Renormalization group effects in astrophobic axion models

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It has been recently pointed out that in certain axion models it is possible to suppress simultaneously both the axion couplings to nucleons and electrons, realizing the so-called astrophobic axion scenarios, wherein the tight bounds from SN1987A and from stellar evolution of red giants and white dwarfs are greatly relaxed. So far, however, the conditions for realizing astrophobia have only been set out in tree-level analyses. Here we study whether these conditions can still be consistently implemented once renormalization group effects are included in the running of axion couplings. We find that axion astrophobia keeps holding, albeit within fairly different parameter space regions, and we provide analytical insights into this result. Given that astrophobic axion models generally feature flavor-violating axion couplings, we also assess the impact of renormalization group effects on axion-mediated flavor-violating observables.

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I. INTRODUCTION

Nonuniversal axion models, in which the Peccei-Quinn (PQ) symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ [1–4] acts on the different Standard Model (SM) fermions in a generation-dependent way, have been often considered in frameworks addressing the SM flavor puzzle (see, e.g., Refs. [5–7]), as well as in more phenomenological contexts. For instance, it was recently pointed out in Ref. [8] that in variants of Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) [9,10] models with two Higgs doublets (2HDM) the nonuniversality of the SM quarks PQ charges is a necessary ingredient to allow a simultaneous suppression of the axion coupling both to protons and neutrons. Nucleophobia can then be obtained in parameter space regions in which the ratio of the two Higgs vacuum expectation values (VEVs) satisfies certain conditions. This allows to relax the tight astrophysical bounds on the decay constant f_a (or on the axion mass m_a) from Supernova (SN) 1987A. Still, the bounds are only marginally loosened because in DFSZ-like models axion couplings to electrons are generically of $\mathcal{O}(1/f_a)$, and then limits from white dwarfs and red giants stars evolution, which are only moderately weaker than the SN1987A bound (see, e.g., Ref. [11] for a recent review) still apply. Axion-electron decoupling can be either obtained at the price of an extra tuning with the flavor structure of the lepton rotation matrices [8] or, more elegantly, it can be implemented together with nucleophobia, and without extra tuning, in a three Higgs doublets model, as detailed in Ref. [12]. In Refs. [8,12] the conditions for nucleo/electrophobia were formulated in terms of tree-level relations (up to small QCD running effects [13]) and it is then mandatory to question whether the resulting suppression of the axion couplings to nucleons and electron can survive after including the effects of radiative corrections.

The full one-loop anomalous dimensions for the $d = 5$ axion effective Lagrangian have been recently computed in Refs. [14,15], while running effects have been systematically investigated, within canonical axion models, in Ref. [16]. For related efforts to include loop effects on flavor-violating axion couplings, with a nontrivial dependence from the UV completion, see Ref. [17]. The purpose of this work is to extend the analysis of the running axion couplings to nonuniversal axion models and to assess, in particular, the radiative stability under the renormalization group (RG) evolution of the nucleo/electrophobic conditions set out in Refs. [8,12]. A remarkable consequence of nonuniversal axion models is the generic occurence of flavor-violating axion couplings, which can be tested in low-energy flavor-changing process, such as, e.g., $K \to \pi a$, that will be probed at current and future experimental facilities [18–20]. We hence complement our study by assessing the relevance of running effects for flavor off-diagonal axion couplings.

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II. ASTROPHOBIC AXIONS

We focus first on a specific nonuniversal axion model introduced in Ref. [12], wherein the nucleo and electrophobic conditions can be elegantly realized within certain regions of the parameter space spanned by the ratios between the VEVs of the Higgs doublets that couple to SM fermions.

The model features three Higgs doublets $H_{1,2,3}$ (hence we will label it as 3HDM) and a SM singlet complex scalar Φ. Under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ the quantum numbers of the scalars are $H_{1,2,3} \sim (1, 2, -1/2)$ and $\Phi \sim (1, 1, 0)$. The SM quarks couple to the first two doublets $H_{1,2}$ and their PQ charges are characterized by a $2+1$ structure, namely the first two generations replicate the same set of charges, while the PQ charges of the third generation differ. The $U(1)_{PQ}$ charges are chosen in such a way that all the entries in the up- and down-type quark Yukawa matrices are allowed, so that there are no texture zeros. In contrast, all the leptons couple to the third doublet H_3 and feature universal PQ charges.[1] The Yukawa sector of the model contains the following operators:

$$
\begin{aligned}
&\bar{q}_1 u_1 H_1, \qquad \bar{q}_3 u_3 H_2, \qquad \bar{q}_1 u_3 H_1, \qquad \bar{q}_3 u_1 H_2, \\
&\bar{q}_1 d_1 \tilde{H}_2, \qquad \bar{q}_3 d_3 \tilde{H}_1, \qquad \bar{q}_1 d_3 \tilde{H}_2, \qquad \bar{q}_3 d_1 \tilde{H}_1, \\
&\bar{\ell}_i e_j \tilde{H}_3, \qquad i, j = 1, 2, 3,
\end{aligned} \tag{1}
$$

where $\tilde{H}_{1,2,3} = i\sigma_2 H^*_{1,2,3}$. Note that the generation label "1" for quarks denotes both the first and second generation, which by assumption have the same PQ charges.

We are interested in the axion couplings to the proton, neutron and electron, which are defined via the effective interaction

$$
\frac{C_\psi}{2f_a} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi, \tag{2}
$$

with $\psi = p,\ n,\ e,\ f_a = f/(2N)$, where f_a is the axion decay constant, f is the scale at which the PQ symmetry is broken, and $2N$ is the coefficient of the PQ-QCD anomaly.[2] The fundamental couplings C_q of the axion to the quarks $q = u, d, \ldots$ are also defined by Eq. (2) with the replacement $\psi \to q$. $C_{p,n}$ can be expressed in terms of C_q using nonperturbative inputs from nucleon matrix elements (see, e.g., [13]). For later purposes it is more convenient to consider the two linear combinations:

$$
C_p + C_n = 0.52(C_u + C_d - 1) - 2\delta_s, \tag{3}
$$

$$
C_p - C_n = 1.28(C_u - C_d - f_{ud}), \tag{4}
$$

where the right-hand sides are obtained by using the expressions for $C_{p,n}$ given in Eqs. (A16) and (A17). In Eq. (4) $f_{ud} = f_u - f_d$, where $f_{u,d} = m_{d,u}/(m_d + m_u)$ are the model-independent contributions induced by the axion coupling to gluons in the physical basis in which the axion is not mixed with π^0. In Eq. (3) is a small $\mathcal{O}(5\%)$ correction dominated by the s-quark contribution (see the Appendix). Neglecting δ_s, the approximate conditions for astrophobia are

$$
C_u + C_d \approx 1, \tag{5}
$$

$$
C_u - C_d \approx f_{ud} \approx \frac{1}{3}, \tag{6}
$$

$$
C_e \approx 0. \tag{7}
$$

At tree level, the relevant couplings $C^0_{u,d} = (\mathcal{X}_{u_1,d_1} - \mathcal{X}_{q_1})/(2N)$ and $C^0_e = (\mathcal{X}_e - \mathcal{X}_\ell)/(2N)$ can be read off from the Yukawa operators in Eq. (1). In terms of the PQ charges $\mathcal{X}_{1,2,3}$ of the three Higgs doublets they read [22]

$$
C^0_u = -\frac{\mathcal{X}_1}{2N}, \quad C^0_d = \frac{\mathcal{X}_2}{2N}, \quad C^0_t = -\frac{\mathcal{X}_2}{2N}, \quad C^0_e = \frac{\mathcal{X}_3}{2N}, \tag{8}
$$

where for later convenience we have listed also the top-quark coupling C^0_t.[3] Due to the particular $2+1$ structure of the quarks PQ charges, the contribution to the PQ anomaly of the third generation cancels against the contribution of one of the two light generations, and it is then straight forward to obtain $2N = \sum_i (\mathcal{X}_{u_i} + \mathcal{X}_{d_i} - 2\mathcal{X}_{q_i}) = \mathcal{X}_2 - \mathcal{X}_1$. This implies that, at tree level, the first condition for nucleophobia Eq. (5) is always satisfied.

Consider now the following terms in the scalar potential, which are needed to break the $U(1)^4$ rephasing symmetry of the kinetic terms of the four scalars down to $U(1)_{PQ} \times U(1)_Y$[4]:

$$
H_3^\dagger H_1 \Phi^2 + H_3^\dagger H_2 \Phi^\dagger. \tag{9}
$$

Normalizing the charges to $\mathcal{X}_\Phi = 1$ we derive the conditions:

$$
\mathcal{X}_1 = \mathcal{X}_3 - 2, \qquad \mathcal{X}_2 = \mathcal{X}_3 + 1, \tag{10}
$$

[1] An alternative Higgs configuration in the lepton sector, leading to a moderately photophobic axion, is discussed in Ref. [21].

[2] For uniformity of notation with studies of running axion couplings [14–16] in the Appendix we will denote the anomaly coefficient as $c_G = 2N$.

[3] In Eq. (8) we have neglected possible corrections to the diagonal quark couplings arising from fermion mixing. Throughout this paper we will assume that these mixing corrections are negligible.

[4] Different choices for the scalar operators are possible, but they do not allow to satisfy simultaneously the nucleo and electrophobic conditions (see Ref. [12]).

which yield $2N = \mathcal{X}_2 - \mathcal{X}_1 = 3$. Substituting the values of $\mathcal{X}_{1,2}$ in Eqs. (6) and (7) we obtain that, in terms of tree-level couplings, astrophobia can be realized if the following conditions on $\mathcal{X}_3$ can be simultaneously satisfied:

$$\mathcal{X}_3 = \frac{1}{2}(1 - 3f_{ud}), \qquad \mathcal{X}_3 = 0. \tag{11}$$

It is a fortunate numerical accident that the actual value of f_{ud} is indeed very close to $1/3$ (corresponding to $m_d/m_u \approx 2$) so that nucleophobia and electrophobia are mutually compatible.

As a final step let us consider the PQ-hypercharge orthogonality condition. Let us parametrize the VEVs as $v_1 = vc_1c_2, v_2 = vs_1c_2, v_3 = vs_2$ with $v^2 = v_1^2 + v_2^2 + v_3^2 \simeq (246\ \mathrm{GeV})^2$, $c_1 \equiv \cos\beta_1$, $c_2 \equiv \cos\beta_2$, etc. By using Eq. (10) we obtain

$$\sum_{i=1,2,3} \mathcal{X}_i v_i^2 = 0 \Rightarrow \mathcal{X}_3 = (3c_1^2 - 1)c_2^2. \tag{12}$$

The condition $\mathcal{X}_3 \approx 0$ then selects a certain region in the (β_1, β_2) plane where the tree-level axion couplings to nucleons and electrons can be conveniently suppressed (see Fig. 1 in Ref. [12]).

A simpler astrophobic model with only two Higgs doublets $H_{1,2}$ in which the $2+1$ structure is extended also to the leptons was originally presented in Ref. [8] (see also Ref. [11]) and it was labeled "model M1." The Yukawa terms for the quarks are as in Eq. (1), while the lepton Yukawas, the operators involving the two scalar doublets and the singlet Φ, and the PQ-hypercharge orthogonality condition now involving only two Higgs doublets (i.e., $\beta_2 = 0$) read, respectively,

$$\bar{\ell}_1 e_1 \tilde{H}_1, \qquad \bar{\ell}_3 e_3 \tilde{H}_2, \qquad \bar{\ell}_1 e_3 \tilde{H}_1, \qquad \bar{\ell}_3 e_1 \tilde{H}_2, \tag{13}$$

$$H_2^\dagger H_1 \Phi \Rightarrow \mathcal{X}_2 = \mathcal{X}_1 + 1, \tag{14}$$

$$\mathcal{X}_1 v_1^2 + \mathcal{X}_2 v_2^2 = 0 \Rightarrow \mathcal{X}_1 = -s_{\beta_1}^2. \tag{15}$$

Since the quarks Yukawa operators are the same as in the previous model, the expression for the quark couplings in Eq. (8) is the same, however now with $2N = \mathcal{X}_2 - \mathcal{X}_1 = 1$. It is now easy to see that, with $f_{ud} \approx 1/3$, the nucleophobic conditions Eqs. (3) and (4) are satisfied at tree level in the parameter space region where $\tan^2\beta_1 \approx 2$. Instead, the electrophobic condition is not satisfied since the charge assignments give $C_e^0 = \mathcal{X}_1 \neq 0$. However, given that in this model the lepton charges are generation dependent, there are corrections to the mass eigenstate couplings due to lepton flavor mixing. Since in the lepton sector mixing effects can be particularly large, as it was pointed out in Ref. [8] electrophobia can still be enforced at the cost of a fine-tuned cancellation yielding $C_e^0 + \delta_e^{\mathrm{mix}} \approx 0$.

III. ASTROPHOBIC AXIONS BEYOND TREE LEVEL

The leading RG effects on the nucleo and electrophobic conditions Eqs. (5)–(7) can be understood from the formulas for the axion running couplings given in Eqs. (A6). The top Yukawa coupling Y_t gives the dominant contribution to the rhs of these equations. For the first generation fermions, in the approximation in which all Yukawa couplings except Y_t are neglected, this contribution appears only through the last term $\beta_\psi \gamma_H$ ($\psi = q_L, u_R, d_R, \ell_L, e_R$). In this approximation the expression for γ_H given in Eq. (A7) reduces to $\gamma_H \approx 6Y_t^2(c'_{t_R} - c'_{t_L}) = 6Y_t^2 c_t^0$, where c_t^0 denotes the axial-vector coupling of the top. We can now combine Eqs. (A6) to obtain RG equations (RGEs) for the u, d, e axial-vector couplings $c_{u,d,e}$. Recalling the definition of the hypercharge ratio $\beta_\psi = Y_\psi/Y_H$, it is easy to see that the γ_H term will appear in these equations, respectively, with coefficients $\beta_u - \beta_q = +1$ and $\beta_d - \beta_q = \beta_e - \beta_\ell = -1$.[5] Hence, in this approximation we can write

$$C_u \approx C_u^0 - \kappa_t C_t^0, \tag{16}$$

$$C_{d,e} \approx C_{d,e}^0 + \kappa_t C_t^0, \tag{17}$$

where $C_{u,d,e} = c_{u,d,e}/(2N)$ are the couplings at the low scale μ, $C_{u,d,e,t}^0 = c_{u,d,e,t}^0/(2N)$ are the couplings at the high scale f defined in terms of the PQ charges in Eq. (8), and the coefficient $\kappa_t \sim 6(Y_t/4\pi)^2 \log(m_{\mathrm{BSM}}/\mu)$ accounts for the running of the couplings from the high scale m_{BSM} where the heavy Higgs components are integrated out, down to the low scale μ.

The first condition for nucleophobia is still satisfied by the running couplings due to the fact that the correction proportional to κ_t cancels in the sum

$$C_u + C_d \approx C_u^0 + C_d^0 = 1. \tag{18}$$

RG effects modify instead the other two conditions Eqs. (6) and (7). It is straightforward to see that now they are, respectively, satisfied for the following values of $\mathcal{X}_3$:

$$\mathcal{X}_3 = \frac{\frac{1}{2}(1 - 3f_{ud}) + \kappa_t}{1 - \kappa_t}, \tag{19}$$

$$\mathcal{X}_3 = \frac{\kappa_t}{1 - \kappa_t}. \tag{20}$$

We see that the same numerical accident that allows to enforce astrophobia with the tree-level relations in Eq. (11) (corresponding to $\kappa_t \to 0$) ensures that the same result still holds after including in the axion couplings the leading RG

[5] The difference between the right-handed (RH) and left-handed (LH) hypercharge ratios is proportional to the weak-isospin of the LH component. This explains the opposite sign between the u and the d, e coefficients.

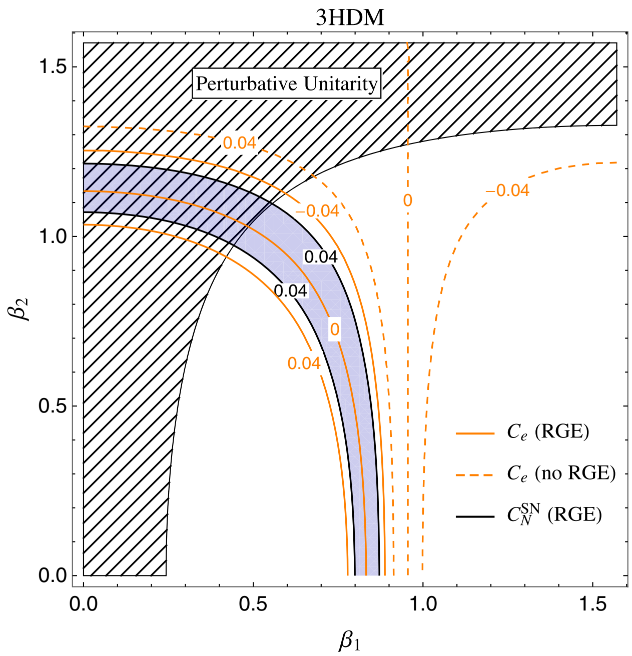

FIG. 1. Contour lines for C_e (orange) and $C_N^{\rm SN}$ (black, see text) in the (β_1, β_2) plane for the astrophobic 3HDM. Solid lines include RG corrections for $m_{\rm BSM} = 10^{10}$ GeV, dashed orange lines correspond to the tree-level results.

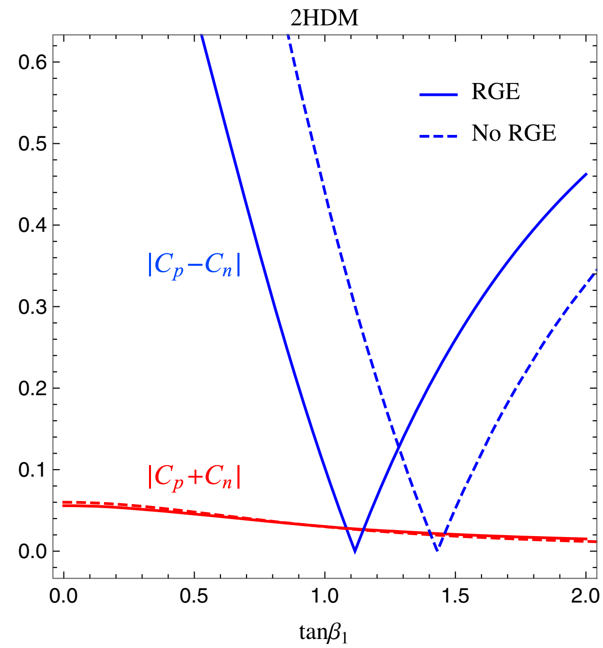

FIG. 2. The values of axion-nucleon couplings, $|C_p + C_n|$ (red) and $|C_p - C_n|$ (blue) in the nucleophobic 2HDM as a function of $\tan\beta_1$. Solid lines include RG corrections for $m_{\rm BSM} = 10^{10}$ GeV, dashed lines depict the tree-level results.

effects. Let us note that this result is independent of the particular value of κ_t, that is, it does not depend on any specific value of the high scale $m_{\rm BSM}$. Only the value of the PQ charges that realize the two conditions is affected by RG corrections, and while at tree level one has $\mathcal{X}_3 \approx 0$, for $\kappa_t \simeq 0.30$ one has instead $\mathcal{X}_3 \approx 0.43$. Of course, since the PQ-hypercharge orthogonality condition in Eq. (12) is now satisfied for a nonvanishing value of $\mathcal{X}_3$, the region in the (β_1, β_2) plane where the axion can exhibit a remarkable degree of astrophobia gets shifted accordingly, see Fig. 1. However, except for this modification in the viable parameter space region, it is a remarkable result that the astrophobic axion model introduced in Ref. [12] still maintains its properties after including RG corrections, without the need of any modification in the theoretical setup. Finally, it goes without saying that the nucleophobic property of the 2HDM model in Ref. [8] are also preserved, but for a different VEVs ratio $\tan^2\beta_1 \approx 1.2$ (see Fig. 2). Also the suppression of the axion-electron coupling can still be engineered, but with a corresponding shift in the value of the mixing correction $\delta_e^{\rm mix}$.

The results of this analysis, based on the approximate expressions Eqs. (16) and (17), are confirmed in Figs. 1 and 2 that are obtained by numerically solving the full RGEs for the axion couplings given in the Appendix. In Fig. 1 we show the contour lines for different values of C_e and $C_N^{\rm SN} = (C_n^2 + 0.61 C_p^2 + 0.53 C_n C_p)^{1/2}$ in the (β_1, β_2) plane. The latter combination of nucleon couplings corresponds to the quadratic form which is bounded by the SN1987A neutrino burst duration [23]. The lowest value corresponds to $C_N^{\rm SN} \simeq 0.02$ which is determined by the correction δ_s in Eq. (3) (for comparison in the Kim-Shifman-Vainshtein-Zakharov [24,25] axion model $C_N^{\rm SN} = 0.36$). The hatched region in Fig. 1 denotes the perturbative unitarity bounds on the Yukawa couplings of the 3HDM (see, e.g., [26,27]) translated in the (β_1, β_2) plane. It is evident from Fig. 1 that, also in the case of running axion couplings, electrophobia and nucleophobia occur in overlapping regions, so that a single choice of the values of the relevant parameters can simultaneously enforce all the astrophobic conditions. Figure 2 instead displays the values of $C_p \pm C_n$ as a function of $\tan\beta_1$ in the 2HDM case. As expected from the approximate expressions in Eqs. (16) and (17), running effects largely cancel out in the combination $C_p + C_n$, while they sizeably change the value of $\tan\beta_1$ for which the couplings combination $C_p - C_n$ is maximally suppressed from $\tan\beta_1 \simeq \sqrt{2}$ to $\tan\beta_1 \simeq 1.1$. Nevertheless the same level of nucleophobia than in the tree-level analysis can still be obtained regardless of the running effects.[6]

[6]We note in passing that also the exponential enhancement of axion-nucleon couplings in the nucleophilic axion models of Ref. [28] is not spoiled by running effects. The reason being that the required cancellation between the QCD anomaly factors of first and second generation quarks holds at all orders.

IV. RUNNING EFFECTS ON FLAVOR-VIOLATING AXION COUPLINGS

Flavor-violating axion couplings are generically expected in axion model with generation dependent PQ charge assignments, and it is therefore important to study the impact of RG corrections on these couplings. We focus for definiteness on the flavor off-diagonal couplings between the axion and the quarks in the 2HDM. Since only the charges of the LH quarks are generation dependent [see Eq. (A4)] and recalling that $c_G = 2N = 1$, using Eqs. (A9) and (A10) we can write[7]

$$(C^V_{d/u})_{i\neq j} \approx -(C^A_{d/u})_{i\neq j} \approx (U_{d_L/u_L} c'_{q_L} U^\dagger_{d_L/u_L})_{ij}, \quad (21)$$

where the LH rotation matrices U_{d_L/u_L} are defined via

$$Y_u = U^\dagger_{u_L} \hat{Y}_u U_{u_R}, \qquad Y_d = U^\dagger_{d_L} \hat{Y}_d U_{d_R}, \quad (22)$$

with $\hat{Y}_{u,d}$ the diagonal Yukawa matrices, and let us recall that U_{d_L/u_L} are related to the Cabibbo-Kobayashi-Maskawa (CKM) matrix via $V_{\rm CKM} = U_{u_L} U^\dagger_{d_L}$. Here we will consider the following two flavor *ansatze*:

$$\text{CKM-}Y_u: Y_u = V^\dagger_{\rm CKM} \hat{Y}_u,\ Y_d = \hat{Y}_d,\ (U_{u_L} = V_{\rm CKM}), \quad (23)$$

$$\text{CKM-}Y_d: Y_u = \hat{Y}_u,\ Y_d = V_{\rm CKM} \hat{Y}_d,\ (U_{d_L} = V^\dagger_{\rm CKM}). \quad (24)$$

In the CKM-Y_u case, $(C^{V,A}_d)_{i\neq j} = 0$ at the tree level and the nonzero $(C^{V,A}_d)_{i\neq j}$ couplings are radiatively generated. We remark that the alignment of the flavor structure in the down sector is not radiatively stable under the RG evolution, and hence processes like $K \to \pi a$ can still occur with a rate sufficiently large to be observable. In the CKM-Y_d case, $(C^{V,A}_u)_{i\neq j} = 0$ at the tree level, and it remains negligible, i.e., at most $O(10^{-9})$ even after including RG effects. For $f_a \gtrsim 10^8$ GeV all the off-diagonal couplings remain well below the experimental limits reported in Table I, where the strongest constraint is $|C^V_{ds}| \leq 3.3 \times 10^{-2} \times (f_a/10^{10}\,\text{GeV})$ from Ref. [29].

In the CKM-Y_u case an interesting feature emerges (see Fig. 3). The $C^{A,V}_{qb}$ ($q = s, d$) couplings are strongly suppressed for $\tan\beta_1 \approx 0.65$. This cancellation can be understood analytically by keeping only leading top-loop effects. Employing the CKM-Y_u structure and neglecting all Yukawa couplings except the top one, the RG evolution of the off-diagonal couplings can be cast in the form

$$\frac{d(c'_{q_L})_{i\neq j}}{d\log\mu} \propto \left[\frac{(c'_{q_L})_{ii}}{2} + \frac{(c'_{q_L})_{jj}}{2} - (c'_{t_R})\right] Y_t^2 (V^\dagger_{\rm CKM})_{i3} (V_{\rm CKM})_{3j}, \quad (25)$$

where only the diagonal couplings of $(c'_{q_L})_{ii}$ have been kept. Since both $(c'_{q_L})_{ii}$ and (c'_{t_R}) are positive, it is possible to cancel the quantity in the square brackets for $i = 3$ or $j = 3$ at a specific value $\tan\beta_1$. The RG corrections to $(c'_{q_L})_{i\neq j}$ are proportional to $(V^\dagger_{\rm CKM})_{i3}(V_{\rm CKM})_{3j}$, which indicates that the off-diagonal axion couplings to the up-quarks do not receive the corrections, given that the CKM factors cancel out due to unitarity.

In the CKM-Y_d case, on the other hand, flavor mixing occurs only through the down-quarks Yukawa couplings, and keeping only the top-loop contribution, the RG

TABLE I. Current experimental bounds on axion flavor-violating couplings. See Ref. [29] for details.

Coupling	Bound $[\times(f_a/10^{10}\ \text{GeV})]$
$\lvert C^V_{uc}\rvert$	$\leq 2.1 \times 10^2$
$\lvert C^V_{ds}\rvert$	$\leq 3.3 \times 10^{-2}$
$\lvert C^V_{db}\rvert$	$\leq 1.8 \times 10^2$
$\lvert C^V_{sb}\rvert$	≤ 61
$\lvert C^A_{uc}\rvert$	$\leq 4.2 \times 10^2$
$\lvert C^A_{ds}\rvert$	$\leq 4.5 \times 10^2$
$\lvert C^A_{db}\rvert$	$\leq 1.5 \times 10^3$
$\lvert C^A_{sb}\rvert$	$\leq 8.7 \times 10^3$

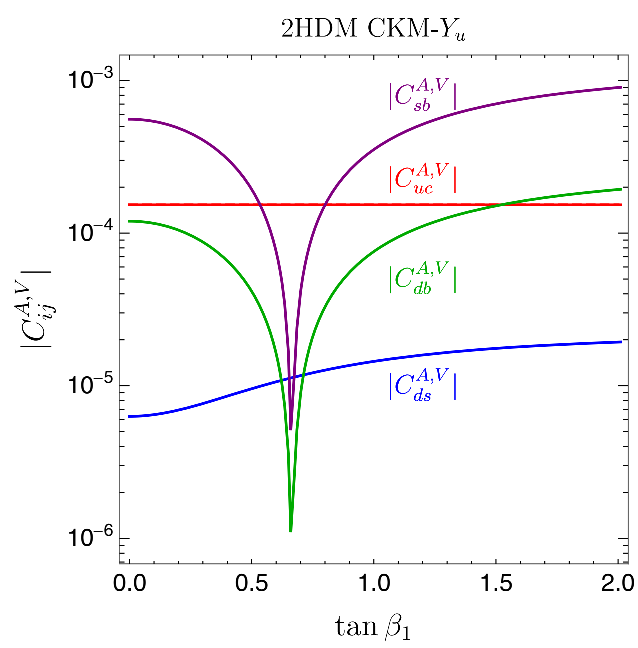

FIG. 3. Flavor off-diagonal axion couplings $|(C^{A,V}_u)_{ij}|$ and $|(C^{A,V}_d)_{ij}|$ with $m_{\rm BSM} = 10^{10}$ GeV in the 2HDM for the CKM Y_u. At tree level $(C^{A,V}_d)_{ij} = 0$ but nonzero values arise radiatively, while $C^{A,V}_{uc} \neq 0$ but it does not receive RG corrections.

[7] Equation (21) is defined at low energy, and thus it holds up to small corrections from RH mixings induced by running [see Eq. (A6)], which lift the universality of the RH couplings. These effects are taken into account in the numerical analysis.

correction to the off-diagonal couplings vanishes, namely $d(c'_{q_L})_{i\neq j}/d\log\mu \approx 0$. RG effects are thus captured solely by the running of the diagonal LH quark couplings $(c'_{q_L})_{ii}$ and matching corrections at the electroweak scale [15], which remain at the level of 1–4%.

V. CONCLUSIONS

In this work we assessed the impact of RG effects on the axion couplings, focusing on the case of nonuniversal axion models. An important application of the RG analysis arises in the context of the so-called astrophobic axions of Refs. [8,12], in which the axion couplings to nucleons and electrons can be simultaneously suppressed, thus allowing to relax the most stringent astrophysical constraints. In the original works the nucleo and electrophobic conditions were only set out at tree level, and it remained an important open question whether the conditions for astrophobia would still hold after including RG effects. In this paper we have shown that, perhaps unexpectedly, the astrophobic features are not spoiled by RG running of the axion couplings. The only effect is a sizeable shift in the parameter space regions in which these conditions are realized.

Since nonuniversal axion models necessarily imply certain flavor-violating axion couplings, we have also assessed the impact of running on these latter couplings. For instance, a tree-level flavor structure aligned in such a way that off-diagonal couplings in the down sector are absent, is not stable under RG evolution, and we have estimated the irreducible contributions to flavor-violating processes arising from this type of effects.

The tools developed in this work could be applied to other problems of phenomenological relevance. For instance, it could be interesting to see whether RG corrections can sizeably modify the fit to the so-called stellar cooling anomalies, improving on the tree-level analysis in Refs. [11,30].

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APPENDIX: RGEs FOR AXION EFTs

In order to take into account running effects it is convenient to adopt the Georgi-Kaplan-Randall (GKR) field basis [31], where the PQ symmetry is realized nonlinearly, so that under a $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry transformation all fields are invariant except the axion field, which changes by an additive constant $a \to a + \alpha f$, that is

$$\begin{aligned}\mathcal{L}_a^{\text{GKR-2HDM}} = {}& \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2}\frac{a}{f} F^A \tilde{F}^A \\ &+ \frac{\partial_\mu a}{f}[c_{H_1} H_1^\dagger i\overleftrightarrow{D^\mu} H_1 + c_{H_2} H_2^\dagger i\overleftrightarrow{D^\mu} H_2 \\ &+ \bar{q}_L c_{q_L}\gamma^\mu q_L + \bar{u}_R c_{u_R}\gamma^\mu u_R + \bar{d}_R c_{d_R}\gamma^\mu d_R \\ &+ \bar{\ell}_L c_{\ell_L}\gamma^\mu \ell_L + \bar{e}_R c_{e_R}\gamma^\mu e_R],\end{aligned} \tag{A1}$$

where $H_{1,2}^\dagger \overleftrightarrow{D^\mu} H_{1,2} \equiv H_{1,2}^\dagger (D^\mu H_{1,2}) - (D^\mu H_{1,2})^\dagger H_{1,2}$ and $c_{q_L}, \ldots$ are diagonal matrices in generation space. Note that in the EFT we have neglected the heavy $\mathcal{O}(f)$ radial mode of Φ and we focused for simplicity on the 2HDM (the generalization to an arbitrary number of Higgs doublets is straightforward). In order to match an explicit axion model to the effective Lagrangian in Eq. (A1) at the high scale $\mu \sim \mathcal{O}(f)$, we perform an axion dependent field redenfinition: $\psi \to e^{-i\mathcal{X}_\psi a/f}\psi$, where ψ spans over all the fields, and $\mathcal{X}_\psi$ is the corresponding PQ charge. Due to $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry, the nonderivative part of the renormalizable Lagrangian is invariant upon this field redefinition, while the $d = 5$ operators in Eq. (A1) are generated from the variation of the kinetic terms and from the chiral anomaly. The couplings are then identified as

$$c_\psi = \mathcal{X}_\psi, \tag{A2}$$

$$c_A = \sum_{\psi_R} 2\mathcal{X}_{\psi_R} \mathrm{Tr} T_A^2(\psi_R) - \sum_{\psi_L} 2\mathcal{X}_{\psi_L} \mathrm{Tr} T_A^2(\psi_L), \tag{A3}$$

where in the second equation $c_{\psi_{R,L}}$ refer to the charges of the chiral fermion fields.[8] For the 2HDM introduced in Sec. II, the charges $\mathcal{X}_\psi$, that can be read off from the Yukawa couplings in Eq. (13) can be set to

$$\begin{aligned}&\mathcal{X}_{q_i} = (0, 0, \mathcal{X}_2 - \mathcal{X}_1), \quad \mathcal{X}_{u_i} = -(\mathcal{X}_1, \mathcal{X}_1, \mathcal{X}_1), \\ &\mathcal{X}_{d_i} = (\mathcal{X}_2, \mathcal{X}_2, \mathcal{X}_2), \quad \mathcal{X}_{\ell_i} = -\mathcal{X}_{q_i}, \quad \mathcal{X}_{e_i} = -\mathcal{X}_{u_i},\end{aligned} \tag{A4}$$

where $\mathcal{X}_1 = -s_{\beta_1}^2$ and $\mathcal{X}_2 = c_{\beta_1}^2$, see Eq. (15), and we have shifted the charges proportionally to B and L to set $\mathcal{X}_{q_{1,2}} = \mathcal{X}_{\ell_{1,2}} = 0$. For the anomaly coefficients in Eq. (A3) one has $(c_G, c_W, c_B) = (1, -2, 8/3)$ and, in particular,

[8] Note that our anomaly coefficients c_A have opposite sign with respect to those in Refs. [14–16]. This is due to the fact that we are using a different convention for the Levi-Civita tensor, namely $\epsilon^{0123} = -1$.

the electromagnetic to QCD anomaly ratio is $E/N \equiv (c_W + c_B)/c_G = 2/3$. For the 3HDM instead the lepton charges are $\mathcal{X}_\ell = 0, \mathcal{X}_e = \mathcal{X}_3$, the corresponding anomaly coefficients read $(c_G, c_W, c_B) = (3, -9, 17)$ and $E/N = 8/3$.

Running effects induced by Yukawa couplings (and in particular by the Yukawa of the top which are the most relevant ones) only occur below the scale of the heavy radial modes of the 2HDM, that will be denoted as $m_{\rm BSM} \simeq m_{H,A,H^\pm}$, with the heavy scalars assumed to be degenerate in the decoupling limit (see, e.g., [32]). This is due to the fact that as long as the complete set of Higgs doublets appear in the EFT, the PQ current is conserved (up to anomalous effects) and thus the couplings, which correspond to PQ charges, do not renormalize. Once the heavy scalar components are integrated out, the sum rule of PQ charges set by $\mathrm{U}(1)_{\rm PQ}$ invariance breaks down, and nonvanishing contributions to the running of the couplings arise (see, e.g., [16]). We can now directly match Eq. (A1) at the scale $\mu = \mathcal{O}(m_{\rm BSM})$ with a GKR basis featuring only one SM-like Higgs doublet

$$\begin{aligned}\mathcal{L}_a^{\text{GKR-SM}} &= \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2}\frac{a}{f} F^A \tilde{F}^A \\ &\quad + \frac{\partial_\mu a}{f}[c_H H^\dagger i\overleftrightarrow{D}^\mu H + \bar{q}_L c_{q_L}\gamma^\mu q_L + \bar{u}_R c_{u_R}\gamma^\mu u_R \\ &\quad + \bar{d}_R c_{d_R}\gamma^\mu d_R + \bar{\ell}_L c_{\ell_L}\gamma^\mu \ell_L + \bar{e}_R c_{e_R}\gamma^\mu e_R], \end{aligned} \tag{A5}$$

where $c_H = c_{H_1} c_\beta^2 + c_{H_2} s_\beta^2$, which follows from the projections on the SM Higgs doublet: $H_1 \to c_\beta H$ and $H_2 \to s_\beta H$, consistently with the definition of $\tan\beta \equiv \tan\beta_1 = v_2/v_1$. In particular, by employing global $\mathrm{U}(1)_Y$ invariance, it is convenient to cast the RG equations in a form that does not depend explicitly on c_H. This can be achieved via the axion-dependent field redefinition: $\psi \to \psi' = e^{-i c_H \beta_\psi a/f}\psi$, with $\beta_\psi = Y_\psi / Y_H$ the ratio of the corresponding hypercharges, which redefines the effective couplings as $c'_\psi = c_\psi - c_H \beta_\psi$ (so in particular $c'_H = 0$). In this basis the RG equations read:

$$\begin{aligned}(4\pi)^2 \frac{dc'_{q_L}}{d\log\mu} &= \frac{1}{2}\{c'_{q_L}, Y_u Y_u^\dagger + Y_d Y_d^\dagger\} - Y_u c'_{u_R} Y_u^\dagger - Y_d c'_{d_R} Y_d^\dagger \\ &\quad + \left(8\alpha_s^2 \tilde{c}_G + \frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{1}{6}\alpha_1^2 \tilde{c}_B\right)\mathbf{1} - \beta_q \gamma_H \mathbf{1}, \\ (4\pi)^2 \frac{dc'_{u_R}}{d\log\mu} &= \{c'_{u_R}, Y_u^\dagger Y_u\} - 2Y_u^\dagger c'_{q_L} Y_u \\ &\quad - \left(8\alpha_s^2 \tilde{c}_G + \frac{8}{3}\alpha_1^2 \tilde{c}_B\right)\mathbf{1} - \beta_u \gamma_H \mathbf{1}, \\ (4\pi)^2 \frac{dc'_{d_R}}{d\log\mu} &= \{c'_{d_R}, Y_d^\dagger Y_d\} - 2Y_d^\dagger c'_{q_L} Y_d \\ &\quad - \left(8\alpha_s^2 \tilde{c}_G + \frac{2}{3}\alpha_1^2 \tilde{c}_B\right)\mathbf{1} - \beta_d \gamma_H \mathbf{1}, \\ (4\pi)^2 \frac{dc'_{\ell_L}}{d\log\mu} &= \frac{1}{2}\{c'_{\ell_L}, Y_e Y_e^\dagger\} - Y_e c'_{e_R} Y_e^\dagger \\ &\quad + \left(\frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{3}{2}\alpha_1^2 \tilde{c}_B\right)\mathbf{1} - \beta_\ell \gamma_H \mathbf{1}, \\ (4\pi)^2 \frac{dc'_{e_R}}{d\log\mu} &= \{c'_{e_R}, Y_e^\dagger Y_e\} - 2Y_e^\dagger c'_{\ell_L} Y_e - 6\alpha_1^2 \tilde{c}_B \mathbf{1} - \beta_e \gamma_H \mathbf{1}, \end{aligned} \tag{A6}$$

where

$$\begin{aligned}\gamma_H &= -2\mathrm{Tr}(3Y_u^\dagger c'_{q_L} Y_u - 3Y_d^\dagger c'_{q_L} Y_d - Y_e^\dagger c'_{\ell_L} Y_e) \\ &\quad + 2\mathrm{Tr}(3Y_u c'_{u_R} Y_u^\dagger - 3Y_d c'_{d_R} Y_d^\dagger - Y_e c'_{e_R} Y_e^\dagger), \\ \tilde{c}_G &= c_G - \mathrm{Tr}(c'_{u_R} + c'_{d_R} - 2c'_{q_L}), \\ \tilde{c}_W &= c_W + \mathrm{Tr}(3c'_{q_L} + c'_{\ell_L}), \\ \tilde{c}_B &= c_B - \mathrm{Tr}\left(\frac{1}{3}(8c'_{u_R} + 2c'_{d_R} - c'_{q_L}) + 2c'_{e_R} - c'_{\ell_L}\right). \end{aligned} \tag{A7}$$

Note that the c_A ($A = G, W, B$) Wilson coefficients in Eq. (A7) do not run, since in the normalization of Eq. (A1) the scale dependence of the operator $aF^A\tilde{F}^A$ is accounted for by the running of the gauge couplings [15,33].

Equation (A5) is matched at the scale $\mu = \mathcal{O}(m_Z)$ with the $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\rm EM}$-invariant axion effective Lagrangian below the electroweak scale

$$\begin{aligned}\mathcal{L}_a &\supset \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G\tilde{G} + \frac{c_\gamma}{c_G}\frac{e^2}{32\pi^2}\frac{a}{f_a} F\tilde{F} \\ &\quad + \sum_{f=u,d,e} \frac{\partial_\mu a}{2f_a}\bar{f}_i \gamma^\mu ((C_f^V)_{ij} + (C_f^A)_{ij}\gamma_5) f_j, \end{aligned} \tag{A8}$$

where we have introduced the standard QCD normalization factor for the $aG\tilde{G}$ term and defined the axion decay constant $f_a = f/c_G$, while $c_\gamma = c_W + c_B$. We further have

$$C_f^V = \frac{1}{c_G}(U_{f_R} c'_{f_R} U_{f_R}^\dagger + U_{f_L} c'_{f_L} U_{f_L}^\dagger), \tag{A9}$$

$$C_f^A = \frac{1}{c_G}(U_{f_R} c'_{f_R} U_{f_R}^\dagger - U_{f_L} c'_{f_L} U_{f_L}^\dagger), \tag{A10}$$

where $U_{f_{L,R}}$ are the unitary matrices that diagonalize the SM fermion mass matrices, and $c'_{u_L} = c'_{d_L} = c'_{q_L}$. After including matching corrections at the weak scale [15], the running for $\mu < m_Z$ is given by

$$(4\pi)^2 \frac{d(C_u^A)_{ii}}{d\log\mu} = -16\alpha_s^2\tilde{c}_G - \frac{8}{3}\alpha_{\rm em}^2\tilde{c}_\gamma,$$
$$(4\pi)^2 \frac{d(C_d^A)_{ii}}{d\log\mu} = -16\alpha_s^2\tilde{c}_G - \frac{2}{3}\alpha_{\rm em}^2\tilde{c}_\gamma,$$
$$(4\pi)^2 \frac{d(C_e^A)_{ii}}{d\log\mu} = -6\alpha_{\rm em}^2\tilde{c}_\gamma, \tag{A11}$$

with

$$\tilde{c}_G(\mu) = 1 - \sum_q C_q^A(\mu)\Theta(\mu - m_q), \tag{A12}$$

$$\tilde{c}_\gamma(\mu) = \frac{c_\gamma}{c_G} - 2\sum_f N_c^f Q_f^2 C_f^A(\mu)\Theta(\mu - m_f), \tag{A13}$$

where $\Theta(x)$ is the Heaviside theta function, while N_c^f and Q_f denote, respectively, the color number and EM charge of the fermion f. Note that the off-diagonal couplings $(C_f^{A,V})_{i\neq j}$ do not run below the electroweak scale, while the diagonal vector couplings $(C_f^V)_{ii}$ can be set to zero thanks to the conservation of the vector current.

The axion-nucleon couplings, neglecting the tiny contributions of the matrix elements $\Delta_{t,b,c}$ of the heavy flavors, can be calculated by using

$$C_p = C_u\Delta_u + C_d\Delta_d + C_s\Delta_s - \left(\frac{m_d\Delta_u}{m_u+m_d} + \frac{m_u\Delta_d}{m_u+m_d}\right), \tag{A14}$$

$$C_n = C_d\Delta_u + C_u\Delta_d + C_s\Delta_s - \left(\frac{m_u\Delta_u}{m_u+m_d} + \frac{m_d\Delta_d}{m_u+m_d}\right), \tag{A15}$$

where $C_{u,d,s} = C_{u,d,s}^A(2\ \mathrm{GeV})$ [we neglect here for simplicity model-dependent tree-level flavor mixing effects—see Eq. (A10)] are evaluated by numerically solving the RG equations, Eqs. (A6) and (A11), starting from the boundary conditions set at the scale f [cf. below Eq. (A3)]. In Eqs. (A14) and (A15), $\Delta_{u,d,s}$ represent the nucleon matrix elements of the light quarks axial-vector current, whose numerical values are $\Delta_u = 0.897(27)$, $\Delta_d = -0.376(27)$, $\Delta_s = -0.026(4)$, while $m_u(2\ \mathrm{GeV})/m_d(2\ \mathrm{GeV}) = 0.48(3)$ [13]. With these inputs, we arrive at

$$C_p = 0.90C_u - 0.38C_d - 0.03C_s - 0.48, \tag{A16}$$

$$C_n = 0.90C_d - 0.38C_u - 0.03C_s - 0.04. \tag{A17}$$

In the calculation, we have employed the two-loop running for gauge and Yukawa couplings, and the input values for the SM Yukawa and CKM mixings are extracted from Ref. [34].

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