\mathbf{z}}$ denote unit vectors). The notation

$$\mathbf{f}_{\mathbf{k}}^{(\lambda)}(x) = N_k e^{-ikt}\mathbf{g}_{\mathbf{k}}^{(\lambda)}(x), \qquad N_k = \sqrt{\frac{2}{(2\pi)^3 k}} \tag{65}$$

will be useful in what follows.

A. Decay process

Let us now consider a decay process, in which the initial state of the atom is an excited state and the EM field is in vacuum,

$$|i_a\rangle = a_l^\dagger|0_a\rangle \quad |i_{\rm EM}\rangle = |0_{\rm EM}\rangle, \tag{66}$$

while final states are

$$|f_a\rangle = |0_a\rangle \quad |f_{\rm EM}\rangle = \alpha_\lambda^\dagger(\mathbf{k})|0_{\rm EM}\rangle. \tag{67}$$

Note that for the electronic transition we have in principle three independent polarizations (not necessarily along the three coordinate axis), so we have to choose a polarization for the excited state of the electron and also for the final state of the EM field.

The matrix elements for the decay process then read

$$\begin{aligned} T_{fi}^{(\rm dec)}(\mathbf{k},l,\lambda) &= \frac{e}{\sqrt{2m\Omega}}\int_{-\infty}^{+\infty} dt e^{-it\Omega}\langle 0_{\rm EM}|\alpha_\lambda(\mathbf{k})\hat{\mathbf{e}}_l\cdot[\mathbf{E}(t,\mathbf{r}(t)) + \dot{\mathbf{r}}(t)\times\mathbf{B}(t,\mathbf{r}(t))]|0_{\rm EM}\rangle \\ &= \frac{e}{\sqrt{2m\Omega}}\int_{-\infty}^{+\infty} dt e^{-it\Omega}\hat{\mathbf{e}}_l\cdot[-\partial_t\mathbf{f}_{\mathbf{k}}^{(\lambda)*}(x) + \dot{\mathbf{r}}(t)\times(\boldsymbol{\nabla}\times\mathbf{f}_{\mathbf{k}}^{(\lambda)*}(x))]_{x=(t,\mathbf{r}(t))}. \end{aligned} \tag{68}$$

We now expand the results up to the second order in $\mathbf{y}(t)$, which is the departure from $\mathbf{r}_0 = (0,0,a)$. Note that, as the matrix elements have a contribution at zeroth order, it is necessary to expand them up to the second order, to compute consistently the decay probabilities beyond the static case. We denote the different orders by

$$T_{fi}^{(\rm dec)} = T_{fi}^{(\rm dec,0)} + T_{fi}^{(\rm dec,1)} + T_{fi}^{(\rm dec,2)}. \tag{69}$$

Performing the expansion, we obtain

$$\begin{aligned} T_{fi}^{(\rm dec,0)}(\mathbf{k},l,\lambda) &= \frac{-2\pi ieN_k k}{\sqrt{2m\Omega}}\delta(\Omega-k)\hat{\mathbf{e}}_l\cdot\mathbf{g}_{\mathbf{k}}^{(\lambda)*}(\mathbf{x})\Big|_{\mathbf{x}=\mathbf{r}_0} \\ T_{fi}^{(\rm dec,1)}(\mathbf{k},l,\lambda) &= \frac{eN_k}{\sqrt{2m\Omega}}\hat{\mathbf{e}}_l\cdot[(-ik)\tilde{y}_j(k-\Omega)\partial_j\mathbf{g}_{\mathbf{k}}^{(\lambda)*}(\mathbf{x}) \\ &\quad - i(k-\Omega)\tilde{\mathbf{y}}(k-\Omega)\times(\nabla\times\mathbf{g}_{\mathbf{k}}^{(\lambda)*}(\mathbf{x}))]\Big|_{\mathbf{x}=\mathbf{r}_0} \\ T_{fi}^{(\rm dec,2)}(\mathbf{k},l,\lambda) &= A_{fi}(\mathbf{k},l,\lambda) + B_{fi}(\mathbf{k},l,\lambda), \end{aligned} \tag{70}$$

with

$$\begin{aligned} A_{fi}(\mathbf{k},l,\lambda) &= \frac{-ieN_k k}{2\sqrt{2m\Omega}}\int_\infty^\infty dt e^{-it(\Omega-k)}y_i(t)y_j(t) \\ &\quad\times\hat{\mathbf{e}}_l\cdot\partial_i\partial_j\mathbf{g}_{\mathbf{k}}^{(\lambda)*}(\mathbf{x})\Big|_{\mathbf{x}=\mathbf{r}_0} \\ B_{fi}(\mathbf{k},l,\lambda) &= \frac{(-1)^\lambda ieN_k k}{\sqrt{2m\Omega}}\int_\infty^\infty dt e^{-it(\Omega-k)}y_j(t)\dot{\mathbf{y}}(t) \\ &\quad\cdot(\hat{\mathbf{e}}_l\times\partial_j\mathbf{g}_{\mathbf{k}}^{(\lambda')*}\left(\mathbf{x}_\parallel, z-\frac{\pi}{2k_z}\right)\Big|_{\mathbf{x}=\mathbf{r}_0}. \end{aligned} \tag{71}$$

Here, we used the notation $\lambda' = 1$ for $\lambda = 2$ and vice versa.

In practice, we expect the experiments not to detect the polarization state of the excited state of the atom; therefore, it makes sense to consider the sum over the three possible values of l when evaluating the probabilities. Namely, we obtain results for the probabilities which depend only the polarization of the photon. Therefore, for an unpolarized initial state, the total decay probability reads

$$dP_{fi}^{\rm dec}(\mathbf{k}) = dP_{fi}^{\rm dec}(\mathbf{k},1) + dP_{fi}^{\rm dec}(\mathbf{k},2) \tag{72}$$

with

$$dP_{fi}^{\rm dec}(\mathbf{k},\lambda) = \frac{1}{3}d^3\mathbf{k}\sum_{l=1}^{3}|T_{fi}^{\rm dec}(\mathbf{k},l,\lambda)|^2. \tag{73}$$

Note that, when computing $|T_{fi}^{\rm dec}(\mathbf{k},l,\lambda)|^2$, there will be a contribution of zeroth order that gives the emission probability for a static atom. By energy conservation, this probability is proportional to $\delta(\Omega-k)$. It is corrected by the second-order contribution to the matrix element. On the other hand, the first-order contribution to the transition amplitude produces an emission probability that, for a center-of-mass oscillation with frequency $\Omega_{\rm cm}$, has lateral peaks at $k = \Omega\pm\Omega_{\rm cm}$. This is the main qualitative change induced by the center-of-mass motion on the spectrum of emitted photons.

We now assume a normal motion for the center-of-mass of the atom, with $\tilde{y}_j = \tilde{y}_\perp\delta_{j3}$. The zeroth-order contribution $T_{fi}^{(dec,0)}$ in Eq. (70) generates the spontaneous emission probability for a static atom. It reads

$$dP_{fi}^{(\rm dec,0)}(\mathbf{k}) = \frac{Te^2N_k^2}{2m\Omega}2\pi\delta(\Omega-k)k^2\sum_\lambda|\mathbf{g}_{\mathbf{k}}^{(\lambda)}(\mathbf{r}_0)|^2d^3\mathbf{k}. \tag{74}$$

The second-order contribution in Eq. (70), when multiplied by the zeroth order, produces a correction to the static probability in Eq. (74) that is given by

$$\begin{aligned} dP_{fi}^{(\rm sta,2)}(\mathbf{k}) &= -\frac{e^2N_k^2}{2m\Omega}2\pi\delta(\Omega-k)k^2k_z^2\sum_\lambda|\mathbf{g}_{\mathbf{k}}^{(\lambda)}(\mathbf{r}_0)|^2 \\ &\quad\times\left(\int_{-\infty}^\infty dty_\perp^2(t)\right)d^3\mathbf{k}. \end{aligned} \tag{75}$$

Note that the position of the peak is independent of the center-of-mass motion of the atom.

We now consider the novel contribution to the emission probability coming from $T_{fi}^{(dec,1)}$ in Eq. (70). It is given by

$$dP_{fi}^{(\mathrm{dyn},2)}(\mathbf{k},1) = \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k-\Omega)|^2\Omega^2 k^3\cos^2(ka\cos\theta)) \times \cos^2\theta\sin\theta d\theta dk \equiv \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k-\Omega)|^2 k^3 p_1(ka,\Omega a,\theta) \times \sin\theta d\theta dk \quad (76)$$

and

$$dP_{fi}^{(\mathrm{dyn},2)}(\mathbf{k},2) = \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k-\Omega)|^2 \times k^3[k^2\sin^2\theta\cos^2\theta\sin^2(ka\cos\theta) + (\Omega - k\sin^2\theta)^2\cos^2(ka\cos\theta)]\sin\theta d\theta dk \equiv \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k-\Omega)|^2 \times k^3 p_2(ka,\Omega a,\theta)\sin\theta d\theta dk. \quad (77)$$

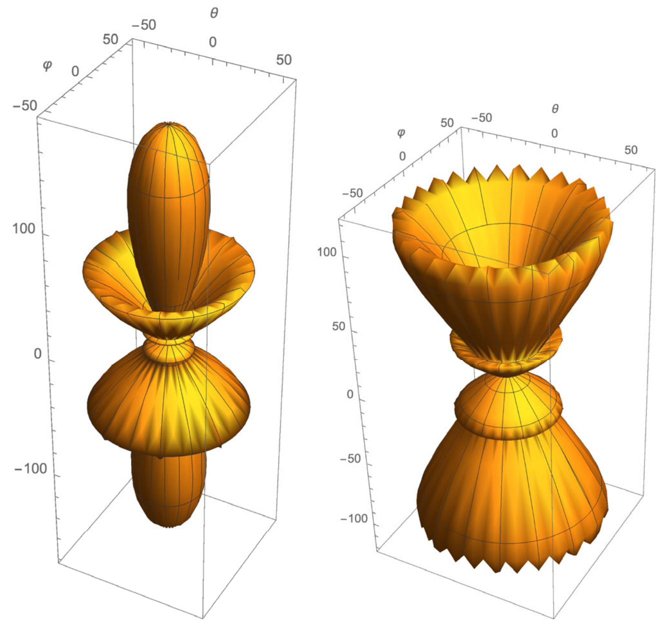
In both equations, we have used spherical coordinates in $\mathbf{k}$ space and integrated over the angle φ (by symmetry, the results do not depend on this angle). As with the correction in Eq. (75), these are of course contributions quadratic in the amplitude of the center-of-mass motion. In Fig. 4, we plot the total contribution to the emission probability $p_1(ka,\Omega a,\theta) + p_2(ka,\Omega a,\theta)$ per unit of solid angle for two different values of ka at a fixed value of $\Omega a = 10$.

The total decay probability can be obtained by summing Eqs. (76) and(77) and integrating the θ angle. The result is

$$\frac{dP_{fi}^{(\mathrm{dyn},2)}}{dk} = \frac{e^2}{144\pi^2 m\Omega a^5}|\tilde{y}_\perp(k-\Omega)|^2 \times \{8a^5k^3(k^2 - 2k\Omega + 2\Omega^2) + 6ak(a^2(k+\Omega)^2 - 6)\cos(2ak) + 3(4a^4k^2\Omega^2 - a^2(9k^2 + 2k\Omega + \Omega^2) + 6) \times \sin(2ak)\}. \quad (78)$$

If we assume that the normal displacement $y_\perp(t)$ is an oscillatory function $y_\perp(t) = y_\perp^0\sin(\Omega_{\mathrm{cm}}t)$, where Ω_{cm} is the frequency of the center of mass, the spectrum of the emitted photons has peaks at $\Omega, \Omega \pm \Omega_{\mathrm{cm}}$. In Fig. 4, we choose two different values of ka [$ka = 9$ in Fig. 4(a) and $ka = 11$ in Fig. 4(b)] in order to show that just beyond adiabatic approximation ($ka \approx \Omega a$) the emission probability is nonsymmetric with respect to the central emission peak.

In a recent work [10], the decay probability of an atom in front of an oscillating mirror has been computed using the adiabatic approximation $\Omega \gg \Omega_{\mathrm{cm}}$. In this limit, our results



(a) $ka = 9, \Omega a = 10$ (b) $ka = 11, \Omega a = 10$

FIG. 4. Total contribution to the emission probability $p_1(ka,\Omega a,\theta) + p_2(ka,\Omega a,\theta)$ per unit of solid angle as a function of spherical angle θ. We show two different values of ka at a fixed value of $\Omega a = 10$.

for the moving atom have the same structure: the spectrum of the emitted photons has the above-mentioned peaks, the decay probability decreases with the distance to the plane, and it shows oscillations with a frequency $2ka \simeq 2\Omega a$. There is a disagreement, however, between the coefficients of the terms appearing in our result and the ones of Ref. [10]. We have verified that this difference comes from the Röntgen term. Indeed, omitting this contribution, both results coincide. For the case of a moving atom, it is well known that this interaction term is crucial for Lorentz covariance. It would be interesting to check if it also appears for the case of a moving mirror and static atom, in the next to leading order of the adiabatic approximation. Note that the boundary conditions for the electromagnetic field on a moving perfect mirror have a velocity-dependent term [13].

The adiabatic approximation is justified when $\Omega \gg \Omega_{\mathrm{cm}}$. It is noteworthy that, for some physical systems, this inequality may be violated: for Rydberg or artificial atoms may have Ω of the order of GHz, and mechanical resonators may attain such frequencies. In this situation, the decay probabilities may be qualitatively different from those obtained when the adiabatic approximation is used.

B. Excitation process

We now consider the probability of excitation of an atom that is initially in its ground state. This excitation is accompanied by the emission of a photon. The initial states read

$$|i_a\rangle = |0_a\rangle \quad |i_{\rm EM}\rangle = |0_{\rm EM}\rangle, \tag{79}$$

while final states are

$$|f_a\rangle = a_l^\dagger|0_a\rangle \quad |f_{\rm EM}\rangle = \alpha_\lambda^\dagger(\mathbf{k})|0_{\rm EM}\rangle. \tag{80}$$

It is not necessary to repeat all calculations. Indeed, the matrix elements for the excitation process can be obtained from those of the decay process just changing the sign of the frequency Ω, which takes into account the changes in the initial and final states of the atom. This change of the sign produces the expected threshold for the center-of-mass frequency, and the excitation occurs only above it. Therefore, the zeroth order in the transition amplitudes is absent for this process.

From Eqs. (76) and (77), we obtain

$$dP_{fi}^{\rm exc}(\mathbf{k},1) = \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k+\Omega)|^2\Omega^2 k^3\cos^2(ka\cos\theta)) \times \cos^2\theta\sin\theta d\theta dk \tag{81}$$

and

$$dP_{fi}^{\rm exc}(\mathbf{k},2) = \frac{e^2}{12\pi^2 m\Omega}|\tilde{y}_\perp(k+\Omega)|^2 \times k^3[k^2\sin^2\theta\cos^2\theta\sin^2(ka\cos\theta) + (\Omega+k\sin^2\theta)^2\cos^2(ka\cos\theta)]\sin\theta d\theta dk \tag{82}$$

As before, in both equations, we have used spherical coordinates in $\mathbf{k}$ space and integrated over the angle φ.

V. CONCLUSIONS

In this paper, we considered the interaction between an accelerated atom near a perfect mirror and the vacuum fluctuations of the electromagnetic field. We have computed the vacuum persistence probability, and then the probabilities for excitation and decay, for an atom that is initially in its ground or first excited state, respectively. The results generalize our previous work in which we studied, as a toy model, a quantum scalar field instead of the full electromagnetic field.

We have compared our results for an atom in perpendicular motion with respect to the mirror, with those in which the atom is at rest and the mirror is oscillating. Up to the lowest-order adiabatic approximation, the Röntgen current does not appear for a moving mirror [10], and this is a source of discrepancy between the results for both situations. It would be interesting to check whether the next-to-leading-order adiabatic correction for the case of a moving mirror restores the equivalence between these two different physical situations or not.

Our results for the moving atom are valid beyond the adiabatic approximation, and we have pointed out that, for artificial or Rydberg atoms, this approximation may be violated. Therefore, one could observe signs of nonadiabaticity in the spectrum of emitted particles.

If the atom has a center-of-mass motion parallel to the mirror, the excitation and deexcitation will depend both on the acceleration and the distance to the mirror. Although in this paper we have not presented an analysis of the transition amplitudes for the parallel motion, the presence of dissipative effects is clear from the computation of the imaginary part of the effective action. These effects have no analogs for a static atom in front of a moving (perfect) mirror.

ACKNOWLEDGMENTS

This research was supported by Agencia Nacional de Promoción Científica y Tecnológica, Consejo Nacional de Investigaciones Científicas y Técnicas, Universidad de Buenos Aires, and Universidad Nacional de Cuyo. The research of F. C. L. was supported in part by the National Science Foundation under Grant No. PHY-1748958.

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