Maxwell field with gauge fixing term in de Sitter space: Exact solution and stress tensor

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The Maxwell field with a general gauge fixing (GF) term is nontrivial: not only the longitudinal and temporal modes are mixed up in the field equations, but also unwanted consequences might arise from the GF term. We derive the complete set of solutions in de Sitter space, and implement the covariant canonical quantization which restricts the residual gauge transformation down to a quantum residual gauge transformation. Then, in the Gupta-Bleuler (GB) physical state, we calculate the stress tensor which is amazingly independent of the gauge fixing constant and is also invariant under the quantum residual gauge transformation. The transverse components are simply the same as those in the Minkowski spacetime, and the transverse vacuum stress tensor has only one UV divergent term ($\propto k^4$), which becomes zero by the zeroth-order adiabatic regularization. The longitudinal-temporal stress tensor in the GB state is zero due to a cancellation between the longitudinal and temporal parts. More interesting is the stress tensor of the GF term. Its particle contribution is zero due to the cancellation in the GB state, and its vacuum contribution is twice that of a minimally coupling massless scalar field, containing k^4 and k^2 divergences. After the second-order adiabatic regularization, the GF vacuum stress tensor becomes zero too, so that there is no need to introduce a ghost field, and the zero GF vacuum stress tensor cannot be a possible candidate for the cosmological constant. Thus, all the physics predicted by the Maxwell field with the GF term will be the same as that without the GF term. We also carry out analogous calculation in the Minkowski spacetime, and the stress tensor is similar to, but simpler than that in de Sitter space.

I. INTRODUCTION

The Maxwell field is well studied as a quantum field in the flat spacetime. The canonical quantization is simple in radiation gauge in which the temporal and longitudinal components are set zero; only two transverse polarizations remain as dynamical variables. However, when a general gauge fixing (GF) term is introduced for a covariant canonical quantization, the longitudinal and temporal modes, A and A_0, are mixed up in their field equations, and the solutions are nontrivial except in the Feynman gauge. In curved spacetimes the mixing-up of A and A_0 occurs even in the Feynman gauge. More seriously, the introduced GF term gives rise to a part of a stress tensor, which would bring about unwanted consequences. Conventionally, a ghost field is introduced [1,2] to cancel out the GF stress tensor, so that the net result will be a zero stress tensor, and no unphysical consequence will occur. In another treatment [3,4], the GF term was used to play a role of the cosmological constant. However, the vacuum GF

stress tensor is UV divergent, and must be regularized before considering its possible physical implication. Reference [5] adopted Dirac's approach to a constrained system to study the Maxwell field (without the GF term) in a general RW spacetime, and calculated the Hamiltonian. But the noncovariant Hamiltonian is not the same as the stress tensor, and the UV divergences and regularization were not addressed either.

In this paper, we shall derive the complete set of solutions of the Maxwell field with a general GF term in de Sitter space, and reveal the interesting structure of the solutions. With these, we shall implement the covariant canonical quantization, and obtain its constraint on the coefficients of solution modes, as well as its restriction on the residual gauge transformation. Then we shall calculate respectively the transverse stress tensor, the longitudinal and temporal stress tensor in the Gupta-Bleuler (GB) state, and the stress tensor due to the GF term in the GB physical state [6–8], and demonstrate the UV divergences of the vacuum stress tensor. Finally, we shall perform the adiabatic regularization on the vacuum stress tensor, and show that the resulting regularized vacuum stress tensor is zero, so that all the predicted physics of the Maxwell

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field in de Sitter space is the same, with or without the GF term.

The paper is organized as follows. In Sec. II we derive, by two methods, the solutions of the Maxwell field with a general GF term in de Sitter space. In Sec. III we present the covariant canonical quantization. In Sec. IV, we calculate all three parts of the stress tensor. In Sec. V, we perform the adiabatic regularization of the vacuum stress tensor, and Sec. VI gives the conclusion and discussions. Appendix A gives the Green's functions for the Maxwell field in the Feynman gauge, and demonstrates its relation to the Green's function of a minimally coupling massless scalar field. In Appendix B, we give analogous calculation of the Maxwell field with a general GF term in the Minkowski spacetime, which has not been fully reported in literature. We shall use the units ($\hbar = c = 1$).

II. THE SOLUTIONS OF THE MAXWELL EQUATIONS WITH ζ IN DE SITTER SPACE

In the free Maxwell field theory, the longitudinal and temporal components, $\mathbf{A}_{\parallel}$ and A_0, are not real radiative dynamical degrees of freedom. A simple treatment is to take the Coulomb (radiation) gauge, in which the longitudinal and temporal components are set to be zero, $\nabla \cdot \mathbf{A} = 0 = A_0$, and the canonical quantization is performed only on the transverse parts. The treatment in the Coulomb gauge is not explicitly covariant. To achieve the covariant canonical quantization, one can introduce a GF term, so that the canonical momenta are not identically zero, and all four components A_μ can be regarded as being dynamical variables without the Lorenz condition. Nevertheless, the GF term will cause A and A_0 to mix up in their field equations, and the solution is nontrivial. In this section we shall derive the solution of A_μ and the corresponding canonical momenta of the Maxwell field with the GF term in de Sitter space.

The metric of a flat Robertson-Walker (RW) spacetime is written as

$$ds^2 = a^2(\tau)[-d\tau^2 + \delta_{ij}dx^i dx^j], \tag{1}$$

which is conformal to the Minkowski spacetime, with τ being the conformal time. The Lagrangian density of the Maxwell field with a GF term in RW spacetimes is [1]

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2\zeta}(A^\nu{}_{;\nu})^2\right), \tag{2}$$

where $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, and ζ is the gauge fixing constant. The field equation of A_μ is

$$F^{\mu\nu}{}_{;\nu} + \frac{1}{\zeta}(A^\nu{}_{;\nu})^{;\mu} = 0. \tag{3}$$

Applying the covariant four-divergence upon Eq. (3) gives $\Box(A^\nu{}_{;\nu}) = 0$, where $\Box \equiv -\frac{1}{a^4}\frac{\partial}{\partial\tau}(a^2\frac{\partial}{\partial\tau}) + \frac{1}{a^2}\nabla^2$, so, $(A^\nu{}_{;\nu})$

satisfies the equation of a minimally coupling massless scalar field. In this paper, all four components A_μ will be formally regarded as dynamical field variables, and the Lorenz condition will not be imposed as a condition on the field operators. The equation (3) is written as

$$\eta^{\sigma\rho}\partial_\sigma\partial_\rho A_\mu + \left(\frac{1}{\zeta} - 1\right)\partial_\mu(\eta^{\rho\sigma}\partial_\sigma A_\rho)$$
$$+ \frac{1}{\zeta}[\delta_{\mu 0}(-D\eta^{\rho\sigma}\partial_\sigma A_\rho + D^2 A_0 - D'A_0) - D\partial_\mu A_0] = 0, \tag{4}$$

where $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, $D \equiv 2a'(\tau)/a(\tau)$. The i-component A_i is decomposed into

$$A_i = B_i + \partial_i A, \tag{5}$$

where $\partial_i B_i = 0$ and A is a scalar function and $\partial_i A$ is the longitudinal. The canonical momenta are defined by

$$\pi_A^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)} = \eta^{\mu\sigma}(\partial_0 A_\sigma - \partial_\sigma A_0) - \frac{1}{\zeta}\eta^{0\mu}(\eta^{\rho\sigma}\partial_\sigma A_\rho - DA_0), \tag{6}$$

its 0 component is contributed by the GF term,

$$\pi_A^0 = \frac{1}{\zeta}a^2 A^\nu{}_{;\nu} = \frac{1}{\zeta}(-(\partial_0 + D)A_0 + \nabla^2 A), \tag{7}$$

and the i component is

$$\pi_A^i = \delta^{ij}(\partial_0 A_j - \partial_j A_0) = w^i + \partial^i \pi_A, \tag{8}$$

where $w^j = \partial_0 B_j$ is transverse, and

$$\pi_A = \partial_0 A - A_0 \tag{9}$$

is a scalar function and its gradient $\partial^i \pi_A$ is the longitudinal. For convenience, in the rest of this section, we shall work with the Fourier k modes of the fields and the canonical momenta, for instance, $B_i(x) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}B_{ik}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}$, etc. To avoid the cumbersome notation of subindex k, we also use $B_i, A, A_0, \pi_A, \pi_A^0$ to represent their k modes whenever no confusion arises in the following. Then, with $\nabla^2 = -k^2$, the k mode of (7) is written as

$$\pi_A^0 = -\frac{1}{\zeta}((\partial_0 + D)A_0 + k^2 A). \tag{10}$$

Equation (4) is decomposed into the following equations in the k space:

$$\partial_0^2 B_i + k^2 B_i = 0, \tag{11}$$

$$-\partial_0^2 A - \frac{1}{\zeta}k^2 A + \left(1 - \frac{1}{\zeta}\right)\partial_0 A_0 - \frac{1}{\zeta}DA_0 = 0, \quad (12)$$

$$-\frac{1}{\zeta}\partial_0^2 A_0 - k^2 A_0 + \frac{1}{\zeta}(D^2 - D')A_0$$
$$+ k^2\left(\left(1 - \frac{1}{\zeta}\right)\partial_0 A + \frac{1}{\zeta}DA\right) = 0, \quad (13)$$

where B_i, A, and A_0 stand for their k modes. The transverse equations (11) are separated from A and A_0, unaffected by the gauge fixing parameter, and, each i component of the k-mode B_i has the positive frequency solution of the following form:

$$B_i(\tau) \propto f_k^{(\sigma)}(\tau) = \frac{1}{\sqrt{2k}}e^{-ik\tau}, \quad (14)$$

where the solution modes $f_k^{(\sigma)}$ are the same for two transverse polarizations $\sigma = 1, 2$ [see (57) (60) for a precise expression of B_i.]

Equation (11) and the solution (14) are independent of the scale factor $a(\tau)$, and hold for a general RW spacetime, including de Sitter space and Minkowski spacetime.

Equations (12) and (13) are the basic second-order differential equations of A and A_0 for a general ζ, in which A and A_0 are mixed up. Even in the Feynman gauge ($\zeta = 1$), (12) and (13) become

$$-\partial_0^2 A - k^2 A - DA_0 = 0, \quad (15)$$

$$-\partial_0^2 A_0 - k^2 A_0 + (D^2 - D')A_0 + k^2 DA = 0, \quad (16)$$

where A and A_0 are still mixed up. [When $D = 0$, Eqs. (15) and (16) reduce to Eqs. (B18) and (B19) in the Minkowski spacetime that is most discussed in literature, and A and A_0 are separate.]

We shall solve Eqs. (12) and (13) with a general ζ in the following. By differentiations and algebraic combinations of (12) and (13), we get two fourth-order differential equations

$$\left[\left(1 - \frac{1}{\zeta}\right)\partial_0 - \frac{1}{\zeta}D\right]\left(\frac{((\zeta - 1)\partial_0^3 + D\partial_0^2 + k^2(\zeta - 1)(2 - \zeta)\partial_0 + (2 - \zeta)k^2 D)A}{(\zeta - 2)D^2 - (\zeta - 1)D' - (\zeta - 1)^2 k^2}\right) - \left(\partial_0^2 + \frac{1}{\zeta}k^2\right)A = 0 \quad (17)$$

and

$$\left[\left(1 - \frac{1}{\zeta}\right)\partial_0 + \frac{1}{\zeta}D\right]\left(\left[(\zeta - 1)D' - D^2 - (\zeta - 1)^2\frac{1}{\zeta}k^2\right]^{-1}\left[(\zeta - 1)\partial_0^3 A_0 - D\partial_0^2 A_0 + (\zeta - 1)\left(D' - D^2 - \frac{1 - 2\zeta}{\zeta}k^2\right)A_0'\right.\right.$$
$$\left.\left.+ (\zeta - 1)(D'' - 2DD')A_0 + D(D^2 - D')A_0 + \frac{1 - 2\zeta}{\zeta}k^2 DA_0\right]\right) - \frac{1}{\zeta}\partial_0^2 A_0 - k^2 A_0 + \frac{1}{\zeta}(D^2 - D')A_0 = 0, \quad (18)$$

which are separate for A and A_0, and valid for $\zeta \neq 1$. [When $D = 0$, Eqs. (17) and (18) reduce to Eqs. (B6) and (B7) in Minkowski spacetime.]

In this paper we consider de Sitter space; the scale factor is

$$a(\tau) = -\frac{1}{H\tau}, \qquad -\infty < \tau \leq \tau_1, \quad (19)$$

where H is a constant and τ_1 is the ending time of de Sitter inflation, $D = -2/\tau$. Dropping an overall factor $\propto (1 - \zeta)^2$, Eqs. (17) and (18) become

$$[(\zeta - 1)^2 k^2\tau^2 - 2(\zeta - 3)]\tau^2 A^{(4)}(\tau) - 4(\zeta - 3)\tau A^{(3)}(\tau) + 2[(\zeta - 1)^2 k^4\tau^4 - (\zeta + 1)(\zeta - 3)k^2\tau^2 + 2(\zeta - 3)]A''(\tau)$$
$$+ 4(\zeta - 2)(\zeta - 3)k^2\tau A'(\tau) + [(\zeta - 1)^2 k^4\tau^4 - 2\zeta(\zeta - 3)k^2\tau^2 - 4(\zeta - 3)]k^2 A(\tau) = 0 \quad (20)$$

and

$$[(\zeta - 1)^2 k^2\tau^2 - 2(\zeta - 3)\zeta]\tau^4 A_0^{(4)}(\tau) - 4(\zeta - 3)\zeta\tau^3 A_0^{(3)}(\tau) + [2(\zeta - 1)^2 k^4\tau^4 + 2(-3\zeta^2 + 6\zeta + 1)k^2\tau^2 + 4(\zeta - 3)\zeta]\tau^2 A_0''(\tau)$$
$$+ 4[(3\zeta - 1)k^2\tau^2 - 2(\zeta - 3)\zeta]\tau A_0'(\tau) + [(\zeta - 1)^2 k^6\tau^6 + 2(-2\zeta^2 + 3\zeta + 1)k^4\tau^4 + 4(-2\zeta^2 + 3\zeta + 1)k^2\tau^2$$
$$+ 8\zeta(\zeta - 3)]A_0(\tau) = 0 \quad (21)$$

where $A^{(4)}(\tau) \equiv \partial^4 A / \partial \tau^4$, $A^{(3)}(\tau) \equiv \partial^3 A / \partial \tau^3$, etc. These are fourth-order differential equations of A_0 and A, valid for a general ζ. Setting $\zeta = 1$, they reduce to the fourth-order differential equations in the Feynman gauge

$$\tau^2 A^{(4)} + 2\tau A^{(3)} + 2(k^2\tau^2 - 1)A'' + 2k^2\tau A'$$
$$+ k^2(k^2\tau^2 + 2)A = 0, \qquad (22)$$

$$\tau^4 A_0^{(4)} + 2\tau^3 A_0^{(3)} + 2\tau^2(k^2\tau^2 - 1)A_0'' + 2(k^2\tau^2 + 2)\tau A_0'$$
$$+ (k^4\tau^4 + 2k^2\tau^2 - 4)A_0 = 0. \qquad (23)$$

The positive frequency solutions of Eqs. (20) and (21) for a general ζ are obtained as

$$A = b_1 \frac{1}{a(\tau)} \frac{i}{k} \left(1 - \frac{i}{k\tau}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau} - b_2 \frac{(3 - \zeta)(k\tau + i)e^{2ik\tau}\mathrm{Ei}(-2ik\tau) - 3i + 2i\zeta}{3k} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (24)$$

$$A_0 = b_1 \frac{1}{a(\tau)} \frac{1}{\sqrt{2k}} e^{-ik\tau} - b_2 \frac{(3i - i\zeta)k^2\tau^2 e^{2ik\tau}\mathrm{Ei}(-2ik\tau) + \zeta(k\tau - i)}{3k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (25)$$

where $\mathrm{Ei}(z) \equiv -\int_{-z}^{\infty} t^{-1} e^{-t} dt$ is the exponential-integral function, and the coefficients b_1, b_2 are dimensionless complex constants. [References [3,4] gave a solution which seems to correspond to the special case $\zeta = 1$ of our (24) and (25).] We have chosen the same set of coefficients (b_1, b_2) for A and A_0 so that they satisfy the basic second-order equations (12) and (13). At the classical level, (b_1, b_2) are arbitrary. The b_1 part will be referred to as the homogeneous solution, and the b_2 part as the inhomogeneous solution, and the terminologies "homogeneous" and "inhomogeneous" will be clear later around (36)–(45). The complex conjugates of (24) and (25) are the independent, negative frequency solutions. Although A and A_0 respectively have four solutions (the Wronskians being nonzero), but A and A_0 in (24) and (25) share the same set (b_1, b_2). We have checked that the respective homogeneous and inhomogeneous parts in (24) and (25) satisfy the basic second-order equations (12) and (13), as well as the fourth-order equations (20) and (21). When setting $\zeta = 1$, (24) and (25) reduce to the solutions of (22) and (23) in the Feynman gauge. Plugging (24) and (25) into the definitions (9) and (10) gives the canonical momenta

$$\pi_A = -b_2 \frac{i}{k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau} = b_2 \frac{iH}{k} a(\tau) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (26)$$

$$\pi_A^0 = b_2 \frac{i - k\tau}{k\tau^2} \frac{1}{\sqrt{2k}} e^{-ik\tau} = b_2 H a(\tau) \left(1 - \frac{i}{k\tau}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (27)$$

which are contributed only by the inhomogeneous part of A and A_0, and are independent of ζ. It should be remarked that the positive frequency ($\propto e^{-ik\tau}$) modes (24), (25), (26), (27) will not evolve into the negative frequency modes ($\propto e^{ik\tau}$) during the de Sitter expansion prescribed by (1) and (19). Note that the dimension $[A_0] = k[A]$ and $[\pi_A^0] = k[\pi_A]$.

The solutions (24) and (25) in the de Sitter space will reduce to the solutions in the Minkowski spacetime. But, if one naively took $a = 1$ and high k in (24) and (25), one would come up with an incorrect claim that the Minkowski limit can be obtained at only for $\zeta = -3$. In fact, $a(\tau)$ and its time derivatives are implicit in (24) and (25). An appropriate procedure of taking the limit of the Minkowski spacetime is the following: Setting $D = 0$ in Eqs. (17) and (18) leads to Eqs. (B6) and (B7) in Minkowski spacetime, and the solutions are listed in Appendix B.

The solutions (24), (25), (26), (27) can also be derived in another way as the following. First, by applying ∂_0 and combinations on the basic equations (12) and (13), we arrive at the equations of π_A and π_A^0,

$$(\partial_0^2 - D\partial_0 + k^2)\pi_A = 0, \qquad (28)$$

$$(\partial_0^2 - D\partial_0 - D' + k^2)\pi_A^0 = 0, \qquad (29)$$

which are independent of ζ. By rescaling $\pi_A = a\bar{\pi}_A$, $\pi_A^0 = a\bar{\pi}_A^0$, Eqs. (28) and (29) become

$$\bar{\pi}_A'' + k^2 \bar{\pi}_A = 0, \qquad (30)$$

$$\bar{\pi}_A^{0\,''} + \left(k^2 - \frac{2}{\tau^2}\right) \bar{\pi}_A^0 = 0, \qquad (31)$$

and the normalized solutions are

$$\bar{\pi}_A = b_2 H \frac{i}{k} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (32)$$

$$\bar{\pi}_A^0 = b_2 H \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}. \qquad (33)$$

Multiplying the above by $a(\tau)$ gives the solutions (26) and (27). Note that Eq. (30) of $\bar{\pi}_A$ is the same as the equation of a rescaled conformally coupling massless scalar field, and Eq. (31) of $\bar{\pi}_A^0$ is the same as the equation of a rescaled minimally coupling massless scalar field [9]. Next, applying ∂_0 on the definitions (9) and (10) and by combinations, we arrive at

$$\partial_0^2 A + D\partial_0 A + k^2 A = (\partial_0 + D)\pi_A - \zeta\pi_A^0, \quad (34)$$

$$\partial_0^2 A_0 + DA_0' + D'A_0 + k^2 A_0 = -(k^2\pi_A + \zeta\partial_0\pi_A^0), \quad (35)$$

which are the second-order differential equations of A and A_0 with the nonhomogeneous term as the source. [The homogeneous equations of (34) (35) are just the equations of A and A_0 of Maxwell theory without the GF term under the Lorenz condition $A^\mu_{;\mu} = 0$.] By rescaling $A_0 = \frac{1}{a}\bar{A}_0$ and $A = \frac{1}{a}\bar{A}$, Eqs. (35) and (34) become

$$\bar{A}'' + \left(k^2 - \frac{2}{\tau^2}\right)\bar{A} = \Pi(\tau), \quad (36)$$

$$\bar{A}_0'' + k^2\bar{A}_0 = \Pi_0(\tau), \quad (37)$$

where the nonhomogeneous terms are

$$\Pi(\tau) \equiv a((\partial_0 + D)\pi_A - \zeta\pi_A^0), \quad (38)$$

$$\Pi_0(\tau) \equiv -a(k^2\pi_A + \zeta\partial_0\pi_A^0), \quad (39)$$

which are known from the given π_A and π_A^0. The homogeneous solutions of (36) and (37) are simply given by

$$\bar{A}_h(\tau) = \frac{i}{k}\frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}, \quad (40)$$

$$\bar{A}_{0h}(\tau) = \frac{1}{\sqrt{2k}}e^{-ik\tau}, \quad (41)$$

which correspond to the b_1 part of the solutions (24) and (25), and the Wronskians are

$$W[\tau] = \bar{A}_h\bar{A}_h^{*\prime} - \bar{A}_h'\bar{A}_h^* = \frac{i}{k^2}, \quad (42)$$

$$W_0[\tau] = \bar{A}_{0h}\bar{A}_{0h}^{*\prime} - \bar{A}_{0h}'\bar{A}_{0h}^* = i. \quad (43)$$

Interestingly, the homogeneous equation (36) and the solution (40) of $\bar{A}_h$ are similar to (31) and (33) of $\bar{\pi}_A^0$, and, the homogeneous equation (37) and the solution (41) of $\bar{A}_{0h}$ are similar to (30) and (32) of $\bar{\pi}_A$ [9]. By the standard formulas of the inhomogeneous equations, we obtain the inhomogeneous solution of (36) and (37)

$$\bar{A}(\tau) = -\bar{A}_h(\tau)\int^\tau d\tau'\frac{\Pi(\tau')\bar{A}_h^*(\tau')}{W} + \bar{A}_h^*(\tau)\int^\tau d\tau'\frac{\Pi(\tau')\bar{A}_h(\tau')}{W},$$
$$= b_2\frac{((3-\zeta)(k\tau+i)e^{2ik\tau}\text{Ei}(-2ik\tau) - 3i + 2i\zeta)}{3Hk\tau}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \quad (44)$$

$$\bar{A}_0(\tau) = \bar{A}_{0h}(\tau)\int^\tau d\tau'\frac{-\Pi_0(\tau')\bar{A}_{0h}^*(\tau')}{W_0} + \bar{A}_{0h}^*(\tau)\int^\tau d\tau'\frac{\Pi_0(\tau')\bar{A}_{0h}(\tau')}{W_0}$$
$$= b_2\frac{((3i-i\zeta)k^2\tau^2 e^{2ik\tau}\text{Ei}(-2ik\tau) + \zeta(k\tau - i))}{3Hk\tau^2}\frac{1}{\sqrt{2k}}e^{-ik\tau}. \quad (45)$$

After rescaling by $1/a(\tau)$, the sum of (40) and (44) recovers the solution A in (24), and the sum of (41) and (45) recovers the solution of A_0 in (25). As we shall see later, the complicated, inhomogeneous parts of A and A_0 will simply cancel in the expectation value of the stress tensor.

We analyze the gauge transformations of the Maxwell field, and examine the consequential changes on the solutions. The Maxwell field without the GF term is invariant under the gauge transformation $A_\mu \to A_\mu' \equiv A_\mu + \theta_{,\mu}$ with θ being an arbitrary scalar function; each component transforms as

$$B_i \to B_i' = B_i,$$
$$A \to A' \equiv A + \theta,$$
$$A_0 \to A_0' \equiv A_0 + \theta_{,0}.$$

When the GF term $\propto (\nabla^\mu A_\mu)^2$ is present, the Lagrangian (2) and the field equation (3) are invariant only under a residual gauge transformation with θ satisfying the following equation:

$$\Box\theta \equiv \nabla^\nu\nabla_\nu\theta = 0. \quad (46)$$

This is also the equation of a minimally coupling massless scalar field [9], and its k-mode equation is

$$\theta_k'' + D\theta_k' + k^2\theta_k = 0. \tag{47}$$

In de Sitter space the k-mode solution is

$$\theta_k(\tau) = C\frac{1}{a(\tau)}\frac{i}{k}\left(1 - \frac{i}{k\tau}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \tag{48}$$

with C being an arbitrary complex constant. The function θ_k in (48) is of the same form as the homogeneous solution A_h of (24), and its time derivative is

$$\theta_{k,0} = C\frac{1}{a(\tau)}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \tag{49}$$

whose form is the same as the homogeneous solution A_{0h} of (25). Thus, under the residual gauge transformation, the longitudinal and temporal k modes transform as

$$A_k \to A_k + C\frac{1}{a(\tau)}\frac{i}{k}\left(1 - \frac{i}{k\tau}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \tag{50}$$

$$A_{0k} \to A_{0k} + C\frac{1}{a(\tau)}\frac{1}{\sqrt{2k}}e^{-ik\tau}. \tag{51}$$

Comparing with the solutions (24) and (25) of A and A_0, the residual gauge transformation (50) and (51) amounts to a change of the homogeneous parts of A and A_0 as the following:

$$b_1 \to b_1' = b_1 + C. \tag{52}$$

Under the residual gauge transformation the canonical momenta are invariant,

$$\pi_A \to \partial_0(A + \theta) - (A_0 + \theta_{,0}) = \pi_A, \tag{53}$$

$$\pi_A^0 \to -\frac{1}{\zeta}(\partial_0(A_0 + \theta_{,0}) + D(A_0 + \theta_{,0}) + k^2(A + \theta)) = \pi_A^0. \tag{54}$$

This invariant property is consistent with the fact that the solutions π_A and π_A^0 in (26) and (27) are contributed only by the inhomogeneous parts of A and A_0, and, therefore, unaffected by any change of the homogeneous parts.

As we shall show in the next section, a consistent covariant canonical quantization requires that the homogeneous part of A and A_0 be nonvanishing, $b_1 \neq 0$, $b_1' \neq 0$. Therefore, at the quantum level, the parameter C of residual gauge transformation will be further restricted.

III. THE COVARIANT CANONICAL QUANTIZATION OF MAXWELL FIELD WITH GENERAL ζ IN DE SITTER SPACE

After obtaining all of the k modes (14) and (24)–(27) for general ζ in de Sitter space, we shall implement the covariant canonical quantization. This procedure will constrain the coefficients for each mode, and restrict the residual gauge transformation as well. The field operators are required to satisfy the equal-time covariant canonical commutation relations,

$$[A_\mu(\tau, \mathbf{x}), \pi_A^\nu(\tau, \mathbf{x}')] = ig^\nu_{\ \mu}\delta(\mathbf{x} - \mathbf{x}'), \tag{55}$$

with $g^\nu_{\ \mu} = \delta^\nu_{\ \mu}$, and the other commutators vanish. The ij component of commutation relations can be decomposed into

$$[A_i, \pi_A^j] = [(B_i + A_{,i}), (w^j + \pi_A^j)]$$
$$= [B_i, w^j] + [\partial_i A, \partial^j \pi_A], \tag{56}$$

where the transverse and longitudinal components are independent, and commute with each other.

The transverse components B_i in de Sitter space are simply the same as in Minkowski spacetime. We write the operators of the transverse fields and canonical momenta as

$$B_i(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\sigma=1}^2 \epsilon_i^\sigma(k)[a_{\mathbf{k}}^{(\sigma)} f_k^{(\sigma)}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{(\sigma)\dagger} f_k^{(\sigma)*}(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}}], \tag{57}$$

$$w^i(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\sigma=1}^2 \epsilon_i^\sigma(k)[a_{\mathbf{k}}^{(\sigma)} f_k^{(\sigma)\prime}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{(\sigma)\dagger} f_k^{(\sigma)*\prime}(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}}], \tag{58}$$

where the modes $f_k^{(1,2)}(\tau)$ are given by (14), and the commutators of the transverse creation and annihilation operators are

$$[a_{\mathbf{k}}^{(\sigma)}, a_{\mathbf{k}'}^{(\sigma')\dagger}] = \eta^{\sigma\sigma'}\delta^{(3)}(\mathbf{k} - \mathbf{k}'), \qquad (\sigma = 1, 2), \tag{59}$$

and the transverse polarizations satisfy

$$\sum_{i=1,2,3} k^i \epsilon_i^\sigma(k) = 0, \qquad \sum_i \epsilon_i^\sigma(k)\epsilon_i^{\sigma'}(k) = \delta^{\sigma\sigma'},$$

$$\sum_{\sigma=1,2} \epsilon_i^\sigma(k)\epsilon_j^\sigma(k) = \delta_{ij} - \frac{k_i k_j}{k^2}. \tag{60}$$

Calculation yields

$$[B_i(\tau, \mathbf{x}), w^j(\tau, \mathbf{x}')]$$
$$= i\delta_{ij}\delta^{(3)}(\mathbf{x} - \mathbf{x}') - i\int \frac{d^3k}{(2\pi)^3}\left(\frac{k_i k_j}{k^2}\right)e^{i\mathbf{k}\cdot\mathbf{x}}e^{-i\mathbf{k}\cdot\mathbf{x}'}. \tag{61}$$

The longitudinal A and temporal A_0 are mixed up in the basic equations (12) and (13) in de Sitter

space, so their field operator expansions are written as follows:

$$A = \int \frac{d^3k}{(2\pi)^{3/2}} [(a_{\mathbf{k}}^0 A_{1k} + a_{\mathbf{k}}^3 A_{2k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}], \quad (62)$$

$$A_0 = \int \frac{d^3k}{(2\pi)^{3/2}} [(a_{\mathbf{k}}^0 A_{01k} + a_{\mathbf{k}}^3 A_{02k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}], \quad (63)$$

where $a_{\mathbf{k}}^{(3)}$ and $a_{\mathbf{k}}^{(0)}$ are the annihilation operator of the respective longitudinal and temporal field and satisfy

$$[a_{\mathbf{k}}^0, a_{\mathbf{k}'}^{0\dagger}] = \eta^{00}\delta^{(3)}(\mathbf{k} - \mathbf{k}') = -\delta^{(3)}(\mathbf{k} - \mathbf{k}'), \quad (64)$$

$$[a_{\mathbf{k}}^3, a_{\mathbf{k}'}^{3\dagger}] = \eta^{33}\delta^{(3)}(\mathbf{k} - \mathbf{k}') = \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \quad (65)$$

Equations (59), (64), (65) together constitute the covariant commutator

$$[a_{\mathbf{k}}^{(\mu)}, a_{\mathbf{k}'}^{(\nu)\dagger}] = \eta^{\mu\nu}\delta^{(3)}(\mathbf{k} - \mathbf{k}'), \quad (66)$$

which is independent of the gauge parameter ζ. The longitudinal and temporal k modes in (63) and (62) are chosen to be

$$A_{1k} = c_1 \frac{1}{a(\tau)} \frac{i}{k} \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} - c_2 \frac{((3-\zeta)(k\tau+i)e^{2ik\tau}\text{Ei}(-2ik\tau) - 3i + 2i\zeta)}{3k} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (67)$$

$$A_{2k} = m_1 \frac{1}{a(\tau)} \frac{i}{k} \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} - m_2 \frac{((3-\zeta)(k\tau+i)e^{2ik\tau}\text{Ei}(-2ik\tau) - 3i + 2i\zeta)}{3k} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (68)$$

$$A_{01k} = c_1 \frac{1}{a(\tau)} \frac{1}{\sqrt{2k}} e^{-ik\tau} - c_2 \frac{((3i - i\zeta)k^2\tau^2 e^{2ik\tau}\text{Ei}(-2ik\tau) + \zeta(k\tau - i))}{3k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (69)$$

$$A_{02k} = m_1 \frac{1}{a(\tau)} \frac{1}{\sqrt{2k}} e^{-ik\tau} - m_2 \frac{((3i - i\zeta)k^2\tau^2 e^{2ik\tau}\text{Ei}(-2ik\tau) + \zeta(k\tau - i))}{3k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (70)$$

where c_1, c_2, m_1, m_2 are dimensionless complex coefficients, and will be subject to some constraints by the canonical quantization. From the expansions (62) and (63) together with (67)–(70) follow the expansions of the canonical momentum operators

$$\pi_A = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} ((a_{\mathbf{k}}^{(0)} \pi_{A1k} + a_{\mathbf{k}}^{(3)} \pi_{A2k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}), \quad (71)$$

$$\pi_A^0 = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} ((a_{\mathbf{k}}^{(0)} \pi_{A1k}^0 + a_{\mathbf{k}}^{(3)} \pi_{A2k}^0) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}), \quad (72)$$

where the k modes of the longitudinal and temporal canonical momenta are found to be

$$\pi_{A1k} = c_2 \frac{-i}{k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (73)$$

$$\pi_{A2k} = m_2 \frac{-i}{k\tau} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (74)$$

$$\pi_{A1k}^0 = c_2 k \left(-\frac{1}{k\tau} + \frac{i}{k^2\tau^2}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (75)$$

$$\pi_{A2k}^0 = m_2 k \left(-\frac{1}{k\tau} + \frac{i}{k^2\tau^2}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau}. \quad (76)$$

These canonical momentum k modes are contributed by only the inhomogeneous part of (67)–(70). There are relations among the modes

$$m_2\pi_{A1k} = c_2\pi_{A2k}, \quad (77)$$

$$m_2\pi_{A1k}^0 = c_2\pi_{A2k}^0, \quad (78)$$

which will be used in Sec. IV to simplify the calculation of the stress tensor. $\pi_A \neq 0$ and $\pi_A^0 \neq 0$ require that $c_2 \neq 0$ and $m_2 \neq 0$.

Substituting the operators (62), (63), (71), (72) into each component of (55), and using the commutator (66), by lengthy calculation, we obtain the following constraints upon the coefficients:

$$|m_1|^2 - |c_1|^2 = 0, \quad (\text{from}[A_0, A_i]), \quad (79)$$

$$|m_2|^2 - |c_2|^2 = 0, \quad (\text{from}[A_0, \pi_A^0]), \quad (80)$$

$$m_2 m_1^* - c_2 c_1^* = -ik/H, \quad (\text{from}[A_0, \pi_A^0]); \quad (81)$$

other commutators give no new constraint. It is seen that $c_1 \neq 0$, $m_1 \neq 0$, $c_2 \neq 0$, $m_2 \neq 0$. This means that both the homogeneous and inhomogeneous parts of the modes (67)–(70) must be present in order to achieve the covariant canonical quantization (55). There are many choices to satisfy the set of constraints (79), (80), (81). For instance, we take the following specific values:

$$c_1 = m_1 = 1, \qquad c_2 = i\frac{k}{2H}, \qquad m_2 = -i\frac{k}{2H}, \qquad (82)$$

which are consistent with those in Minkowski spacetime.

Another implication of the constraints (79), (80), (81) is that, in order to ensure the nonvanishing homogeneous part of A and A_0, the residual gauge transformation will be further restricted. Under the residual gauge transformation (50) and (51), the k modes (A_{1k}, A_{01k}) and (A_{2k}, A_{02k}) change as

$$A_{1k} \to \tilde{A}_{1k} = A_{1k} + C\frac{1}{a}\frac{i}{k}\left(1 - \frac{i}{k\tau}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (83)$$

$$A_{01k} \to \tilde{A}_{01k} = A_{01k} + C\frac{1}{a}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (84)$$

$$A_{2k} \to \tilde{A}_{2k} = A_{2k} + M\frac{1}{a}\frac{i}{k}\left(1 - \frac{i}{k\tau}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (85)$$

$$A_{02k} \to \tilde{A}_{02k} = A_{02k} + M\frac{1}{a}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (86)$$

where C and M are two constants and shift only the coefficients of the homogeneous parts

$$c_1 \to c_1' = c_1 + C, \qquad (87)$$

$$m_1 \to m_1' = m_1 + M. \qquad (88)$$

In analogy to the constraints (79)–(81), the new coefficients also obey the following constraints:

$$|m_1'|^2 - |c_1'|^2 = 0, \qquad (89)$$

$$|m_2'|^2 - |c_2'|^2 = 0, \qquad (90)$$

$$m_2'm_1'^* - c_2'c_1'^* = -i\frac{k}{H}, \qquad (91)$$

which leads to the following restriction on the constants C and M:

$$|M|^2 - |C|^2 + 2\mathrm{Re}(m_1^*M - c_1^*C) = 0, \qquad (92)$$

$$m_2 M^* - c_2 C^* = 0. \qquad (93)$$

For the choice (82), the restriction (92) and (93) becomes

$$C = -M = ir, \qquad (94)$$

where r is an arbitrary real number. As a result, the homogeneous parts will not be transformed to zero

$$c_1' = 1 + ir \neq 0, \qquad m_1' = 1 - ir \neq 0. \qquad (95)$$

We call the residual gauge transformation with the restriction (92) and (93), or (94), the quantum residual gauge transformation. It is required by the covariant canonical quantization, and is only a subset of the residual gauge transformation (50) and (51) at the classical level.

IV. THE STRESS TENSOR OF THE MAXWELL FIELD WITH GUAGE FIXING TERM IN DE SITTER SPACE

The stress tensor serves as the source of the Einstein equation. Given the action $S[A^\mu] = \int \mathcal{L}d^4x$, the stress tensor is defined by $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$, which is covariant. Variation gives the stress tensor of the Maxwell field with the GF term,

$$T_{\mu\nu} = F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\sigma\lambda}F^{\sigma\lambda}$$
$$+ \frac{1}{\zeta}\left[\frac{1}{2}g_{\mu\nu}(A^\sigma{}_{;\sigma})^2 + g_{\mu\nu}A^\lambda A^\sigma{}_{;\sigma\lambda} - A^\sigma{}_{;\sigma\mu}A_\nu - A^\sigma{}_{;\sigma\nu}A_\mu\right], \qquad (96)$$

and the trace of the stress tensor is $T^\mu{}_\mu = \frac{2}{\zeta}(A^\lambda A^\sigma{}_{;\sigma})_{;\lambda}$ which is contributed by the GF term only. The corresponding energy density and pressure consist of three parts:

$$\rho = -T^0{}_0 = \rho^{\mathrm{TR}} + \rho^{\mathrm{LT}} + \rho^{\mathrm{GF}}, \qquad (97)$$

$$p = \frac{1}{3}T^j{}_j = p^{\mathrm{TR}} + p^{\mathrm{LT}} + p^{\mathrm{GF}}. \qquad (98)$$

The transverse stress tensor is

$$\rho^{\mathrm{TR}} = 3p^{\mathrm{TR}} = \frac{1}{2}a^{-4}(B_j'B_j' + B_{i,j}B_{i,j}), \qquad (99)$$

which has an extra factor a^{-4} to that in Minkowski spacetime. This part corresponds to the Maxwell field without the gauge term in the Coulomb gauge. Since B_j is independent of the gauge fixing constant ζ and invariant under the residual gauge transformation, so are ρ^{TR} and p^{TR}. The longitudinal-temporal (LT) stress tensor is

$$\rho^{\mathrm{LT}} = 3p^{\mathrm{LT}} = \frac{1}{2}a^{-4}(A_{,j}'A_{,j}' + A_{0,j}A_{0,j} - 2A_{0,j}A_{,j0})$$
$$= \frac{1}{2}a^{-4}\partial_i\pi_A\partial^i\pi_A, \qquad (100)$$

which is written in terms of the longitudinal canonical momentum π_A. Since π_A is independent of ζ and invariant under the residual gauge transformation (53), so are ρ^{LT} and p^{LT}. The GF stress tensor is

$$\rho^{GF} = \frac{1}{a^4}\left[-\frac{1}{2}\zeta(\pi_A^0)^2 - A_0(\partial_0\pi_A^0 - D\pi_A^0) - A_{,j}\pi_{A,j}^0\right], \quad (101)$$

$$p^{GF} = \frac{1}{a^4}\left[\frac{1}{2}\zeta(\pi_A^0)^2 - A_0(\partial_0\pi_A^0 - D\pi_A^0) + \frac{1}{3}A_{,j}\pi_{A,j}^0\right]. \quad (102)$$

This part comes from a variant of the GF term $-\frac{1}{2\zeta}\sqrt{-g}(A^\nu_{,\nu})^2$ with respect to the metric $g_{\mu\nu}$, and involves π_A^0, A, and A_0. At the classical level, ρ^{GF} and p^{GF} in (101) and (102) apparently depend on ζ. Besides, since A and A_0 vary under the residual gauge transformation, ρ^{GF} and p^{GF} seem to vary too. Later we shall see that the expectation values of the operators ρ^{GF} and p^{GF} in the GB state are independent of ζ, and invariant under the quantum residual gauge transformation.

In the above the stress tensor of the Maxwell field is still a quantum operator. To be a source of the Einstein equation, its expectation value in quantum states is pertinent [10–14]. We now calculate the expectation value of the stress tensor. In a state $|\phi\rangle$ of the transverse field, using the property (60) of transverse polarizations, we obtain the expectation value of the transverse stress tensor

$$\langle\phi|\rho^{TR}|\phi\rangle = 3\langle\phi|p^{TR}|\phi\rangle = \frac{1}{2}a^{-4}\langle\phi|(B'_jB'_j + B_{i,j}B_{i,j})|\phi\rangle$$
$$= \int_0^\infty \rho_k^{TR}\frac{dk}{k} + \int\frac{dk}{k}\rho_k^{TR}\sum_{\sigma=1,2}\langle\phi|a_{\mathbf{k}}^{(\sigma)\dagger}a_{\mathbf{k}}^{(\sigma)}|\phi\rangle, \quad (103)$$

where the first term is the vacuum part, the second term is the photon part, and the spectral energy density and pressure in de Sitter space is

$$\rho_k^{TR} = 3p_k^{TR} = \frac{k^3}{2\pi^2 a^4}[|f_k^{(1)\prime}(\tau)|^2 + k^2|f_k^{(1)}(\tau)|^2] = \frac{k^4}{2\pi^2 a^4}, \quad (104)$$

where the transverse mode $f_k^{(1)}$ is given by (14). If the photon part during de Sitter inflation is in thermal equilibrium approximately, the photon number distribution will be described by $\langle\phi|a_{\mathbf{k}}^{(\sigma)\dagger}a_{\mathbf{k}}^{(\sigma)}|\phi\rangle \propto 1/(e^{k/T}-1)$, and the integration over k yields the photon part of the transverse energy density

$$\int\frac{dk}{k}\rho_k^{TR}\sum_{\sigma=1,2}\langle\phi|a_{\mathbf{k}}^{(\sigma)\dagger}a_{\mathbf{k}}^{(\sigma)}|\phi\rangle = \frac{\pi^2}{15}\left(\frac{T}{a(\tau)}\right)^4, \quad (105)$$

which is convergent, and diluting as a^{-4} with the cosmic expansion. We are more interested in the vacuum part.

The transverse vacuum spectral stress tensor (104) has only one UV divergent k^4 term, which is similar to that in the Minkowski spacetime [see (B41) in Appendix B]. Since the solution (14) of B_i holds for a general RW spacetime, so does the transverse stress tensor (104), which also respects the conservation law in a general RW spacetime

$$\rho_k^{TR\prime} + 3\frac{a'}{a}(\rho_k^{TR} + p_k^{TR}) = 0. \quad (106)$$

The LT stress tensor should be removed since the longitudinal and temporal fields are not radiative dynamical degrees of freedom. This is conventionally implemented by adopting the GB physical state [6–8]. For the longitudinal and temporal fields, the GB physical states $|\psi\rangle$ are defined as the following. The positive frequency part of the temporal canonical momentum operator π_A^0 of (72) annihilates the state $|\psi\rangle$,

$$\pi_A^{0(+)}|\psi\rangle = 0 \rightarrow (c_2 a_{\mathbf{k}}^{(0)} + m_2 a_{\mathbf{k}}^{(3)})|\psi\rangle = 0. \quad (107)$$

This GB condition on the physical state is weaker than the Lorenz condition ($\nabla^\nu A_\nu = 0$) on the field operators. By the choice (82), $c_2 = -m_2$, (107) can be written as

$$[a_{\mathbf{k}}^{(0)} - a_{\mathbf{k}}^{(3)}]|\psi\rangle = 0, \quad (108)$$

which also implies

$$\langle\psi|a_{\mathbf{k}}^{(0)\dagger}a_{\mathbf{k}}^{(0)}|\psi\rangle = \langle\psi|a_{\mathbf{k}}^{(3)\dagger}a_{\mathbf{k}}^{(3)}|\psi\rangle, \quad (109)$$

i.e., the number of temporal and longitude photons are equal in the GB physical state. Together with the transverse state $|\phi\rangle$, the complete state of the Maxwell field can be denoted as a direct product $|\phi,\psi\rangle = |\phi\rangle \otimes |\psi\rangle$. It is known that the GB condition (107) may not hold for a general RW spacetime [15], where the positive frequency modes in the asymptotic in-region may evolve into a combination of positive and negative frequency modes in the asymptotic out-region. This generally happens when the cosmic expansion consists of several stages of power-law expansion [16,17]. However, during the de Sitter expansion (1) and (19), the positive frequency modes (24)–(27) remain $\propto e^{-ik\tau}$ for the whole range of τ, so that the GB condition (107) can be imposed consistently.

The expectation of the LT stress tensor in the GB physical state is

$$\langle\psi|\rho^{LT}|\psi\rangle = 3\langle\psi|p^{LT}|\psi\rangle = \frac{1}{2}a^{-4}\langle\psi|\partial_i\pi_A\partial^i\pi_A|\psi\rangle. \quad (110)$$

Substituting the operator π_A of (71) into the above gives

$$\langle\psi|\rho^{LT}|\psi\rangle = 3\langle\psi|p^{LT}|\psi\rangle = \int\rho_k^{LT}\frac{dk}{k}, \quad (111)$$

where

$$
\begin{aligned}
\rho_{\mathrm{LT}}^{k} = {} & \frac{k^{5}}{4\pi^{2}a^{4}} (2\langle\psi|a_{\mathbf{k}}^{(0)\dagger}a_{\mathbf{k}}^{(0)}|\psi\rangle|\pi_{A1k}|^{2} + 2\langle\psi|a_{\mathbf{k}}^{(3)\dagger}a_{\mathbf{k}}^{(3)}|\psi\rangle|\pi_{A2k}|^{2} - |\pi_{A1k}|^{2} + |\pi_{A2k}|^{2}) \\
& + \frac{k^{5}}{4\pi^{2}a^{4}} (2\langle\psi|a_{\mathbf{k}}^{(3)\dagger}a_{\mathbf{k}}^{(0)}|\psi\rangle\pi_{A2k}^{*}\pi_{A1k} + 2\langle\psi|a_{\mathbf{k}}^{(0)\dagger}a_{\mathbf{k}}^{(3)}|\psi\rangle\pi_{A1k}^{*}\pi_{A2k}) \\
& + \frac{k^{5}}{4\pi^{2}a^{4}} (\langle\psi|a_{\mathbf{k}}^{(0)}a_{-\mathbf{k}}^{(0)}|\psi\rangle\pi_{A1k}^{2} + \langle\psi|a_{\mathbf{k}}^{(3)}a_{-\mathbf{k}}^{(0)}|\psi\rangle\pi_{A2k}\pi_{A1k} \\
& + \langle\psi|a_{\mathbf{k}}^{(0)}a_{-\mathbf{k}}^{(3)}|\psi\rangle\pi_{A1k}\pi_{A2k} + \langle\psi|a_{\mathbf{k}}^{(3)}a_{-\mathbf{k}}^{(3)}|\psi\rangle\pi_{A2k}^{2} + \mathrm{H.c.}).
\end{aligned}
\tag{112}
$$

Applying the GB condition (107) and (109) and the mode relation (77) with $c_2 = -m_2$, we find that the longitudinal and temporal contributions cancel each other, and (112) becomes

$$
\rho_k^{\mathrm{LT}} = 3p_k^{\mathrm{LT}} = 0, \tag{113}
$$

including the photon and vacuum parts. Thus, the LT stress tensor is vanishing in the GB state even before regularization. This result is independent of ζ. The longitudinal-temporal cancellation occurs in the GB state as long as the modes π_{A1k} and π_{A2k} satisfy the relation (77), regardless the concrete functions π_{A1k} and π_{A2k}. We have also checked that the LT stress tensor is zero also for the radiation dominant stage ($a \propto \tau$). So, it might be expected that the LT stress tensor will be zero for a general power-law expansion with $a \propto \tau^n$. But this may not hold in a general RW spacetime consisting of several stages of power-law expansion.

More interesting is the GF stress tensor which is less studied in literature. The expressions (101) and (102) in the GB physical state give

$$
\begin{aligned}
\langle\psi|\rho^{\mathrm{GF}}|\psi\rangle = {} & \frac{1}{a^{4}}\langle\psi|\left(-\frac{1}{2}\zeta(\pi_A^0)^2 - A_0(\partial_0\pi_A^0 - D\pi_A^0)\right. \\
& \left. - A_{,j}\pi_{A,j}^0\right)|\psi\rangle,
\end{aligned}
\tag{114}
$$

$$
\begin{aligned}
\langle\psi|p^{\mathrm{GF}}|\psi\rangle = {} & \frac{1}{a^{4}}\langle\psi|\left(\frac{1}{2}\zeta(\pi_A^0)^2 - A_0(\partial_0\pi_A^0 - D\pi_A^0)\right. \\
& \left. + \frac{1}{3}A_{,j}\pi_{A,j}^0\right)|\psi\rangle.
\end{aligned}
\tag{115}
$$

It can be shown that the expectation value $\langle\psi|(\pi_A^0)^2|\psi\rangle = 0$ in the GB physical state, so (114) and (115) reduce to

$$
\langle\psi|\rho^{\mathrm{GF}}|\psi\rangle = \frac{1}{a^{4}}\langle\psi|(-A_0(\partial_0\pi_A^0 - D\pi_A^0) - A_{,j}\pi_{A,j}^0)|\psi\rangle,
\tag{116}
$$

$$
\langle\psi|p^{\mathrm{GF}}|\psi\rangle = \frac{1}{a^{4}}\langle\psi|\left(-A_0(\partial_0\pi_A^0 - D\pi_A^0) + \frac{1}{3}A_{,j}\pi_{A,j}^0\right)|\psi\rangle.
\tag{117}
$$

Substituting the operators (62), (63), (72) into (116), using the commutators (64) and (65), the mode relation (78), the coefficient constraint (80), and the GB condition (107), we obtain

$$
\langle\psi|\rho^{\mathrm{GF}}|\psi\rangle = \int \rho_k^{\mathrm{GF}}\frac{dk}{k}, \tag{118}
$$

$$
\langle\psi|p^{\mathrm{GF}}|\psi\rangle = \int p_k^{\mathrm{GF}}\frac{dk}{k}, \tag{119}
$$

where the GF spectral energy density and pressure are

$$
\begin{aligned}
\rho_k^{\mathrm{GF}} = {} & \frac{k^{3}}{2\pi^{2}a^{4}}\left[\langle\psi|a_{\mathbf{k}}^{(0)\dagger}a_{\mathbf{k}}^{(0)}|\psi\rangle\left(\left(\frac{c_2}{m_2}A_{02k} - A_{01k}\right)(\partial_0 - D)\pi_{A1k}^{0*} + k^2\left(\frac{c_2}{m_2}A_{2k} - A_{1k}\right)\pi_{A1k}^{0*}\right)\right. \\
& + \langle\psi|a_{\mathbf{k}}^{(3)\dagger}a_{\mathbf{k}}^{(3)}|\psi\rangle\left(\left(\frac{m_2}{c_2}A_{01k} - A_{02k}\right)(\partial_0 - D)\pi_{A2k}^{0*} + k^2\left(\frac{m_2}{c_2}A_{1k} - A_{2k}\right)\pi_{A2k}^{0*}\right) \\
& \left. + (A_{01k}(\partial_0 - D)\pi_{A1k}^{0*} - A_{02k}(\partial_0 - D)\pi_{A2k}^{0*} + k^2 A_{1k}\pi_{A1k}^{0*} - k^2 A_{2k}\pi_{A2k}^{0*})\right],
\end{aligned}
\tag{120}
$$

$$p_k^{\text{GF}} = \frac{k^3}{2\pi^2 a^4} \left[\langle \psi | a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)} | \psi \rangle \left(\left(\frac{c_2}{m_2} A_{02k} - A_{01k} \right)(\partial_0 - D)\pi_{A1k}^{0*} - \frac{1}{3} k^2 \left(\frac{c_2}{m_2} A_{2k} - A_{1k} \right)\pi_{A1k}^{0*} \right) \right.$$
$$+ \langle \psi | a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)} | \psi \rangle \left(\left(\frac{m_2}{c_2} A_{01k} - A_{02k} \right)(\partial_0 - D)\pi_{A2k}^{0*} - \frac{1}{3} k^2 \left(\frac{m_2}{c_2} A_{1k} - A_{2k} \right)\pi_{A2k}^{0*} \right)$$
$$\left. + \left(A_{01k}(\partial_0 - D)\pi_{A1k}^{0*} - A_{02k}(\partial_0 - D)\pi_{A2k}^{0*} - \frac{1}{3} k^2 A_{1k}\pi_{A1k}^{0*} + \frac{1}{3} k^2 A_{2k}\pi_{A2k}^{0*} \right) \right], \tag{121}$$

each consisting of three contributions: the temporal photons, the longitudinal photons, and the vacuum. Substituting the modes A_{1k}, A_{2k}, A_{01k}, A_{02k}, of (67)–(70) and the modes π_{A1}^0, π_{A2}^0 of (75) and (76) into (120) and (121), we do lengthy calculations. As we notice, the inhomogeneous parts of A_0 cancel in each of the following combinations: $(\frac{c_2}{m_2} A_{02k} - A_{01k})$, $(\frac{m_2}{c_2} A_{01k} - A_{02k})$, $A_{01k}(\partial_0 - D)\pi_{A1k}^{0*} - A_{02k}(\partial_0 - D)\pi_{A2k}^{0*}$; and similarly, the inhomogeneous parts of A cancel in the following: $(\frac{c_2}{m_2} A_{2k} - A_{1k})$, $(\frac{m_2}{c_2} A_{1k} - A_{2k})$, $(A_{1k}\pi_{A1k}^{0*} - A_{2k}\pi_{A2k}^{0*})$. So, only the homogeneous parts contribute to (120) and (121), yielding

$$\rho_k^{\text{GF}} = \frac{k^4}{(2\pi^2)a^4} \left[\left(\langle \psi | a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)} | \psi \rangle - \langle \psi | a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)} | \psi \rangle \right) \left(1 + \frac{1}{2k^2\tau^2} \right) \right] + \frac{k^4}{2\pi^2 a^4} \left(1 + \frac{1}{2k^2\tau^2} \right), \tag{122}$$

$$p_k^{\text{GF}} = \frac{k^4}{(2\pi^2)a^4} \frac{1}{3} \left[\left(\langle \psi | a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)} | \psi \rangle - \langle \psi | a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)} | \psi \rangle \right) \left(1 - \frac{1}{2k^2\tau^2} \right) \right] + \frac{k^4}{(2\pi^2)a^4} \frac{1}{3} \left(1 - \frac{1}{2k^2\tau^2} \right). \tag{123}$$

By $\langle \psi | a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)} | \psi \rangle = \langle \psi | a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)} | \psi \rangle$, the longitudinal and temporal photons cancel each other, and only the vacuum part remains,

$$\rho_k^{\text{GF}} = \frac{k^4}{2\pi^2 a^4} \left(1 + \frac{1}{2k^2\tau^2} \right), \tag{124}$$

$$p_k^{\text{GF}} = \frac{k^4}{2\pi^2 a^4} \frac{1}{3} \left(1 - \frac{1}{2k^2\tau^2} \right). \tag{125}$$

This GF vacuum part is independent of ζ too, because the ζ-dependent, inhomogeneous parts of the k modes of A and A_0 have canceled. The GF vacuum stress tensor also respects the conservation law

$$\rho_k^{\text{GF}\prime} + 3\frac{a'}{a}(\rho_k^{\text{GF}} + p_k^{\text{GF}}) = 0 \tag{126}$$

but contributes a nonzero trace

$$-\rho_k^{\text{GF}} + 3p_k^{\text{GF}} = -\frac{k^4}{2\pi^2 a^4} \frac{1}{(k^2\tau^2)} \neq 0. \tag{127}$$

The form of (124) and (125) is the same as twice the vacuum stress tensor of the minimally coupling massless scalar field [9,14]. It contains two UV divergent terms: the k^4 term is dominant and corresponds to the UV divergence in the Minkowski spacetime [see (B46) in Appendix B], and the k^2 term reflects the effect of the cosmic expansion and is absent in Minkowski spacetime.

The transverse stress tensor and the LT stress tensor are invariant under the residual gauge transformation even at the classical level. Now we examine the behavior of the GF vacuum stress tensor (124) and (125) under the quantum residual gauge transformation. Firstly, according to (95), $c_1' \neq 0$ and $m_1' \neq 0$, the homogeneous part of A and A_0 will not be transformed to zero under the quantum residual gauge transformation. As a result, the vacuum GF stress tensor will not be transformed to zero since it is contributed by the homogeneous part. More than that, the GF stress tensor in the GB state is actually invariant under the quantum residual gauge transformation. This fact can be shown by a direct calculation of the variation of the GF spectral stress tensor (120) and (121)

$$\delta\rho_k^{\text{GF}} = \frac{k^3}{2\pi^2 a^4} (c_2^* C - m_2^* M) iH \left[\left(\langle \psi | a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)} | \psi \rangle - \langle \psi | a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)} | \psi \rangle \right) \left(1 + \frac{1}{2k^2\tau^2} \right) + \left(1 + \frac{1}{2k^2\tau^2} \right) \right], \tag{128}$$

$$\delta p_k^{\text{GF}} = \frac{k^3}{2\pi^2 a^4} \frac{1}{3} (c_2^* C - m_2^* M) iH \left[\left(\langle \psi | a_{\mathbf{k}}^{3\dagger} a_{\mathbf{k}}^{3} | \psi \rangle - \langle \psi | a_{\mathbf{k}}^{0\dagger} a_{\mathbf{k}}^{0} | \psi \rangle \right) \left(1 - \frac{1}{2k^2\tau^2} \right) + \left(1 - \frac{1}{2k^2\tau^2} \right) \right]. \tag{129}$$

According to the constraint $m_2 M^* - c_2 C^* = 0$ of (93), the above is vanishing

$$\delta\rho_k^{\text{GF}} = 0, \qquad \delta p_k^{\text{GF}} = 0. \tag{130}$$

V. THE REGULARIZATION OF STRESS TENSOR OF MAXWELL FIELD IN DE SITTER SPACE

So far three parts of the vacuum stress tensor have been derived in de Sitter space. The LT stress tensor (113) is zero in the GB state, no need for regularization. The transverse vacuum stress tensor and the GF vacuum stress tensor both contain UV divergences, which need to be regularized as the following.

The transverse vacuum stress tensor (104) has only one quartic k^4 divergent term, so the zeroth-order adiabatic regularization is sufficient to remove the UV divergence [9,14,16–18]. The equation of two transverse modes is Eq. (11) and the exact solution is $f_k^{(\sigma)}(\tau)$ in (14). The adiabatic transverse modes are the same for two polarizations ($\sigma = 1, 2$), given by the Wenzel-Kramers-Brillouin (WKB) solution of (11) as the following [9–11,14,16,17]:

$$f_k(\tau) = (2W(\tau))^{-1/2} \exp\left[-i \int^\tau W(\tau')d\tau'\right], \quad (131)$$

where the effective frequency is

$$W(\tau) = \left[\omega^2 - \frac{1}{2}\left(\frac{W''}{W} - \frac{3}{2}\left(\frac{W'}{W}\right)^2\right)\right]^{1/2}, \quad (132)$$

which will be solved iteratively. The zeroth-order frequency and mode are

$$W_{0th} = \omega = k, \quad (133)$$

$$f_{k0th}(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau} = f_k^{(\sigma)}. \quad (134)$$

In fact, all adiabatic orders for the transverse modes are the same

$$W_{0th} = W_{2nd} = W_{4th} = \ldots = k, \quad (135)$$

$$f_{k0th}(\tau) = f_{k2nd}(\tau) = f_{k4th}(\tau) = \ldots = f_k^{(\sigma)}, \quad (136)$$

like a conformally coupling massless scalar field [9,14]. Substituting the zeroth-order mode f_{k0th} of (134) into (104) to replace $f_k^{(1)}$ yields

$$\rho_{k0th}^{TR} = 3p_{k0th}^{TR} = \frac{k^3}{2\pi^2 a^4}[|f'_{k0th}(\tau)|^2 + k^2|f_{k0th}(\tau)|^2]$$
$$= \frac{k^4}{2\pi^2 a^4} = \rho_k^{TR} = 3p_k^{TR}, \quad (137)$$

i.e., the zeroth-order adiabatic subtraction term for the transverse spectral stress tensor is just equal to the exact spectral stress tensor (104). Hence, by subtraction, the zeroth-order regularized transverse vacuum spectral stress tensor is vanishing,

$$\rho_{k\,reg}^{TR} \equiv \rho_k^{TR} - \rho_{k0th}^{TR} = 0, \quad (138)$$

$$p_{k\,reg}^{TR} \equiv p_k^{TR} - p_{k0th}^{TR} = 0. \quad (139)$$

The results (137)–(139) hold also for a general RW spacetime. This is because B_i of (14) and its adiabatic modes (136) hold for a general RW spacetime [9].

The GF vacuum stress tensor (124) and (125) has the k^4 and k^2 divergent terms, so the second-order adiabatic regularization is sufficient to remove the UV divergences [9,14,16–18]. To calculate the second-order adiabatic subtraction terms of the stress tensor, we need also respectively the second-order adiabatic modes of π_{Ak}^0, A_k, and A_{0k}.

The equation of rescaled $\bar{\pi}_A^0$ is given by (31) and the solution is given by (33). The WKB solution of (31) is

$$\bar{\pi}_{A\,nth}^0 = (2W(\tau))^{-1/2} \exp\left[-i \int^\tau W(\tau')d\tau'\right], \quad (140)$$

where the effective frequency is

$$W(\tau) = \left[\omega^2 - \frac{2}{\tau^2} - \frac{1}{2}\left(\frac{W''}{W} - \frac{3}{2}\left(\frac{W'}{W}\right)^2\right)\right]^{1/2}, \quad (141)$$

which will be solved iteratively. The zeroth-order is $W_{0th} = \omega = k$, and the second-order and the higher orders are found as

$$W_{2nd} = W_{4th} = \ldots = k - \frac{1}{k\tau^2}, \quad (142)$$

so the second-order and all higher order adiabatic modes are the same, and given by

$$\bar{\pi}_{A\,2nd}^0 = \bar{\pi}_{A\,4th}^0 = \bar{\pi}_{A\,6th}^0 = \ldots$$
$$= \frac{1}{\sqrt{2k}}\left(1 + \frac{1}{2}\frac{1}{(k\tau)^2}\right)\exp\left[-ik\int^\tau\left(1 - \frac{1}{(k\tau')^2}\right)d\tau'\right]$$
$$\simeq \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}, \quad (143)$$

which is equal to the exact mode $\bar{\pi}_A^0$ in (33). Multiplying by $a(\tau)$, one has $\pi_{A\,2nd}^0 = \pi_A^0$, i.e., the second and higher order adiabatic modes are equal to the exact modes (75) and (76).

The WKB approximation of A and A^0 can be derived, in principle, from their fourth-order differential equations, but the calculation will be more involved. Actually we can directly get their second-order adiabatic modes from high k expansions of the exact modes (67)–(70). Moreover, as mentioned earlier, the inhomogeneous part of A and A_0 do not contribute to the GF stress tensor, so we need only the homogeneous parts of (67)–(70) as the following:

$$A_{1k} = c_1 \frac{1}{a(\tau)}\frac{i}{k}\left(1 - \frac{i}{k\tau}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \quad (144)$$

$$A_{2k} = m_1 \frac{1}{a(\tau)} \frac{i}{k} \left(1 - \frac{i}{k\tau}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (145)$$

which are of the second adiabatic order already, and

$$A_{01k} = c_1 \frac{1}{a(\tau)} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (146)$$

$$A_{02k} = m_1 \frac{1}{a(\tau)} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad (147)$$

which are of the zeroth adiabatic order, and are also equal to all higher order homogeneous modes. [Similarly, for π_A, the adiabatic modes of all orders are equal to the exact mode (26). Here we shall not need these for regularization.] Substituting these adiabatic modes into the expressions (120) and (121) to replace π_{A1k}^0, π_{A2k}^0, A_{1k}, A_{2k}, A_{01k}, A_{02k}, we obtain

$$\rho_{k\,2nd}^{\mathrm{GF}} = \frac{k^4}{2\pi^2 a^4} \left(1 + \frac{1}{2k^2\tau^2}\right) = \rho_k^{\mathrm{GF}}, \qquad (148)$$

$$p_{k\,2nd}^{\mathrm{GF}} = \frac{1}{3} \frac{k^4}{2\pi^2 a^4} \left(1 - \frac{1}{2k^2\tau^2}\right) = p_k^{\mathrm{GF}}. \qquad (149)$$

As expected, the second-order adiabatic subtraction term for the GF spectral stress tensor is equal to the exact GF spectral stress tensor. By subtraction, the second-order regularized GF vacuum stress tensor is zero,

$$\rho_{k\,\mathrm{reg}}^{\mathrm{GF}} \equiv \rho_k^{\mathrm{GF}} - \rho_{k\,2nd}^{\mathrm{GF}} = 0, \qquad (150)$$

$$p_{k\,\mathrm{reg}}^{\mathrm{GF}} \equiv p_k^{\mathrm{GF}} - p_{k\,2nd}^{\mathrm{GF}} = 0, \qquad (151)$$

and the regularized trace is also zero,

$$-\rho_{k\,\mathrm{reg}}^{\mathrm{GF}} + 3 p_{k\,\mathrm{reg}}^{\mathrm{GF}} = 0. \qquad (152)$$

So, there is no need to introduce a ghost field to cancel the vanishing GF vacuum stress tensor (150) and (151), and this vanishing vacuum stress tensor cannot be a candidate for the cosmological constant [3,4]. (Instead, the regularized vacuum stress tensor of a massive scalar field, either minimally or conformally coupling, does give rise to the cosmological constant [14,18]). Putting the three parts together, the total regularized vacuum stress tensor of a Maxwell field with a general GF term is zero,

$$\rho_{\mathrm{reg}} = p_{\mathrm{reg}} = 0, \qquad (153)$$

and there is no trace anomaly. This result is independent of ζ, and also invariant under the quantum residual gauge transformation. Reference [1] adopted the point-splitting regularization [18–20], and also arrived at the zero vacuum stress tensor of the Maxwell field in the Feynman gauge, at the price of introducing a ghost field to cancel the GF stress tensor. The trace anomaly has been regarded as a consensus since the 1970s; nevertheless our calculation shows no trace anomaly for the Maxwell field. References [21,22] claimed the trace anomaly under the assumption that the Green's function contains a boundary term $w(x, x')$ which is unsymmetric in (x, x'). But, as we show, the exact Green's functions (A8) and (A9) in de Sitter space do not contain such an unsymmetric boundary term [9,14,18].

VI. CONCLUSION AND DISCUSSIONS

We have studied the Maxwell field with a general gauge fixing term in de Sitter space. All four components A_μ are formally treated as independent variables, and no Lorenz condition is imposed. The introduction of the GF term restricts the gauge invariance of the Maxwell field down to a residual gauge invariance given by (48). Furthermore, the covariant canonical quantization restricts further the residual gauge invariance down to the quantum residual gauge invariance specified by Eq. (94).

The transverse components B_i are separated from other components, independent of the gauge fixing constant ζ, and represent real dynamical degrees of freedom, and their equation (11) and solution (14) hold for a general RW spacetime including de Sitter space. The transverse stress tensor (103) consists of the particle parts (105) and the vacuum part (104) with a UV divergent term $\propto k^4$.

The longitudinal and temporal components A and A_0 are mixed up in the ζ-dependent equations (12) and (13). We have obtained their solutions (24) and (25) in two different ways. In particular, in the second way, via the inhomogeneous equations (34) and (35), the nontrivial structure of the solutions A and A_0 is revealed, each being a sum of the homogeneous and inhomogeneous solutions. The canonical momenta are contributed only by the inhomogeneous solutions of A and A_0, and only the homogeneous parts will vary under the residual gauge transformation (50) and (51). For a consistent covariant canonical quantization, both the homogeneous and inhomogeneous k modes of A and A_0 need to be present in the operator expansions. Moreover, the homogeneous k modes of A and A_0 will not go vanishing under the quantum residual gauge transformation. The LT stress tensor (110) is independent of ζ, and invariant under the quantum residual gauge transformation. Further, its expectation (113) is zero in the GB physical state due to the longitudinal and temporal cancellation.

More interesting is the GF stress tensor, which is less studied in literature. At the classical level the GF stress tensor (101) and (102) depends upon ζ; nevertheless, its expectation value (122) and (123) in the GB physical state is independent of ζ, and also is invariant under the quantum residual gauge transformation. Moreover, its particle part is zero due to the longitudinal and temporal cancellation; only the vacuum part (124) and (125) remains, which contains

two UV divergent terms, $\propto k^4$, k^2, and is equal to twice the vacuum stress tensor of the minimally coupling massless scalar field.

To remove the UV divergences of the vacuum stress tensor, we have carried out the adiabatic regularization. The transverse vacuum stress tensor becomes zero under the zeroth-order adiabatic regularization, and, respectively, the GF vacuum stress tensor becomes zero under the second-order adiabatic regularization. Thus, there is no need to introduce a ghost field to cancel the GF stress tensor, and the vanishing vacuum GF stress tensor of the Maxwell field cannot be a possible candidate for the cosmological constant. Instead, the regularized vacuum stress tensor of a (minimally or conformally coupling) massive scalar field corresponds to the cosmological constant that drives the de Sitter inflation [14,18].

In summary, for the Maxwell field with a general GF term in de Sitter space described by (1) and (19), the total regularized vacuum stress tensor in the GB state is zero, and only the photon part of the transverse stress tensor (105) remains, and all the predicted physics will be the same as that of the Maxwell field without the GF term.

We have also carried out analogous calculations in the Minkowski spacetime, attached in the Appendix B. The outcome is similar to de Sitter space, except that the GF vacuum stress tensor has only one k^4 term, which can be made zero by the normal ordering.

ACKNOWLEDGMENTS

APPENDIX A: GREEN'S FUNCTIONS FOR MAXWELL FIELD IN THE FEYNMAN GAUGE

Reference [19] proposed the following relations (see also Ref. [1] for the application)

$$G^{(1);\nu}_{\nu\sigma'} = -G_{S,\sigma'}, \qquad (A1)$$

$$G^{(1);\sigma'}_{\nu\sigma'} = -G_{S,\nu}, \qquad (A2)$$

where

$$G^{(1)}_{\nu\sigma'}(x,x') = \langle 0|(A_\nu(x)A_{\sigma'}(x') + A_{\sigma'}(x')A_\nu(x))|0\rangle \qquad (A3)$$

is the Hadamard type Green's function for the Maxwell field in the Feynman gauge ($\zeta = 1$), and

$$G_S(x,x') = \langle 0|\phi(x)\phi(x') + \phi(x')\phi(x)|0\rangle \qquad (A4)$$

is the Green's function for a minimally coupling massless scalar field where $\phi(x)$ is the scalar field operator. Note that

$G^{(1)}_{\nu\sigma'}(x,x')$ is not an ordinary tensor, but a bivector at x and at x' respectively. Similarly, $G_S(x,x')$ is a biscalar at x and at x' respectively. In the following we check the relation (A1) in de Sitter space.

Write the operator ϕ as

$$\phi(x) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} (a_{\vec{k}}\phi_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\vec{k}}\phi_k(\tau)^* e^{-i\mathbf{k}\cdot\mathbf{x}}), \qquad (A5)$$

where the k mode of ϕ in de Sitter space is [9,14]

$$\phi_k(\tau) = \frac{1}{a(\tau)}\frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}. \qquad (A6)$$

Simple calculation yields the Green's function of the scalar field

$$G_S = \int \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}((-i + k\tau)(i + k\tau') \\ \times e^{-ik(\tau-\tau')} + \text{c.c.})e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \qquad (A7)$$

After the k integration, (A7) becomes [18]

$$G_S(x,x') = -\frac{H^2}{8\pi^2}\left[\frac{1}{\sigma} + \ln\left(-\frac{2\tau\tau'}{\tau_0^2}\sigma\right)\right] \qquad (A8)$$

with $\sigma \equiv \frac{1}{(2\tau\tau')}[(\tau - \tau')^2 - |\mathbf{x} - \mathbf{x}'|^2]$ and τ_0 being a constant. For the conformally coupling massless scalar field the Green's function is

$$G(x,x') = -\frac{H^2}{8\pi^2}\frac{1}{\sigma}, \qquad (A9)$$

which is relevant to the case in Refs. [1,21,22]. Both (A8) and (A9) are symmetric in (x,x'). For an extension of (A8) to vacuum states other than the Bunch-Davies vacuum state, see Ref. [23].

The time and spatial derivatives of (A7) are

$$G_{S,0'} = \int \frac{d^3k}{(2\pi)^3}\frac{(H\tau)(H\tau')}{2} \\ \times \left(\left(i + \frac{1}{k\tau}\right)e^{-ik(\tau-\tau')} + \text{c.c.}\right)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}, \qquad (A10)$$

$$G_{S,i'} = -\int \frac{d^3k}{(2\pi)^3}(ik_{i'})\frac{H^2}{2k^3}((-i + k\tau)(i + k\tau') \\ \times e^{-ik(\tau-\tau')} + \text{c.c.})e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \qquad (A11)$$

From the solutions (14), (24), (25), of $A_\mu(x)$ in de Sitter space, we obtain each component of the Green's functions of the Maxwell field as the following:

$$G_{00'}^{(1)} = \int \frac{d^3k}{(2\pi)^3} \frac{e^{-ik(\tau+\tau')}}{6k^3\tau\tau'} [-i\zeta(\tau^2 e^{2ik\tau} + \tau'^2 e^{2ik\tau'}) + k\zeta\tau\tau'(\tau e^{2ik\tau} + \tau' e^{2ik\tau'}) - \zeta\tau^2(i+k\tau')e^{2ik\tau'} - \zeta\tau'^2(i+k\tau)e^{2ik\tau}$$
$$- ik^2(-3+\zeta)\tau^2\tau'^2(Ei(2ik\tau) + Ei(2ik\tau') + e^{2ik(\tau+\tau')}(Ei(-2ik\tau) + Ei(-2ik\tau')))]e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x'})}, \tag{A12}$$

$$G_{0i'}^{(1)} = \int \frac{d^3k}{(2\pi)^3} \frac{ik_{i'}}{6k^3\tau} [(e^{-ik(\tau+\tau')}(1+ik\tau')(k^2\tau^2(-3+\zeta)Ei(2ik\tau) + e^{2ik\tau}\zeta(1-ik\tau))$$
$$+ e^{ik(\tau+\tau')}e^{-2ik\tau}k^2\tau^2(3-2\zeta + e^{-2ik\tau'}(-3+\zeta)(1+ik\tau')Ei(2ik\tau')))$$
$$- (e^{ik(\tau+\tau')}(-1+ik\tau')(k^2\tau^2(-3+\zeta)Ei(-2ik\tau) + e^{-2ik\tau}\zeta(1+ik\tau))$$
$$+ e^{-ik(\tau+\tau')}e^{2ik\tau}k^2\tau^2(-3+2\zeta + e^{2ik\tau'}(-3+\zeta)(-1+ik\tau')Ei(-2ik\tau')))]e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x'})}, \tag{A13}$$

$$G_{li'}^{(1)} = \int \frac{d^3\vec{k}}{2k(2\pi)^3} \left(\delta_{li'} - \frac{k_l k_{i'}}{k^2}\right)(e^{-ik(\tau-\tau')} + e^{-ik(\tau'-\tau)})e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x'})}$$
$$+ \int \frac{d^3k}{(2\pi)^3} \frac{k_l k_{i'}}{6k^5} e^{-ik(\tau+\tau')}[ik^2(-i+k\tau)(e^{2ik\tau'}(-3+2\zeta) - i(-3+\zeta)(-i+k\tau')Ei(2ik\tau'))$$
$$- ik^2 e^{2ik\tau'}(i+k\tau')(-3+2\zeta + ie^{2ik\tau}(-3+\zeta)(i+k\tau)Ei(-2ik\tau))$$
$$+ ik^2(-i+k\tau')(e^{2ik\tau}(-3+2\zeta) - i(-3+\zeta)(-i+k\tau)Ei(2ik\tau))$$
$$- ik^2 e^{2ik\tau}(i+k\tau)(-3+2\zeta + ie^{2ik\tau'}(-3+\zeta)(i+k\tau')Ei(-2ik\tau'))]e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x'})}. \tag{A14}$$

Each contains the exponential-integration function Ei. In a homogeneous and isotropic RW spacetime, there is a symmetry

$$G_{\mu\nu'}^{(1)}(x,x') = G_{\nu'\mu}^{(1)}(x',x),$$

so that

$$G_{i'0}^{(1)}(x',x)|_{x\leftrightarrow x'} = G_{0i'}^{(1)}(x,x')|_{x\leftrightarrow x'}.$$

Since $G_{\nu\sigma'}^1(x,x')$ is a vector at the point x, the $0'$ component of the four divergence is calculated as

$$G_{\nu0'}^{(1);\nu} = g^{\mu\nu}G_{\nu0';\mu}^{(1)} = g^{\mu\nu}(G_{\nu0',\mu}^{(1)} - \Gamma_{\nu\mu}^\alpha G_{\alpha0'}^{(1)})$$
$$= a^{-2}\left(-G_{00',0}^{(1)} + G_{i0',i}^{(1)} - 2\frac{a'}{a}G_{00'}^{(1)}\right) \tag{A15}$$

with $\Gamma_{00}^0 = \frac{a'}{a}$, $\Gamma_{ij}^0 = \delta_{ij}\frac{a'}{a}$, $\Gamma_{0j}^i = \frac{a'}{a}\delta_{ij}$. Substituting (A12)–(A14) with $\zeta = 1$ into the above yields,

$$G_{\nu0'}^{(1);\nu} = -G_{S,0'}, \tag{A16}$$

where $G_{S,0'}$ is given by (A10), and the Ei function has been canceled. Similarly, the i' component of the four divergence is

$$G_{\nu i'}^{(1);\nu} = \frac{1}{a(\tau)^2}\left(-G_{0i',0}^{(1)} - 2\frac{a(\tau)'}{a(\tau)}G_{0i'}^{(1)} + G_{li',l}^{(1)}\right). \tag{A17}$$

Calculation shows that

$$G_{\nu i'}^{(1);\nu} = -G_{S,i'}, \tag{A18}$$

where $G_{S,i'}$ is given by (A11). So, the relation (A1) in the Feynman gauge is verified. Similarly, (A2) can be also checked. Note that (A1) and (A2) are not valid for a general ζ.

APPENDIX B: MAXWELL FIELD WITH A GAUGE FIXING TERM IN MIKOWSKI SPACETIME

Although the Maxwell field in the Minkowski spacetime is well known, the Maxwell field with a general GF term is nontrivial, and has not been adequately reported in literature [8]. The procedure of calculation is analogous to that in de Sitter space. In the following we shall report briefly the results. Setting $D = 0$ in (4) gives the field equation

$$\eta^{\sigma\rho}\partial_\sigma\partial_\rho A_\nu + \left(\frac{1}{\zeta}-1\right)\partial_\nu(\eta^{\rho\sigma}\partial_\sigma A_\rho) = 0. \tag{B1}$$

Setting $D = 0$ in (12) and (13) gives the following basic second-order equations:

$$-\partial_0^2 A - \frac{1}{\zeta}k^2 A + \left(1 - \frac{1}{\zeta}\right)\partial_0 A_0 = 0, \qquad (B2)$$

$$-\frac{1}{\zeta}\partial_0^2 A_0 - k^2 A_0 + k^2\left(1 - \frac{1}{\zeta}\right)\partial_0 A = 0, \qquad (B3)$$

where A_0 and A are mixed up, and B_i has the same equation and solution as (11) and (14) in de Sitter space. The decomposition is similar to (5)–(10), but the temporal canonical momentum is

$$\pi_A^0 = -\frac{1}{\zeta}(\partial_0 A_0 + k^2 A). \qquad (B4)$$

Equations (28) and (29) reduce to $(\partial_0^2 + k^2)\pi_A = 0$, and $(\partial_0^2 + k^2)\pi_A^0 = 0$ in Minkowski spacetime; the positive frequency solutions are

$$\pi_A = d_1 \frac{1}{\sqrt{2k}}e^{-ikt}, \qquad \pi_A^0 = d_2(ik)\frac{1}{\sqrt{2k}}e^{-ikt}, \qquad (B5)$$

where d_1 and d_2 are arbitrary coefficients. By differentiation and combination of Eqs. (B2) and (B3), we obtain the fourth-order differential equations

$$(\partial_0^2 + k^2)^2 A = 0, \qquad (B6)$$

$$(\partial_0^2 + k^2)^2 A_0 = 0, \qquad (B7)$$

which are separate, and independent of ζ, unlike (20) and (21) in the de Sitter space. The positive frequency solutions of (B6) and (B7) are

$$A(\tau) = b\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau} + c\frac{i}{k}\frac{(1 + 2ik\tau)}{\sqrt{2k}}e^{-ik\tau}, \qquad (B8)$$

$$A_0(\tau) = b_0\frac{1}{\sqrt{2k}}e^{-ik\tau} + c_0\frac{(1 + 2ik\tau)}{\sqrt{2k}}e^{-ik\tau}, \qquad (B9)$$

where b, b_0, c, c_0 are arbitrary constants. Substituting (B8) and (B9) into the basic equations (B2) and (B3) to constraint (c, c_0, b, b_0), we obtain, for $\zeta \neq -1$,

$$A(\tau) = b\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau} + \frac{1}{2}(b - b_0)\frac{\zeta - 1}{\zeta + 1}\frac{i}{k}\frac{(1 + 2ik\tau)}{\sqrt{2k}}e^{-ik\tau}, \qquad (B10)$$

$$A_0(\tau) = b\frac{1}{\sqrt{2k}}e^{-ik\tau} + (b - b_0)$$
$$\times \left(-1 + \frac{1}{2}\frac{\zeta - 1}{\zeta + 1}(1 + 2ik\tau)\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B11)$$

$$\pi_A = \frac{2(b - b_0)}{\zeta + 1}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B12)$$

$$\pi_A^0 = \frac{2(b - b_0)}{\zeta + 1}(-ik)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B13)$$

where the canonical momenta are contributed only by the $(b - b_0)$ part of (B10) and (B11). Similarly, for $\zeta \neq 1$, we obtain

$$A(\tau) = b\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau} + c(1 + 2ik\tau)\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B14)$$

$$A_0(\tau) = b\frac{1}{\sqrt{2k}}e^{-ik\tau} + c\left((1 + 2ik\tau) - 2\frac{\zeta + 1}{\zeta - 1}\right)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B15)$$

$$\pi_A = c\frac{4}{\zeta - 1}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B16)$$

$$\pi_A^0 = c\frac{4}{\zeta - 1}(-ik)\frac{1}{\sqrt{2k}}e^{-ik\tau}. \qquad (B17)$$

In the Feynman gauge Eqs. (B2) and (B3) with $\zeta = 1$ reduce to

$$(\partial_0^2 + k^2)A = 0, \qquad (B18)$$

$$(\partial_0^2 + k^2)A_0 = 0, \qquad (B19)$$

which are already separated for A and A_0, and the solutions (B10)–(B13) reduce to

$$A(\tau) = b\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad A_0(\tau) = b_0\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad (B20)$$

$$\pi_A = (b - b_0)\frac{1}{\sqrt{2k}}e^{-ik\tau}, \qquad \pi_A^0 = (b - b_0)(-ik)\frac{1}{\sqrt{2k}}e^{-ik\tau}. \qquad (B21)$$

The Feynman gauge is commonly used in text books, whereas a general gauge is less addressed.

The solutions of A and A_0 can be rederived by another way. Setting $D = 0$ in (34) and (35) leads to the following inhomogeneous equations:

$$\partial_0^2 A + k^2 A = \partial_0 \pi_A - \zeta \pi_A^0, \qquad (B22)$$

$$\partial_0^2 A_0 + k^2 A_0 = -(k^2 \pi_A + \zeta \partial_0 \pi_A^0). \qquad (B23)$$

Since π_A and π_A^0 are known in (B5), we get the solutions of (B22) and (B23),

$$A = b\frac{i}{k}\frac{1}{\sqrt{2k}}e^{-ik\tau} - \frac{1}{4}(d_1 + d_2\zeta)\frac{i}{k}\frac{(1 + 2ik\tau)}{\sqrt{2k}}e^{-ik\tau}, \qquad (B24)$$

$$A_0 = b_0 \frac{1}{\sqrt{2k}} e^{-ik\tau} - \frac{1}{4}(d_1 + d_2\zeta) \frac{(1+2ik\tau)}{\sqrt{2k}} e^{-ik\tau}. \quad \text{(B25)}$$

Substituting (B24) and (B25) into (B2) and (B3) leads to the constraints on the coefficients

$$(d_1 + \zeta d_2) = -2(b - b_0) \frac{(\zeta - 1)}{(\zeta + 1)} \quad (\zeta \neq -1), \quad \text{(B26)}$$

$$(b - b_0) = -\frac{(\zeta + 1)}{2(\zeta - 1)}(d_1 + \zeta d_2) \quad (\zeta \neq 1). \quad \text{(B27)}$$

This gives (B10) and (B11) for $\zeta \neq -1$ and (B14) and (B15) for $\zeta \neq 1$, respectively.

Given these solutions, we perform the canonical quantization for a general ζ. The quantization of the transverse fields B_i is the same as (57)–(61) in de Sitter space. The longitudinal and temporal operators A, A_0, π_A and π_A^0 for a general ζ are the same as (62), (63), (71), (72), but with the k modes (for $\zeta \neq 1$)

$$A_{1k}(\tau) = b_1 \frac{i}{k} \frac{1}{\sqrt{2k}} e^{-ik\tau} + c_1 \frac{i}{k}(1 + 2ik\tau) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B28)}$$

$$A_{2k}(\tau) = b_2 \frac{i}{k} \frac{1}{\sqrt{2k}} e^{-ik\tau} + c_2 \frac{i}{k}(1 + 2ik\tau) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B29)}$$

$$A_{01k}(\tau) = b_1 \frac{1}{\sqrt{2k}} e^{-ik\tau} + c_1\left((1+2ik\tau) - 2\frac{\zeta+1}{\zeta-1}\right) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B30)}$$

$$A_{02k}(\tau) = b_2 \frac{1}{\sqrt{2k}} e^{-ik\tau} + c_2\left((1+2ik\tau) - 2\frac{\zeta+1}{\zeta-1}\right) \times \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B31)}$$

$$\pi_{A1k} = c_1 \frac{4}{\zeta-1} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B32)}$$

$$\pi_{A2k} = c_2 \frac{4}{\zeta-1} \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B33)}$$

$$\pi_{A1k}^0 = c_1 \frac{4}{\zeta-1}(-ik) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B34)}$$

$$\pi_{A2k}^0 = c_2 \frac{4}{\zeta-1}(-ik) \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \text{(B35)}$$

where (b_1, c_1), (b_2, c_2) are two sets of coefficients. [In the Feynman gauge, the conventional one-operator expansion of A and A_0 will be used since their equations are separated as (B18) and (B19).] We impose the covariant canonical commutation relations

$$[A^\mu(\tau, \mathbf{x}), \pi_A^\nu(\tau, \mathbf{y})] = i\eta^{\mu\nu}\delta(\mathbf{x} - \mathbf{y}). \quad \text{(B36)}$$

Substituting the operators (63), (62), (72), (71) into each $(\mu\nu)$ component of (B36), using the commutation relations (66), we obtain the following constraints upon the coefficients (for $\zeta \neq 1$):

$$(c_{2*}b_2 - c_{1*}b_1) = \frac{1}{4}(\zeta - 1), \quad \text{(B37)}$$

$$|c_2|^2 - |c_1|^2 = 0, \quad \text{(B38)}$$

$$|b_2|^2 - |b_1|^2 = 1. \quad \text{(B39)}$$

There are infinite many choices to satisfy the above constraints. For instance, a simple choice is $c_1 = -c_2 = 1$, $b_1 = \frac{2}{\zeta-1} - \frac{\zeta-1}{8}$, $b_2 = -\frac{2}{\zeta-1} - \frac{\zeta-1}{8}$.

The stress tensor is not actually used in the Minkowski spacetime since gravity is not considered. Here, in analogy to that in de Sitter space, we calculate the stress tensor in Minkowski spacetime. The transverse, LT, and GF stress tensors are defined similar to the expressions (99)–(102) with $a = 1$ and $D = 0$. We list the main results. The transverse stress tensor is

$$\langle\phi|\rho^{TR}|\phi\rangle = 3\langle\phi|p^{TR}|\phi\rangle = \int_0^\infty \rho_k^{TR} \frac{dk}{k} + \int \frac{dk}{k} \rho_k^{TR} \sum_{\sigma=1,2} \langle\phi|a_{\mathbf{k}}^{\dagger(\sigma)} a_{\mathbf{k}}^{(\sigma)}|\phi\rangle, \quad \text{(B40)}$$

where the transverse spectral stress tensor is

$$\rho_k^{TR} = \frac{k^3}{2\pi^2}[|f_k^{(1)\prime}(\tau)|^2 + k^2|f_k^{(1)}(\tau)|^2] = \frac{k^4}{2\pi^2} = 3p_k^{TR}. \quad \text{(B41)}$$

The first term of (B40) is the UV divergent vacuum energy density in Minkowski spacetime, which is routinely removed by normal ordering of the creation and annihilation operators. The LT stress tensor in the GB state $|\psi\rangle$ is

$$\langle\psi|\rho^{LT}|\psi\rangle = 3\langle\psi|p^{LT}|\psi\rangle = \frac{1}{2}\langle\psi|\partial_i\pi_A\partial^i\pi_A|\psi\rangle = \int \rho_k^{LT}\frac{dk}{k}. \quad \text{(B42)}$$

where

$$\rho_k^{\mathrm{LT}} = 3p_k^{\mathrm{LT}} = 0. \tag{B43}$$

The GF stress tensor in the GB state is

$$\langle\psi|\rho^{\mathrm{GF}}|\psi\rangle = \int \rho_k^{\mathrm{GF}} \frac{dk}{k}, \qquad \langle\psi|p^{\mathrm{GF}}|\psi\rangle = \int p_k^{\mathrm{GF}} \frac{dk}{k}, \tag{B44}$$

where

$$\rho_k^{\mathrm{GF}} = 3p_k^{\mathrm{GF}} = \frac{k^4}{2\pi^2} [\langle\psi|a_{\mathbf{k}}^{(3)\dagger} a_{\mathbf{k}}^{(3)}|\psi\rangle - \langle\psi|a_{\mathbf{k}}^{(0)\dagger} a_{\mathbf{k}}^{(0)}|\psi\rangle + k]. \tag{B45}$$

[It is remarked that the trace of the GF part is zero in Minkowski spacetime, unlike the nonzero trace (127) in de Sitter space.] By the GB condition (109), the photon part cancels, and only the vacuum part remains,

$$\rho_k^{\mathrm{GF}} = 3p_k^{\mathrm{GF}} = \frac{k^4}{2\pi^2}, \tag{B46}$$

which has only one divergent k^4 term, corresponding to the dominant UV divergent terms of (124) and (125) in de Sitter space. The UV divergence of (B46) in Minkowski spacetime can be removed by normal ordering also, yielding a zero GF stress tensor. The expectation values of all three parts of the stress tensor are independent of ζ, and the regularized vacuum stress tensor is zero. Thus, the properties of the stress tensor of the Maxwell field with the GF term in Minkowski spacetime are similar to those in de Sitter space.

The above calculations are based on the modes (B28)–(B35) for $\zeta \neq 1$. We may as well use the modes (B10)–(B13) for $\zeta \neq -1$, implement the covariant canonical quantization, and get the same stress tensor.

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