Neutrino physics in TeV scale gravity theories

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In this paper the general features of the neutrino sector in TeV scale quantum gravity theories, such as Arkani-Hamed-Dimopoulos-Dvali (ADD) model and "many species" theory, are investigated. This class of theories has an inherent way of generating small neutrino masses. After reviewing this mechanism it is generalized to a realistic three-flavor case. Furthermore, a procedure is presented on how to diagonalize a mass matrix which is generated by this class of theories and how one can find the Standard Model flavor eigenstates. The developed general approach is applied to two specific scenarios within ADD and many species theory and possible effects on neutrino oscillations and on unitarity of the lepton mixing matrix are calculated. Finally, a short overview of phenomenology which can be potentially testable by the current neutrino experiments is presented.

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I. INTRODUCTION

The Standard Model (SM) of particle physics is one of the most successful theories because it fully accounts for all the processed data from high-energy particle physics accelerators.

Nevertheless, there are several hints that the SM is not complete. In particular, it produces two outstanding puzzles that are the subject of the present paper: 1) the origin and inexplicable smallness of the neutrino mass; 2) the hierarchy problem.

The hierarchy problem is perhaps the most prominent naturalness puzzle of the SM and gravity plays a defining role in its essence due to the following [1]. The Higgs mass is quadratically sensitive towards the cutoff of the theory, but the ultimate cutoff is provided by gravity in the form of the Planck mass. This cutoff is fully nonperturbative since the Planck mass is an absolute upper boundary on the mass of elementary particles. Indeed, any elementary object much heavier than the Planck scale is a classical black hole.

This raises the question of what keeps the observed value of the Higgs mass-term by some 34 orders of magnitude smaller than the expected upper limit. This hierarchy strongly hints toward some new stabilizing physics not far from the weak scale.

One mechanism for stabilization is based on lowering the fundamental scale of quantum gravity. In this framework, the Planck mass M_P still sets the coupling strength of graviton at large distances. However, the actual scale M_*, at which the quantum gravitational effects are strong, is much lower. Correspondingly, in such a scenario the cutoff-sensitive corrections to the Higgs mass are regulated by the scale M_* and not by M_P. This idea was originally proposed in the Arkani-Hamed-Dimopoulos-Dvali (ADD) model [2,3]. (see, [4] for string theory realization).

In this setup the fundamental scale of gravity is lowered due to a large volume of extra dimensions. The reason is that due to universal nature of gravity, the graviton wave function spreads over the entire volume of extra space and gets effectively "dilute". As a result, the coupling scale of a graviton M_P is hierarchically larger than the fundamental scale of quantum gravity M_*.

Remarkably, more recently it was shown in [5,6] (for various aspects, see, [7–11]) that the effect of lowering the cutoff M_* relative to Planck mass M_P is a universal property of any theory with a large number of particle species. Correspondingly, a general solution to the hierarchy problem based on this mechanism was proposed in [5] and further studied in subsequent papers. We shall refer to this as the "many species" framework.

As explained in [5], the ADD model of large extra dimensions represents a particular manifestation of this very general phenomenon. There the role of the species is assumed by the Kaluza-Klein (KK) excitations of graviton. This connection enables us to understand the dilution of graviton wave function in the extra space of ADD as a dilution in what is called in [7] the "space of species". In the latter paper, it was argued that, due to unitarity and other general consistency properties, the "space of species" in many respects behaves as an ordinary geometric space.

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In a particularly interesting realization of the“many species” solution the role of the species is played by the identical copies of the SM [5,6] and various phenomenological aspects of this proposal were studied in [9].

Soon after the invention of the low-scale quantum gravity idea, it was realized, first [12,13] within the ADD model and later in [9] within the many species theory, that this general framework, as a bonus, offers a universal solution to the neutrino mass problem in the SM. Namely, the same mechanism that explains the hierarchy between the weak and Planck scales, is responsible for the suppression of the neutrino mass.

Naturally, this fact boosts the motivation for the above class of theories, since the origin and the hierarchy of the neutrino mass is a fundamental open question in SM. Due to the phenomenon of neutrino oscillations [14], we know that they have a mass and this is now a field of several ongoing experiments. So far, neutrino mass has not been detected directly; just the upper bound of roughly 1 eV have been given [15]. It is also not known whether the neutrino mass is of Majorana or of Dirac nature as this is the case with all other fermions of the SM. We also do not know why neutrinos are so much lighter than charged fermions.

If neutrino masses are of Dirac type and originate from an ordinary Higgs mechanism, an unusually small Yukawa coupling would be required for generating masses smaller than eV.

Lack of explanation for this smallness, prompted thinking that perhaps the neutrino mass is of Majorana type. In such a case, the mass term must be generated from an effective high-dimensional operator [16] and the smallness can be attributed to the high cutoff scale. In the traditional seesaw mechanism [17–21], such an effective operator is generated by integrating out a neutrino's hypothetical right-handed partner with a large Majorana mass.

However, after the formulation of theories with low-scale gravity, it became clear that they offer an alternative possibility in form of naturally small Dirac neutrino masses. Originally, this idea was realized in [12,13] within the ADD framework [2] and deeper analysis in [22–29]. We shall refer to this as ADDM model. Later a complementary mechanism of suppressed neutrino mass was introduced by Dvali-Redi (DR) [9] within the “many species” framework with identical copies of the SM.[1]

Although complementary, the above two scenarios are based on one and the same fundamental mechanism of the suppression of the neutrino Yukawa coupling, very similar to the suppression of the coupling of the graviton. In both cases, this can be viewed as a consequence of the dilution of the wave function of the sterile neutrino into the bulk of the extra space of species. This dilution is identical to the dilution of the wave function of the graviton in the same space. In ADD this space is also organized as a real coordinate space but this does not change the essence of the dilution. In summary, the theories with low M_* can solve both the hierarchy and the neutrino mass problems by the same mechanism. Both hierarchies are controlled by the ratio M_*/M_P.

The main focus of the present work is implications for neutrino physics. The above class of theories predict certain universal features of phenomenological interest. In particular as already discussed in [13] for ADDM and in [9] for DR scenarios, the mixing with the tower of sterile neutrino species results into oscillations of neutrinos into the hidden sector. This implies nonconservation of the neutrino number within the SM and, correspondingly, a seeming violation of unitarity. This is obviously of potential experimental interest.

We will work out a general framework for how neutrino physics can be treated in this class of theories. Moreover, we will generalize this framework to a realistic three-flavor case and investigate their effects on low-energy phenomena and observables such as neutrino oscillations into the hidden modes and possible deviations from the Standard Model Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix which can be tested experimentally. This is still an ongoing project [31].

This paper is organized as follows. In Sec. II the structures of DR and ADDM models are presented and reviewed. In Sec. III we formulate a general approach how neutrino masses are induced in these kinds of theories and how we can guarantee the smallness of their mass. In Sec. IV we present a generalization of the mass matrices to a realistic three-flavor case and in Sec. V we investigate the special case of highly-symmetric mass matrices which are of interest for the DR model. In Sec. VI the phenomenology of TeV scale gravity theories in neutrino physics is investigated. In Sec. VII we wrap up our findings and give an outlook on possible experimental tests.

II. ADD AND “MANY SPECIES THEORY”

A. Many species theory

Originally the idea of many mirror copies of the SM has been proposed in [5,6] as the framework for solving the hierarchy problem. In this work, it has been shown that through introducing N particle species, the fundamental scale of gravity M_* in lowered relative to Planck scale M_P in the following way,

$$M_p^2 \geq N M_*^2. \quad (1)$$

This result has been obtained using the well-established properties of black hole physics and is fully nonperturbative. In order to lower the scale of gravity to TeV energies, N must be of the order of 10^{32}. Since the bound (1) is

[1]Such scenarios have other potential bonuses. For example, particles of other copies could just interact with each other gravitationally and are good candidates for dark matter [9,30].

independent of the nature of the particles, the species of various types can be used for lowering the gravitational cutoff. A particular version introduced in [5,6], assumes that the species are identical copies of the SM. Phenomenological aspects including the generation of neutrino mass were discussed in [9].

In this work, it is assumed that the species obey the full permutation group $P(N)$ initially. This means that all species are equidistant in what one can call the "space of species". This gives a certain predictive power to the theory. Alternative choices such as cyclic symmetry are also possible. It was shown that many species framework can give various phenomenological signatures, including micro-black holes, in the region of TeV energies. In the present work, we shall focus on the implications for neutrino masses.

We shall study generalizations of the mechanism, originally introduced in [9], which allows the generation of small neutrino masses in many species framework. This mechanism represents an infrared alternative to seesaw which cannot be used in frameworks with a low cutoff.

Let us briefly review the results of [9]. As already said, the framework represents N identical copies of the SM. The copies are permuted under $P(N)$. It is useful to visualize the copies as placed on equidistant sites in the space of species. Fermions of each sector are charged under their own gauge group. The exceptions are sterile neutrinos, which represent the right-handed partners of corresponding active left-handed neutrinos. We shall denote them by ν_{Rj}, where $j = 1, 2, \ldots, N$ is the label of the SM copy. These particles do not carry any charges under the SM gauge groups. Thus the notion of "belonging" is defined by their transformation properties under the permutation group $P(N)$ as well as by their couplings to particles of specific SM copies. In particular, the gauge charges do not forbid sterile neutrinos to interact with neutrinos of the other copies. One can say that sterile neutrinos are not confined to specific sites in the space of species. The most generic renormalizable coupling has the following structure,

$$(HL)_i \lambda_{ij} \nu_{Rj} + \text{H.c.}, \tag{2}$$

where H and L stand for the Higgs and lepton doublets of the ith copy. Here λ_{ij} is a $N \times N$ Yukawa matrix interaction in the space of species. This Yukawa coupling matrix is restricted by the permutation symmetry group $P(N)$ and has the following form:

$$\lambda_{ij} = \begin{pmatrix} a & b & b & \ldots \\ b & a & b & \ldots \\ b & b & a & \ldots \\ \ldots & \ldots & \ldots & \ddots \end{pmatrix}. \tag{3}$$

For the calculation of the mass matrix of neutrinos, one has to have a closer look at the Higgs doublet H_i. The simplest case for calculation is when the permutation symmetry is unbroken by the electroweak vacua. This means, that the vacuum expectation value (VEV) of the Higgs doublet in every copy of the SM takes the same value v. In this section we shall focus on this case. The generalization to the case of broken permutation symmetry will be given later.

For now, let us, therefore, take v as the VEV of the Higgses for all copies. Then, the mass matrix takes the form $m_{ij} = \lambda_{ij} v$.

This mass matrix has the eigenvalues

$$m'_1 = (a - b)v, \tag{4}$$

$$m_H = [a + (N-1)b]v, \tag{5}$$

corresponding to the eigenvectors

$$\nu'_1 = \sqrt{\frac{N-1}{N}}\nu_1 - \frac{1}{\sqrt{N}}\nu_h, \tag{6}$$

and

$$\nu_H = \frac{1}{\sqrt{N}}\nu_1 + \sqrt{\frac{N-1}{N}}\nu_h. \tag{7}$$

It is worth noticing for later convenience that the light eigenvalue is $N - 1$ times degenerated. Because

$$b \leq \frac{1}{\sqrt{N}}, \tag{8}$$

and $a \approx 100b$ we see that the mass of the neutrino is suppressed by the number of species. The mechanism presented here can explain the smallness of the neutrino mass but has no phenomenological implications which can be tested by experiments due to the huge mass of the heavy state which scales with the number of species which is of order $N \approx 10^{32}$.

B. The ADDM model

The ADDM model [2–4] is based on the idea that in addition to observed 3 space dimensions, there exist d additional compact-space ones with radii R_i, $i = 1, 2, \ldots, d$ below tenths of a millimetre.

The role of the gravitational cutoff in this theory is played by the fundamental Planck mass of the $(4 + d)$-dimensional theory, M_f. The two Planck scales are related via,

$$M_P = M_f \sqrt{M_f^d V_d}, \tag{9}$$

where

$$V_d = (2\pi)^d R_1 \ldots R_d, \tag{10}$$

is the volume of the extra-dimensional space.

Like the DR, this theory provides a solution to the hierarchy problem by lowering the cutoff M_f relative to Planck mass due to the large volume of extra space. $M_f \sim$ TeV requires that the volume of the extra space, measured in units of the fundamental Planck mass, be about $M_f^d V_d \sim 10^{32}$.

As noticed in [5] the lowering of the cutoff in ADD can be understood as a particular case of many species effect. This is because the quantity $M_f^d V_d$ measures the number of Kaluza-Klein species of the graviton. Thus, Eq. (9) represents a particular manifestation of a more general relation (1).

According to this theory, the Standard Model particles are localized on a 3-dimensional hypersurface (brane) which is embedded in the bulk of d large extra dimensions. The graviton propagates into the entire high-dimensional space. Together with gravity, the bulk is a natural habitat for all possible particles that carry no gauge quantum numbers under the Standard Model group.

Notice that [2] the bulk particles cannot carry any quantum numbers under the SM gauge group. This is a consistency requirement that follows from the gauge invariance and is an intrinsic feature of the localization mechanism of the gauge field on the brane [32]. Correspondingly, the localization of SM gauge fields on the brane automatically forbids the existence of any bulk modes with such charges. Only the particles carrying no SM gauge quantum numbers are permitted to represent bulk modes. In particular, such are sterile neutrinos that play the role of the right-handed partners of the ordinary left-handed neutrinos of the Standard Model.

This setup generates a naturally small Dirac mass for neutrinos [12,13]. This mass originates from the mixing of the right-handed component of bulk sterile neutrino ν with the Standard Model left-handed neutrino ν_L which is localized on the brane. In the approximation of a zero-width brane, the part of the action responsible for this mixing can be written as

$$\int d^4x \frac{h}{M_f^{d/2}} H(x_\mu)\bar{\nu}_L(x_\mu)\nu(x_\mu, y_i = 0) + \text{H.c.}, \tag{11}$$

where x_μ stands for ordinary 4-dimensional spacetime coordinates and $y_i, i = 1, 2, \ldots, d$ are the extra ones. The brane location is taken at $y_i = 0$ point. The canonically normalized $(4 + d)$-dimensional fermion field $\nu(x, y)$ has dimensionality $(3 + d)/2$. Correspondingly, the coupling constant has dimensionality $-d/2$. We have parametrized this coupling constant in terms of the fundamental scale M_f and an order-one dimensionless constant h.

From the point of view of 4-dimensional theory, $\nu(x, y)$ represents a tower of Kaluza-Klein modes with their masses quantized in units of the inverse radii $m^2 = \sum_i n_i^2/R_i^2$ where n_i are integers.

Notice that, a high dimensional fermion field ν, viewed from the point of view of a 4-dimensional theory, has no chirality. That is, at each Kaluza-Klein level of mass m it contains 4-dimensional fermions of both chiralities, $\nu_R^{(m)}$ and $\nu_L^{(m)}$. The 4-dimensional reduction of the coupling (11) gives

$$\frac{hM_f}{M_P} H\bar{\nu}_L \sum_m \nu_R^{(m)} + \text{H.c.}, \tag{12}$$

where the factor $1/M_P$ comes from the canonical normalization of the Kaluza-Klein modes. Notice that only the right-handed components $\nu_R^{(m)}$ of the Kaluza-Klein modes mix with SM neutrino.

After taking into account the VEV of the Higgs field, $\langle H \rangle \equiv v$, the above couplings translate as the Dirac-type mass terms

$$m_D \bar{\nu}_L \sum_m \nu_R^{(m)} + \text{H.c.}, \tag{13}$$

with $m_D \equiv \frac{hvM_f}{M_P}$. This mixing generates a Dirac mass of the SM neutrino. Below, for evaluating the mass matrix we shall restrict ourselves to the case of a single extra dimension. In this case, the masses of Kaluza-Klein excitations are labeled by a single integer $m = n/R$.

Taking into account the Dirac mass terms of Kaluza-Klein modes coming from the mixing between their left- and right-handed components,

$$\sum_{n=-\infty}^{\infty} \frac{n}{R} \bar{\nu}_{nR} \bar{\nu}_{nL}, \tag{14}$$

the resulting mass matrix has the form

$$M = \begin{pmatrix} m_D & \sqrt{2}m_D & \sqrt{2}m_D & \ldots & \sqrt{2}m_D \\ 0 & \frac{1}{R} & 0 & \ldots & 0 \\ 0 & 0 & \frac{2}{R} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & \frac{k}{R} \end{pmatrix}. \tag{15}$$

After the diagonalization of the mass matrix, one can express a neutrino of a specific flavor with the following expression,

$$\nu = \frac{1}{\Omega}\left(\nu_0 + \xi \sum_{n=1} \frac{1}{n} \tilde{\nu}_n\right), \tag{16}$$

with

$$\xi = \frac{\sqrt{2}vM_f Rh}{M_P}. \tag{17}$$

The normalization parameter is $\Omega^2 = 1 + \frac{\pi^2}{6}\xi^2$.

The mass of the lightest eigenstate ν_0 is

$$m_D = \frac{hvM_f}{M_P}. \tag{18}$$

The other eigenstates have

$$m_n \approx \frac{n}{R}. \tag{19}$$

One of the important phenomenological implications of this scenario is the oscillation of active neutrino species into the KK neutrinos [13]. The effect takes place already for a single-flavor case. We shall review this later and compare it with the case of three flavors of active neutrino species.

III. GENERALIZATION OF NEUTRINO MASSES

We have seen that in ADDM and in DR one can generate small neutrino masses by introducing a sterile neutrino which is uncharged under the SM gauge group and can therefore propagate into an additional space which was introduced in this class of theories. In the case of ADDM, this space is represented by the bulk of large extra dimensions and in the DR it is described as the "space of species". In both cases, the neutrino mass is suppressed by the large effective volume of this extra space.

This common structure we want to investigate further. We shall make a rather general assumption of the existence of an extra space in which the sterile neutrino can propagate. Also, we assume that this extra space lowers the scale of gravity via

$$M_*^2 = \frac{M_P^2}{\Lambda}, \tag{20}$$

where Λ is the volume of the extra space measured in fundamental units. In ADDM the size of the extra space is a function of R $\Lambda(R)$ and in DR of N $\Lambda(N)$.

It is assumed that the particles that are not charged under the SM gauge symmetries can propagate in this extra space. That is, the couplings of such particles are more or less uniformly spread over this space. Correspondingly, the coupling to individual copies is suppressed.

Within the known framework there are two candidates for such particles. The first one is of course the graviton since gravity interacts universally. The second natural candidate is a sterile neutrino. Currently, it is not known whether the neutrino is a purely Majorana particle. If it is not, then there necessarily exists a sterile partner ν_R that together with an ordinary left-handed neutrino forms a Dirac state. This sterile neutrino carries no gauge quantum numbers under the Standard Model group. Correspondingly, it has no obligation to be confined to the site where our Standard Model is located. Instead, just like gravity, such particles can spread over the entire extra space, regardless of whether this space stands for extra-space dimensions or the space of species. This spread naturally suppressed the coupling of the sterile fermion to SM neutrino, thereby resulting in a small Dirac mass. The suppression of the coupling with many mixing partners results from the principle of unitarity and was shown in [7]. This is the key mechanism behind the small neutrino mass both in ADDM [12] as well as in DR [9].

A possible operator for neutrino mass of the SM neutrino is the Dirac operator

$$yH\bar{\nu}_L\nu_R, \tag{21}$$

where y is a Yukawa coupling and H is the SM Higgs doublet. In this framework, the left-handed neutrinos of the SM can mix with different types of right-handed neutrinos which are inhabitants of the extra space. So ν_R is a superposition of all possible mixing partners

$$\nu_R = \frac{1}{\Lambda}\sum_n c_n \nu_{nR}. \tag{22}$$

Of course, the superposition has to be normalized and this depends on the size of the extra space the right-handed neutrinos live in. Therefore the different contributions of all mixing partners have to be divided by the volume of space in which they can propagate. The resulting form of (21) is then

$$yH\bar{\nu}_L\nu_R = \frac{yv}{\Lambda}\bar{\nu}_L\sum_n c_n \nu_{nR}. \tag{23}$$

With (20) one gets

$$\frac{M_f}{M_P} yv\bar{\nu}_L \sum_n c_n \nu_{nR}. \tag{24}$$

The factor in front of the operator represents the effective Dirac mass of neutrino which we can denote by

$$m_D = \frac{M_f}{M_P} yv. \tag{25}$$

Here we want to point out that this prefactor is suppressed by the Planck mass. We see that it induces a small Dirac mass for neutrinos. This captures a universal essence of generating a small neutrino mass in ADDM [12] and in DR [9] formulated in a theory-independent way. It follows that a small mass of neutrinos is a natural property of this class of theories. The new feature is that the suppression of the mass of the neutrino comes from the size of the extra space to which the sterile neutrino can propagate. This is very different from the introduction of a heavy Majorana particle as this is the case in the seesaw model. In other words, the spirit of the solution for the smallness of the neutrino mass we presented here is an infrared solution, and not an ultraviolet solution, by introducing a very heavy particle.

Of course, such mixing can also occur between ν_{Rj} and the left-handed inhabitants, ν_{Li}, of the extra space. Therefore, we also include the mass terms of the following form,

$$m_{ij}\bar{\nu}_{Li}\nu_{Rj}. \tag{26}$$

Let us label the neutrino of the SM with $i = 1$ and redefine the Yukawa coupling as $y = yc_1$. Moreover, let us assume that the interactions among certain pairs of neutrinos are stronger than the mixing with other types. We shall organize such mass terms as the diagonal entries m_{ii}. Correspondingly the off-diagonal entries μ_{ij} will denote mixings with other species. The resulting mass matrix is

$$\begin{pmatrix} m_D & \mu_{12} & \cdots & \cdots \\ \mu_{21} & m_{22} & \mu_{23} & \cdots \\ \vdots & \ldots & \ddots & \ldots \end{pmatrix}, \tag{27}$$

with $\mu_{1i} = c_i m_D$, and we ordered the diagonal entries according to their hierarchy

$$m_D < m_{22} < \ldots < m_{kk}. \tag{28}$$

Assuming that the mixing angles, due to off-diagonal entries, are small, we can split this matrix into the diagonal and off-diagonal parts and treat the latter one as a perturbation

$$\begin{pmatrix} m_D & \mu_{12} & \cdots & \cdots \\ \mu_{21} & m_{22} & \mu_{23} & \cdots \\ \vdots & \ldots & \ddots & \ldots \end{pmatrix} = \begin{pmatrix} m_D & 0 & \cdots & \cdots \\ 0 & m_{22} & 0 & \cdots \\ \vdots & \ldots & \ddots & \ldots \end{pmatrix} + \begin{pmatrix} 0 & \mu_{12} & \cdots & \cdots \\ \mu_{21} & 0 & \mu_{23} & \cdots \\ \vdots & \ldots & \ddots & \ldots \end{pmatrix}, \tag{29}$$

and we denote

$$V \equiv \begin{pmatrix} 0 & \mu_{12} & \cdots & \cdots \\ \mu_{21} & 0 & \mu_{23} & \cdots \\ \vdots & \ldots & \ddots & \ldots \end{pmatrix}. \tag{30}$$

With this, we find that the eigenvalues do not become corrected in the first order in mixing

$$m_i = m_{ii} + \langle n_i|V|n_i\rangle = m_{ii} + \mathcal{O}^2. \tag{31}$$

The correction to the mass eigenstates has the following form

$$|m_1\rangle = |1^{(0)}\rangle + \sum_{k=2} \frac{\mu_{1k}}{m_1^{(0)} - m_k^{(0)}} |k^{(0)}\rangle, \tag{32}$$

where the $|n\rangle$ are the eigenstates of the unperturbed matrix. Of course, one has to normalize the expression with

$$\mathrm{Norm}^2 = 1 + \sum_{k\neq n} \left(\frac{\mu_{nk}}{m_n^{(0)} - m_k^{(0)}}\right)^2. \tag{33}$$

This leads then to the following expression for the mass eigenstates

$$|\vec{m}\rangle = \begin{pmatrix} 1 & \frac{\mu_{12}}{m_1-m_2} & \cdots \\ \frac{\mu_{21}}{m_2-m_1} & 1 & \cdots \\ \vdots & \ldots & \ddots \end{pmatrix} |\vec{n}\rangle, \tag{34}$$

symbolically

$$|\overrightarrow{m}\rangle = U|\overrightarrow{n}\rangle. \tag{35}$$

Now one has to invert U in order to find the expression for the space states. In order to invert the matrix U, we use the equation

$$(A + X)^{-1} = A^{-1} + Y, \tag{36}$$

with

$$Y = -A^{-1}XA^{-1}, \tag{37}$$

and X being in this case the perturbation matrix V. One, therefore, gets for U^{-1}

$$U^{-1} = \begin{pmatrix} 1 & -\frac{\mu_{12}}{m_1-m_2} & \cdots \\ -\frac{\mu_{21}}{m_2-m_1} & 1 & \cdots \\ \vdots & \ldots & \ddots \end{pmatrix}. \tag{38}$$

This is how the mixing with the states of extra space takes place in the case of a single flavor of SM neutrino. In particular, the above reproduces the results of such mixings in ADDM [12] and in DR [9] for the case of a single flavor.

IV. GENERALIZATION TO THREE-FLAVOR CASE

We now generalize the discussion for the case of three flavors of SM neutrinos. The simplest (but unrealistic) case is if all three-flavor neutrinos have their own mixing partners in the extra space. In such a case the mass matrix has the following block-diagonal form

$$\mathcal{M} = \begin{pmatrix} M_e & 0 & 0 \\ 0 & M_\mu & 0 \\ 0 & 0 & M_\tau \end{pmatrix}, \tag{39}$$

where the M_α stands for the mass matrices of the different flavors. Each of them has a form analogous to (27). Of course, we must take mixing among the different flavors into account. This is necessary for phenomenological consistency. In particular, to make the SM three flavor neutrino oscillations possible. In order to incorporate this phenomenon we have to depart from the above block-diagonal structure. We therefore write

$$\mathcal{M} = \begin{pmatrix} M_e & e\mu & e\tau \\ e\mu & M_\mu & \mu\tau \\ e\tau & \mu\tau & M_\tau \end{pmatrix}, \tag{40}$$

where we denote with the $\alpha\beta$ ($\alpha, \beta = e, \mu, \tau$) entries the mixing matrices among the different space state partners of different flavors. In order to increase the precision of the perturbative calculation, we treat the mixing of the flavor ground states (i.e., the direct mixing among SM neutrinos) as part of the perturbed matrix and not as a part of the perturbation matrix V. This leads to the following expressions for the mass eigenstates of the three active neutrinos (we denoted the entries of the SM-like mixing elements as U_{ei}^{-1})

$$|m_1^e\rangle = U_{e1}^{-1}|e\rangle + U_{e2}^{-1}|\mu\rangle + U_{e3}^{-1}|\tau\rangle + \sum_{k=2} \frac{U_{e1}^{-1}\mu_{1k}^e + U_{e2}^{-1}e\mu_{1k} + U_{e3}^{-1}e\tau_{1k}}{m_1^e - m_k^e}|k_1^e\rangle + \sum_{k=2} \frac{U_{e1}^{-1}e\mu_{1k} + U_{e2}^{-1}\mu_{1k}^\mu + U_{e3}^{-1}\mu\tau_{1k}}{m_1^e - m_k^\mu}|k_1^\mu\rangle + \sum_{k=2} \frac{U_{e1}^{-1}e\tau_{1k} + U_{e2}^{-1}\mu\tau_{1k} + U_{e3}^{-1}\mu_{1k}^\tau}{m_1^e - m_k^\tau}|k_1^\tau\rangle. \tag{41}$$

This is the expression for the lightest mass eigenstate and we identify it with the dominant mass eigenstate for the electron neutrino. We have to invert this expression now in an analogous way as in the one-flavor case and in order to do so we assume that

$$U_{e1} \gg U_{e2}, \qquad U_{e3} \gg e\mu_{1i}, e\tau_{1i}. \tag{42}$$

Then we can write the interaction eigenstate approximately as

$$|\nu_e\rangle = U_{e1}|m_1^e\rangle + U_{e2}|m_1^\mu\rangle + U_{e3}|m_1^\tau\rangle - U_{e1}\left(\sum_{k=2} \frac{U_{e1}^{-1}\mu_{1k}^e + U_{e2}^{-1}e\mu_{1k} + U_{e3}^{-1}e\tau_{1k}}{m_1^e - m_k^e}|m_k^e\rangle + \sum_{k=2} \frac{U_{e1}^{-1}e\mu_{1k} + U_{e2}^{-1}\mu_{1k}^\mu + U_{e3}^{-1}\mu\tau_{1k}}{m_1^e - m_k^\mu}|m_k^\mu\rangle + \sum_{k=2} \frac{U_{e1}^{-1}e\tau_{1k} + U_{e2}^{-1}\mu\tau_{1k} + U_{e3}^{-1}\mu_{1k}^\tau}{m_1^e - m_k^\tau}|m_k^\tau\rangle\right). \tag{43}$$

If we assume that U_{ei} are already normalized, the normalization looks as follows:

$$N_e^2 = 1 + U_{e1}\left(\sum_{k=2} \frac{U_{e1}^{-1}\mu_{1k}^e + U_{e2}^{-1}e\mu_{1k} + U_{e3}^{-1}e\tau_{1k}}{m_1^e - m_k^e}\right)^2 + \left(\sum_{k=2} \frac{U_{e1}^{-1}e\mu_{1k} + U_{e2}^{-1}\mu_{1k}^\mu + U_{e3}^{-1}\mu\tau_{1k}}{m_1^e - m_k^\mu}\right)^2 + \left(\sum_{k=2} \frac{U_{e1}^{-1}e\tau_{1k} + U_{e2}^{-1}\mu\tau_{1k} + U_{e3}^{-1}\mu_{1k}^\tau}{m_1^e - m_k^\tau}\right)^2. \tag{44}$$

We can simplify the expression for the flavor neutrino a little bit further by assuming that the masses of the bulk states in the diagonal entries are the same for all flavors. This means that

$$m_k^e = m_k^\mu = m_k^\tau = m_k. \tag{45}$$

We also want to assume that different cross-mixing elements among different flavors have the same structure as the mixing of bulk states with their own flavor. This means that also the mixing parts $\alpha\beta_{1k}$ and μ_{1k}^α look like

$$\mu_{1k}^{\alpha} = \mu f(m_D^{\alpha}), \tag{46}$$

with the same overall constant μ and the same function f depending on the induced Dirac mass just differing by the argument. This leads then to the following expression for the flavor eigenstate

$$|\nu_e\rangle = U_{e1}|m_1^e\rangle + U_{e2}|m_1^{\mu}\rangle + U_{e3}|m_1^{\tau}\rangle - U_{e1}\sum_{\alpha=1}^{3}\sum_{k=1}\frac{\mu_{1k}^e U_{e1}^{-1} + \mu_{1k}^{\mu}U_{e2}^{-1} + \mu_{1k}^{\tau}U_{e3}^{-1}}{m^e - m_k}|k^{\alpha}\rangle. \tag{47}$$

Now let us drop the assumption (42) and give for the simplified equation (47) the expression for a larger cross mixing among the SM neutrinos which is a more realistic scenario. Then the equation gets modified in the following way,

$$|\nu_e\rangle = U_{e1}|m_1^e\rangle + U_{e2}|m_1^{\mu}\rangle + U_{e3}|m_1^{\tau}\rangle - \sum_{\alpha=1}^{3}\sum_{k=1}\frac{\overrightarrow{U_e}\,\overrightarrow{C}}{m^e - m_k}|k^{\alpha}\rangle, \tag{48}$$

with

$$\overrightarrow{U_e} = \begin{pmatrix} U_{e1} \\ U_{e2} \\ U_{e3} \end{pmatrix}, \tag{49}$$

and

$$\overrightarrow{C} = \begin{pmatrix} \mu_{1k}^e U_{e1}^{-1} + \mu_{1k}^{\mu}U_{e2}^{-1} + \mu_{1k}^{\tau}U_{e3}^{-1} \\ \mu_{1k}^e U_{\mu1}^{-1} + \mu_{1k}^{\mu}U_{\mu2}^{-1} + \mu_{1k}^{\tau}U_{\mu3}^{-1} \\ \mu_{1k}^e U_{\tau1}^{-1} + \mu_{1k}^{\mu}U_{\tau2}^{-1} + \mu_{1k}^{\tau}U_{\tau3}^{-1} \end{pmatrix}. \tag{50}$$

In an analogous way, Eq. (43) can get modified.

With these developed tools we can now calculate a general expression for a flavor eigenstate of a neutrino which has mixing with a large number of extra states and also includes mixing with the other flavor states. The investigated case of nondegenerated nonperturbed eigenstates can be used for the ADDM scenario and via a cross-check we can reproduce the one-flavor equation obtained in [13]. In the following section we show how one can calculate the flavor states for a highly-degenerated mass matrix which are important for the DR scenario.

V. HIGHLY-SYMMETRIC MASS MATRICES

So far we investigated the case of a very general mass matrix which contains mixing with all the states of the extra space, but the cases where these mass matrices have a specific structure and are highly symmetric are of interest also. One specific example of this is the "many species theory" with exact copies of the SM.

We now want to present a way to deal with these kind of matrices when they are blockwise grouped in their mass matrix. Without loss of generality, we illustrate this on the example of the DR scenario. A grouping of the different copies of the SM can occur according to the VEV of the Higgs doublets. Notice that even if copies obey a strict permutation symmetry, this symmetry can be spontaneously broken by the VEVs of the Higgs doublets. This is because, due to the low cutoff and the cross-couplings among different doublets, the potential can admit vacua in which Higgs doublets of different copies take different VEVs, $\langle H_j\rangle = v_j$.

Also, because in principle a Majorana mass term for neutrinos is not forbidden either by gauge or by permutation symmetry, we will investigate the common Dirac operator

$$(HL)_i\lambda_{ij}\nu_{Rj}, \tag{51}$$

also a Weinberg operator of the form

$$(\overline{L^c}i\sigma_2 H)_i\lambda_{ij}(Hi\sigma_2 L)_j, \tag{52}$$

where the indices i and j label different copies and L being the SU(2) doublet and σ_2 acting in this space. As previously, we assume that Yukawa couplings obey the $P(N)$-symmetry and therefore have the form of (3).

Notice that the operators (52) break the global lepton number symmetries explicitly.

The key now is to assign different Higgs VEVs to different SM copies. We group the copies with the same VEVs in diagonal blocks of the neutrino mass matrix.

Let us consider a minimal case of this sort in which the VEVs take two possible values v and v'. We take a subgroup of size $N < N_{\mathrm{TOTAL}}$ and assign the VEV v. To the rest of the species $M = N_{\mathrm{TOTAL}} - N$ we assign the VEV v'. This assignment can be expressed as

$$v_i = \begin{cases} v & \text{for } i \leq N \\ v' & \text{for } i > N. \end{cases} \tag{53}$$

Taking this into account and plugging it into the operators (51) and (52) one gets the following mass matrices

$$M^{\text{Majorana}} = \begin{pmatrix} av^2 & bv^2 & bv^2 & \dots & bv^2 & bv'v & & \dots & & bv'v \\ bv^2 & av^2 & bv^2 & \dots & bv^2 & \vdots & & \ddots & & \vdots \\ \vdots & & \ddots & & \vdots & \vdots & & \ddots & & \vdots \\ bv^2 & & \dots & & av^2 & bv'v & & \dots & & bv'v \\ bv'v & & \dots & & bv'v & av'^2 & bv'^2 & bv'^2 & \dots & bv'^2 \\ \vdots & & \ddots & & \vdots & bv'^2 & av'^2 & bv'^2 & \dots & bv'^2 \\ \vdots & & \ddots & & \vdots & \vdots & & \ddots & & \vdots \\ bv'v & & \dots & & bv'v & bv'^2 & & \dots & & av'^2 \end{pmatrix}, \tag{54}$$

and

$$M^{\text{Dirac}} = \begin{pmatrix} av & bv & bv & \dots & bv & bv & & \dots & & bv \\ bv & av & bv & \dots & bv & \vdots & & \ddots & & \vdots \\ \vdots & & \ddots & & \vdots & \vdots & & \ddots & & \vdots \\ bv & & \dots & & av & bv & & \dots & & bv \\ bv' & & \dots & & bv' & av' & bv' & bv' & \dots & bv' \\ \vdots & & \ddots & & \vdots & bv' & av' & bv' & \dots & bv' \\ \vdots & & \ddots & & \vdots & \vdots & & \ddots & & \vdots \\ bv' & & \dots & & bv' & bv' & & \dots & & av' \end{pmatrix}. \tag{55}$$

The diagonalization of the above mass matrices will be performed in the next section.

A. Diagonalizing of the Majorana mass matrices

In this part, the Majorana mass matrices will be diagonalized. Because the resulting expressions are rather complex the diagonalization procedure will be done within certain limits. The two limits which will be discussed are $v' \gg v$ and vice versa.

1. The symmetric-breaking limit of the mass matrix

Here the focus lies on the Majorana mass matrix (54) and we make the assumption that the breaking of $P(N)$ is into two equally large sectors, $M = N$. In order to simplify the resulting equations even further, we will also assume that $v' \gg v$. we put the value v' close to the cutoff of the theory (∼TeV). This will lead to very interesting phenomenological implications.

We start diagonalizing (54) noticing that it is a 2×2 block matrix. As the first step, we multiply the matrix with the following transformation matrix

$$U' = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}, \tag{56}$$

where S is the diagonalization matrix of a matrix of just ones (a matrix with the same entry everywhere)

$$S = \begin{pmatrix} 1 & -1 & \dots & \dots & \\ \vdots & 1 & 0 & \dots & \\ \vdots & 0 & \ddots & 0 & \dots \end{pmatrix}. \tag{57}$$

This leads then to the following expression

$$U'^{-1} M^{\text{Majorana}} U' = U'^{-1} \begin{pmatrix} A & B \\ C & D \end{pmatrix} U' = \begin{pmatrix} S^{-1}AS & S^{-1}BS \\ S^{-1}CS & S^{-1}DS \end{pmatrix}, \tag{58}$$

where the matrices A, B, C, and D denote the block entries of the mass matrix. One can separate the diagonal entries of the matrices A and D from the rest of the matrix and turn this one into a matrix with just the same entry

$$A = v\lambda_{ij} = \begin{pmatrix} (a-b)v^2 & 0 & \dots \\ 0 & \ddots & 0 \\ \vdots & \dots & (a-b)v^2 \end{pmatrix} + \begin{pmatrix} bv^2 & \dots & \dots \\ \vdots & \ddots & \vdots \\ \vdots & \dots & bv^2 \end{pmatrix}. \tag{59}$$

The diagonal part commutes with S and one is therefore left with the following matrix

$$\begin{pmatrix} (a-b)v^2+Nbv^2 & 0 & \dots & Nbvv' & 0 & \dots & 0 \\ 0 & (a-b)v^2 & 0 & \dots & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 & & \dots & 0 \\ Nbvv' & 0 & \dots & (a-b)v'^2+Nbv'^2 & 0 & \dots & 0 \\ 0 & \dots & & 0 & (a-b)v'^2 & \dots & 0 \\ 0 & \dots & & \dots & 0 & \ddots & 0 \end{pmatrix}. \tag{60}$$

Now one can take out the diagonal element and can bring it down to a 2×2 matrix of the following form,

$$\begin{pmatrix} Nbv^2 & Nbvv' \\ Nbvv' & Nbv'^2+(a-b)(v'^2-v^2) \end{pmatrix}. \tag{61}$$

In order to find the mass eigenstates, one has to manipulate (60) further with the following rotation matrix,

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}, \tag{62}$$

with the rotation angle

$$\theta = \frac{1}{2}\arctan\left(2\frac{vv'}{v'^2-v^2}\right). \tag{63}$$

The rotation matrix multiplied with the U' matrix gives the transformation matrix of the mass matrix. The result is

$$U = \begin{pmatrix} \cos(\theta) & -1 & \dots & -1 & \sin(\theta) & 0 & & \dots \\ \cos(\theta) & 1 & 0 & \dots & \sin(\theta) & 0 & & \dots \\ \vdots & 0 & \ddots & & \vdots & 0 & & \dots \\ -\sin(\theta) & 0 & \dots & 0 & \cos(\theta) & -1 & & \dots \\ \vdots & 0 & \dots & 0 & \vdots & 1 & 0 & \dots \end{pmatrix}. \tag{64}$$

From here we can see that just two states are affected by the symmetry breaking and the rest stays degenerated with the eigenvalues $(a-b)v^2$ and $(a-b)v'^2$. Therefore, we can rewrite the new heavy states in terms of the heavy states of the unbroken permutation subset, which we already encountered in Eqs. (6) and (7). Again in order to simplify the rotation angle (63), we use the limit $v' \gg v$. The result is then the following:

$$n_H^b = n_H - \frac{v}{v'}\tilde{n}_H, \tag{65}$$

$$\tilde{n}_H^b = \tilde{n}_H + \frac{v}{v'}n_H, \tag{66}$$

where we used tilde for the v' sector. When one now solves for the species states of the two different sectors one gets the following two expressions,

$$n_1 = \sqrt{\frac{N-1}{N}}n_1' + \frac{1}{\sqrt{N}}n_H^b + \frac{1}{\sqrt{N}}\frac{v}{v'}\tilde{n}_H^b, \tag{67}$$

(notice that for the sake of simplicity the overall normalization factor is suppressed)

$$n_{N+1} = \sqrt{\frac{N-1}{N}}\tilde{n}_1' + \frac{1}{\sqrt{N}}\tilde{n}_H^b - \frac{1}{\sqrt{N}}\frac{v}{v'}n_H^b, \tag{68}$$

with the eigenvalues of the mass eigenstates,

$$m_1' = (a-b)v^2, \tag{69}$$

$$\tilde{m}_1' = (a-b)v'^2, \tag{70}$$

$$m_H = 2(a-b)v^2, \tag{71}$$

$$\tilde{m}_H > M_P. \tag{72}$$

This is a rather interesting result for phenomenology which we will take a closer look at later. We want to point out that the common heavy eigenstate n_H has a mass, independent of N, which was not the case in the original mechanism. This means that the common heavy eigenstate is not super heavy and neutrino oscillations into this state are therefore possible.

2. *Asymmetric breaking pattern with a large heavy sector*

One can also break the symmetry in a way that the sectors include different amounts of copies, $N \neq M$, where N stands for the sector with a VEV of v and M for v'.

In order to keep the expressions for the final results in a simple form, we take the limit $Mv'^2 \gg Nv^2$. After repeating the same diagonalization procedure, the matrix (60) in this case has the following form,

$$\begin{pmatrix} (a-b)v^2+Nbv^2 & 0 & \dots & Mbvv' & 0 & \dots & 0 \\ 0 & (a-b)v^2 & 0 & \dots & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 & & \dots & 0 \\ Nbvv' & 0 & \dots & (a-b)v'^2+Mbv'^2 & 0 & \dots & 0 \\ 0 & \dots & & 0 & (a-b)v'^2 & \dots & 0 \\ 0 & \dots & & \dots & 0 & \ddots & 0 \end{pmatrix}. \quad (73)$$

Before we can perform the rotation, we have to make an intermediate step which brings the off-diagonal entries to the same value. Therefore one applies another transformation matrix of the form

$$\begin{pmatrix} 1 & 0 & \dots & \dots & \dots & \dots \\ 0 & \ddots & 0 & \dots & \dots & \dots \\ \vdots & \dots & \kappa & 0 & \dots & \dots \\ \vdots & \dots & 0 & 1 & 0 & \dots \\ \vdots & \dots & \dots & 0 & \ddots & 0 \end{pmatrix}, \quad (74)$$

with κ being

$$\kappa = \sqrt{\frac{N}{M}}. \quad (75)$$

After this procedure the off-diagonal entries are equal and one can perform the rotation like in the symmetric case. Correspondingly, one gets a mixing angle of the form

$$\theta = \frac{1}{2}\arctan\left(-2\frac{\sqrt{N}\sqrt{M}bvv'}{Nbv^2 - Mbv'^2}\right). \quad (76)$$

The resulting transformation matrix is

$$U = \begin{pmatrix} \cos(\theta) & -1 & \dots & -1 & \sin(\theta) & 0 & & \dots \\ \cos(\theta) & 1 & 0 & \dots & \sin(\theta) & 0 & & \dots \\ \vdots & 0 & \ddots & & \vdots & 0 & & \dots \\ -\kappa\sin(\theta) & 0 & \dots & 0 & \kappa\cos(\theta) & -1 & & \dots \\ \vdots & 0 & \dots & 0 & \vdots & 1 & 0 & \dots \end{pmatrix}, \quad (77)$$

and θ simplified to

$$\theta = \sqrt{\frac{N}{M}}\frac{v}{v'}. \quad (78)$$

The resulting mass eigenstates are then

$$n_H^b = n_H - \frac{N}{M}\frac{v}{v'}\tilde{n}_H, \quad (79)$$

$$\tilde{n}_H^b = \tilde{n}_H + \frac{v}{v'}n_H, \quad (80)$$

with the eigenvalues

$$m_H = (a-b)v^2, \quad (81)$$

$$\tilde{m}_H > M_P. \quad (82)$$

The corresponding copy eigenstates are

$$n_1 = \sqrt{\frac{N-1}{N}}n_1' + \frac{1}{\sqrt{N}}n_H^b + \frac{1}{\sqrt{N}}\frac{N}{M}\frac{v}{v'}\tilde{n}_H^b, \quad (83)$$

$$n_{N+1} = \sqrt{\frac{M-1}{M}} + \frac{1}{\sqrt{M}}\tilde{n}_H^b - \frac{1}{\sqrt{M}}\frac{v}{v'}n_H^b. \quad (84)$$

We see that the mass m_H is the same as for the degenerated mass eigenstates.

3. Asymmetric breaking pattern with a large light sector

One can also investigate the case with a large light sector $Nv^2 \gg Mv'^2$. In this case, the procedure is the same and (73) stays untouched. The resulting mixing angle is

$$\theta = -\sqrt{\frac{M}{N}}\frac{v'}{v}. \quad (85)$$

The eigenvalues are

$$m_H = (a-b)v'^2, \quad (86)$$

$$\tilde{m}_H > M_P. \tag{87}$$

The corresponding eigenstates are given by

$$\tilde{n}_H^b = \frac{v}{v'} n_H + \tilde{n}_H, \tag{88}$$

$$n_H^b = \tilde{n}_H - \frac{Mv'}{Nv} n_H. \tag{89}$$

The copy eigenstates are

$$n_1 = \sqrt{\frac{N-1}{N}} n_1' - \frac{1}{\sqrt{N}} \frac{v'}{v} n_H^b + \frac{1}{\sqrt{N}} \frac{v'}{v} \tilde{n}_H^b, \tag{90}$$

$$n_{N+1} = \sqrt{\frac{M-1}{M}} \tilde{n}_1' + \frac{1}{\sqrt{M}} n_H^b + \frac{1}{\sqrt{M}} \frac{M}{N} \left(\frac{v'}{v}\right)^2 \tilde{n}_H^b. \tag{91}$$

Now the situation is reversed. The m_H goes to the eigenvalues of the degenerated states of the heavy sector. Taking v' close to the cutoff (∼TeV) the estimated values of m_H could be up to ∼keV.

B. Diagonalizing of the Dirac mass matrix

Let us now turn to diagonalization of the Dirac mass matrix which results from the operator (51). The procedure is similar but some details differ from the Majorana case.

1. The symmetric-breaking limit of the Dirac mass matrix

After the first steps, similar to the ones taken for the Majorana case, the matrix has the form

$$\begin{pmatrix} (a-b)v + Nbv & 0 & \dots & Nbv & 0 & \dots & 0 \\ 0 & (a-b)v & 0 & \dots & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 & & \dots & 0 \\ Nbv' & 0 & \dots & (a-b)v' + Nbv' & 0 & \dots & 0 \\ 0 & \dots & & 0 & (a-b)v' & \dots & 0 \\ 0 & \dots & & \dots & 0 & \ddots & 0 \end{pmatrix}. \tag{92}$$

Now the situation is different because the matrix (92) is not symmetric (60). Because of this one has to introduce the auxiliary parameter κ already in the symmetric breaking limit

$$\kappa = \sqrt{\frac{v'}{v}}, \tag{93}$$

and the rotation angle is

$$\theta = \sqrt{\frac{v}{v'}}. \tag{94}$$

The resulting heavy eigenstates are

$$n_H^b = \frac{1}{\sqrt{2}} n_H - \frac{1}{\sqrt{2}} \tilde{n}_H, \tag{95}$$

$$\tilde{n}_H^b = \tilde{n}_H + \frac{v}{v'} n_H, \tag{96}$$

with the eigenvalues

$$m_H = 2(a-b)v, \tag{97}$$

$$\tilde{m}_H > M_P. \tag{98}$$

Solving for the species states leads to

$$n_1 = \sqrt{\frac{N-1}{N}} n_1' + \sqrt{\frac{2}{N}} n_H^b + \frac{1}{\sqrt{N}} \tilde{n}_H^b, \tag{99}$$

$$n_{N+1} = \sqrt{\frac{N-1}{N}} n_1' - \sqrt{\frac{2}{N}} \frac{v}{v'} n_H^b + \frac{1}{\sqrt{N}} \tilde{n}_H^b. \tag{100}$$

2. Asymmetric breaking pattern with a large heavy sector

Now we turn again to the cases of asymmetric breaking of the permutation group. We investigate the scenario with $M \gg N$. In order to do so, in the matrix (92) we replace N with M for one sector like in the Majorana case. In this scenario the auxiliary parameter becomes

$$\kappa = \sqrt{\frac{Nv'}{Mv}}, \tag{101}$$

and the resulting rotation angle is

$$\theta = \sqrt{\frac{Nv}{Mv'}}. \tag{102}$$

The mass eigenstates are

$$n_H^b = n_H - \frac{N}{M}\tilde{n}_H, \tag{103}$$

$$\tilde{n}_H^b = \frac{v}{\sqrt{v^2 + v'^2}} n_H + \frac{v'}{\sqrt{v'^{2+v^2}}}\tilde{n}_H, \tag{104}$$

with the eigenvalues

$$m_H = (a - b)v, \tag{105}$$

$$\tilde{m}_H > M_P. \tag{106}$$

The species states are

$$n_1 = \sqrt{\frac{N-1}{N}} n_1' + \frac{1}{\sqrt{N}} n_H^b + \frac{1}{\sqrt{N}}\frac{N}{M}\tilde{n}_H^b, \tag{107}$$

$$n_{N+1} = \sqrt{\frac{M-1}{M}}\tilde{n}_1' + \frac{1}{\sqrt{M}}\tilde{n}_H^b - \frac{1}{\sqrt{M}}\frac{v}{v'} n_H^b. \tag{108}$$

Again the oscillation in our copy has an extremely small frequency because the Δm goes to 0 but, on the other hand it is suppressed as $1/N$, but N in the present case is not large.

3. Asymmetric breaking pattern with a large light sector

Finally, let us investigate the case with $N \gg M$ and $v' \gg v$. The auxiliary parameter κ stays the same as in Eq. (101). The rotation angle is

$$\theta = -\sqrt{\frac{Mv'}{Nv}}, \tag{109}$$

with the eigenstates

$$n_H^b = \tilde{n}_H - \frac{M}{N} n_H, \tag{110}$$

$$\tilde{n}_H^b = \tilde{n}_H + \frac{v}{v'} n_H. \tag{111}$$

The eigenvalues are

$$m_H = (a - b)v', \tag{112}$$

$$\tilde{m}_H > M_P. \tag{113}$$

The corresponding species states are

$$n_1 = \sqrt{\frac{N-1}{N}} n_1' - \frac{1}{\sqrt{N}}\frac{v'}{v} n_H^b, \tag{114}$$

$$n_{N+1} = \sqrt{\frac{M-1}{M}}\tilde{n}_1' + \frac{1}{\sqrt{M}} n_H^b. \tag{115}$$

VI. PHENOMENOLOGY

We now want to turn to the phenomenological implications of the theoretical framework we built up in the previous sections. We will do this within a specific theory. First, we want to point out that the first steps in this topic were already done in [13] for ADDM and [9] in DR, but in both cases, just the one-flavor case of the SM neutrino was investigated. We now aim to generalize this analysis to the three-flavor case using the general framework which we presented before.

A. Phenomenology of ADDM model

First, we want to discuss the phenomenology of the ADDM scenario in a realistic three-flavor setting. In order to do so, we want to use the framework of Sec. III and apply our generally derived formulas to the ADDM case. First, we have to define the mass matrix we are investigating. For this, we take the ansatz from [13] and generalize it to the three-flavor case. To write down the resulting mass matrix we assume that the flavor symmetry is preserved in the bulk. This leads to the effect that the mixing among bulk states is diagonal and the resulting mass matrix is

$$\begin{pmatrix}
m_{ee} & \sqrt{2}m_{ee} & \cdots & \cdots & m_{e\mu} & \sqrt{2}m_{ee} & \cdots & m_{e\tau} & \sqrt{2}m_{ee} & \cdots \\
0 & \frac{1}{R} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \ddots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \frac{k}{R} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\
m_{\mu e} & \sqrt{2}m_{\mu\mu} & \cdots & \sqrt{2}m_{\mu\mu} & m_{\mu\mu} & \sqrt{2}m_{\mu\mu} & \cdots & m_{\mu\tau} & \sqrt{2}m_{\mu\mu} & \cdots \\
0 & \cdots & \cdots & \cdots & 0 & \frac{1}{R} & 0 & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots & 0 & \ddots & 0 & \cdots & \cdots \\
m_{\tau e} & \sqrt{2}m_{\tau\tau} & \cdots & \cdots & m_{e\tau} & \sqrt{2}m_{\tau\tau} & \cdots & m_{e\tau} & \sqrt{2}m_{\tau\tau} & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \frac{1}{R} & \cdots
\end{pmatrix}. \tag{116}$$

In order to perform the diagonalization of this mass matrix, one has to define the parametrization of the U_{PMNS} matrix

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}. \tag{117}$$

With this PMNS-matrix parametrization, we can use the formula (48) to calculate the expression for e.g., the muon neutrino. The result is

$$|\nu_\mu\rangle = U_{\mu 1}|m_1^e\rangle + U_{\mu 2}|m_1^\mu\rangle + U_{\mu 3}|m_1^\tau\rangle + \overrightarrow{U_\mu}\,\overrightarrow{C}\sum_\alpha\sum_k \frac{1}{k}|k_\alpha\rangle, \tag{118}$$

with $\overrightarrow{C}_{ADD}$

$$\overrightarrow{C}_{ADD} = \begin{pmatrix} \xi^e U_{e1}^{-1} + \xi^\mu U_{e2}^{-1} + \xi^\tau U_{e3}^{-1} \\ \xi^e U_{\mu 1}^{-1} + \xi^\mu U_{\mu 2}^{-1} + \xi^\tau U_{\mu 3}^{-1} \\ \xi^e U_{\tau 1}^{-1} + \xi^\mu U_{\tau 2}^{-1} + \xi^\tau U_{\tau 3}^{-1} \end{pmatrix}, \tag{119}$$

and the normalization

$$N_\mu^2 = 1 + \frac{\pi^2}{2}(\overrightarrow{U_\mu}\,\overrightarrow{C})^2. \tag{120}$$

Notice that the parameters ξ^α are related to each other via

$$\xi^e \propto m_e \approx \mathcal{O}(1)m_\mu \approx \mathcal{O}'(1)m_\tau, \tag{121}$$

and therefore the key parameter in this expression is just the size of the dominant extra dimension R.

In order to get an impression of the dependence of this deviation of the composition of a muon neutrino from the Standard Model composition, one can calculate the survival probability. We assume that just the lowest modes of the KK towers contribute to the oscillations since the higher modes get averaged out due to large mass splittings. Then the survival probability reads as

$$P(\nu_\mu \to \nu_\mu) = \frac{1}{|N_\mu|^4}\left[\sum_i\sum_j |U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2 - m_j^2)}{2E}} + 3|\vec{U}_\mu\vec{C}|^4\left(\frac{\pi^4}{90} - 1\right)\right], \tag{122}$$

with E being the energy of the investigated neutrino. This can be compared to the original result in [13] for the one-flavor case

$$P = \frac{1}{(1 + (\pi^2/6)\xi^2)^2}\left[(1+\xi^2)^2 + \left(\frac{\pi^4}{90} - 1\right)\xi^4 - \xi^2\sin^2\frac{(m_n^2 - m_D^2)t}{4E}\right]. \tag{123}$$

From these two equations, one can see that some properties of the one-flavor case also appear in a modified way in the three-flavor equation. Particularly interesting is how in ADDM models the averaged out modes influence the survival probability by a term proportional to $(\frac{\pi^4}{90} - 1)$ if just the lowest mode is not averaged out. Of course, the experimental setup and the specific mass splitting determine how many modes can be resolved in the oscillations. As more Kaluza-Klein modes participate as less important the contribution of the averaged-out modes is.

For comparison of the three-flavor scenario with SM prediction, we take the latest results of the NuFIT Collaboration [33] which are

$$\theta_{12} = 33{,}44°, \quad \theta_{23} = 49{,}0°, \quad \theta_{13} = 8{,}57°, \quad \delta_{CP} = 195°. \tag{124}$$

With this data, one can calculate the survival probability of a muon neutrino depending on the parameter ξ of the ADDM model. Figure 1 shows the result of this calculation and how it deviates from the SM case. Thus, the precision measurements of neutrino oscillations can put bounds on the critical value ξ of the ADDM model. This can be directly related to R which has been searched for by several different experiments [34–45] with the strongest bound being $R < 0.81$ μm. From Fig. 1 we get a feeling of how sensitive neutrino probes can be and we see that a size of $R = 0.4$ μm still has quite strong deviations from the SM, and therefore modern neutrino experiments can hope for giving bounds around $R < 0.4$ μm or even smaller.

An interesting distinguishing property of neutrino experiments is that these measure the size of the largest extra

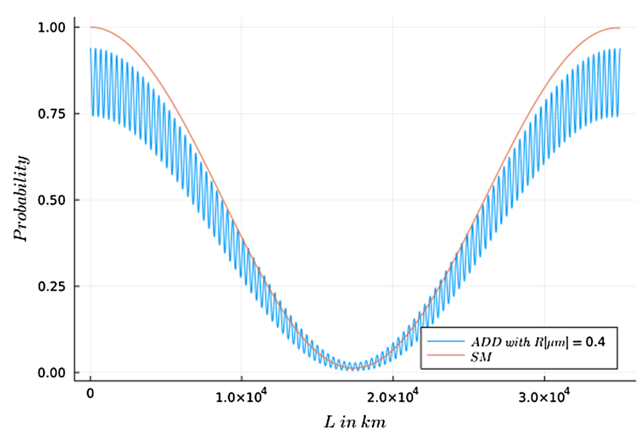

FIG. 1. Survival probability of a muon neutrino in a three-flavor ADD mixing scenario

dimension, meanwhile fifth force, collider, and astrophysical experiments give bounds on the fundamental scale of gravity M_*. According to [46] the bounds on M_* are $M_* > 4$ TeV for tabletop experiments, $M_* > 5.9$–11.2 TeV for collider signals, and $M_* > 1700$ TeV for neutron star estimations. How these bounds translate into the actual size of the extra dimensions depends heavily on the number of assumed extra dimensions and if one allows a different scale among them. Therefore, measuring R via neutrino experiments is a complementary way to test the ADDM scenario.

Moreover, this deviation from the SM case also affects the unitarity of the lepton mixing matrix. If the familiar three flavors of the SM neutrinos exhaust the spectrum of neutral leptons, the 3×3 mixing matrix we measure in neutrino experiments must be unitary. This does not hold if there exist more than three neutrinos. In this case, the lepton mixing matrix is not a 3×3 matrix anymore but a $(3+n) \times (3+n)$ matrix where n is the number of additional neutrinos.

Nevertheless, in the experiments that are sensitive to active species, we would still measure only the 3×3 part of the full lepton mixing matrix. Since this restricted part, in general, will not be unitary we will effectively register a deviation from unitarity. This happens also in the ADDM model where neutrinos can oscillate into the KK modes. Using the above results, the 3×3 part of the full lepton mixing matrix would get modified in the following way

$$\begin{pmatrix} U_{ee}\frac{1}{N_e} & U_{e\mu}\frac{1}{N_e} & U_{e\tau}\frac{1}{N_e} \\ U_{\mu e}\frac{1}{N_\mu} & U_{\mu\mu}\frac{1}{N_\mu} & U_{\mu\tau}\frac{1}{N_\mu} \\ U_{\tau e}\frac{1}{N_\tau} & U_{\tau\mu}\frac{1}{N_\tau} & U_{\tau\tau}\frac{1}{N_\tau} \end{pmatrix}. \tag{125}$$

Now the task is to measure the parameters of the well-known PMNS matrix very precisely and look for possible deviations from unitarity. Of course, this feature is not unique to ADDM and something similar can be realized in other models too. However, the above example provides us with concrete motivated framework for setting bounds on the unitarity-violating parameters and using them for discriminating between the different models. We are going to see this explicitly in the next section when we discuss the phenomenology of the DR model and confront it with the ADDM framework.

B. Phenomenology of the Dvali-Redi model

The generalization of the mass matrix in the DR scenario goes as follows. The general structure of the mass matrix is again similar to (40) but this time the off-diagonal block matrices have the following form

$$M_{\alpha\beta} = \begin{pmatrix} m_{\alpha\beta} & 0 & 0 & \dots \\ 0 & m_{\alpha\beta} & 0 & \dots \\ \vdots & 0 & \ddots & \vdots \end{pmatrix}. \tag{126}$$

This specific structure comes from the fact that in this theory the mixing among the different flavors can happen within a single copy since it is determined by the physics of the SM. This leads to the following electron neutrino eigenstate

$$|\nu_e\rangle = \sqrt{\frac{N-1}{N}}(U_{e1}|m_1\rangle + U_{e2}|m_2\rangle + U_{e3}|m_3\rangle) + \frac{1}{\sqrt{N}}(U_{e1}|m_1^H\rangle + U_{e2}|m_2^H\rangle + U_{e3}|m_3^H\rangle). \tag{127}$$

The key parameter is the number of active species. Above we showed how we can group the total number of species into light and heavy sectors; now one can investigate different scenarios with sectors which contain different numbers of copies. Because we have access predominantly to our copy of the SM, for us the scenarios with a small number of active species in the sector our copy belongs to are of special interest. Due to this reason, we focus on the scenarios with large heavy sectors that bring down the number of active species in our sector.

Taking the two expressions we found earlier for the Weinberg and Dirac operator (83) and (107) and comparing them with each other, one sees that the oscillation into the other sector is suppressed by the number of active species in the large sector. Therefore, one can safely ignore this contribution, especially in the Weinberg case, since there it is further suppressed by the scale of the larger VEV v'. Then, the probability of survival in the one-flavor case [9] is given by

$$P(t) = 1 - \frac{4}{N}\sin^2\left(\frac{\Delta m^2 t}{2E}\right). \tag{128}$$

In this scenario, the problem of observing the effect is shifted from the large suppression by the amplitude into the extremely low frequency which comes from very small splitting among the mass eigenstates. Nevertheless, this case is still of high potential interest for long-baseline experiments of neutrino oscillations. Astrophysical sources of high-neutrino fluxes could be useful candidates for testing such scenarios. Of course, detection of deviations from the expected neutrino flux in pure SM requires an understanding of the operation mechanisms of these sources to sufficiently high accuracy.

1. Integrating out scenario

We observed that in small light sector scenarios the suppression of the amplitude goes contrary to the frequency of the oscillations. A scenario that can bring both parameters to the range of easier experimental accessibility is the "integrating out" scenario which we will now consider. The goal is to combine the advantages of the different above-studied scenarios into a unique setup. Let us assume that the permutation symmetry is broken very heavily among the two sectors; one sector contains a large number M of copies and another sector contains a smaller number N. (This is the case which we have already discussed above.) However, let us now assume that due to additional breaking of the permutation symmetry, the smaller sector is further split into two sectors with the numbers N' and M'. Obviously, the primary breaking of perturbation symmetry into the M and N sectors is still dominant and the secondary breaking does not affect physics up to effects of order $\mathcal{O}(\frac{N}{M})$ which is already negligibly small. Due to this reason, this sector can be considered as effectively decoupled from the other sectors.

Let us now turn our attention to the leftover copies that are broken down into two smaller sectors N' and M'. Here we have a choice to which sector our SM copy belongs. In particular, we can assume that the number of copies in our sector N' is much larger than the other sector M'. This does not decrease the suppression of the amplitude very much but allows us to liberate the value of the common heavy eigenstate m_H in which the neutrinos of both sectors oscillate and can make Δm large enough for bringing the frequency to a value comparable to the ordinary oscillations of the SM. This scenario of splitting is analogous to the large light-sector scenario.

Overall this integrating out scenario enables us to free both parameters of the theory. It allows us to bring down the number of copies and the corresponding oscillation frequencies to a scale that makes it observable for experiments.

We can now calculate the oscillation in the three-flavor case with an equal-size splitting scenario. The equation for the survival probability can be written down as

$$P(\nu_\mu \to \nu_\mu) = \sum_{i=1}^{6}\sum_{j=1}^{6}|U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)}{2E}}. \tag{129}$$

First, we want to point out that in this expression no modes are averaged out like in the ADDM scenario. The reason for this is that just three additional mass eigenstates have to be included; meanwhile, in ADDM scenarios the KK tower can inhabit a very large number of additional mass eigenstates. To analyze Eq. (129) further we split it up in the following way

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) = {} & \left(\frac{N-1}{N}\right)^2 \sum_{i=1}^{3}\sum_{j=1}^{3}|U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)}{2E}} \\ & + \frac{N-1}{N^2}\sum_{i=1}^{3}\sum_{j=4}^{6}|U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)}{2E}} \\ & + \frac{N-1}{N^2}\sum_{i=4}^{6}\sum_{j=1}^{3}|U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)}{2E}} \\ & + \frac{1}{N^2}\sum_{i=4}^{6}\sum_{j=4}^{6}|U_{\mu i}|^2|U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)}{2E}}. \end{aligned} \tag{130}$$

The first term in this expression represents the oscillations within the flavors which are already known in the SM. For large N these oscillations are just slightly modified. One also sees that the dominant contributions are coming from oscillations into the hidden species of order $\frac{1}{N}$ like in the one-flavor case in Eq. (129). The contributions of solely the BSM terms are suppressed by $\frac{1}{N^2}$. Figure 2 shows the result of the calculations for a muon neutrino. In this figure we see that the difference compared to the SM can be also quite severe and therefore we expect that current neutrino experiments can restrict the number of species to $N > 10 - 100$. Using neutrinos to test extra species is quite an exciting result because the LHC gives a lower bound on M_* and therefore an upper bound to the number of species meanwhile neutrino experiments give a lower bound on N. Testing the DR scenario with neutrinos is therefore complementary to the bounds that LHC gives us.

Let us also show the unitarity violation in the SM lepton mixing matrix which is expected by the DR scenario. For this, we have to look into the formula (127). Picking out the 3×3 block matrix in the upper-left corner of the resulting mixing matrix we can write

$$\begin{pmatrix} \sqrt{\frac{N-1}{N}}U_{ee} & \sqrt{\frac{N-1}{N}}U_{e\mu} & \sqrt{\frac{N-1}{N}}U_{e\tau} \\ \sqrt{\frac{N-1}{N}}U_{\mu e} & \sqrt{\frac{N-1}{N}}U_{\mu\mu} & \sqrt{\frac{N-1}{N}}U_{\mu\tau} \\ \sqrt{\frac{N-1}{N}}U_{\tau e} & \sqrt{\frac{N-1}{N}}U_{\tau\mu} & \sqrt{\frac{N-1}{N}}U_{\tau\tau} \end{pmatrix}. \tag{131}$$

This is the matrix which is measured by experiments. One can immediately see that unitarity is violated by the overall factor of $\sqrt{\frac{N-1}{N}}$. This is a characteristic signature of the theory and it comes from the democratic oscillation into the common heavy eigenstates of the neutrino matrix. The idea of testing the violation of unitarity experimentally by using available experimental data is still ongoing work [31].

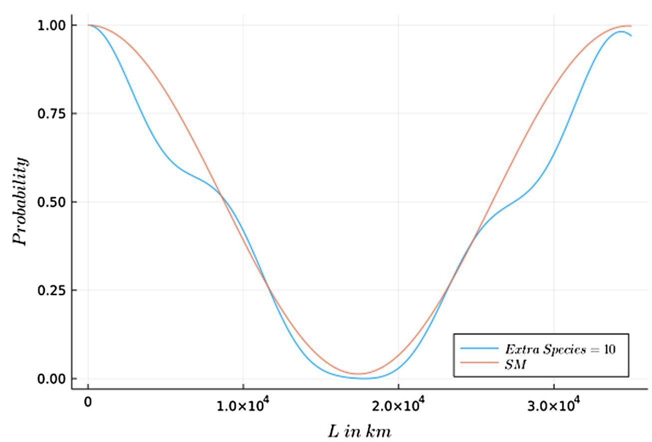

FIG. 2. Survival probability of a muon neutrino in a three-flavor case in DR model with an equal size splitting scenario.

At the end of the phenomenological section we want to briefly address matter effects for such theories. To calculate these effects one can use the standard paradigm for additional sterile neutrinos and therefore, the effective Hamiltonian can be written as

$$H_{\mathrm{eff}} = \frac{1}{2E}\left[U\begin{pmatrix} m_e & 0 & 0 & 0 & 0 \\ 0 & m_\mu & 0 & 0 & 0 \\ 0 & 0 & m_\tau & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & \ddots \end{pmatrix}U^\dagger + \begin{pmatrix} A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A' & 0 \\ 0 & 0 & 0 & 0 & \ddots \end{pmatrix}\right], \tag{132}$$

with $A = 2\sqrt{2}G_F N_e E$, $A' = -\sqrt{2}G_F N_n E$ and N_e, N_n being the densities of electrons, neutrons respectively. The consequence is that resonances for BSM modes can appear and change the oscillation pattern. Even though it is not guaranteed that these resonance effects appear in experiments which are heavily influenced by matter effects like IceCube [47] there is still a chance that a favorable combination of experimental parameters can increase the sensitivity of the experiment.

VII. CONCLUSIONS

In this paper, we have focused on neutrino masses in the class of theories in which gravity cutoff is lowered down to ~TeV scale. The two main frameworks accomplishing this lowering of the cutoff are ADD [2,3] and “many species” [5,6] theories. In both cases, the decrease of the gravitational cutoff can be understood as a result of the “dilution” of the graviton wave function in a certain space labeled by a new coordinate. In both scenarios, the volume of this space can be measured by the number of particle species. Correspondingly, the role of the coordinate can be played by a species label. As shown in [5], in case of ADD [2] the species represent the Kaluza-Klein excitations. Correspondingly, the extra space has an actual geometric meaning of large extra spatial dimensions. On the other hand, in the “many species” solution to the hierarchy problem [5,6], the species can be arbitrary particles.

Previously it has been suggested that in both scenarios the small neutrino masses emerge naturally due to the dilution of the wave function of the sterile (right-handed) neutrino in the extra space. Within ADDM this idea was introduced [12] and its phenomenological implications were studied in [13]. In this case, the wave function of the sterile neutrino is diluted in the actual geometric extra space. This results in a highly suppressed Yukawa coupling between the sterile and the active neutrino of the SM, thereby, generating a tiny neutrino mass. As shown in [13], due to the mixing of an active left-handed neutrino with the KK tower of the sterile partner, a nontrivial oscillation pattern emerges.

More recently it has been shown [9] that a similar suppression mechanism of the neutrino mass works in the DR scenario [5,6] in which species represented the identical copies of the SM and the role of the extra coordinate is played by their label. Using this framework, it was shown in [9] that the dilution of the wave function of the sterile neutrino in the space of species results in a small neutrino mass. However, as discussed there the phenomenological aspects of this scenario are very different from the case of [12] which relies on ADDM framework.

In this paper, we have generalized the above original proposals in certain directions. In particular, we included a more realistic case of three SM neutrino flavors. We adopted the universal language of species which allows capturing some general aspects of the neutrino mixing matrix and confront different scenarios.

We calculated an approximate formula for the flavor eigenstates of a general mass matrix using perturbation theory in the three-flavor case. Next, we showed how highly symmetric mass matrices can be calculated in an exact manner and we investigated different symmetry breaking patterns of these highly symmetric mass matrices. We gave explicit expressions for flavor eigenstates for each case.

Our further step was to apply these generally derived formulas to the explicit theories of neutrino masses such as the proposal of ADDM [12,13] within ADD and the one of DR [9] within the many species frameworks respectively. Here we used the derived formulas and gave a three-flavor solution that depends on the parameters of the specific theories.

As it was already pointed out in [13,9] within ADDM and DR frameworks, the generic prediction of both scenarios is the nonconservation of neutrino number within the SM; this is due to the mixing of SM active neutrinos with the tower of sterile partners. This mixing results in the oscillations of neutrinos into hidden species as well as in seeming violation of unitarity within the SM lepton sector.

Thus, our calculations of these effects for three-flavor case have important phenomenological implications in both directions. First is the account of deviations of neutrino oscillations from the case of SM. Second, is the parametrization of violation of unitarity of the PMNS matrix.

The structures of unitarity-violation in the two different theories (ADDM [12,13] of ADD versus Dvali-Redi [9] of many species) differ from each other. Our analyses therefore has a discriminating power between these two theories.

In general, we can say that small neutrino mass generation via mixing with a large number of extra species is an exciting field with different phenomenological effects on low-energy neutrino physics. These effects can be searched for both in current neutrino experiments, such as IceCube, as well as in the planned ones like JUNO [48]. Here, the violation of unitarity can be tested and one can use their results to give bounds on the parameters of the theories, such as the size of the extra dimensions in ADDM or the number of sterile neutrino species to which our neutrino mixes within many species scenario.

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[1] G. Dvali, Subnucl. Ser. **53**, 189 (2017).
[2] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
[3] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999).
[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).
[5] G. Dvali, Fortschr. Phys. **58**, 528 (2010).
[6] G. Dvali and M. Redi, Phys. Rev. D **77**, 045027 (2008).
[7] G. Dvali, Int. J. Mod. Phys. A **25**, 602 (2010).
[8] G. Dvali and C. Gomez, Phys. Lett. B **674**, 303 (2009).
[9] G. Dvali and M. Redi, Phys. Rev. D **80**, 055001 (2009).
[10] G. Dvali and C. Gomez, arXiv:1004.3744.
[11] G. Dvali, arXiv:2103.15668.
[12] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and J. March-Russell, Phys. Rev. D **65**, 024032 (2001).
[13] G. R. Dvali and A. Y. Smirnov, Nucl. Phys. **B563**, 63 (1999).
[14] G. Fantini, A. G. Rosso, F. Vissani, and V. Zema, Adv. Ser. Dir. High Energy Phys. **28**, 37 (2018).
[15] M. Aker, K. Altenmller, M. Arenz, M. Babutzka, J. Barrett, S. Bauer, M. Beck, A. Beglarian, J. Behrens, T. Bergmann *et al.*, Phys. Rev. Lett. **123**, 221802 (2019).
[16] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
[17] P. Minkowski, Phys. Lett. **67B**, 421 (1977).
[18] M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C **790927**, 315 (1979), https://inspirehep.net/literature/9686.
[19] T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980).
[20] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[21] R. Mohapatra, *SEESAW25: International Conference on the Seesaw Mechanism and the Neutrino Mass* (2004), p. 29, arXiv:hep-ph/0412379v1.
[22] K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999).
[23] R. N. Mohapatra, S. Nandi, and A. Perez-Lorenzana, Phys. Lett. B **466**, 115 (1999).
[24] A. Ioannisian and A. Pilaftsis, Phys. Rev. D **62**, 066001 (2000).
[25] R. N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. **B576**, 466 (2000).
[26] R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000).
[27] R. N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. **B593**, 451 (2001).
[28] K. R. Dienes and I. Sarcevic, Phys. Lett. B **500**, 133 (2001).
[29] A. De Gouvea, G. F. Giudice, A. Strumia, and K. Tobe, Nucl. Phys. **B623**, 395 (2002).
[30] G. Dvali, I. Sawicki, and A. Vikman, J. Cosmol. Astropart. Phys. 08 (2009) 009.
[31] A. Caldwell, G. Dvali, P. Eller, and M. Ettengruber (to be published).
[32] G. R. Dvali and M. A. Shifman, Phys. Lett. B **396**, 64 (1997); **407**, 452(E) (1997).
[33] I. Esteban, M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, J. High Energy Phys. 09 (2020) 178.
[34] P. A. N. Machado, H. Nunokawa, and R. Zukanovich Funchal, Phys. Rev. D **84**, 013003 (2011).
[35] P. A. N. Machado, H. Nunokawa, F. A. P. dos Santos, and R. Z. Funchal, Phys. Rev. D **85**, 073012 (2012).
[36] V. S. Basto-Gonzalez, A. Esmaili, and O. L. G. Peres, Phys. Lett. B **718**, 1020 (2013).
[37] I. Girardi and D. Meloni, Phys. Rev. D **90**, 073011 (2014).
[38] W. Rodejohann and H. Zhang, Phys. Lett. B **737**, 81 (2014).
[39] J. M. Berryman, A. de Gouvêa, K. J. Kelly, O. L. G. Peres, and Z. Tabrizi, Phys. Rev. D **94**, 033006 (2016).
[40] M. Carena, Y.-Y. Li, C. S. Machado, P. A. N. Machado, and C. E. M. Wagner, Phys. Rev. D **96**, 095014 (2017).
[41] G. V. Stenico, D. V. Forero, and O. L. G. Peres, J. High Energy Phys. 11 (2018) 155.
[42] C. A. Argüelles *et al.*, Rep. Prog. Phys. **83**, 124201 (2020).
[43] B. Abi *et al.* (DUNE Collaboration), Eur. Phys. J. C **81**, 322 (2021).
[44] V. S. Basto-Gonzalez, D. V. Forero, C. Giunti, A. A. Quiroga, and C. A. Ternes, Phys. Rev. D **105**, 075023 (2022).
[45] C. A. Argüelles *et al.*, in 2022 Snowmass Summer Study (2022), arXiv:2203.10811.
[46] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[47] M. Ahlers, K. Helbing, and C. Prez de los Heros, Eur. Phys. J. C **78**, 924 (2018).
[48] F. An *et al.*, J. Phys. G **43**, 030401 (2016).