Roles of $a_0(980)$, $\Lambda(1670)$, and $\Sigma(1385)$ in the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay

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Recently, the Belle Collaboration has measured the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay and reported the $\eta\Lambda$ and $\Lambda\pi^+$ invariant mass distributions, which show the clear signals of the resonances $\Lambda(1670)$ and $\Sigma(1385)$, respectively. Based on our previous works [Eur. Phys. J. C **76**, 496 (2016) and Phys. Rev. D **95**, 074024 (2017)], we reanalyze this process by considering the S-wave $\eta\Lambda$ and $\eta\pi^+$ final state interactions within the chiral unitary approach, which dynamically generate the $\Lambda(1670)$ and $a_0(980)$, respectively. Our results are in agreement with the Belle measurements of the $\eta\Lambda$ and $\pi^+\Lambda$ invariant mass distributions. In addition, the $\pi^+\eta$ invariant mass distribution is also calculated and a cusp structure of $a_0(980)$ is clearly shown around the $K\bar{K}$ mass threshold.

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I. INTRODUCTION

Compared to the two-body hadronic decays, multibody hadronic decays of heavy hadrons could be used to explore the nature of the intermediate resonances, since those processes have much larger phase space, and involve the strong interactions of the final states [1]. In the last decades, many hadron resonances are discovered in multibody decays of heavy hadrons by different experiments, such as the $X(3872)$, P_c, P_{cs}, and T_{cc} [2–7]. The baryon Λ_c^+, as the first observed charmed baryon [8], has a large number of decay modes reported by the BESIII, Belle, *BABAR*, CLEO, and LHCb Collaborations [7,9–13], which offers an excellent laboratory for testing the theoretical predictions of the light mesons and light baryons [14–23], such as the processes $\Lambda_c^+ \to pK^+K^-$, $p\pi^+\pi^-$ [20], $\Lambda_c^+ \to p\eta\pi^0$ [21], $\Lambda_c^+ \to \pi^0\phi p$ [22], and $\Lambda_c^+ \to \bar{K}^0\eta p$ [23].

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The process of $\Lambda_c^+ \to \Lambda\eta\pi^+$ has attracted much attentions from theoreticians and experimentalists, since it provides the fruitful information about the intermediate states. In Ref. [24], it is proposed to study the $a_0(980)$ and $\Lambda(1670)$ resonances in the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay via the final-state interactions of the $\eta\pi^+$ and $\Lambda\eta$ pairs. In addition, the process $\Lambda_c^+ \to \Lambda\eta\pi^+$ is used to study the resonances $\Lambda(1405)$ and $\Lambda(1670)$ in the $\Lambda\eta$ final state [25]. Reference [26] has also suggested that this process could be used to search for the missing baryon $\Sigma^*(1/2^-)$ which is interesting theoretically [17,27–33].

Experimentally, the decay $\Lambda_c^+ \to \Lambda\eta\pi^+$ has been measured by the CLEO Collaboration in 1995 [34] and 2003 [35]. Later in 2009, the BESIII Collaboration has presented an improved measurement of the absolute BFs of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ and studied the intermediate state $\Sigma(1385)^+$ in the three-body decay [36]. Recently, the Belle Collaboration has also measured the process $\Lambda_c^+ \to \eta\Lambda\pi^+$, and presented the $\eta\Lambda$ and $\Lambda\pi^+$ invariant mass distributions, which show the significant signals of the $\Lambda(1670)$ and $\Sigma(1385)^+$ resonances [37]. Furthermore, the Belle Collaboration has also reported the contribution of $a_0(980)^+$ to the final state $\eta\pi^+$.

Both the $\Lambda(1670)$ and $a_0(980)$ have generated a lot of interests in their nature. For the baryon $\Lambda(1670)$, although the three-quark explanation is supported by the analysis of the high precision data on $K^-p \to \eta\Lambda$ [38], and the study of $K^- p \to \pi^0 \Sigma^0$ within the chiral quark model [39], the S-wave meson-baryon interactions within the chiral unitary approach can dynamically generate the resonance $\Lambda(1670)$ [28,40,41]. Indeed, the actual shape of the $\Lambda(1670)$ strongly depends on the process. For instance, the $\Lambda(1670)$ peak is shifted to about 1.7 GeV in the modulus squared of the $\bar{K}N$ or $\pi\Sigma$ elastic scattering amplitudes with isospin $I = 0$, due to the strong distortion induced by the $\eta\Lambda$ channel, and its shape appears as a clear strong enhancement, not the symmetric Breit-Wigner structure, in the $\eta\Lambda$ mass distribution [41], which is confirmed by the Belle measurements [37]. In addition, the light scalar meson $a_0(980)$ also has been explained to be either a molecular state, a tetraquark state, a conventional $q\bar{q}$ meson, or the mixing of different components [42–45]. In the chiral unitary approach, $a_0(980)$ could be dynamically generated from the S-wave interactions of the coupled channels $K\bar{K}$ and $\pi\eta$ [43,46], which could be used to successfully interpret many experimental measurements [1,20,47].

In this work, we reanalyze the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay by taking into account the $\Sigma(1385)^+$ decaying into $\pi^+\Lambda$ in P wave. The dynamically generated states $\Lambda(1670)$ and $a_0(980)$ are also considered. Within the chiral unitary approach, we calculate the invariant mass distributions for the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay. It is found that the experimental measurements of the Belle Collaboration [37] can be well reproduced.

This article is organized as follows. In Sec. II, we present the theoretical formalism of the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay, and numerical results and discussions are presented in Sec. III, followed by a short summary in the last section.

II. FORMALISM

In this section, we will introduce the theoretical formalism for the process $\Lambda_c^+ \to \eta\Lambda\pi^+$. We first present the mechanism of the intermediate states $\Lambda(1670)$ and $a_0(980)$ production in Sec. II A. We will also consider the $\Sigma(1385)$ contribution in the final state interaction of the $\pi\Lambda$ system in Sec. II B. Finally, we give the formalism of the invariant mass distributions for the process $\Lambda_c^+ \to \eta\Lambda\pi^+$ in Sec. II C.

A. Contributions of $\Lambda(1670)$ and $a_0(980)$

In analogy to Refs. [24–26], the process $\Lambda_c^+ \to \eta\Lambda\pi^+$ proceeds in the following three steps:

(1) Weak decay: the c quark of Λ_c^+ weakly decays into an s quark plus a W^+ boson, then the W^+ boson goes into a $u\bar{d}$ pair, as shown in Fig. 1.

(2) Hadronization: the $u\bar{d}$ pair from the W^+ decay will hadronize into π^+, and the quark s from the c decay, and the ud pair of the initial Λ_c^+, together with the $\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$ created from vacuum, hadronize into a pseudoscalar meson and a baryon, as depicted in Fig. 2. In addition, the $u\bar{d}$ pair from the W^+ decay could hadronize into a pair of pseudoscalar mesons, together with the $\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$ created from vacuum, and the quark s from the quark c decay, together with the ud pair of the initial Λ_c^+, will form the baryon Λ, as shown in Fig. 3.

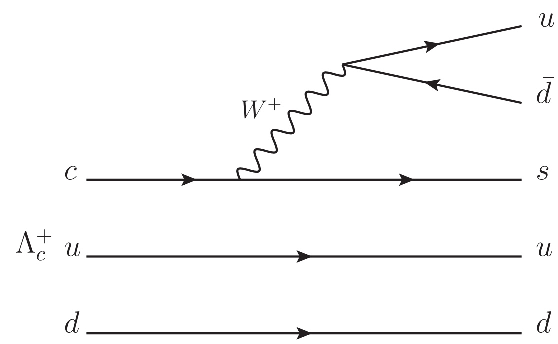

FIG. 1. Feynman diagram at the quark level for the weak process $\Lambda_c^+ \to W^+ + s + ud \to u\bar{d} + s + ud$.

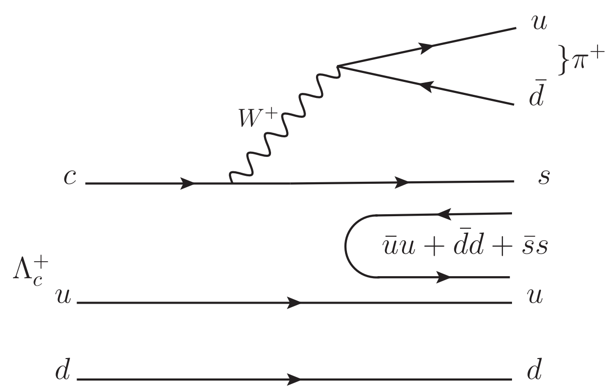

FIG. 2. Hadronization of the process $\Lambda_c^+ \to \pi^+ MB$. The $u\bar{d}$ pair from the W^+ decay will hadronize into π^+, and the quark s from the c decay, and the ud pair of the initial Λ, together with the $\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$ created from vacuum, hadronize into a pseudoscalar meson and a baryon.

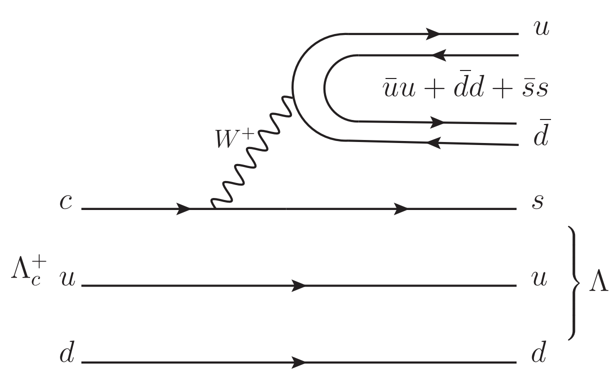

FIG. 3. Hadronization of the process $\Lambda_c^+ \to MM\Lambda$. The $u\bar{d}$ pair from the W^+ decay could hadronize into a pair of pseudoscalar mesons, together with the $\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$ created from vacuum, and the quark s from the quark c decay, together with the ud pair of the initial Λ_c^+, will form the baryon Λ.

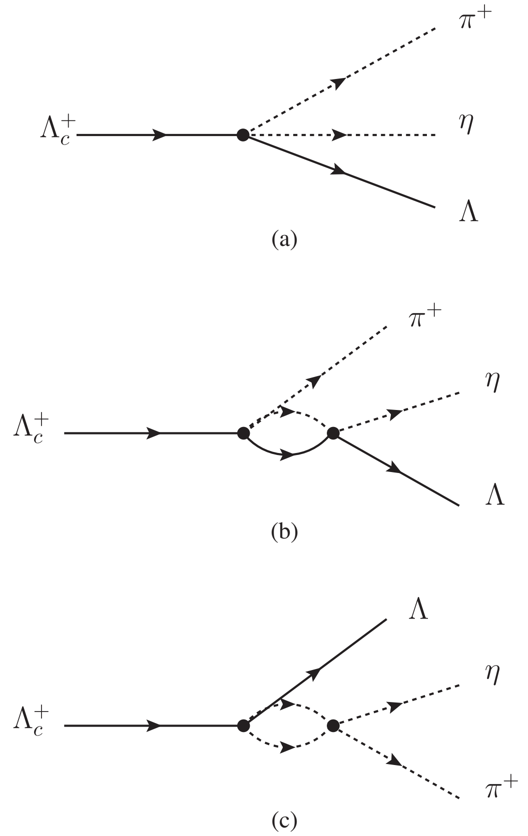

FIG. 4. Mechanism for tree diagram (a), the meson-baryon final-state interaction (b), and the meson-meson final-state interaction (c) for the process $\Lambda_c^+ \to \eta\Lambda\pi^+$.

(3) Final-state interactions: within the chiral unitary approach, the *S*-wave interactions of the meson-baryon and meson-meson pairs will dynamically generate the $\Lambda(1670)$ and $a_0(980)$, respectively, as shown in Fig. 4.[1]

According to Eq. (6) of Ref. [24], we can write the meson-baryon interaction amplitude of the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay as follows:

$$T^{\mathrm{MB}}(M_{\eta\Lambda}) = V_P\bigg\{-\frac{\sqrt{2}}{3} + G_{K^-p}(M_{\eta\Lambda})t_{K^-p\to\eta\Lambda}(M_{\eta\Lambda}) + G_{\bar{K}^0n}(M_{\eta\Lambda})t_{\bar{K}^0n\to\eta\Lambda}(M_{\eta\Lambda}) - \frac{\sqrt{2}}{3}G_{\eta\Lambda}(M_{\eta\Lambda})t_{\eta\Lambda\to\eta\Lambda}(M_{\eta\Lambda})\bigg\}, \tag{1}$$

where V_P accounts for the weak decay and hadronization strength, and is approximate independent of the final state interaction [1,48]. In the above equation, G_{MB} denotes the one-meson-one-baryon loop function, which depends on the invariant mass of the $\eta\Lambda$ system, $M_{\eta\Lambda}$. The meson-baryon scattering amplitudes $t_{MB\to\eta\Lambda}$ are obtained in the chiral unitary approach, which also depends on $M_{\eta\Lambda}$, and the details can be found in Refs. [40,49].

According to Eq. (13) of Ref. [24], the meson-meson transition amplitude is given as follows:

$$T^{\mathrm{MM}}(M_{\pi^+\eta}) = V'_P\frac{2\sqrt{2}}{3}\bigg\{1 + G_{\pi^+\eta}(M_{\pi^+\eta})t_{\pi^+\eta\to\pi^+\eta}(M_{\pi^+\eta}) + \frac{\sqrt{3}}{2}G_{K^+\bar{K}^0}(M_{\pi^+\eta})t_{K^+\bar{K}^0\to\pi^+\eta}(M_{\pi^+\eta})\bigg\}, \tag{2}$$

where G_{MM} is the loop function of the two-meson propagators [46] and $t_{MM\to\pi^+\eta}$ are the meson-meson scattering amplitudes obtained in Ref. [46], which depend on the $\pi^+\eta$ invariant mass $M_{\pi^+\eta}$. Here V'_P is the weak and hadronization strength for the mechanism of meson-meson interaction. We will take $V'_P = 0.38V_P$, which is obtained with the assumption that the amplitudes of Eqs. (1) and (2) give rise to the same width for the process $\Lambda_c^+ \to \eta\Lambda\pi^+$ (see more details in Ref. [24]).

B. Contribution of $\Sigma(1385)^+$

For the contribution of $\Sigma(1385)^+$ decaying into $\pi^+\Lambda$ in the process of $\Lambda_c^+ \to \eta\Lambda\pi^+$, the Feynman diagram is shown in Fig. 5. In the nonrelativistic reduction [50–52], the invariant decay amplitude of Fig. 5 can be written as follows[2]:

$$T^{\Sigma^*}(M_{\pi^+\Lambda}) = V''_P\frac{|\vec{p}_\pi|\cdot|\vec{p}_\eta|\cdot\cos\theta}{M_{\pi^+\Lambda} - M_{\Sigma^*} + i\frac{\Gamma_{\Sigma^*}}{2}}, \tag{3}$$

where V''_P is the relative strength of the contribution of $\Sigma(1385)^+$, while $\vec{p}_\pi$ and $\vec{p}_\eta$ are the momenta of π^+ and η in the $\pi^+\Lambda$ rest frame, respectively, and θ is the angle between π^+ and η in the center of mass frame of the $\pi^+\Lambda$ system [52], which are given by

$$|\vec{p}_\pi| = \frac{\lambda^{\frac{1}{2}}(M^2_{\pi^+\Lambda}, m^2_{\pi^+}, M^2_\Lambda)}{2M_{\pi^+\Lambda}}, \tag{4}$$

$$|\vec{p}_\eta| = \frac{\lambda^{\frac{1}{2}}(M^2_{\Lambda_c}, m^2_\eta, M^2_{\pi\Lambda})}{2M_{\pi\Lambda}}, \tag{5}$$

$$\cos\theta = \frac{M^2_{\eta\Lambda} - M^2_{\Lambda_c} - m^2_{\pi^+} + 2P^0_{\Lambda_c}P^0_\pi}{2|\vec{p}_\pi||\vec{p}_\eta|}, \tag{6}$$

[1]The exact unitary condition of the two-body scattering amplitudes can be incorporated through the Bethe-Salpeter equation, inverse amplitude method, or N/D method, which could generate the poles for the resonances/bound states in the unitarized scattering amplitudes. In this work, the two-body unitary is realized through the Bethe-Salpeter equation in the meson-meson and meson-baryon scattering amplitudes, as discussed in Refs. [40,46].

[2]In this part we use Σ^* to represent $\Sigma(1385)$.

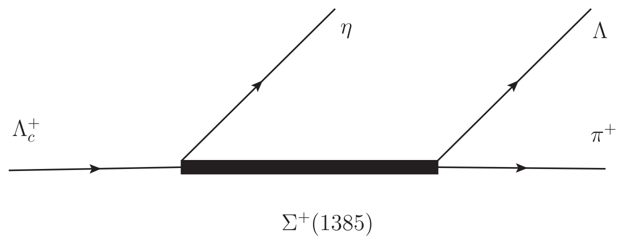

FIG. 5. Feynman diagram for the contribution of $\Sigma(1385)^+$ to the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay.

with $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$. In the $\pi\Lambda$ rest frame, $\vec{p}_{\Lambda_c} = \vec{p}_\eta$, $\vec{p}_\pi = -\vec{p}_\Lambda$, and the Λ_c and π energies are

$$P^0_{\Lambda_c} = \sqrt{M^2_{\Lambda_c} + |\vec{p}_{\Lambda_c}|^2} = \sqrt{M^2_{\Lambda_c} + |\vec{p}_\eta|^2},$$
$$P^0_\pi = \sqrt{m^2_{\pi^+} + |\vec{p}_\pi|^2}.$$

It should be pointed out that the amplitude of Eq. (3) does not fulfil the two-body unitary. Since the $\Sigma(1385)$ can be well explained as a conventional qqq baryon, the Breit-Wigner amplitude for the $\Sigma(1385)$ has been widely used in the literature [29,53,54]. Considering that the unitary coupled-channel model for $\Sigma(1385)$ proposed in Ref. [55] includes many parameters of the coupled channels, we use the Breit-Wigner form for the $\Sigma(1385)$ amplitude in this work.

C. Invariant mass distributions of the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay

With all the ingredients obtained in the previous section, one can write down the modulus squared of the total amplitude as [24]

$$|T^{\text{total}}|^2 = |T^{\text{MB}} + T^{\text{MM}}|^2 + |T^{\Sigma^*}|^2. \tag{7}$$

The invariant mass distributions for the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay can be written as [7]

$$\frac{d^2\Gamma_{\Lambda_c^+\to\eta\Lambda\pi^+}}{dM^2_{\pi^+\eta}dM^2_{\eta\Lambda}} = \frac{1}{(2\pi)^3}\frac{M_\Lambda}{8M^2_{\Lambda_c^+}}|T^{\text{total}}|^2. \tag{8}$$

Then the $\pi^+\eta$ and $\eta\Lambda$ mass distributions can be obtained by integrating over the other invariant mass in Eq. (8), and the $\pi^+\Lambda$ mass distributions can be obtained by substituting $M_{\eta\Lambda}$ with $M_{\pi^+\Lambda}$ in Eq. (8). In addition, for a given value of M_{12}, the range of M^2_{23} is defined as [7]

$$(M^2_{23})_{\max} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2}\right)^2,$$
$$(M^2_{23})_{\min} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2}\right)^2,$$

where E_2^* and E_3^* are the energies of particles 2 and 3 in the rest frame of particles 1 and 2, respectively,

$$E_2^* = \frac{M^2_{12} - m_1^2 + m_2^2}{2M_{12}},$$
$$E_3^* = \frac{M^2_{\Lambda_c} - M^2_{12} - m_3^2}{2M_{12}},$$

where m_1, m_2, and m_3 are the masses of particles 1, 2, and 3, respectively.[3] All the masses and widths of the particles involved in this work are taken from the review of particle physics [7].

Indeed, including the contributions from the three-body unitaries for the scattering amplitudes [56–59], the decay amplitudes would become more complex due to additional parameters, which are difficult to be determined. Taking into account that the three-body dynamics does not significantly modify the invariant mass distributions of the two-body systems around the resonant peaks [60], we neglect the three-body dynamics in this work.

III. NUMERICAL RESULTS AND DISCUSSIONS

With the above formalism, we calculate the invariant mass distributions of the process $\Lambda_c^+ \to \eta\Lambda\pi^+$. There are three free parameters to be obtained by fitting to the experimental data: (1) V_p for the weak and hadronization strength related to the meson-baryon interaction of Fig. 3; (2) V_p'' for the weight of the contribution of $\Sigma(1385)$; (3) $a_{K\Xi}$ for the subtraction constant in the $K\Xi$ channel of Ref. [40].

In this work we take $a_{K\Xi}$ as a free parameter, because the position of the dynamically generated state $\Lambda(1670)$ is quite sensitive to the value of $a_{K\Xi}$ and only moderately sensitive to $a_{\bar{K}N}$, $a_{\pi\Sigma}$, and $a_{\eta\Lambda}$, as discussed in Ref. [40]. For instance, the pole position is $M_R = 1708 + i21$ MeV for $a_{K\Xi} = -2.52$, and $M_R = 1680 + i2$ MeV for $a_{K\Xi} = -2.67$ Ref. [40]. Thus we would like to constrain the pole position of $\Lambda(1670)$ by fitting the parameter $a_{K\Xi}$ to the Belle measurements [37].

With the model presented above, we perform the fit to the Belle measurements of the $\eta\Lambda$ and $\Lambda\pi^+$ mass distributions [37]. The obtained $\chi^2/\text{d.o.f.}$ is $815.4/(91+162-3) = 3.26$, and the fitted parameters are $V_P = 1.32 \pm 0.01$, $V_P'' = (3.46 \pm 0.02) \times 10^{-4}$, and $a_{K\Xi} = -2.72 \pm 0.01$.[4] It should be stressed that the obtained $a_{K\Xi} = -2.72$, which is close to the values used in Ref. [40]. With this value, we present the modules squared of the transition amplitudes $|T_{K\Xi-K\Xi}|^2$, $|T_{K\Xi-\eta\Lambda}|^2$, and $|T_{K\Xi-KN}|^2$ in Fig. 6, where one can find that the position of the pole is about 1680 MeV.

[3] In this work, we label the final states π^+, η, or Λ by 1, 2, and 3.

[4] In this work, we adopt the MINUIT program to perform the fit. Our $\chi^2/\text{d.o.f.}$ is larger than 1, because we have only considered the contributions from the $a_0(980)$, $\Lambda(1670)$, and $\Sigma(1385)$ resonances, while the $\Lambda(1670)$ and $a_0(980)$ states are dynamically generated within the chiral unitary approach and there are no free parameters for them.

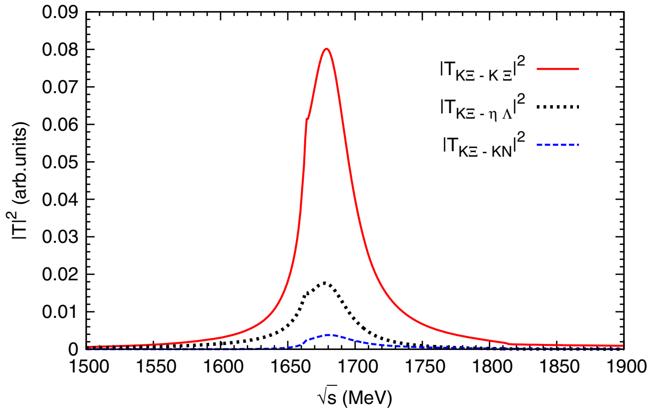

FIG. 6. Modulus squared of the transition amplitudes $|T_{K\Xi-K\Xi}|^2$, $|T_{K\Xi-\eta\Lambda}|^2$, and $|T_{K\Xi-KN}|^2$, respectively.

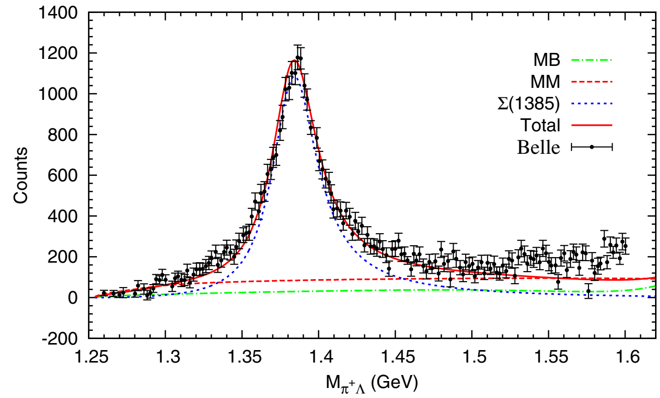

FIG. 8. The $\pi^+\Lambda$ invariant mass distribution for the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay. The explanations of the curves are the same as those of Fig. 7.

With the fitted values of the parameters, we calculate the $\eta\Lambda$ and $\pi^+\Lambda$ mass distributions, as shown in Figs. 7 and 8, respectively, where we show the contribution from meson-baryon (MB) interaction corresponding to $\Lambda(1670)$, the contribution from meson-meson (MM) interaction corresponding to $a_0(980)$, $\Sigma(1385)$, the total contribution (Total), and the Belle measurements (Belle) [37]. For both mass distributions, our results are in good agreement with the Belle measurements.

From Fig. 7 one can find that the peak close to the threshold in the $\eta\Lambda$ mass distribution is the signal of the $\Lambda(1670)$, which is dynamically generated from the S-wave meson-baryon interaction, and the interference effect between $\Lambda(1670)$ and $a_0(980)$ is destructive, which is consistent with Ref. [24]. The contribution of $\Sigma(1385)$ in the $\Lambda(1670)$ peak is very small.

In Fig. 8, our model calculations can describe well the reported $\pi^+\Lambda$ invariant mass distribution where the $\Sigma(1385)^+$ is clearly shown. It is found that the other contributions are rather small to produce the peak of $\Sigma(1385)$. It should be pointed out that the $\Sigma(1385)$ mainly contributes to the region of $M_{\eta\Lambda} > 1.9$ GeV, beyond the region considered for the $\Lambda(1670)$. In addition, the $\Lambda(1670)$ mainly contributes to the high energy region of the $\pi^+\Lambda$ invariant mass distribution, beyond the energy region of the Belle data points of Fig. 8 for the $\Sigma(1385)$ peak. Both of them could be easily understood from the Dalitz plot measured by the Belle Collaboration, as shown in Fig. 5 of Ref. [37].

Next, we turn to the $\pi^+\eta$ invariant mass distribution. Our numerical results are shown in Fig. 9. One can find a clear cusp structure around the $K\bar{K}$ threshold, which is consistent with our previous results [24]. However, the reflection effect of the $\Sigma(1385)^+$ simultaneously appears in the low energy and high energy regions, which could be easily understood from the Dalitz plot of the process $\Lambda_c^+ \to \eta\Lambda\pi^+$ measured by Belle [37]. Indeed, the cusp structure of the $a_0(980)$ has been observed in many processes, such as $\chi_{cJ} \to \eta\pi\pi$ measured by BESIII [61], which is mainly due to its strong coupling to the $K\bar{K}$ channel. Thus, the cusp

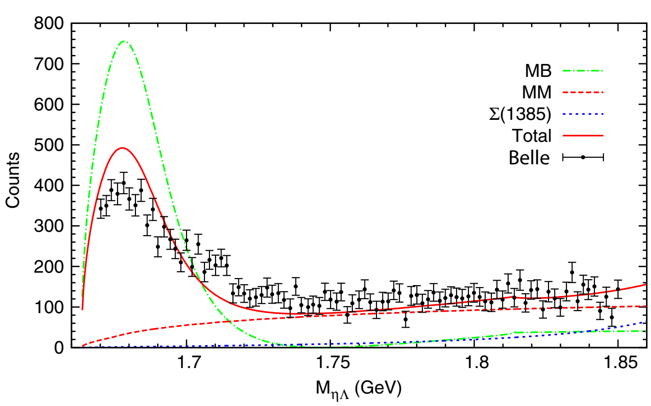

FIG. 7. The $\eta\Lambda$ invariant mass distribution for the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay. The curves labeled as "MB," "MM," and "Σ(1385)" show the results obtained with $T^{\rm MB}$, $T^{\rm MM}$, and T^{Σ^*} respectively; "Total" curve corresponds to the total contribution of Eq. (7). The Belle data are taken from Ref. [37].

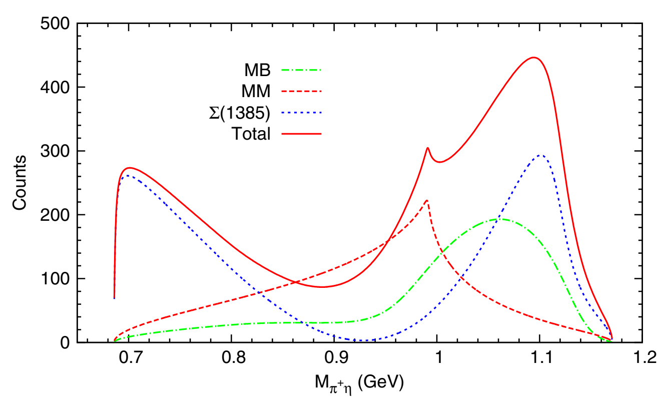

FIG. 9. The $\eta\pi^+$ invariant mass distribution for the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay. The explanations of the curves are the same as those of Fig. 7.

structure of $a_0(980)$, as one feature of its molecular nature, is expected to be observed in the $\eta\pi^+$ invariant mass distribution.

In addition, we have already investigated this reaction considering only the contributions of the $\Sigma(1385)(3/2^+)$ and the predicted $\Sigma^*(1/2^-)$ in Ref. [26]. As shown in Fig. 7, the $\Sigma(1385)$ gives a smooth contribution in the $\eta\Lambda$ invariant mass distribution. Thus, one expects that the structure of the $\Lambda(1670)$ and $a_0(980)$ will not appear if the final state interactions of the $\eta\Lambda$ and $\pi^+\eta$ are neglected.

IV. SUMMARY

In this work, we have reanalyzed the process of $\Lambda_c^+ \to \eta\Lambda\pi^+$ by taking into account the P-wave contribution from the $\Sigma(1385)$, and the S-wave meson-meson and meson-baryon interactions provided by the chiral unitary approach, which dynamically generate the resonances $a_0(980)$ and $\Lambda(1670)$. Our calculated $\eta\Lambda$ and $\Lambda\pi^+$ invariant mass distributions are in agreement with the Belle measurement, which implies that the peak structure close to the $\eta\Lambda$ threshold in the $\eta\Lambda$ invariant mass distribution could be explained by the dynamically generated resonance $\Lambda(1670)$.

On the other hand, we also plot the $\pi^+\eta$ invariant mass distribution with the fitted parameters, which shows a clear cusp structure of the $a_0(980)$, as one feature of its molecular nature. Since the $a_0(980)$ is tied to the S-wave pseudoscalar-pseudoscalar interaction within the chiral unitary approach, the predicted cusp structure of $a_0(980)$ should be reliable, and could be tested by future measurements. Due to the P-wave coupling of $\Sigma(1385)^+ \to \Lambda\pi^+$, the reflection effect of the $\Sigma(1385)^+$ mainly appears in both the low energy region and high energy region of the $\pi^+\eta$ invariant mass distribution, which could be easily understood from the Dalitz plot of the process $\Lambda_c^+ \to \eta\Lambda\pi^+$ measured by Belle [37].

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